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A Stochastic Mixed-Integer Convex Programming Model for Long-Term Distribution System Expansion Planning Considering Greenhouse Gas Emission Mitigation

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Abstract

This paper proposes a multistage convex distribution system planning model to find the best reinforcement plan over a specified horizon. This strategy determines planning actions such as reinforcement of existing substations, conductor replacement of overloaded feeders, and siting and sizing of renewable and dispatchable distributed generation units. Besides, the proposed approach aims at mitigating the greenhouse gas emissions of electric power distribution systems via a monetary form. Inherently, this problem is a non-convex optimization model that can be an obstacle to finding the optimal global solution. To remedy this issue, convex envelopes are used to recast the original problem into a mixed integer conic programming (MICP) model. The MICP model guarantees convergence to optimal global solution by using existing commercial solvers. Moreover, to address the prediction errors in wind output power and electricity demands, a two-stage stochastic MICP model is developed. To validate the proposed model, detail analysis is carried out over various case studies of a 34-node distribution system under different conditions, while to show its potential and effectiveness a 135-node system with two substations is used. Numerical results confirm the effectiveness of the proposed planning scheme in obtaining an economic investment plan at the presence of several planning alternatives and to promote an environmentally committed electric power distribution network.

Keywords: Conic programming, distributed energy, multistage distribution system expansion planning, renewable energy sources, stochastic programming.

Nomenclature

Sets and Indexes

Ω_a	Set of conductor types.
$\Omega_b/\ \Omega_d^b$	Set of time blocks/load scenarios at time block b.
$\Omega_g/\ \Omega_r$	Set of candidate nodes for gas/wind DG units allocation.
$\Omega_l/\ \Omega_n$	Set of circuits/nodes.
$\Omega_d^w/\ \Omega_w^b$	Set of wind and load scenarios/wind scenarios at block time b.
$\Omega_p/\ \Omega_t$	Set of investment periods/operating years.
$\Omega_s/\ \Omega_k^{SS}$	Set of substation buses/substation expansion types.
$\Omega_k^G/\ \Omega_k^W$	Set of gas/wind turbine types.
$N(i)$	Set of nodes connected to the bus i by a branch.
a	Index of conductors.
b	Index of time blocks.
$c/\ d$	Index of scenarios/demand scenarios.
$g/\ r$	Index of gas/wind DG candidate buses.
k	Index of investment alternatives.
s	Index of investment substation buses.
$(l;ij)/\ i$	Index of branches/buses.
$p/\ t$	Index of investment periods/ operating years.

Parameters

a_0^l	Conductor type existing in branch l.
$b_{ij,a}/\ b_{ij,a}^{sh}$	Series/shunt susceptance in the π-model of branch ij for conductor type a
$C_k^{I,G}/\ C_k^{I,W}$	Installation cost of gas/wind turbine type k.
$C_k^{I,SS}$	Reinforcement cost of substation with transformer type k.
$C_{a_0^l,a}^{I,L}$	Cost of replacement conductor type a_0 with conductor type a in branch l.
C_t^C	CO_2 emission tax rate in year t [\$/h].
$C_{c,t}^E$	Energy cost in scenario c in year t [\$/kWh].
$C_k^G/\ C_k^W$	Operating cost of the gas turbine/wind turbine [\$/kWh].
C^U	Cost of unserved energy [\$/kWh].
$e^{gr}/\ e_k^{gs}$	Emission factor of the main grid/gas turbine type k [kg CO_2 /kWh].
$f_c^D/\ f_{k,c}^W$	Demand level factor/wind level for turbine type k in scenario c.
$g_{ij,a}$	Conductance in the π-model of branch ij, for conductor a.

