Surface-to-Propagating Wave Conversion Using Metasurfaces: Exact Solution

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Abstract – We investigate surface wave to propagating wave conversion. The exact solution of the boundary problem for an impedance surface which fully transforms a given surface wave into a propagating inhomogeneous plane wave (with a polarization transformation) is found, discussed, and numerically demonstrated. The results of the study are of importance for creating leaky-wave antennas with the ultimate efficiency, and can potentially lead to many novel applications, from microwave techniques to nanophotonics.

I. INTRODUCTION

Surface wave to propagating wave conversion (or vice versa) is one of the classical problems in antenna theory and techniques. Such conversions can be performed by using conventional leaky-wave antennas [1], based on periodically modulated open or partially open waveguides. However, even if the surface wave and the space wave modes are ideally phase-matched, unwanted higher-order Floquet modes are excited. Recently, a new way for the conversion efficiency improvement has been proposed for the case of one TM-polarized surface wave and TE-polarized homogeneous plane wave [2], where polarization change is applied to avoid power coupling of the waves with each other [3]. However, the case of the most desirable lossless and reciprocal solution can be only found using an approximation, which imposes essential limitations on the size of the radiating structure. Here, we present and discuss an exact solution for full conversion between one TM-polarized surface wave and one TE-polarized inhomogeneous plane wave. This new scenario allows the theoretically ideal surface-wave antenna performance.

II. PROBLEM STATEMENT

Figure 1 schematically depicts the problem under consideration: the ideal case (without dissipation or scattering losses) of surface-to-inhomogeneous plane waves conversion in free space. The surface wave is launched from the left port and propagates along an impenetrable metasurface characterized by a periodic tensor impedance \( Z(x) \) with period \( d \). Radiated inhomogeneous plane wave is propagating along the normal to the surface, while its amplitude is decaying along \( x \). The aim here is to find such surface impedance distribution which ensures perfect transformation of the surface wave into the propagating wave.

First of all, fields in the presence of a flat periodically modulated surface can be characterized using Floquet modes, which are plane waves. Let us assign the surface wave described as a plane wave bounded to the surface and decaying along the direction of propagation, and inhomogeneous plane wave radiated from the surface (namely, leaky wave) to be 0- and \(-1\) Floquet modes, respectively. The complex wavenumbers of the
0-mode in this case are denoted as
\[ k_{x}^{\text{lw}} = \beta_{x}^{\text{lw}} - j\alpha_{x}, \quad k_{y}^{\text{lw}} = \beta_{y}^{\text{lw}} - j\alpha_{y}, \]
where \( \beta_{x}^{\text{lw}} > k_{0} \) and \( \beta_{y}^{\text{lw}} < 0 \) are the propagation constants, \( \alpha_{x} > 0 \) and \( \alpha_{y}^{\text{lw}} > 0 \) are the attenuation constants (\( x \)- and \( y \)-components, respectively) of the surface wave, and \( k_{0} \) is the free-space wavenumber. Using the Floquet analysis, the complex wavenumbers of the \(-1\)-mode are
\[ k_{x}^{\text{lw}} = \beta_{x}^{\text{lw}} - j\alpha_{x} - 2\pi/d = \beta_{x}^{\text{lw}} - j\alpha_{x}, \quad k_{y}^{\text{lw}} = \beta_{y}^{\text{lw}} - j\alpha_{y}^{\text{lw}}, \]
where \( \beta_{x}^{\text{lw}} \approx 0 \) and \( \beta_{y}^{\text{lw}} > 0 \) are the propagation constants (\( x \)- and \( y \)-components, respectively), \( \alpha_{x} > 0 \) and \( \alpha_{y}^{\text{lw}} \approx 0 \) are the attenuation constants of the inhomogeneous plane wave propagating along the normal to the surface. All the constants should satisfy the free-space dispersion relation.

Secondly, the wave coupler as a point-wise lossless impenetrable metasurface can be designed only when superposition of the surface and propagating waves total fields satisfy the zero net penetration power condition across the \( xy \)-plane everywhere. It leads to the following ratio of magnetic field magnitude \( H_{0}^{\text{lw}} \) of the surface wave to the electric field magnitude \( E_{0}^{\text{lw}} \) of the inhomogeneous plane wave:
\[ \frac{H_{0}^{\text{lw}}}{E_{0}^{\text{lw}}} = \frac{1}{\eta_0} \sqrt{\frac{\beta_{y}^{\text{lw}}}{\beta_{y}^{\text{sw}}}}, \]
where \( \beta_{y}^{\text{lw}} \) is negative. This relation assures that all the power from the surface wave is radiated as the inhomogeneous plane wave. The next step is to find the impedance of such surface.

