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Correction of local deformations in free vibration analysis of ship deck structures by equivalent single layer elements

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Equivalent single layer (ESL) elements provide easy and computationally effective way to model stiffened plates in Finite Element Analysis of ship structures. Secondary stiffeners are incorporated into the plate or shell formulation in a way that it results in equally divided equivalent stiffness of the element. In the free vibration analysis, these elements do not take into account inertia induced local deformation of plating between the secondary stiffeners. Oscillating motion causes inertia induced body load that locally deforms the plate. This local deformation may have significant effect on the global modal frequencies of a deck structure. The effect is highest when local plate natural frequency is close to the frequency of the studied global mode of the deck. In ship structures such cases can appear e.g. in cabin decks with thin plates and in areas with large non-structural masses. This paper presents a method for correcting ESL modal frequencies by modifying generalized mass and stiffness of the modes. The modification is based on the kinetic and strain energies of the local deformations. Energy components are derived from local consideration of plate in cylindrical bending under enforced support vibration. Local deformation is assumed to be induced only by translational vibration of the ESL elements in their normal direction. The method is validated in case study of ship deck structure against shell mesh results, and good agreement is found.

Keywords: free vibration; equivalent single layer; equivalent element; finite element method; stiffened panel; deck

Nomenclature

A	Amplitude / Membrane stiffness
B	Membrane bending coupling stiffness
D	Bending stiffness
DQ	Shear Stiffness

E	Young's modulus
f	Frequency in Hz
G	Shear Modulus
h	Height
K	Generalized stiffness
L	Length
m	mass of unit area
M	Generalized mass / Moment
N	Number of deck nodes / Normal force
Q	Shear force
r	Response
s	Local coordinate
S	Stiffener spacing
t	Time
t_p	Plate thickness
T	Kinetic energy
U	Strain energy
u,v,w	Displacements in x, y, and z directions

x,y,z Coordinates

Greek Symbols

α Effective area

γ Shear strain

δ Convergence limit

ε Normal strain

θ Slope

κ Curvature

υ Poisson’s ratio

ξ Generalized coordinate

ρ Mass density per volume

Ψ Deflection mode shape in z-direction

ω Frequency in rad-1

Subscripts and superscripts

0 Layer mid plane

C Constant / Not directly function of local deformation

d Dynamic

D Deck plate

ESL	Equivalent Single Layer element result
f	Flange
G	Global
i	Iteration step number / index
L	Local
m	Mode number
n	Node number
NS	Non-structural
p	plate
peak	Peak value
R	Relative quantity
s	Static
w	web

1. Introduction

During the last decades the passenger ships' size and complexity has increased, as so has the passengers' expectations for comfort on board. Therefore, in addition to ship quasi-static response also the vibration performance needs to be evaluated, and advanced numerical methods are needed as the ship geometry is too complex for analytical models to handle. A complete vibration analysis of a passenger ship does not only include the calculation of global vibration modes, but also detailed examination of local structures. Global and local behaviours are often coupled and therefore a method is needed that can assess both at the same time. 3D-FEA is effective tool to handle this interaction, but suffers for computational cost especially and conceptual design stage where time is limited and design space defined multiple materials and structural topologies is to be explored. Therefore, there is a need for a method that combines the speed of analytical methods and flexibility of FEA.

Free vibration of the structure is the foundation of all modal type vibration analyses, and thus has great practical value. Physics of harmonic vibrations has been well known since presented by Lord Rayleigh (1877), whose work was based on the assumption that total energy (potential + kinetic) of the system remains constant. Exact natural frequencies and mode shapes of free vibration were solved from differential equations of motions for beams and plates by direct and approximate methods (known as Rayleigh's method). For more complex structures, these classical solution methods soon become cumbersome. Therefore, attention has been paid for iterative algorithms to efficiently solve discretized modal eigenvalue problem. The classical works in this respect are by Ritz (1909), Krylov (1931) and Lanczos (1950); the latest due to its performance has become popular in modal analysis of large structures by present day Finite Element Method.

