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Electronic and Vibrational Properties of TiS$_2$, ZrS$_2$, and HfS$_2$: Periodic Trends Studied by Dispersion-Corrected Hybrid Density Functional Methods

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Abstract

The electronic and vibrational properties of TiS$_2$, ZrS$_2$, and HfS$_2$ have been studied using dispersion corrected hybrid density functional methods. The periodic trends in electronic band structures, electronic transport coefficients, IR and Raman spectra, and phonon dispersion relations were investigated. Comparison to the available experimental data shows that the applied DFT methodology is suitable for the investigation of the layered transition metal dichalcogenide materials with weak interlayer Van der Waals interactions. The choice of damping function in the D3 dispersion correction proved to have surprisingly large effect. Systematic investigation of the periodic trends within group 4 disulfides reveals that TiS$_2$ shows many differences to ZrS$_2$ and HfS$_2$ due to the more covalent M–S bonding in TiS$_2$. ZrS$_2$ and HfS$_2$ mainly show differences for properties where the atomic mass plays a role. All three compounds show similar Seebeck coefficients, but clear differences in the relative electrical conductivity of cross- and in-plane directions. The transport and vibrational properties of thin TiS$_2$ single crystals were also investigated experimentally.

Introduction

Group 4 transition metal disulfides TiS$_2$, ZrS$_2$, and HfS$_2$ adopt a trigonal, layered CdI$_2$-type structure (Figure 1, space group $P\overline{3}m1$). The intralayer metal-chalcogen bonding in the $ab$-plane (in-plane) is ionic/covalent, while the layers are stacked via weak Van der Waals interactions along the $c$ axis (cross-plane). Each metal atom is surrounded by six sulfur atoms in trigonal prismatic coordination and each S atom is connected to three metal atoms. The fact that TiS$_2$, ZrS$_2$, and HfS$_2$ possess similar CdI$_2$-type structure, opens the possibility for systematic computational studies into their periodic, groupwise trends. The Hf atom is of rather similar size as Zr due to lanthanoid contraction, and Zr and Hf often show rather similar electronic properties. However, the two elements show a clear mass difference, which can be expected to affect the periodic trends of vibrational and thermal properties of Zr and
Hf compounds.

![Crystal structure of MS2 compounds](image)

Figure 1: The crystal structure of the studied MS$_2$ compounds (M = Ti, Zr, Hf). Left: Side view, Right: top view. Blue: M, yellow: S.

The layered structure of group 4 transition metal disulfides allows an immense number of possible intercalation reactions, where with guest atoms or molecules are accommodated in the interlayer space. The intercalation reactions make it possible to tune the properties of the parent MS$_2$ lattice, resulting for example in lower thermal conductivity and higher thermoelectric efficiency.

Thermoelectric materials can be used to produce electrical energy from temperature gradients that can be found for example from industrial settings, microelectronics, or even human body/ambient air interface. The growing need to harvest low-grade waste heat to stimulates studies towards cheaper and environmentally more friendly thermoelectric (TE) materials that could be used to substitute most commonly used TE materials such as Bi$_2$Te$_3$ or PbTe.$^{1,2}$ Combining experimental synthesis and engineering of TE materials with \emph{ab initio} quantum chemical methods with predictive power provides the means for rational design of novel TE materials.$^{3}$

The performance of thermoelectric materials is evaluated by dimensionless figure of merit $ZT$, which can be expressed by equation (1):

$$ZT = S^2\sigma T/\kappa,$$  \hfill (1)
where $S$ is Seebeck coefficient (thermopower), $\sigma$ is electrical conductivity, $\kappa$ is thermal conductivity, and $T$ is the temperature. $\kappa$ can be considered as the sum of electronic contribution ($\kappa_{el}$) and phonon contribution ($\kappa_l$):

$$\kappa = \kappa_{el} + \kappa_l$$

(2)

In insulators and semiconductors, $\kappa_l$ typically dominates over $\kappa_{el}$.

Intercalated group 4 transition metal disulfides are potential TE materials due to the performance improvements arising from the intercalation.\textsuperscript{4-8} For example, intercalating TiS$_2$ by Cu results in significant improvement in thermoelectric efficiency and also decreases the anisotropy of $ZT$ in the in-plane and cross-plane directions.\textsuperscript{9} In fact, TiS$_2$ is typically synthesized in nonstoichiometric form Ti$_{1+x}$S$_2$, where the excess Ti atoms are intercalated between the TiS$_2$ sheets.\textsuperscript{10} On the other hand, intercalation by organic molecules has been shown to lead in major reduction of lattice thermal conductivity and improvement of $ZT$ in a number of studies.\textsuperscript{11,12} Furthermore, it is possible to derive superlattices such as (SnS)$_{1.2}$(TiS$_2$)$_2$ by intercalating the TiS$_2$ lattice by layers from other layered materials such as SnS. This approach also reduces the lattice thermal conductivity significantly, resulting in improved $ZT$.\textsuperscript{13}