$\overline{I}_a$	Maximum current capacity of conductor a.
INV_p	Investment budget for period p.
l_l	Length of branch l [km].
$P^D_{i,t}/\ Q^D_{i,t}$	Active and reactive power load at node i at year t.
$\overline{P^G_k}/\ \overline{P^W_k}$	Maximum real power output of gas/wind turbine type k.
$R_{l,a}$	Resistance of branch l and conductor type a.
$S^o_s/\ S^{'}_{s,k}$	Apparent power limit of existent substation/ transformer k for upgrading substation s.
T_c	Time in hours of the scenario c.
$\tan(\varphi^G_i/\varphi^W_i)$	Tangent angle of DG gas/wind technology.
$\overline{V}_i/\ \underline{V}_i$	Upper and lower voltage bounds at node i.
λ	Years of planning horizon per period.
ρ^b_D/ρ^b_W	Probability of the load and wind level in time block b.
ρ_c	Probability of the scenario c.
$\hat{\rho}^b_D/\hat{\rho}^b_W$	Average load and wind speed level in time block b.
τ	Interest rate.
ϑ	Maximum level of DG penetration.
ζ	Life cycle for devices.
Variables	
$I^{sqrt}_{l,a,c,t}$	Square of current in branch l, conductor type a, scenario c and year t.
$y^L_{l,a,p}$	Binary investment variable that defines the conductor type a on branch l and period p.
$y^G_{g,k,p}/\ y^W_{r,k,p}$	Binary investment variable that defines the installation of gas/wind turbines type k in bus g/r and period p.
$y^{SS}_{s,k,p}$	Binary investment variable that defines the installation of transformer type k in substation s at period p .
$z^L_{l,a,t}$	Binary utilization variable that defines the conductor type a on branch l and year t.
$P^{SS}_{i,c,t},\ Q^{SS}_{i,c,t}$	Active and reactive power purchased by the substation at bus i, scenario c, and year t.
$P^G_{i,k,c,t},\ Q^G_{i,k,c,t}$	Active and reactive power injected by the gas turbine type k in bus i, scenario c, and year t.

$P^U_{i,c,t},\ Q^U_{i,c,t}$	Unserved active and reactive power at bus i, scenario c and year t.
$P^W_{i,k,c,t},\ Q^W_{i,k,c,t}$	Active and reactive power injected by the wind turbine k in bus i, scenario c and year t.
$P_{ij,c,t},\ Q_{ij,c,t}$	Active and reactive power flow through branch ij, in scenario c at year t.
$X_{ij,a,c,t},\ Y_{ij,a,c,t}$	Auxiliary Variables associated with branch ij in the conic model, for conductor a, and scenario c and year t.
$\delta^l_{i,a,c,t}$	Auxiliary variable associated with voltage in bus i, branch l in the conic model for conductor a, and scenario c at year t.
$\delta_{i,c,t}$	Auxiliary variable associated with voltage in bus i, in the conic model for scenario c and year t.

1. Introduction

The integration of distributed generation (DG) units resulted in new challenges for the planning and operation of electrical distribution systems (EDSs). These challenges are accentuated with the high emergence of renewable energy-based technologies. In this regard, the distribution companies are looking for appropriate methodologies to cope with the inherently intermittent behavior of renewable energy sources (RESs) and guaranteeing the network performance. In literature, sophisticated approaches have been proposed to handle such intermittent energy production. Two most common approaches to handle the uncertainties are the robust and stochastic programming frameworks. A robust optimization is an effective way to cope with uncertainties associated with the electric power system operation, however, it tends to be conservative to guarantee feasible constraints [1]. It mainly addresses the uncertainties without making proper probability distributions analyses, and consequently, the uncertain parameters are assumed to belong to predefined deterministic uncertainty sets [2]. On the other hand, in stochastic programming approaches, to determine a feasible and realistic solution, a suitable probability distribution function is required to model the uncertain behavior via appropriate scenarios, however such information cannot be easily accessed in most real-world situations, either it does not exist or privacy issues prevent access to such information [3], [4]. The robust- and stochastic-based methods due to uncertain behavior present significant challenges to obtaining a precise and realistic optimization model [5]. However, these methods due to using different strategies to handle the existing uncertainties provide different outcomes and therefore are not comparable.