3D fine mesh finite element analysis (FEA) is regarded as the most reliable method to model structural behaviour; however it yields to computationally very expensive analyses and therefore is only utilized in small ships; see for example, Boote et al. (2013), Macchiavello and Tonelli (2015) and Lin et al. (2009). In larger vessels, this approach becomes computationally very expensive due to significant modelling efforts and computational time. It also becomes difficult to find global and local modes from the large FE-mesh and these modes are important to identify from the engineering perspective (vibration control). Therefore, the global FE-model is created using coarse mesh, where primary stiffeners i.e. girders and frames are modelled using off-set beam elements and secondary stiffeners are incorporated into the shell element formulations (Hughes, 1988) or lumped in to the neighbouring nodes (DNV-GL 2016). This approach gives the global modes, while the local modes, at smaller length-scale, are solved by smaller models representing the representative structural units; these smaller units can be solved effectively with analytical methods in certain cases. However, the equivalent element techniques consider only the extensional stiffness of the stiffened panel. Therefore, they neglect the effect of bending, length-scale interaction and are unable to evaluate the vibration performance of a local structure, where bending stiffness has significant effect to the total response. In order to remove this deficiency equivalent single layer theory can be used where the global and panel level vibrations are solved with FEA and local vibration of the rectangular plate between the stiffeners analytically.

Avi, Lillemäe, et al. (2015) developed the Equivalent Single Layer (ESL) Mindlin-element for a stiffened panel which includes the membrane, membrane-bending coupling, bending and out-of-plane shear stiffness based on the work of Romanoff and Varsta (2007). They also presented the interaction modelling between the primary stiffeners by offset beams. Case studies showed that it is possible to capture static response with excellent accuracy and the free vibration results agreed very well with the 3D fine mesh model. However, in vibration modes where plate between the stiffeners participates in the fine mesh global mode, see Fig.1, larger differences in frequency were observed. This effect of inertia induced local plate deformations was included in analytical study of cabin decks by Laakso et al. (2013). They showed that adding the local deformations into model can have significant effect on the global deck frequencies for certain geometries. The problem with ESL arises from the fact that the homogenized stiffness properties are used, which means that only the averaged response of the panel is considered and the local behaviour between the discrete stiffeners is neglected. For static case this limitation can be corrected using simple superposition principle as was shown by (Avi, Lillemäe, et al. 2015), but in vibration the problem is more complex. Existing equivalent elements used for vibration analysis of sandwich panels: Kolsters and Wennhange (2009), Lok and Cheng (2000) and Lok and Cheng (2001) do not consider local deformation effects on vibration either. In order to correct this, Avi, Laakso, et al. (2015) proposed approach to correct natural frequencies of the equivalent element results by combining them with the local plate frequencies, which were separately calculated using sub-modelling technique. The combination was done using assumption that stiffeners with plate between them act as springs and masses in series. The method showed very good agreement with 3D fine mesh validation models. However, the study was limited to stiffened panels only, and is not easily extensible for deck structures including girders and other external structures. Laakso et al. (2017) applied analytically a local deformation correction for thin-walled beams, which is utilized by modifying modal generalized mass and stiffness based on the energy involved in the local deformations. This approach requires iteration of local response which introduces a slight additional computational cost, but adds rapidly accuracy for the coupled problems. The benefit of the method is that iterations happen mainly in analytical calculations and that the correction can be precomputed for simple shapes. This accelerates the design process.

The aim of this paper is to develop a correction method for ESL equivalent element free vibration results, which follows the principles presented in Laakso et al. (2017) and which can be utilized in analysis of ship deck structure.

[Figure 1 near here]

1.1. Equivalent Single Layer Element– Relation between Stress Resultants and Strains

According to (Avi, Lillemäe, et al. 2015), a stiffened panel can be modelled with three-layered laminate element. Plate layer has the thickness of the plate t_p, web layer thickness is equal to the height of the stiffener web h_w and the flange layer has the height equal to the stiffener flange h_f.

Displacement relations are given in (Avi, Lillemäe, et al. 2015) as follows:

$$
\begin{aligned}
u_i &= u_0(x,y) - z_i\theta_x(x,y),\ i \in \{p,w,f\} \\
v_i &= v_0(x,y) - z_i\theta_y(x,y),\ i \in \{p,w,f\} \\
w &= w(x,y)
\end{aligned}
\tag{1}
$$

In Eq. (1) u, v, and w are the displacements in x, y and z-directions respectively, and θ the slope around x, and y axis as marked by sub-indexes. Sub-index 0 denotes the layer mid plane, and the layers are referenced through index i as: p is the plate layer, w is the web layer, and f is the flange layer. These variables are shown in Fig. 2.

