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Polarizability and Light Scattering by Subwavelength Graded-Index Plasmonic Spheres

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***Abstract*—Light scattering by a subwavelength sphere exhibiting radially inhomogeneous permittivity is here presented. The theoretical foundations describing the scattering response are given in a simple manner and the concept of polarizability is generalized incorporating the inhomogeneity effects. Two illustrative examples are briefly discussed, i.e., a power-law and a Drude-like inhomogeneous profile, exposing the physical mechanisms of their scattering response. The presented results can open the path towards the implementation of graded-index subwavelength particles in modern nanoantenna applications.**

I. Introduction

Scattering by a subwavelength homogeneous sphere is a simple, yet fundamental, problem that can be found in the core of many modern power control/harvesting applications [1], especially at the terahertz-optics regime. Its universality emerges from the fact that the scattering response of a small dielectric sphere can be rigorously quantified by a (normalized) polarizability expression $\alpha = \frac{\varepsilon_1-\varepsilon_h}{\varepsilon_1+2\varepsilon_h}$ [2]. This simple expression reveals a wealth of physical phenomena, such as the position of the localized surface plasmon (LSP) or plasmonic resonances ($\varepsilon_1 = -2\varepsilon_h$).

In this work the generalized concept of the polarizability is presented by assuming the case of a sphere with a radially inhomogeneous (graded-index) permittivity profile. This kind of profiles occur either naturally [3] or as a result of sophisticated engineering processes [4]. Previous studies expanded both Mie and electrostatic scattering theory for certain cases of inhomogeneous profiles (see for example [5], [6], [7]). Here we present some simple analytical formulas for several cases of profiles such as the power-law, exponential, and inhomogeneous Drude profile. Special attention to the LSP resonances is given, exposing the underlying scattering mechanisms of the aforementioned spheres.

II. Theory & Results

Let us assume a sphere (subscript 0 for external and 1 for internal domain) of radius r_1 (Fig. 2) subject to a uniform electrostatic potential $\Phi_0(r,\theta) = -E_0 r$, causing a scattering potential of dipolar character, $\Phi_s(r,\theta) = \frac{B_0}{r^2}\cos\theta$, and an internal potential $\Phi_1(\mathbf{r}) = f(r)\cos\theta$ [8]. Requiring that the electric flux density be divergence-less at the internal region we have, $\nabla\cdot\mathbf{D}_1 = \nabla\cdot(\varepsilon_r(r)\nabla\Phi_1(r,\theta)) = 0$, resulting to the following O.D.E

$$f''(r) + \left(\frac{2}{r} + \frac{\varepsilon_r'(r)}{\varepsilon_r(r)}\right) f'(r) - \frac{2}{r^2} f(r) = 0 \tag{1}$$

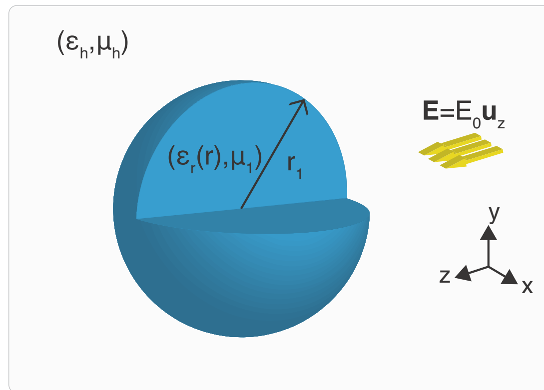

Fig. 1: A radially inhomogeneous sphere immersed in a host medium subject to a constant excitation field.

The main focus here is to study the cases where Eq. (1) obtains a closed form solution, hence extracting an analytical expression of the unknown scattering amplitudes. For these cases one can express the radial function as $f(r) = A_1 A(r) + B_1 B(r)$, where $A(r)$ is a constant-type and $B(r)$ is a dipole-type solutions of the radial function. It is tacitly assumed that function $A(r)$ is well-behaving at the origin, while $B(r)$ contains a singularity. This is, however, not always true and a special mathematical treatment is required, e.g., introduction of a singularity subtracting region at the center. Assuming $A(r)$ being a regular, non-singular, function the potential in the internal region is $\Phi_1(r,\theta) = A_1 A(r)\cos\theta$.

The unknown scattering coefficients, B_0 and A_1, can be calculated by enforcing the required boundary conditions, i.e., $\partial_\theta\Phi_0(r,\theta) = \partial_\theta\Phi_1(r,\theta)$, and $\varepsilon_h\partial_r\Phi_0(r,\theta) = \varepsilon_r(r)\partial_r\Phi_1(r,\theta)$, at $r = r_1$. After some simple algebra, the polarizability (external coefficient) reads

$$B_0 = \frac{C\varepsilon_r(r_1) - \varepsilon_h}{C\varepsilon_r(r_1) + 2\varepsilon_h} r_1^3 E_0 \tag{2}$$

with $C = r_1\frac{A'(r_1)}{A(r_1)}$ being the inhomogeneity parameter. Interestingly, this parameter is a function of $A(r)$ evaluated at the surface of the sphere, $r = r_1$, hence an effective permittivity of this inhomogeneous inclusion can be extracted *à la* Maxwell Garnett [9]. Inspecting Eq. (2) we observe that the inhomogeneity elegantly modifies the polarizability expression, generalizing the plasmonic resonance condition in a simple manner, i.e.,

$$\varepsilon_r(r_1) = -\frac{2\varepsilon_h}{C} \tag{3}$$

Apparently, the homogeneous case is when $A(r) = r$ and $C = 1$.