The fundamental electronic properties of the group 4 disulfides have been investigated in a number of publications and all of them are known to be n-type semiconductors. For the band gap of TiS$_2$, there are several conflicting experimental reports. In a number of papers, TiS$_2$ has been described as a semiconductor with band gap varying from 0.2 to 1.5 eV,\textsuperscript{14-18} while few other studies describe it as a semimetal.\textsuperscript{19-21} The discrepancy can be explained by taking into account structural defects and the well-known tendency of TiS$_2$ to be nonstoichiometric.\textsuperscript{22} Possibly due to the difficulties in synthesis of good quality single crystals, there are fewer experimental works on the band gap of ZrS$_2$ and HfS$_2$. However, ZrS$_2$ and HfS$_2$ are generally accepted as semiconductors with band gap range of 1.68–1.78 eV and 1.96–2.85 eV, respectively.\textsuperscript{16,23,24}

The majority of previous computational studies on group 4 transition metal disulfides
have focused on the properties of TiS$_2$. Allan et al. investigated the effect of pressure on the crystal structure and electronic properties of TiS$_2$ utilizing density functional theory in local density approximation (DFT-LDA). According to their results, TiS$_2$ is a semiconductor with a band gap of about 1.9 eV at ambient conditions. Fang et al. carried out DFT-LDA calculations for TiS$_2$ and TiSe$_2$, reporting that single-layered TiS$_2$ is a semiconductor with about 1 eV band gap, which then decreases as the number of monolayers increases and eventually turns to a semimetal in bulk. Reshak et al. studied the electronic and optical properties of MX$_2$ (M = Ti, Zr; X = S, Se, Te) with DFT-LDA, concluding that TiS$_2$ and ZrS$_2$ are semimetallic and semiconducting, respectively (1.4 eV band gap for ZrS$_2$). In two other studies, Hubbard $U$ correction was used (GGA+$U$ and LDA+$U$) to study the electronic and vibrational properties of TiS$_2$. In both studies, TiS$_2$ was found to be a semiconductor with a band gap of 1 eV.

To our knowledge, the periodic trends of electronic and vibrational properties of group 4 transition metal disulfides TiS$_2$, ZrS$_2$, and HfS$_2$ have not been examined previously. Jiang investigated the electronic and vibrational properties of ZrS$_2$ and HfS$_2$ with various local and semilocal DFT methods and many-body perturbation theory (GW approximation). The band gaps calculated within the GW approximation were generally in good agreement with the experiment, showing an overestimation of 0.1–0.3 eV depending on the experimental reference. Yumnam et al. studied the electronic and thermal transport properties of ZrS$_2$ and HfS$_2$ using GGA-PBE and modified Becke Johnson TB-mBJ functionals. They found ZrS$_2$ and HfS$_2$ to be semiconductors with band gaps of 1.3 and 1.7 eV, respectively, being about 0.26–0.4 eV smaller than the experimental values.

Recently, the vibrational and dielectric properties of various transition metal dichalcogenides including TiS$_2$ and ZrS$_2$ have been studied with DFT and DFPT by Pike et al. The main computational tool in the work was DFT-PBE functional, combined with Grimme’s dispersion correction (DFT-D3). The work of Pike et al. clearly highlights the importance of using a computational method that can properly describe the weak interlayer Van der
Waals interactions. The data obtained with PBE-D3 were reported as the most accurate with respect to the experiment and the dispersion correction had an effect on both in the cross-plane $c$-direction and the $ab$-plane.

Here we investigate the periodic trends of the electronic, vibrational, and thermoelectric properties of group 4 transition metal disulfides TiS$_2$, ZrS$_2$, and HfS$_2$. We apply dispersion-corrected hybrid density functional methods and Gaussian-type local basis sets to describe both the crystal structure and electronic structure of all three group 4 disulfides. Finding a level of theory that can consistently reproduce the experimentally observed periodic trends is also a prerequisite for further computational studies into the intercalation of group 4 transition metal disulfides.

**Computational details**

The DFT calculations were carried out with the CRystal14$^{34}$ program using PBE0$^{35}$ hybrid density functional. The results discussed in the main paper have been obtained with a Gaussian-type triple-$\zeta$-valence+polarization basis set (TZVP) derived from the molecular Karlsruhe basis sets.$^{36}$ Comparisons to smaller split-valence+polarization (SVP) and larger TZVP basis sets as well as detailed basis set listings are included as Supporting information. The weak interlayer interactions were taken into account using Grimme’s empirical D3 dispersion correction.$^{37}$ The influence of the damping function in the D3 correction was evaluated by calculating the lattice parameters both with zero-damping (ZD) and Becke-Jones (BJ) damping.$^{38}$ After initial benchmark calculations, all production calculations were carried out at the DFT-PBE0-D3(ZD)/TZVP level of theory.

The structures of the studied compounds were fully optimized within the $P\bar{3}m1$ space group. Default optimization convergence criteria and DFT integration grid of CRystal14 were applied. Based on the results of convergence tests, the reciprocal space was sampled using a Monkhorst-Pack type $8 \times 8 \times 6$ $k$-mesh.$^{39}$ All Brillouin zone paths for electronic band
structures and phonon dispersions were taken from ref. The majority of the calculations were conducted using very tight Coulomb and exchange integral tolerance factors of 8, 8, 8, 8, and 16 (TOLINTEG). In the case of TiS$_2$ phonon dispersion calculations, even tighter values of 9, 9, 9, 9, and 18 were needed to avoid numerical inaccuracies in the force constant calculation.