The main reason for giving too much attention to the DG operation-planning problem and jointly exploring with the EDS reinforcement problem is due to the capability of attending several requirements of power system simultaneously. The solution to this problem determines an optimal investment plan to fulfill the electricity demand while satisfying the physical, economic, and operational constraints over a predefined horizon (long- or short term). However, due to the complexity of this problem, heuristic approaches have been widely applied to solve the corresponding models. A particle swarm optimization (PSO) was developed in [6] to solve EDS reinforcement planning considering dispatchable DG units allocation where the uncertainties in load and energy prices were handled via probability distribution function (PDF) and Monte-Carlo simulation (MCS). Similarly, in [7] an EDS reinforcement planning considering upgrading and/or installation of substations, feeders, and dispatchable DG was solved by using a harmony search (HS) algorithm. A probabilistic hybrid approach was presented in [8] to determine the optimal placement and sizing of wind-based DG sources and reactive support devices in an EDS planning problem where the model was handled via a hybrid technique based on a Tabu Search (TS) algorithm and a Chu-Beasley Genetic algorithm (CBGA). A non-dominated genetic algorithm II (NSGA-II) was presented in [9] to handle a multi-objective DG planning model under correlated uncertainties. This paper presented a decision-making analysis where a set of Pareto solutions was used to find a trade-off between the investment cost and power losses. The authors in [10] applied a hybrid approach, dierential evolution algorithm as well as tree structure encoding-partheno genetic algorithm and primal-dual interior point method, to solve a stochastic three-layer DG expansion planning, for which the performance of these planning decisions was simultaneously evaluated under demand-side management and network reconfiguration. Nevertheless, heuristic-based approaches do not guarantee the optimal global solution. To address this drawback, researchers have focused on developing solver-based models and mainly mixed-integer linear programming (MILP) models [11]-[19]. A multi-stage long-term distribution planning model was presented in a two-part paper [12] and [13] to maximize the hosting capacity of the renewable energy-based sources considering the reactive power support in the network. The given model was solved via commercial solvers, while e heuristic-based reduction also applied to alleviate the computational burden. A multistage distribution planning model considering energy storage systems was proposed in [14] where the long-term investments were validated using short-term operation strategies and practical daily load curves. From another perspective, a multi-load-scenario distribution expansion planning model was developed in [15] considering the co-optimization of existence microgrids. A bilevel model was presented in [16] in which the upper and lower levels were respectively minimizing the renewable generation investment costs and the customers' payment over time-varying prices via a demand response program. A two-stage robust programming model to determine a solid plan for feeders reinforcement, renewable, and dispatchable DG units allocation was presented in [17]. In a similar way, a reinforcement expansion planning considering dispatchable DG units allocation was presented in [18] where uncertainties in electricity price were approached using information-gap decision theory. A probabilistic chance constrained MILP model was developed in [19] aiming at placing the electric vehicles charging stations and upgrading the EDS. The main issue with the MILP models is increasing the number of variables, the auxiliary variables used in the linearization process. An alternative solution approach to address this drawback is using conic models. In the literature, a few works handled the EDS planning problems via mixed-integer conic programming (MICP) models [20], [21]. In [20], a second-order conic chance-constrained model to represent the EDS planning problem considering circuit and substation upgrading, and reactive power support devices allocation was presented. In [21], an MICP model was used to solve one stage of the EDS planning at the presence of wind generation where the authors compared the results of a commercial solver with a heuristic-aided commercial solver. All the aforementioned works disregarded considering carbon emission control and management, which is a significant challenge in all levels of a power system.

In recent years, many countries intend to apply environmental policies in order to attend global warming targets [22], [23]. The primary environmental policies at the distribution level are the carbon tax and carbon cap. The first policy is a price-based tool that imposes a fixed tax to the emitted CO_2 (per ton), while in the second policy a quantity of emission is imposed on the emitted units. To promote an EDS with low carbon emission, these policies have been proposed via the integrated planning of renewable energy sources. However, at the distribution level, a few studies have explored a planning method regarding the application of carbon policies. A planning scheme via an integrated approach considering renewable energy-based sources, demand response, and carbon taxation were presented in [24]. To handle the proposed model, an interior-point-method-embedded discrete genetic algorithm (IPM-DGA) was used. An integrated planning scheme considering dispatchable and renewable energy-based sources was proposed in [25] and the environmental aspects were addressed by introducing performance indices where through of solar- and wind-based sources these indices were reduced. To find a solution to this model, a PSO algorithm was applied. In the same way, to find the best allocation for combined renewable energy-based sources targeting at CO_2 emission reduction, a strategic planning was proposed in [26] where a PSO algorithm along with fuzzy decision-making criteria was used to solve the model. A deterministic approach to limit the emissions at the distribution level considering renewable DG units, reactive support, and energy storage devices was proposed in [27], while a two-stage stochastic mixed-integer linear programming (MILP) model was developed in [28] to address long-term renewable sources allocation planning problem considering carbon emission cost and renewable incentives. On the other hand, a robust two-stage short-term planning problem with multiples alternatives and sizing and placement of renewable DG units was proposed in [29] where the environmental issues were also considered to promote a low carbon emission system. In this paper, planning actions aimed at maximizing the efficiency of an EDS via an MILP model while mitigating the carbon emission via carbon cap policy.