[Figure 2 near here]

Strain vector $\{\varepsilon\}$ is given in (Avi, Lillemäe, et al. 2015) in terms of mid-plane strain vectors $\{\varepsilon\}^0$ and vector of curvature $\{\kappa\}$ as follows:

$$
\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_i = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix}_i + z_i \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix},\ i \in \{p,w,f\}
\tag{2}
$$

Strains and curvatures of the Eq. (2) in terms of displacement are given in Eq. (3) as taken from (Avi, Lillemäe, et al. 2015):

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \partial u_0/\partial x \\ \partial u_0/\partial x \\ \partial u_0/\partial y + \partial v_0/\partial x \end{Bmatrix} \;\&\; \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = \begin{Bmatrix} -\partial^2 w_G/\partial x^2 \\ -\partial^2 w_G/\partial y^2 \\ -2\partial^2 w_G/\partial x \partial y \end{Bmatrix} \tag{3}$$

In order to properly model the stiffness couplings and mass distribution between stiffened panel and T-girder, the reference plane of the laminate element should be offset to top of the deck plate where the 3D-product model reference plane is, see Fig. 3. The laminate element follows ESL theory presented by Reddy and Ochoa (1992), where the relationship between homogenized internal forces, strain and curvatures are presented as a single matrix form:

$$\begin{Bmatrix} N \\ M \\ Q \end{Bmatrix} = \begin{bmatrix} A & B & 0 \\ B & D & 0 \\ 0 & 0 & D_Q \end{bmatrix} \begin{Bmatrix} \varepsilon \\ \kappa \\ \gamma \end{Bmatrix} \tag{4}$$

Normal force {N}, moment {M} and shear force {Q} vectors in Eq. (4) are related to strain {ε}, curvature {κ} and out of plane shear strain {γ} vectors by stiffness matrix that consists of membrane [*A*], membrane-bending [*B*], bending [*D*], and out-of-plane shear [D_Q] stiffness matrices. The stiffness matrixes for stiffened panel are obtained from Eqs. (5), (6), (7) and (8) respectively:

$$[A] = \int_{-tp}^{0} [E]_p \,\mathrm{dz} + \int_{0}^{hw} [E]_w \,\mathrm{dz} + \int_{hw}^{hw+hf} [E]_f \,\mathrm{dz} \tag{5}$$

$$[B] = \int_{-tp}^{0} [E]_p \,\mathrm{zdz} + \int_{0}^{hw} [E]_w \,\mathrm{zdz} + \int_{hw}^{hw+hf} [E]_f \,\mathrm{zdz} \tag{6}$$

$$[D] = \int_{-tp}^{0} [E]_p \,z^2\mathrm{dz} + \int_{0}^{hw} [E]_w \,z^2\mathrm{dz} + \int_{hw}^{hw+hf} [E]_f \,z^2\mathrm{dz}, \tag{7}$$

In Eqs (5-7) the plate layer elasticity matrix $[E]_p$ is described as a 2D isotropic shell element. The web and flange layers are described as 2D orthotropic shell elements, where the components of the elasticity matrix $[E]_w$ and $[E]_f$ are found by applying the Rule of Mixtures. The out-of-plane shear stiffness matrix $[D_Q]$ contains the shear stiffness in stiffener direction D_{Qx} and transverse to stiffener direction D_{Qy}:

$$[D_Q] = \begin{bmatrix} D_{Qx} & 0 \\ 0 & D_{Qy} \end{bmatrix}, D_{Qx} = k_{xz}(G_p t_p + G_w h_w + G_f h_f), D_{Qy} = k_{yz}(G_p t_p) \tag{8}$$

In Eq. (8), k_{xz} and k_{yz} are shear correction factors in xz-plane and yz-plane respectively. G_p is the shear modulus for the plate layer and G_w and G_f are the shear moduli for web and flange layers, respectively, which are calculated according to Rule of Mixtures, see (Avi, Lillemäe, et al. 2015).

Free vibration modes of ESL model are found by solving the eigenvalues and vectors of displacement based equations of motion presented by Reddy and Ochoa (1992). Algorithms to solve the eigenvalue problem, such as Lanczos (1950), are available in many commercial computer programs such as Abaqus, Ansys and Nastran.

[Figure 3 near here]

2. Correction method

2.1. Assumptions and limitations

This study considers ship deck structures that can include, in addition to the deck plate and its stiffeners, also T-girders, pillars, adjacent bulkheads and other structures. However, the present correction is limited to only include local deformations induced by translational inertia of the deck plate in its normal direction. Coordinate system is defined so that the deck reference plane coincides with the xy-plane. Coordinate x is pointing in the stiffener direction, y is perpendicular to x, and z is normal to the deck plate. An additional coordinate s is used for describing local transversal distance from stiffeners. The used coordinate system is illustrated in Fig. 4.

Small amplitude free vibration of a linear system is assumed. Stiffener and plate connections are assumed to follow global reference plane obtained by the ESL FEM. The study is limited to correct global modes where global flexural waves are long in comparison of local plate field dimensions. This limitation leads to an assumption that, in local consideration, the deck plate is supported by the global reference plane only through the stiffener and plate connections.