The vibrational frequencies at the $\Gamma$ point were obtained within the harmonic approximation by calculating the numerical second derivatives of the potential energy using the scheme implemented in CRYSTAL. IR and Raman intensities for simulating spectra were obtained with the Coupled Perturbed Hartree-Fock method implemented in CRYSTAL. The IR absorbances are reported in units of km mol$^{-1}$, while the Raman intensities are in arbitrary units and normalized to the strongest intensity peak. The IR spectra were broadened by using Lorentzian lineshape and FWHM of 16 cm$^{-1}$. The Raman spectra were broadened using pseudo-Voigt peak profile (50:50 Lorenzian:Gaussian) and FWHM of 16 cm$^{-1}$.

Phonon dispersion calculations were carried out within the harmonic approximation using Phonopy code, which utilizes supercell approach coupled with finite displacement method. In all vibrational frequency and phonon calculations, the convergence criterion of the total energy was set to $10^{-10}$ a.u. to obtain accurate forces. The second order force constants were calculated for ZrS$_2$ and HfS$_2$ using $3 \times 3 \times 2$ supercell and atomic displacement amplitude of 0.03 Å. TiS$_2$ phonon dispersion calculation required a larger $5 \times 5 \times 3$ supercell (see Supporting Information for convergence tests). Additionally, a diagonal finite displacement with an amplitude of 0.09 Å had to be used. Such relatively large atomic displacements have been utilized previously for Mo- and W-based layered dichalcogenides and proven to be reasonable. According to our benchmarks presented in the Supporting Information, the displacement amplitude does not have a significant effect on overall phonon dispersion picture, but improves the numerical stability in the case of TiS$_2$. Non-analytical correction at $\mathbf{q} \rightarrow 0$ was taken into account by using the formulation of Wang et al.
The Seebeck coefficient and electrical conductivity with respect to the relaxation time were calculated with the BoltzTraP code\textsuperscript{49} which is based on Fourier expansion of electron band energies combined with Boltzmann transport theory. The transport coefficients are solved within the constant relaxation time and rigid band approximations. CRYSTAL was used to calculate the wavefunctions for BoltzTraP. For the transport coefficient calculations, we used a dense $k$-mesh of $58 \times 58 \times 29$.

**Experimental**

To obtain experimental data on the transport and vibrational properties of TiS$_2$, we synthesized single crystals \textit{via} vapor transport method using iodine as the transport agent (5 mg/cm$^3$).\textsuperscript{50,51} The original procedure reported in the literature was slightly modified in order to achieve higher purity. The quartz ampoule used in the synthesis was carefully cleaned and evacuated within a Schlenk line to prevent oxidation reactions. Stoichiometric amounts of Ti and S powders were mixed in the reaction ampoule, which was sealed and placed in the furnace at 650°C for 15 hours with heating rate of 1.25 °C/min. After cooling down, the reaction ampoule was relocated to a tube furnace with a linear temperature gradient from 800 to 900°C and heated for 3 days. Eventually, yellow-brownish thin crystals with metallic luster were obtained. The Raman spectrum of TiS$_2$ was recorded with a Horiba LabRAM HR Raman microscope system using wavelength of 632.8 nm. The in-plane Seebeck coefficient ($S$), DC electrical resistivity ($\rho$), and Hall coefficient ($R_H$) together with carrier concentration ($n$) were measured using Quantum Design Physical Property Measurement System.
Results

Structural parameters

As discussed above, TiS$_2$, ZrS$_2$, and HfS$_2$ possess an anisotropic, layered crystal structure where the cross-plane $c$-parameter of the unit cell is governed by weak Van der Waals interactions and the $a$ parameter is determined by the intralayer M–S structure and bonding. To evaluate the importance of dispersion correction, we conducted geometry optimizations using a) plain PBE0 with no dispersion correction, b) PBE0-D3 with Becke-Jones damping, and c) PBE0-D3 with zero-damping (Table 1). Using plain PBE0 with no dispersion corrections resulted in a strong overestimation of the $c$ parameter compared to the experimental values. The error in the predicted $c$ parameter ranged from 8.97% to 14.23%, with Ti showing the smallest error and Hf the largest. We also tested the performance of plain GGA-PBE functional with no dispersion corrections. The predicted lattice constants are rather similar to plain PBE0, with large errors in the $c$ parameter. Furthermore, when using PBE, TiS$_2$ becomes metallic and the band gap of ZrS$_2$ and HfS$_2$ is two times smaller compared to the PBE0 results. Consequently, PBE was not used in any further calculations (the PBE benchmark results are listed in the Supporting Information).