### III. Solution and Results

The surface located at \( y = 0 \) can be characterized by a surface impedance tensor \( \overline{Z} \), which relates the total tangential electric and magnetic fields as
\[ E_{t} = \overline{Z} \cdot (y \times H_{t}), \]
where \( E_{t} \) and \( H_{t} \) are the tangential electric and magnetic fields, respectively. From the applications point of view, the most desirable solution is a lossless and reciprocal metasurface. In this case, all the elements of the impedance matrix are purely imaginary and the matrix is skew-Hermitian. It leads to the exact impedance of the metasurface which performs the perfect conversion from 0 into \(-1\) Floquet modes and is found to be
\[ \overline{Z} = j \left[ \frac{\eta_0}{k_{0}} \left( \alpha_{y} - \beta_{y}^{\text{lw}} \cot \beta_{x}^{\text{lw}} x \right) \right] \left[ \begin{array}{c} \eta_0 \frac{\sin \beta_{y}^{\text{lw}} x}{\beta_{y}^{\text{lw}}} \sqrt{\frac{\beta_{y}^{\text{lw}}}{\beta_{y}^{\text{sw}}}} \\ -\frac{\sin \beta_{x}^{\text{lw}} x}{\beta_{x}^{\text{lw}}} \sqrt{\frac{\beta_{x}^{\text{lw}}}{\beta_{x}^{\text{sw}}}} \end{array} \right], \]
where \( \eta_0 \) is the free-space wave impedance.

Using full wave simulations with COMSOL Multiphysics [4], the conversion characteristics of the surface defined by (5) can be evaluated. The model set-up has a rectangular cross section in the \( xy \)-plane with the length \( L = 20 \lambda \) and height \( H = 5 \lambda \), where \( \lambda \) is the wavelengths at the operational frequency \( f = 10 \) GHz (results are scalable to any frequency). An impedance boundary condition models the surface, implemented by impressing electric currents specified in terms of electric fields and the surface admittance matrix. On the left side of the model box an active port condition which launches the surface wave was defined. The magnitude \( H_{0}^{\text{lw}} \) is found by setting \( E_{0}^{\text{lw}} = 1 \) V/m and the surface wave propagating constants are chosen to be \( \beta_{x}^{\text{lw}} = 1.1k_{0}, \beta_{y}^{\text{lw}} = -0.1k_{0} \), while the tangential component of the leaky wave propagating constant is negligibly small : \( \beta_{x}^{\text{lw}} \approx 0 \). Therefore, the rest of the wave parameters can be found from the free-space dispersion relation: \( \alpha_{x} = 0.0425k_{0}, \alpha_{y}^{\text{lw}} = 0.4671k_{0}, \beta_{x}^{\text{lw}} \approx k_{0}, \) and \( \alpha_{y}^{\text{lw}} \approx 0 \). On the right side and on the top, the perfectly matched layer boundary condition is set.

Figures 2(a) and (c) show snapshots of the tangential components of the total magnetic and electric fields, respectively. The stable to the mesh density conversion efficiency of 99.7% is obtained over the twenty wavelengths of the metasurface. Visualization of the desired perfect field distribution [shown in Figs. 2(b),(d)] was obtained using MATLAB [5]. We can conclude that the metasurface with the found surface reactance (5) realizes a nearly perfect leaky-wave antenna.
Fig. 2: (a) Simulated and (b) ideal field distributions of tangential component of the total magnetic field. (c) Simulated and (d) ideal field distributions of tangential component of the total electric field.

IV. CONCLUSION

The problem of ideal conversion of a surface wave into a propagating plane wave has been examined theoretically and numerically. It is important to note that the exact solution of the boundary value problem for an impedance surface does not include any higher-order Floquet modes nor asymptotic expansions. It means that there is no energy stored by the higher-order modes: all the power carried by the surface wave is used for the leaky wave creation. This observation shows that this solution may open a way to enhance leaky-wave antenna bandwidth. By varying the components of the complex wavenumbers one can reach faster or slower leakage from the surface.

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