[Figure 4 near here]

2.2. Modes as generalized single degree of freedom systems

Free harmonic vibration modes of undamped linear vibration systems are orthogonal; see e.g. (Feeny and Kappagantu 1998). Thus each individual mode can be considered separately as single degree of freedom system with generalized stiffness K_m, and generalized mass M_m, as shown in Fig.5.

[Figure 5 near here]

Displacement of deck plate in z-direction w of each mode m in free vibration can be defined by a harmonic function of time, such as sine:

$$w_m(x, y, \omega_m, t) = \Psi_m(x, y, \omega_m)\sin(\omega_m t) \tag{9}$$

In Eq. (9) the mode shape Ψ_m defines the relative amplitude distribution within the structure. In this study, it consists of a global part obtained by modal FE analysis for the ESL model, and a local part, which is considered analytically. The local part of the mode shape is a function of frequency. The mode shape is considered closer in Chapter 2.3.

Let us now define a generalized coordinate ξ_m:

$$\xi_m(t) = w_m(x_0, y_0, \omega_m, t) \tag{10}$$

Such point (x_0, y_0) is chosen that Eq. (11) holds.

$$\Psi_m(x_0, y_0, \omega_m) \neq 0 \tag{11}$$

The modal vibration Eq. (9) can now be written as motion of the generalized degree of freedom, Eq. (10), as follows:

$$\xi_m(t) = A_m \sin(\omega_m t) \tag{12}$$

In Eq. (12) A_m is amplitude, which depends on selection of the generalized coordinate, and the mode shape Ψ_m. Generalized properties of the single degree of freedom system Eq.(12) can now be defined. Modal angular frequency ω_m is defined by the square root of the ratio between the generalized stiffness K_m and generalized mass M_m, as written in Eq. (13).

$$\omega_m = \sqrt{\frac{K_m(\omega_m)}{M_m(\omega_m)}} \tag{13}$$

Generalized mass and stiffness are derived from kinetic and strain energies respectively, as presented in Chapters 2.4, and 2.5. Iterative approach to find the angular frequency from Eq. (13) is presented in Chapter 2.6.

2.3. Mode shape

Free vibration of global mode m is considered. Displacement as function of time was given in Eq. (9). Displacement mode shape, $\Psi_m(x, y, \omega_m)$, can be divided into sum of global reference plane (ESL) mode shape Ψ_{Gm}, and local plate deformation shape Ψ_{Lm} as illustrated in Fig. 6:

$$\Psi_m(x, y, s, \omega_m) = \Psi_{Gm}(x, y) + \Psi_{Lm}(x, y, s, \omega_m) \quad (14)$$

According to Clough and Penzien (1993), response of the assumed linear vibration is directly proportional to the excitation amplitude. Due to the long wavelength assumption, the local plate in location x, y is supported (and excited) only by the global reference plane through deck plate and stiffener connection points. Therefore the local amplitude is directly proportional to the amplitude of the global reference plane in those points which equals the global ESL mode shape $\Psi_{Gm}(x, y)$. Local part of the mode shape, Ψ_{Lm} in Eq (14), can thus be written as:

$$\Psi_{Lm}(x, y, s, \omega_m) = \Psi_{Gm}(x, y)\Psi_{LR}(s, \omega_m) \quad (15)$$

In Eq. (15), Ψ_{LR} is the local plate mode shape relative to the global reference plane amplitude i.e. the deformation caused by unit amplitude enforced displacement of global reference plane. By combining Eq. (14) and Eq. (15), total mode shape gets the form:

$$\Psi_m(x, y, s, \omega_m) = \Psi_{Gm}(x, y)[1 + \Psi_{LR}(s, \omega_m)] \quad (16)$$

It should be noticed that global reference plane mode shape $\Psi_{Gm}(x, y)$ is not function of frequency, and remains constant during the correction process.