The structural parameters shown in Table 1 illustrate how the DFT-PBE0-D3(BJ) gives too strong interlayer interaction and results in underestimated $c$ parameter. The DFT-PBE0-D3(ZD) approach shows smaller deviations in comparison to the experimental values of the $c$ parameter. Related to our findings here, Pike et al. have recently compared DFT-PBE-D3(ZD) and DFT-PBE-D3(BJ) for several layered transition metal dichalcogenides and reported that DFT-PBE-D3(ZD) gave better agreement with experimental low-energy phonon branches. Based on our geometry optimization benchmarks and additional basis set convergence tests described in the Supporting Information, we chose DFT-PBE0-D3(ZD)/TZVP level of theory for studying the properties of TiS$_2$, ZrS$_2$, and HfS$_2$.

In addition to the structural parameters, we also investigated the interlayer binding
Table 1: DFT-optimized and experimental\textsuperscript{52,53} lattice constants \(a\) and \(c\) (in Å) for TiS\(_2\), ZrS\(_2\), and HfS\(_2\). The numbers in parentheses show the deviation from experimental values.

<table>
<thead>
<tr>
<th>Param.</th>
<th>TiS(_2)</th>
<th>ZrS(_2)</th>
<th>HfS(_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PBE0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a)</td>
<td>3.41 (+0.0%)</td>
<td>3.68 (+0.5%)</td>
<td>3.68 (+1.3%)</td>
</tr>
<tr>
<td>(c)</td>
<td>6.22 (+9.0%)</td>
<td>6.43 (+10.5%)</td>
<td>6.67 (+14.2%)</td>
</tr>
<tr>
<td></td>
<td>PBE0-D3(BJ)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a)</td>
<td>3.36 (-1.4%)</td>
<td>3.63 (-0.8%)</td>
<td>3.64 (+0.1%)</td>
</tr>
<tr>
<td>(c)</td>
<td>5.51 (-3.3%)</td>
<td>5.62 (-3.4%)</td>
<td>5.65 (-3.1%)</td>
</tr>
<tr>
<td></td>
<td>PBE0-D3(ZD)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a)</td>
<td>3.40 (-0.4%)</td>
<td>3.66 (-0.1%)</td>
<td>3.66 (+0.6%)</td>
</tr>
<tr>
<td>(c)</td>
<td>5.67 (-0.7%)</td>
<td>5.73 (-1.5%)</td>
<td>5.73 (-1.8%)</td>
</tr>
<tr>
<td></td>
<td>Experimental data</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a)</td>
<td>3.41</td>
<td>3.66</td>
<td>3.64</td>
</tr>
<tr>
<td>(c)</td>
<td>5.71</td>
<td>5.81</td>
<td>5.84</td>
</tr>
</tbody>
</table>

The interlayer binding energy \(E_b\) is defined here as \(E_b = E(\text{bulk}) - E(\text{monolayer})\). The structure of each MS\(_2\) monolayer was fully relaxed. The interlayer binding energies for TiS\(_2\), ZrS\(_2\), and HfS\(_2\) are 29.4, 28.9, and 28.2 kJ/mol, respectively. Taking the surface area of each monolayer into account, the \(E_b\) values become 26.5, 22.4, and 21.8 meV/Å\(^2\). This compares very well with the \(E_b\) values of 28.6, 24.5, and 24.2 meV/Å\(^2\) obtained by Björkman \textit{et al.} using Vydrov-van Voorhis (VV10) density functional method. The trend shows that the monolayers are slightly more strongly bound in the case of TiS\(_2\) in comparison to ZrS\(_2\) and HfS\(_2\), where the interlayer binding energies are very similar.\textsuperscript{54}

\textbf{Band structures}

The electronic band structures of TiS\(_2\), ZrS\(_2\), and HfS\(_2\) are illustrated in Figure 2. The general features of the band structure are rather similar and all three materials are semiconductors. The valence bands are dominated by sulfur 3\(p\) orbitals whereas the conduction bands are dominated by transition metal \(d\)-orbitals. When moving from Ti to Hf, the conduction bands increase in energy and the band gap increases for TiS\(_2\) \(\rightarrow\) ZrS\(_2\) \(\rightarrow\) HfS\(_2\) as 1.4 eV \(\rightarrow\)
2.7 eV $\rightarrow$ 3.3 eV, respectively. As discussed in the introduction, the available experimental data vary as 0.2–1.5 eV $\rightarrow$ 1.68–1.78 eV $\rightarrow$ 1.96–2.85 eV, the largest band gap values corresponding to the most stoichiometric phases. The DFT-PBE0 and gaps thus seem to be slightly overestimated in comparison to the available experimental values, in particular for ZrS$_2$. Unfortunately, accurate estimates of the sample stoichiometry are not available in the case of ZrS$_2$.

**Population analyses**

We investigated the nature of the intralayer M–S bonding by carrying out Mulliken population analysis. The partial atomic charges are as Ti$^{+0.48}$S$^{-0.24}$, Zr$^{+0.93}$S$^{-0.47}$, and Hf$^{+0.94}$S$^{-0.47}$ which suggests more ionic bonding for ZrS$_2$ and HfS$_2$ in comparison to TiS$_2$. The Mulliken bond population of the M–S bond behaves as ZrS$_2$ $\approx$ HfS$_2$ $<$ TiS$_2$ with values of 0.18, 0.18, and 0.20 e$^-$, respectively. In conclusion, the M–S bonding in TiS$_2$ can be considered to possess the most covalent character of the studied three materials. This is also in agreement with the trend of the Pauling electronegativities (1.54, 1.33, and 1.3 for Ti, Zr, and Hf, respectively).