III. Discussion & Conclusions

As an illustrative example we present the results for two different cases: the power-law profile and the inhomogeneous Drude-like profile. Note that the O.D.E. in Eq. (1) obtains a closed-form analytical solution for certain permittivity profiles such as power-law [7], polynomial [10], and exponential profiles. In our analysis the results are validated by implementing an iterative code of a multilayer sphere exhibiting the same profile [6]. For the case of the power-law profile, i.e., $\varepsilon_r(r) = \varepsilon_1 r^n$, the radial function reads

$$f(r) = A_1 r^{p_1} + B_1 r^{p_2}, \tag{4}$$

with $p_{1,2} = -\frac{1}{2}\left(n + 1 \mp \sqrt{(n+1)^2 + 8}\right)$, where n is a the power factor [7]. The inhomogeneity parameter in this case is $C = p_1$. The plot of the polarizability as a function of both permittivity and power factor can be found in Fig. 2 reveals a shift in the polarization enhancement as a function of n.

The second case of inhomogeneous Drude-like profile [11], $\varepsilon_r(\omega, r) = \varepsilon_\infty - \frac{\omega_p^2}{\omega^2}(1 + br^n)$, where ω_p is the plasma frequency, n is the power factor, and b is the inhomogeneity factor. For this case function $A(r)$ can be expressed as a hypergeometric function ${}_2F_1(\alpha, \beta; c; z)$ [12, Ch. 15] with the arguments

$$A(r) = r\, a_r\, {}_2F_1\left(\nu_1, \nu_2; 1 + \frac{3}{n}; -\frac{ab}{a - \varepsilon_\infty} r^n\right) \tag{5}$$

where $a = \frac{\omega_p^2}{\omega^2}$, $a_r = \left(\frac{ab}{n^2(a - \varepsilon_\infty)}\right)^{1/n} n^{2/n}$, and $\nu_{1,2} = \frac{1}{2n}\left(n + 3 \mp \sqrt{(n+1)^2 + 8}\right)$. The inhomogeneity parameter is a function of ω, ω_p, n, and the inhomogeneity factor b. This compact description enables the study of inhomogeneous structures with dispersive characteristics, such as inhomogeneities in metals or other materials [13] at the optical-IR frequencies.

Summarizing, the expression of Eq. (2) generalizes the concept of a homogeneous polarizability, allowing us to rigorously explore the non-trivial physical mechanisms for a whole new family of graded-index particles. Additionally, this simplified description can be used to reverse-engineer the inhomogeneity coefficient C fitting the experimental data, suggesting an alternative explanation to the experimentally observed deviations of plasmonic resonances on deeply subwavelength spheres [14].

The introduced description can be also implemented for a wide range of practical cases such as the accurate modeling of inhomogeneous structures (stratified spheres, transformation optics [13]), the implementation of temperature gradients (via a varying permittivity profile), the modeling of diffusive effects especially between interfaces or for extremely small particles [14], where interfaces are not hard but rather follow a radially dependent distribution. It is envisioned that the presented study will stimulate novel power control/harvesting ideas for nanophotonic applications, such as the implementation of subwavelength plasmonic particles exhibiting Luneburg, Eaton, or more exotic graded-index profiles.

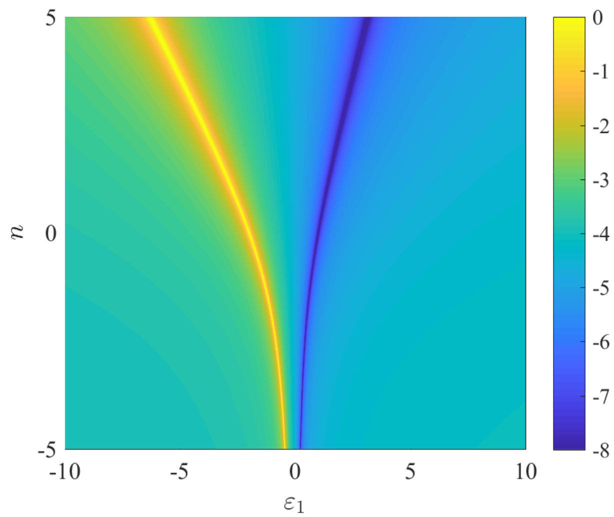

Fig. 2: Polarizability (in logarithmic scale) of a sphere with a power-law profile as a function of the power factor and the permittivity parameter.

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