[Figure 6 near here]

2.4. Generalized mass from kinetic energy

Kinetic energy T_m of the generalized single degree of freedom system can be defined by generalized mass M_m as follows:

$$T_m(t) = \frac{1}{2} M_m \dot{\xi}_m^{\,2} = \frac{1}{2} A_m{}^2 \omega_m{}^2 M_m \cos^2(\omega_m t) \quad (17)$$

The generalized mass can thus be found by considering the peak value of kinetic energy of the vibration mode. That occurs at the time when $\cos^2\omega_m t = 1$. The generalized mass can be solved from Eq. (17). It is written in Eq. (18):

$$M_m = \frac{2T_m^{peak}}{{A_m}^2{\omega_m}^2} \tag{18}$$

The translation kinetic energy of the vibration mode T_m can be written as sum of kinetic energy of z-directional translation of the deck plate T_{Dz} and kinetic energy of other structures and directions T_C.

$$T_m(\omega_m, t) = T_{Cm}(\omega_m, t) + T_{Dzm}(\omega_m, t) \tag{19}$$

The term T_C in Eq. (19) represents the kinetic energy that does not depend directly on the local deformation part of the mode shape. However, it is a function of frequency due to the fact that velocity changes with the frequency. This effect is taken into account by Eq. (38) in Chapter 2.6.

The deck plate z-direction part of the kinetic energy T_{Dz} is more interesting as it includes the kinetic energy of the local deformation. The energy term is by definition:

$$T_{Dzm}(\omega_m, t) = \oiint \left[\frac{1}{2} m(x, y) \left(\frac{\partial w_m(x,y,\omega_m,t)}{\partial t}\right)^2\right] \tag{20}$$

In Eq. (20) z-velocity can be written in terms of mode shape of Eq. (9) yielding:

$$\frac{\partial w_m(x,y,\omega_m,t)}{\partial t} = \omega_m \Psi_m(x, y, \omega_m)\cos(\omega_m t) \tag{21}$$

Peak velocity of Eq. (21) occurs when $\cos(\omega_\mathrm{m}) = 1$. Now, peak value for the kinetic energy of Eq. (19) can be written:

$$T_m^{peak}(\omega_m) = T_{Cm}^{peak}(\omega_m) + \frac{{\omega_m}^2}{2} \oiint [m(x, y)\Psi_m(x, y, \omega_m)^2] \tag{22}$$

By assuming constant mass distribution, and inserting mode shape of Eq. (16) into Eq. (22):

$$T_m^{peak}(\omega_m) = T_{Cm}^{peak}(\omega_m) + \frac{\omega_m{}^2 m}{2} \oiint \left[\Psi_{Gm}(x,y)^2 \int_0^S \left(1 + \Psi_{LR}(s,\omega_m)\right)^2 ds\right] \qquad (23)$$

In Eq. (23), global mode shape Ψ_{Gm} is the only function of the global location (x, y) inside the integral. The part independent of location can now be defined as local kinetic energy factor T_{LR}. It represents kinetic energy of unit area of the deck under enforced excitation of unit amplitude of the global reference plane:

$$T_{LR}(\omega_m) = \frac{\omega_m{}^2 m}{2S} \int_0^S (1 + \Psi_{LR}(s,\omega_m))^2 ds \qquad (24)$$

Closed form solution for local kinetic energy factor of Eq. (24) is defined in Appendix A. Alternatively; it can be calculated by a sub model. The generalized mass of Eq. (18) now gets the form:

$$M_m(\omega_m) = \frac{2}{\omega_m{}^2 A_m{}^2} \left[T_{Cm}^{peak}(\omega_m) + T_{LR}(\omega_m) \oiint \left(\Psi_{Gm}(x,y)\right)^2\right] \qquad (25)$$

For application with Finite elements method, area integral in Eq. (25) is replaced by area weighted summation of nodal z-translation values of the deck plate nodes in ESL model. The resulting definition for generalized mass is written:

$$M_m(\omega_m) = \frac{2}{\omega_m{}^2 A_m{}^2} \left[T_{Cm}^{peak}(\omega_m) + T_{LR}(\omega_m) \sum_{n=1}^{N} (\alpha_n \Psi_{Gmn}{}^2)\right] \qquad (26)$$

In Eq. (26), α_n is effective area of deck plate of node n, Ψ_{Gmn} is the nodal z-translation value of the global mode shape vector in node n, and N the number of nodes connected to the deck plate.

2.5. Generalized stiffness from strain energy

Generalized stiffness K_m of mode m is considered. Starting point of the consideration is the definition of strain energy U_m for generalized single degree of freedom system in Eq. (27).

$$U_m(t) = \frac{1}{2} K_m {\xi_m}^2 = \frac{1}{2} {A_m}^2 K_m \sin(\omega_m t) \tag{27}$$

Generalized stiffness can now be solved as function of peak strain energy:

$$K_m = \frac{2}{{A_m}^2} U_m^{peak} \tag{28}$$

Strain energy Eq. (27) for the considered deck structure can be presented as sum of strain energy of local deformation U_{Lm} and a constant part of strain energy U_{Cm} . The constant part U_{Cm} includes all strain energy of the mode shape received by FE analysis of the ESL model as the global reference plane deformation shape is constant in the correction.