**Electronic transport coefficients**

As the group 4 disulfides are prospective thermoelectric materials, we investigated their Seebeck coefficient (thermopower) and electrical conductivity at 300 K. The experimental transport coefficients are usually available only in the $ab$-plane because the thin single crystals make it difficult to determine the cross-plane transport coefficients. All group 4 disulfides are known to be n-type semiconductors, but we have investigated the Seebeck coefficient for both n- and p-type doping and for both in-plane and cross-plane. Figure 3 illustrates the $ab$-plane Seebeck coefficient $S_{xx}$ for n-type TiS$_2$, ZrS$_2$, and HfS$_2$ within relevant doping levels. There is practically no difference in the n-type $S_{xx}$ values between the three materials. This is reasonable as the Seebeck coefficient arises from the electronic band structure and the
Figure 2: Band structure and density of states (DOS) at the DFT-PBE0-D3(ZD)/TZVP level of theory: a) TiS$_2$; b) ZrS$_2$; c) HfS$_2$. The top of the valence bands is at 0 eV.
band structures in Figure 2 show how similar the conduction bands of the three materials are. Concerning the anisotropy of the Seebeck coefficient for n-doping, the $S_{xx}$ coefficient is larger than the $S_{zz}$ coefficient in the studied doping range. At an experimentally relevant carrier concentration of $2.5 \times 10^{20}$, the $S_{xx}$ coefficient is larger by about 10, 15, and 20 $\mu$V/K for TiS$_2$, ZrS$_2$, and HfS$_2$, respectively. In relative terms, this means about 10% anisotropy for TiS$_2$ at 300 K. The anisotropy is expected to arise from the different in-plane and cross-plane contributions to the electronic density of states at this doping level. However, due to the relatively small anisotropy, this is not easy to quantify from the band structure and DOS plots. The anisotropy of $S$ has been observed in previous experimental studies for textured polycrystalline TiS$_2$, which showed $S_{xx}$ to be larger than $S_{zz}$. In the case of p-type doping, the overall $|S_{xx}|$ values are smaller than for n-type doping and we did not pursue more detailed investigation in this direction.

The transport properties of TiS$_2$ in both single crystal and polycrystalline form have been studied extensively. Often the experimental results have been presented as a function of temperature instead of doping level, but the effect of the carrier concentration has also been investigated in a number of studies. Here we focus on the experimental results obtained for TiS$_2$ single crystals at 300 K. The lack of experimental transport data for ZrS$_2$ and HfS$_2$ prevents us from presenting any comparisons for them.

The pioneering study of Imai et al. reported in-plane Seebeck coefficient of $-251 \mu$V/K with carrier concentration of $2.8 \times 10^{20}$ cm$^{-3}$. Later, Koumoto et al. reported an in-plane Seebeck coefficient of $-171 \mu$V/K with doping level $3.4 \times 10^{20}$ cm$^{-3}$. The fact that 20% higher charge carrier concentration results in a clearly lower Seebeck coefficient can be explained by non-stoichiometry effect: the excess of Ti atoms inside the interlayer space leads to higher carrier concentration and shifting of the Fermi level towards 3d metal bands, decreasing the Seebeck coefficient. Daou et al. recently investigated the in-plane transport properties of TiS$_2$ single crystals, reporting Seebeck coefficient of $-97 \mu$V/K and carrier concentration of $1.8 \times 10^{21}$ cm$^{-3}$. The TiS$_2$ single crystals synthetized in this study show a
carrier concentration of $1.3 \times 10^{21}$ cm$^{-3}$, fairly close to the value reported by Daou et al. For this carrier concentration, we measured an in-plane Seebeck coefficient of $-106$ $\mu$V/K. Our $|S_{xx}|$ is slightly larger than for Daou et al., which is fully understandable due to the lower carrier concentration in our sample. Considering our predicted values, a doping level of $1.3 \times 10^{21}$ cm$^{-3}$ results in $S_{xx}$ of $-50$ $\mu$V/K, and a lower doping level of $2.6 \times 10^{20}$ cm$^{-3}$ produces $S_{xx}$ of $-103$ $\mu$V/K. The predicted $|S_{xx}|$ is thus underestimated in comparison to the experimental values. The agreement could perhaps be improved by using higher level of theory, but it is not easy to improve further from the hybrid DFT methods applied here. Another important reason for the underestimation may be the rigid band approximation used in the transport calculations: the effect of doping is considered to just cause a rigid shift of the Fermi level in the band structure, while in reality the rather high carrier concentrations typically observed in TiS$_2$ may break the rigid band approximation. Despite the underestimation of the absolute values of $S_{xx}$, the trend of $S_{xx}$ vs. carrier concentration appears to be reproduced rather well. When the carrier concentration is increased first from $2.8 \times 10^{20}$ to $1.3 \times 10^{21}$ cm$^{-3}$ and then further to $1.8 \times 10^{21}$ cm$^{-3}$, $S_{xx}$ first decreases by 53% and then by 11%. In the experimental studies discussed above, the corresponding decrease in $S_{xx}$ was 57% and 8%.