All in all, the planners are willing to use a model that not only considers the practical constraint but also at the same time stay tractable. This means that finding a proper trade-off between the complexity and tractability is the most critical issue to be attended in proposing an operation-planning model. Therefore, as can be seen from the above literature review, the heuristic-based approaches and MILP model that used to be widely applied in EDS planning, nowadays have been replaced with the MICP models due to the capability of capturing the critical nonlinear factors of AC optimal power flow (OPF), yet the MICP models require to be developed to cover more practical aspects. Therefore, the primary objective of this work is to develop an MICP model to integrate multiple planning alternatives aiming at finding the most appropriate carbon mitigating long-term plan. To this end, the proposed model takes the advantages of using an exact convex formulation of AC OPF that without losing useful information guarantees to find the optimal global solution [30]. This problem is formulated considering alternatives such as a) reinforcement of existing substations, b) conductor replacement of overloaded circuits and c) siting and sizing of wind- and gas-based DG technologies. The proposed environment- and economic-based strategy is inherently a non-convex mixed integer nonlinear programming (MINLP) problem. To solve this problem via classical optimization techniques and to guarantee the convergence to the global solution, this non-convex MINLP model is recast into an exact convex model by using a conic programming approach. Furthermore, in order to handle with uncertainties in electricity demand and wind-based power generation, this problem is extended into a two-stage stochastic conic programming model. To show its potential and applicability under different conditions, this uncertainty-based model is validated using a 34-node test distribution system and a 135-nodes real network. In summary, the main contributions of this work are categorized as follows:

- Proposing an integrated reinforcement planning strategy by developing a second-order conic programming model that aims at addressing the environmental and economic goals of a project simultaneously by finding the optimal investment plan.
- Finding a strategy that proposes CO_2 emission mitigation at distribution level via siting and sizing of renewable energy-based sources and carbon taxation.
- Proposing a multi-period two-stage stochastic scenario-based convex programming model that guarantees the optimal global solution of the planning problem via mature commercial solvers. This way, the proposed optimization model also provides the optimal operational state of the system by using an OPF tool.

The remainder of this paper is organized as follows. Section II presents the problem formulation of the

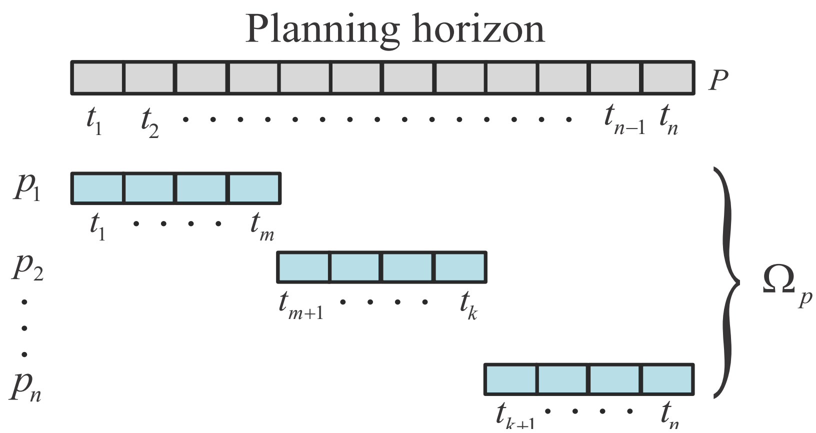

Fig. 1: Proposed planning horizon scheme

optimization problem in details. Assumptions for the test cases are shown in section III. Numerical studies, results discussions, and comments are reported in section IV. Section V contains the concluding remarks.