$$U_m(\omega_m, t) = \left[U_{Lm}^{peak}(\omega_m) + U_{Cm}^{peak}\right] \sin(\omega_m t) \tag{29}$$

Peak values occur when sin($\omega_m t$)= 1:

$$U_m^{peak}(\omega_m) = U_{Lm}^{peak}(\omega_m) + U_{Cm}^{peak} \tag{30}$$

In Eq. (30) peak strain energy induced by the local deformation needs closer consideration:

$$U_{Lm}^{peak}(\omega_m) = \frac{1}{2} \oiint \frac{1}{S} \int_0^S D(s) \left[\frac{\partial^2 [\Psi_{Lm}(x,y,s,\omega_m)]}{\partial s^2}\right]^2 ds \tag{31}$$

In Eq. (31) D is local bending stiffness of the deck plate, s is the local coordinate, and S the stiffener spacing. Local mode shape Ψ_{Lm} is given in Eq. (15):

$$U_{Lm}^{peak}(\omega_m) = \frac{1}{2}\oiint \frac{1}{S}\int_0^S D(s)\left[\frac{\partial^2[\Psi_{Gm}(x,y)\Psi_{LR}(s,\omega_m)]}{\partial s^2}\right]^2 ds \tag{32}$$

By assuming constant local bending stiffness, the peak local strain energy of Eq. (32) simplifies into:

$$U_{Lm}^{peak}(\omega_m) = \frac{D}{2S}\int_0^S \left[\frac{\partial^2[\Psi_{LR}(s,\omega_m)]}{\partial s^2}\right]^2 ds \oiint\left(\Psi_{Gm}(x,y)\right)^2 \tag{33}$$

In order to simplify Eq. (33), factor U_{LR} is defined as peak strain energy of unit area of the deck under enforced excitation of unit amplitude of the global reference plane.

$$U_{LR}(\omega_m) = \frac{D}{2S}\int_0^S \left[\frac{\partial^2[\Psi_{LR}(s,\omega_m)]}{\partial s^2}\right]^2 ds \tag{34}$$

Closed form solution for Eq. (34) is presented in Appendix A. Alternatively U_{LR} can be calculated by a sub model. Modal stiffness of Eq. (28) can now be written:

$$K_m(\omega_m) = \frac{2}{{A_m}^2}\left[U_{Cm}^{peak} + U_{LR}(\omega_m)\oiint\left(\Psi_{Gm}(x,y)\right)^2\right] \tag{35}$$

For application with Finite elements method, area integral in Eq. (35) is replaced by area weighted summation of nodal z-translation values of the deck plate nodes in ESL model. This is done in Eq. (36).

$$K_m(\omega_m) = \frac{2}{{A_m}^2}\left[U_{Cm}^{peak} + U_{LR}(\omega_m)\sum_{n=1}^{N}(\alpha_n {\Psi_{Gmn}}^2)\right] \tag{36}$$

2.6. Frequency by iteration

Angular frequency of mode m can now be calculated from generalized mass and stiffness by Eq. (13). However, in Eq. (13) generalized stiffness and mass are both functions of the frequency itself. Iterative approach is thus needed to find the angular frequency separately for each mode *m*. Angular frequency for iteration step *i*+1 is obtained from previous step *i* by solving following equation:

$$\omega_{i+1} = \sqrt{\frac{K_m(\omega_i)}{M_m(\omega_i)}} \tag{37}$$

Modal mass and stiffness in Eq. (37) are solved from Eqs. (26) and (36). Initial guess of frequency is needed for $i = 0$. The guess is made based on ESL frequency ω_{ESL} and local frequency ω_L as presented in Table 1. These are educated guesses and there is not in depth study behind the values. However, same guesses were applied successfully for beams by Laakso et al. (2017).

[Table 1 near here]

In equation (26) there is term for kinetic energy T_{Cm} that represents all kinetic energy of the model other than *z*-component of the deck plate. As the term is function of frequency which changes due to iteration, also that term should change accordingly. Kinetic energy is proportional to square of the velocity, thus the term changes in relation of squares of the frequencies as written in Eq. (38).

$$T_{Cm}^{peak}(\omega_{i+1}) = \frac{{\omega_i}^2}{{\omega_{ESL}}^2} T_{Cm}^{peak}(\omega_{ESL}) \tag{38}$$

Now all necessary information is available for carrying out iterative procedure to find corrected modal frequency. The iterative procedure is presented in Figure 7. Iteration is continued until desired convergence is found. There are only closed form formulae inside the iteration loop which makes the process very effective.