Concerning the electrical conductivity, BoltzTraP can presently calculate it only with respect to the electron relaxation time $\tau$, which is assumed to be direction-independent. In many computational studies of thermoelectric properties, $\tau$ is set to about $10^{-14}$ s, but this may lead in overestimation of the TE performance. We have used here shorter electronic relaxation time of $10^{-15}$ s to avoid any overestimation. Middle figure in Figure 3 shows the electrical conductivity trends for all studied systems. For TiS$_2$, Imai et al. and Daou et al. have reported resistivities of 1.7 and 0.8 m$\Omega$cm, respectively. For the TiS$_2$ single crystals synthetized in this study, we measured in-plane electrical resistivity of 1.3 m$\Omega$cm. At the experimental carrier concentration of $1.3 \times 10^{21}$ cm$^{-3}$, the electrical conductivity predicted here for TiS$_2$ is 91480 S/m, which corresponds to 1.2 m$\Omega$cm.

The electrical conductivity values predicted for the $ab$-plane and cross-plane directions
Figure 3: Top: Seebeck coefficient for n-type doped TiS$_2$, ZrS$_2$, and HfS$_2$ in $ab$-plane ($S_{xx}$) direction. Middle: Electrical conductivity in $ab$-plane ($\sigma_{xx}$) and cross-plane direction ($\sigma_{zz}$) with constant electronic relaxation time of $10^{-15}$ s. Bottom: Ratio of $ab$-plane and cross-plane electrical conductivity.
are clearly different. To investigate the anisotropy in detail, we have plotted $\sigma_{xx}/\sigma_{zz}$ in the bottom part of Figure 3. The anisotropy increases with increasing carrier concentration and also when moving from TiS$_2$ to HfS$_2$. Considering the layered structure of the studied materials, it is not surprising that the $ab$-plane electrical conductivity is significantly larger than in the cross-plane direction. However the anisotropy observed here is significantly smaller in comparison to the 750-fold anisotropy ratio reported by Imai et al. for TiS$_2$ single crystal at 300 K. Possible reason for this discrepancy could be the constant relaxation time approximation applied in our calculation, which means that we assume similar electronic relaxation time for the $ab$-plane and cross-plane direction. Interestingly, in textured ceramic TiS$_2$ samples, the anisotropy ratio has been shown to be about 2.$^4$

**IR/Raman spectra**

Vibrational properties of a material play a key role in understanding various thermodynamic and transport properties of solids, including lattice thermal conductivity.$^{57}$ The phonon modes at the $\Gamma$-point of the first Brillouin zone can be examined by Raman and IR spectroscopies. The primitive cell of TiS$_2$, ZrS$_2$, and HfS$_2$ consists of two chalcogen atoms and one metal atom with trigonal prismatic coordination, resulting in three acoustic modes and six optical modes, the latter of which split in three $g$-modes and three $u$-modes due to the presence of inversion center:

$$\Gamma = A_{1g} + E_g + 2A_{2u} + 2E_u.$$  \hspace{1cm} (3)

$A_{1g}$ and $E_g$ modes are Raman active, while $A_{2u}$ and $E_u$ modes are IR active. One $A_{2u}$ mode and one $2E_u$ mode correspond to the acoustic modes. The $A$ modes represent the atomic displacement along the Cartesian $z$ axis (out-of-plane modes) and the doubly degenerate $E$-modes represent atomic displacements along both $x$ and $y$ axes (in-plane modes). The atomic displacements are illustrated in Figure 4 and the calculated vibrational frequen-
cies are summarized in Table 2. In the case of ZrS$_2$ and HfS$_2$, the calculated vibrational frequencies are in good agreement with the experimental values. For TiS$_2$, the differences between the theory and experiment are larger, especially for the lowest energy $E_u$ mode. We don’t have any conclusive explanation why the $E_u$ mode shows such large difference, but it could be related to the high tendency for non-stoichiometry in TiS$_2$ single crystals. Previous studies of non-stoichiometric TiS$_2$ have mostly focused on its effect on the electronic properties and less on the vibrational properties. Friend et al. did investigate the effect of excess Ti atoms for the high energy $A_{1g}$ optical mode, observing a small shift towards smaller wavenumbers caused by increasing carrier concentration, along with broadening of the peak towards higher energy (appearance of a shoulder).$^{22,58}$ Sandoval et al. suggested the broadening to arise from overtone and/or summation processes.$^{59}$

![Figure 4: Atomic displacements in the IR- and Raman-active modes of TiS$_2$, ZrS$_2$, and HfS$_2$. Blue square and yellow circles represent metal and sulfur atoms, respectively.](image)