2. Problem formulation and solution framework

This section describes the multi-stage long-term planning problem as well as several hypotheses adopted to formulate this problem. The solution of this planning problem defines a multi-choice strategy to obtain the most appropriate long-term EDS plan. Inherently, this combinatorial decision process is a non-convex mixed integer nonlinear programming (MINLP) model. Therefore, to remedy this non-convexity, this MINLP is recast to a mixed integer conic programming model (MICP) that is presented in 2.2.

2.1. Planning scheme methodology

In this work, to formulate the optimization problem, uncertainties correlated with the electricity load and wind speed are taken into account. This uncertainty-based approach is characterized via a two-stage stochastic programming model where the first stage defines the planning decision, for which these are made before the uncertainty realization. On the other hand, in the second stage, the expected values of the stochastic variables are calculated when the scenarios and investment decisions are known. To properly formulate and to obtain a realistic approach, the planning horizon is divided into the investment periods ps in the set Ω_p, as can be seen in Fig. 1. From Fig. 1, each p is represented by n-years (n-ts) to be analyzed. Therefore, the long-term horizon planning is represented by n short-term periods and in each short-term period, investment actions can be realized to find the best EDS plan.

In order to consider the uncertainty in data, a year t is represented by several scenarios of load and wind speed considering a time-resolution of 8760h. In such representation, seasonal characteristics and weather factors are duly represented where historical data are used to generate a set of combined scenarios between

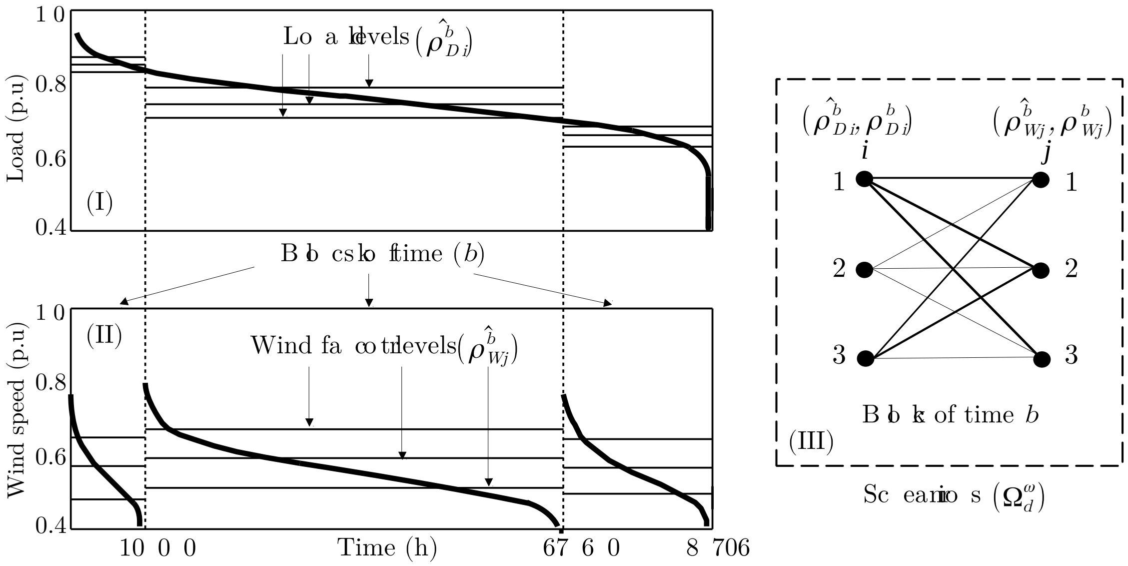

Fig. 2: Load (I) and wind speed (II) duration curves, and combined stochastic scenarios for one block of time (III)