[Figure 7 near here]

3. Case study

Case study is performed by using commercial Finite Elements solver NX Nastran 10.0.

Pre- and post-processing of the models are done by Femap 11.3 software. The presented correction method is applied for correcting results obtained by FE model, where deck plate and its stiffeners are modelled by ESL elements. Model with shell element deck plate with discretely modelled stiffeners is used for validation purposes. Mesh size and structures other than the studied stiffened deck plate are similar for ESL and validation models. Normalization of the mode shapes is made so that generalized mass of the FE results equal unity.

3.1. Passenger ship deck with pillars and T-girders

Deck structure from (Avi, Lillemäe, et al. 2015) is chosen for the case study as it represents typical passenger ship cabin area. The structure consists of 19.2 meters long and 10.88 meters wide steel deck with 7 transversal and one longitudinal T-440x7+FB150x10 T-girders, and 3 vertical D150x15 pillars in the middle, see Fig. 8. The deck plate is 6 mm thick and is stiffened by HP 100x6 stiffeners with spacing 0.68 m. Only structural mass with density 7850 kg/m^3 is applied.

[Figure 8 near here]

General mesh size of 100 mm is used, and exactly the same nodes are used for both models: the ESL model, and the validation model. Girders are modelled as offset beam elements. Pillars are modelled as beam elements. Only the elements of deck plate are different between the two models: In the ESL model the stiffened deck plate is modelled by ESL elements only, and in the validation model as combination of shell elements for the plate and offset beam elements for the stiffeners. This makes comparison reasonable for validation purpose, as stiffeners are connected to T-girders and constraints only through the nodes in deck reference plane in both models. Both models are constrained in the boundaries by putting all 6 degrees of freedom to zero in the boundary nodes, including the pillar end nodes above and below the deck.

Modes that represent global deck vibration are selected for detailed validation of the method. The selected 5 modes are denoted by letters as A, B, C, D and E. Their properties and frequencies by different methods are compared in Table 2. Four of these modes (A, B, D and E) were also studied by (Avi, Lillemäe, et al. 2015). With long wavelengths, these modes are in line with the assumptions made for this study. Modes A and B appear at mode numbers 1 and 4 respectively in both ESL and Shell models. Mode C appears at mode number 11 in the ESL model, and at mode number 28 in shell model. Reason for different mode numbers is that several purely local plate modes appear in the shell mesh model below this mode. For the same reason, also modes D and E appear at different numbers in the ESL model and the shell model.

[Table 2 near here]

Mode shapes of the modes A, B, C, D and E by the validation shell model, and the ESL model are presented in Table 3. By visual comparison of the shapes, it is possible to verify that they represent physically same modes in both models. It is also possible to see the local deformation as part of the shell model mode shape and that ESL shapes are lacking the local deformation. This local deformation is further visualized in Fig. 9, which shows enlarged view of mode D shapes by the 2 models.

[Figure 9 near here]

The ESL frequencies are corrected by adding the missing effect of the local deformations by the method presented in this paper. Convergence of the iteration of Fig. 7 is very fast and is shown in Fig 10. Frequencies of all modes find $\delta < 10^{-5}$Hz convergence already in step $i = 4$.

[Figure 10 near here]

Difference in frequencies by corrected ESL and validation model vary between -1.99 % and +0.05 %. Improvement in accuracy from uncorrected ESL results is significant as the difference between frequencies of uncorrected ESL, and the validation models are between 4% and 10 %. The correction method tends to slightly underestimate the frequency of modes with shorter wavelength in x-direction (modes B and A), while being very precise for modes with longer wavelengths (modes D and E). Likely reason for this is that assumption of long global waves was made, in order to assume that plate is locally supported by stiffeners only. The assumption is less valid for modes with shorter wavelengths leading to underestimated of stiffness.

[Table 3 near here]

4. Conclusion

This study presented a correction method for improving accuracy of modal free vibration results of ship deck structures obtained by Equivalent Single Layer (ESL) Finite Element model as defined by (Avi, Lillemäe, et al. 2015). Generalized mass and stiffness of the modes are redefined while accounting for kinetic and strain energies involved in local plate deformation between the stiffeners. The correction method is based on the classical assumption of conservation of energy in free undamped vibration (Rayleigh 1877), and follows similar principles as correction method for beams by Laakso et al. (2017). Correction is carried out by iteration of closed form equations, thus it is computationally extremely light.