**Table 2: Calculated vibrational frequencies of the IR and Raman active optical modes in TiS$_2$, ZrS$_2$, and HfS$_2$ (cm$^{-1}$ units). The values in parentheses show the difference to the experimental spectra.$^{60,61}$**

<table>
<thead>
<tr>
<th>Mode</th>
<th>TiS$_2$</th>
<th>ZrS$_2$</th>
<th>HfS$_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_u$</td>
<td>115 (-35.8%)</td>
<td>168 (-6.4%)</td>
<td>169 (+1.6%)</td>
</tr>
<tr>
<td>$E_g$</td>
<td>258 (+9.8%)</td>
<td>263 (+4.7%)</td>
<td>270 (+3.2%)</td>
</tr>
<tr>
<td>$A_{1g}$</td>
<td>361 (+7.8%)</td>
<td>340 (-2.7%)</td>
<td>346 (+2.6%)</td>
</tr>
<tr>
<td>$A_{2u}$</td>
<td>408 (+9.8%)</td>
<td>346 (-1.2%)</td>
<td>319 (-1.9%)</td>
</tr>
</tbody>
</table>

We also calculated the IR and Raman intensities and the simulated IR and Raman spectra for TiS$_2$, ZrS$_2$, and HfS$_2$ are presented in Figures 5 and 6. Looking at the predioidic trends in
the simulated spectra, we can observe that energy gap between the optical modes decreases when moving from Ti to Hf and the energy gap for TiS$_2$ is clearly larger than for ZrS$_2$ and HfS$_2$. In the Raman spectra, the intensity of the in-plane $E_g$ mode is about three times smaller than that for cross-plane $A_{1g}$ mode. This is in line with the experimental Raman spectra reported for ZrS$_2$ and HfS$_2$.$^{61}$ For the IR active modes, the situation is reversed and the in-plane $E_u$ mode has a much larger absorbance in comparison to the cross-plane $A_{2u}$ mode.

![Figure 5: Calculated Raman spectra for TiS$_2$, ZrS$_2$, and HfS$_2$.](image)

We also have compared the predicted Raman spectra of TiS$_2$ with the experimental Raman spectrum measured for the single crystals prepared for this work (Figure 7). To facilitate comparisons with the experiment, the calculated intensities of TiS$_2$ were scaled by a factor of 18.18. Similar to our TiS$_2$ Raman spectrum in Figure 7, other groups have reported broadening and the appearance of a shoulder for the $A_{1g}$ Raman peak for all three group 4 disulfides.$^{61-63}$ Unlike for TiS$_2$, the shoulder is significantly smaller for ZrS$_2$ and HfS$_2$ and appears on the left side of the $A_{1g}$ peak. Earlier studies have attributed the
Figure 6: Calculated IR spectra for TiS$_2$, ZrS$_2$, and HfS$_2$. Note that y-axis scale for TiS$_2$ is different.

shoulder to the coupling of IR and Raman active modes due to a resonance effect, or, as mentioned above, to non-stoichiometry.$^{22,61,63,64}$ More recent studies have suggested non-harmonic effects such as the emission of low-energy acoustic phonon by optical phonon as the reason to the broadening.$^{62}$

For the IR active $E_u$ and $A_{2u}$ modes, the splitting in Longitudinal optical (LO) and Transverse optical (TO) modes can also be investigated. The LO–TO splitting has been observed experimentally, primarily for the $E_u$ mode, but has not been examined systematically computationally.$^{65,66}$ We have calculated the LO-TO splitting of the $E_u$ and $A_{2u}$ modes and summarized the results in Table 3 together with the available experimental data. The predicted LO-TO splittings for the $E_u$ mode are in a reasonable agreement with the experimental data. The LO–TO splitting is much larger for the low-energy $E_u$ mode in comparison to the $A_{2u}$ mode.

To predict the LO–TO splittings, high frequency dielectric constants ($\varepsilon_\infty$) and Born effective charges were also calculated. Direct comparisons of the dielectric constants with
Figure 7: Simulated and experimental Raman spectrum of TiS$_2$ (both spectra from this work).

Table 3: Calculated and experimental$^{61,63,66}$ LO–TO splitting for TiS$_2$, ZrS$_2$, and HfS$_2$ (cm$^{-1}$ units).

<table>
<thead>
<tr>
<th>Mode</th>
<th>TiS$_2$</th>
<th>ZrS$_2$</th>
<th>HfS$_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_u$ (DFT)</td>
<td>276</td>
<td>199</td>
<td>165</td>
</tr>
<tr>
<td>$E_u$ (exp.)</td>
<td>248</td>
<td>170</td>
<td>155</td>
</tr>
<tr>
<td>$A_{2u}$ (DFT)</td>
<td>20</td>
<td>22</td>
<td>20</td>
</tr>
</tbody>
</table>
experiments are complicated by the wide range of experimental data reported. For example, in case of TiS$_2$, $\varepsilon_\infty$ varies in the range 13.7–19.8$^{20,67}$. The overall trend of decreasing $\varepsilon_\infty$ for TiS$_2$ (15.7) $\rightarrow$ ZrS$_2$ (8.3) $\rightarrow$ HfS$_2$ (7.6) was reproduced in our calculations.