load and wind speed. To reduce the problem complexity, uncertainty in data is handled by characteristic scenarios c using the load and wind duration curves methodology illustrated in Fig. 2 [31]. In this context, a year t is divided into several blocks of time (bs), where load and wind speed scenarios are estimated at different levels based on cumulative distribution functions. As can be seen from Fig. 2, information is represented in pairs $(\hat{\rho}^b_{D_i}, \rho^b_{D_i})$ and $(\hat{\rho}^b_{W_j}, \rho^b_{W_j})$ that corresponds to the load and wind speed average levels considering the respective probabilities in b. Therefore, each scenario within Ω^w_d is created considering the combination of load and wind speed scenarios in the same b as $\Omega^w_d = \Omega^b_d, \Omega^b_w$ for $b \in \Omega_b$. In this regard, Fig. 2.III shows that the probability of this combined scenario is represented by ρ_c, which is the product of load and wind speed probabilities ($\rho_c = \rho^b_D \rho^b_W$). Taking into account this information, wind speeds $\hat{\rho}^b_W$ are converted into the level factors of power generation limits f^W_c; this value depends on the wind turbine (WT) characteristics that, in this work, is considered as a linear approximation based on the wind output power curve. Finally, the load factor of each scenario, f^D_c, is equal to the correspondent average load level factors $\hat{\rho}^b_D$. It is worth mentioning that for the sake of simplicity, it is assumed that all locations of WT sources are subject to the same weather and seasonal conditions. Analogously, it is considered that all load nodes are subject to the same variability.

2.2. Stochastic MICP model

A stochastic-based model for the long-term EDS planning problem is presented in (1)-(37). The solution of this model determines a) reinforcement of substation, b) the optimal conductor replacement of overloaded circuits, thereby, suggesting the respective new conductor, and c) siting and sizing of wind and gas technology-based DG units. Stochastic variables determine the expected operating costs considering a) the power purchase in the market, b) operating condition of the wind and gas generator installed in the system, c) power losses, d) expected unserved energy, and e) carbon tax for the main grid and gas turbines. In order to deal with load and wind power uncertainties, this problem is represented via a stochastic MICP model.

The proposed stochastic MICP model defines an economic and environmental strategy to obtain the best long-term expansion plan for an EDS. To formulate this problem, the following assumptions are taken into account:

- The planning scheme follows the centralized context, for which all the technologies are owned and operated by the distribution company. Therefore, a pre-analysis is considered to define a set of candidate buses.
- DG units are based on the WT and gas technologies.
- The wind energy production is considered as a stochastic variable and depends on the wind speed of a specific region and the mechanical characteristics of WT.
- The gas turbine is considered as a dispatchable DG unit where the energy request of the system controls the output power.
- The distribution company can define an investment budgetary to obtain a proper expansion plan.
- The presented planning problem considers an existing radial topology where the conductors of the circuits of the EDS can be reinforced. It is worth mentioning that the construction of new circuits and network reconfiguration are out of the scope of this work.

$$\min \sum_{p\in\Omega_p} \frac{1}{\tau(1+\tau)^{(p-1)\lambda}} \left(IC_p^{SS} + IC_p^{L} + IC_p^{DG}\right) + \sum_{t\in\Omega_t} \frac{1}{(1+\tau)^{(t-1)}} [\sum_{c\in\Omega_c} \rho_c (OC_{c,t}^{SS} + OC_{c,t}^{DG} \tag{1}$$

$$+ C_{c,t}^{LS} + C_{c,t}^{U} + CC_{c,t}^{SS} + CC_{c,t}^{GS})]$$

where,

$$IC_p^{SS} = \sum_{s\in\Omega_s} \sum_{k\in\Omega_k^{SS}} y_{s,k,p}^{SS} C_k^{I,SS} \frac{\tau(1+\tau)^{\zeta_{SS}}}{(1+\tau)^{\zeta_{SS}} - 1} \qquad \forall(p \in \Omega_p) \tag{2}$$

$$IC_p^{L} = \sum_{l\in\Omega_l} \sum_{a\in\Omega_a} y_{l,a,p}^{L} C_{a_0^l,a}^{I,L} l_l \frac{\tau(1+\tau)^{\zeta_a}}{(1+\tau)^{\zeta_a} - 1} \qquad \forall(p \in \Omega_p) \tag{3}$$

$$IC_p^{DG} = \sum_{r\in\Omega_r}\sum_{k\in\Omega_k^W} y_{r,k,p}^W C_k^{I,W}\frac{\tau(1+\tau)^{\zeta_W}}{(1+\tau)^{\zeta_W}-1}+$$

$$\sum_{g\in\Omega_g}\sum_{k\in\Omega_k^G} y_{g,k,p}^G C_k^{I,G}\frac{\tau(1+\tau)^{\zeta_G}}{(1+\tau)^{\zeta_G}-1} \qquad \forall(p\in\Omega_p) \tag{4}$$