Methods that take the local deformations between the stiffeners into account have been presented before by Laakso et al. (2013) in analytical consideration of cabin deck free vibration and (Avi, Laakso, et al. 2015) for correcting ESL free vibration results of stiffened panels. Advantage of the present method is its applicability to more general structures, e.g. girders and pillars can be included in the model while the correction is made for the stiffened deck. In comparison to fine mesh 3D FEM, main advantage of present ESL based model is the possibility to use coarser mesh and thus significantly reduce computational effort.

ESL with the applied correction provides very good accuracy in global modal frequencies. Error in comparison of fine mesh is reduced to less than 2 % from 4-10 % error of uncorrected ESL results. The method seems to slightly underestimate the modal frequencies, but is much more accurate than methods which consider only the orthotropic ESL layer response (e.g. Lok and Cheng, 2000). Magnitude of the error grows slightly towards modes with shorter global wavelengths. Similar trend was observed for thin-walled beams in (Laakso et al. 2017). Reason for this is that assumption of long flexural waves is less valid.

The study was limited to cases where correction was only made for one in-plane stiffened deck plate with constant stiffener spacing, mass and stiffness. Generalizing the correction into new applications is left for further work. These could include application with sandwich panels such as those studied by Kolsters and Wennhage (2009), and Romanoff and Varsta (2007). Another direction for further work would be generalizing the method to be used with more complex structural applications where local deformations are occurring simultaneously in multiple stiffened panels with different properties and orientations.

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Appendix A. Local response and energy factors

This appendix presents definition of kinetic and strain energy factors of local plate deformation for unit area of the stiffened plate excited by unit amplitude enforced support motion. For simplicity cylindrical bending of clamped-clamped strip is assumed as illustrated in Fig. A1.

[Figure A1 near here]

Response is calculated by modal approach assuming that only lowest clamped-clamped mode is active. Mode shape is given in (Blevins 1979). Response of the mode is the following:

$$\Psi_{LR}(s,\omega) = \frac{r_d(\omega)}{B}\left(\sigma\sin\left(\frac{\beta s}{S}\right) - \sigma\sinh\left(\frac{\beta s}{S}\right) - \cos\left(\frac{\beta s}{S}\right) + \cosh\left(\frac{\beta s}{S}\right)\right) \tag{A1}$$

Parameter values for $\sigma \approx 0.982502215$ and $\beta \approx 4.73004074$ are given in (Blevins 1979) for the considered mode. Normalization factor B is solved by setting mid span response equal to dynamic response in that point r_d. It follows:

$$B = \sigma\sin\left(\frac{\beta}{2}\right) - \sigma\sinh\left(\frac{\beta}{2}\right) - \cos\left(\frac{\beta}{2}\right) + \cosh\left(\frac{\beta}{2}\right) \approx 1.58814626 \tag{A2}$$

Numeric solutions can be found for the energy factors: Kinetic energy factor from Eq. (21):

$$T_{LR}(\omega) = \frac{\omega^2 m}{2S}\int_0^S (1 + \Psi_{LR}(s,\omega))^2 ds \tag{A3}$$

$$\approx \omega^2 m\left(\frac{1}{2} + 0.523164\, r_d(\omega) + 0.198239\, {r_d}^2(\omega)\right) \tag{A4}$$

Strain energy factor from Eq. (30):

$$U_{LR}(\omega) = \frac{D}{2S}\int_0^S \left[\frac{\partial^2[\Psi_{LR}(s,\omega)]}{\partial s^2}\right]^2 ds \tag{A5}$$

$$\approx 99.23127\frac{D{r_d}^2(\omega)}{S^4} \tag{A6}$$

Dynamic response of the mid span $r_d(\omega)$ is needed. It can be written as dynamically amplified static response r_s as follows:

$$r_d(\omega) = \frac{r_s(\omega)}{1 - \frac{\omega}{\omega_L}} \tag{A7}$$

where modal frequency of the clamped-clamped lowest mode ω_L is the following (Blevins 1979):

$$\omega_L = \frac{\beta^2}{S^2}\sqrt{\frac{D}{m}} \tag{A8}$$

Static response r_s can be found by considering static bending in mid span that uniform inertia load of unit amplitude motion would cause. According to Newton's 2nd law, and tabulated solution (Parnes 2001):

$$r_S(\omega) = \frac{m\omega^2 S^4}{384D} \tag{A9}$$

Finally bending stiffness D:

$$D = \frac{{t_p}^3 E}{12(1-\upsilon^2)} \tag{A10}$$

and mass for unit area of deck plate:

$$m = t_p\rho + m_{NS} \tag{A11}$$

where ρ is material density and m_{NS} non-structural mass per area.