Phonon dispersions

The calculated phonon dispersion relations and phonon density of states (DOS) for TiS$_2$, ZrS$_2$, and HfS$_2$ are illustrated in Figure 8. The group velocity of the longitudinal acoustic (LA) mode in the $\Gamma - A$ direction is rather similar for TiS$_2$ (3.7 km/s) and ZrS$_2$ (3.4 km/s) and somewhat smaller in HfS$_2$ (2.7 km/s). This direction corresponds to the cross-plane direction in the real space and is governed by the weak van der Waals interactions. In the $\Gamma - M$ direction the group velocities of the LA mode are clearly higher in comparison to the $\Gamma - A$ direction. At the same time, the group velocities clearly decrease when moving from TiS$_2$ (6.5 km/s) to ZrS$_2$ (5.1 km/s) and HfS$_2$ (4.2 km/s). This direction corresponds to the $ab$-plane in the real space and corresponds to intralayer M–S bonding. For TiS$_2$, there is also some experimental data available on the longitudinal (5.3 km/s) and transverse (3.3 and 2.8 km/s) sound velocities.$^{13}$ The corresponding phonon group velocities from our calculations are 6.5, 4.4, and 2.4 km/s.

The decrease of phonon energy and group velocity when moving from TiS$_2$ to HfS$_2$ suggests that the M–S bonding in TiS$_2$ is more covalent than in ZrS$_2$ and HfS$_2$, in agreement with the population analysis and Pauling electronegativity differences discussed above. At the same time, the phonon gap between acoustic and optical phonons increases when moving from TiS$_2$ to HfS$_2$, showing clear difference also between Zr and Hf. The appearance of an phonon band gap between the acoustic and optical modes may have an effect on the phonon scattering processes and hence the lattice thermal conductivity of a material. From the phonon DOS we can conclude that acoustic phonons are mostly dominated by displacements of the transition metal atoms, while displacements of the sulfur atoms dominate the high-energy optical phonons. In addition to the phonon dispersion relations, predicting the
lattice thermal conductivity $\kappa_l$ and total thermoelectric figure of merit $ZT$ would require the calculation of third-order (anharmonic) force constants.

The TiS$_2$ phonon dispersion plot shows a soft phonon mode between $\Gamma$ and $M$ wave vectors. The mode softening has been suggested to be due to a charge density wave distortion (CDW), that is, electron density modulation caused by a disruption in the periodic lattice.$^{30,68,69}$ CDWs are known to occur for metallic low-dimensional solids at low temperatures. However, ideal TiS$_2$ is semiconducting and it is normally not classified as a CDW material.$^{70,71}$ For instance, Van Bakel et al. described TiS$_2$ as a ”dirty” small band gap semiconductor which does not show any significant CDW nature at any temperature.$^{72}$ In accordance with this, we do not observe CDW behavior since TiS$_2$ is a semiconductor in our calculations. Our benchmark calculations, discussed in Supporting Information, show that a relatively large supercell is required to properly describe the TiS$_2$ acoustic phonon modes between $\Gamma - M$ and $A - L$ with the DFT-PBE0-D3 method.

**Conclusion**

We have investigated the periodic trends in the electronic and vibrational properties of group 4 disulfides TiS$_2$, ZrS$_2$, and HfS$_2$ using dispersion-corrected hybrid density functional methods. DFT-PBE0-D3(ZD)/TZVP level of theory performs well for a broad variety of electronic and vibrational properties without any system-dependent parametrization. The choice of the damping function for the D3 correction had a surprisingly large effect in the studied systems, the zero-damping approach giving better results in comparison to Becke-Johnson damping. Systematic investigation of the periodic trends within group 4 disulfides reveals that TiS$_2$ shows many differences to ZrS$_2$ and HfS$_2$, the more covalent M–S bonding in TiS$_2$ clearly contributing to the differences. Concerning the thermoelectric properties of the materials, all three compounds show similar Seebeck coefficients, but clear differences in the relative electrical conductivity of cross- and in-plane directions. To fully understand the
Figure 8: Phonon dispersion relations and phonon density of states (in arbitrary units) for TiS$_2$ (top), ZrS$_2$ (middle), and HfS$_2$ (bottom). Non-analytical correction at $q \to 0$ has been taken into account in the phonon dispersion relations.$^{48}$ Note that the DOS scale is different for all three materials.
thermoelectric performance of the group 4 disulfides, the periodic trends of the lattice thermal conductivity must also be investigated. The DFT-PBE0-D3(ZD)/TZVP level of theory describes the group 4 disulfides with such accuracy that it appears to be a very reasonable computational approach for future studies on the lattice thermal conductivity. The phonon gap that arises between acoustic and optical phonons for Hf may have a noticeable effect on the phonon scattering processes and the thermal conductivity of HfS$_2$.

**Supporting Information description**

Additional computational results on the performance of GGA-PBE, basis set convergence tests, and phonon supercell convergence tests. Complete basis set details and the structural data in CRYSTAL input format.

**Acknowledgement**

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**References**


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