$$OC_{c,t}^{SS} = T_c\sum_{s\in\Omega_s} C_{c,t}^E(P_{s,c,t}^{SS}) \qquad \forall(c\in\Omega_c, t\in\Omega_t) \tag{5}$$

$$OC_{c,t}^{DG} = T_c\sum_{r\in\Omega_r}\sum_{k\in\Omega_k^W} C_k^W P_{r,k,c,t}^W+$$

$$T_c\sum_{g\in\Omega_g}\sum_{k\in\Omega_k^G} C_k^G P_{g,k,c,t}^G \qquad \forall(c\in\Omega_c, t\in\Omega_t) \tag{6}$$

$$C_{c,t}^{LS} = T_c\sum_{l\in\Omega_l}\sum_{a\in\Omega_a} C_{c,t}^E I_{l,a,c,t}^{sqrt} R_{l,a} l_l \qquad \forall(c\in\Omega_c, t\in\Omega_t) \tag{7}$$

$$C_{c,t}^U = T_c\sum_{i\in\Omega_n} C^U(P_{i,c,t}^U + Q_{i,c,t}^U) \qquad \forall(c\in\Omega_c, t\in\Omega_t) \tag{8}$$

$$CC_{c,t}^{SS} = T_c C_t^C e^{gr}\sum_{s\in\Omega_s}(P_{s,c,t}^{SS}) \qquad \forall(c\in\Omega_c, t\in\Omega_t) \tag{9}$$

$$CC_{c,t}^{GS} = T_c C_t^C\sum_{g\in\Omega_g}\sum_{k\in\Omega_k^G}(e_k^{gs} P_{g,k,c,t}^G) \qquad \forall(c\in\Omega_c, t\in\Omega_t) \tag{10}$$

Subject to:

$$\sum_{j\in N(i)} P_{ij,c,t} = P_{i,c,t}^{SS} + \sum_{k\in\Omega_k^W} P_{i,k,c,t}^W + \sum_{k\in\Omega_k^G} P_{i,k,c,t}^G + P_{i,c,t}^U - f_c^D P_{i,t}^D \qquad \forall(i\in\Omega_n, c\in\Omega_c, t\in\Omega_t) \tag{11}$$

$$\sum_{j\in N(i)} Q_{ij,c,t} = Q_{i,c,t}^{SS} + \sum_{k\in\Omega_k^W} Q_{i,k,c,t}^W + \sum_{k\in\Omega_k^G} Q_{i,k,c,t}^G + Q_{i,c,t}^U - f_c^D Q_{i,t}^D \qquad \forall(i\in\Omega_n, c\in\Omega_c, t\in\Omega_t) \tag{12}$$

$$P_{ij,c,t} = \sum_{a\in\Omega_a}\Big(\sqrt{2}\delta_{i,a,c,t}^l g_{ij,a} - X_{ij,a,c,t} g_{ij,a} - Y_{ij,a,c,t} b_{ij,a}\Big) \qquad \forall(ij\in\Omega_l, c\in\Omega_c, t\in\Omega_t) \tag{13}$$

$$Q_{ij,c,t} = \sum_{a\in\Omega_a}\Big(-\sqrt{2}\delta_{i,a,c,t}^l b_{ij,a} + X_{ij,a,c,t} b_{ij,a} - Y_{ij,a,c,t} g_{ij,a,c,t}\Big) \qquad \forall(ij\in\Omega_l, c\in\Omega_c, t\in\Omega_t) \tag{14}$$

$$0 \le P_{r,k,c,t}^W \le \sum_{p\in\Omega_p}^{(p-1)Y<t} f_{k,c}^W \overline{P_k^W} y_{r,k,p}^W \qquad \forall(r\in\Omega_r, k\in\Omega_k^W, c\in\Omega_c, t\in\Omega_t) \tag{15}$$

$$-P_{r,k,c,t}^W\tan(\varphi_c^W) \le Q_{r,k,c,t}^W \le P_{r,k,c,t}^W\tan(\varphi_i^W) \qquad \forall(r\in\Omega_r, k\in\Omega_k^W, c\in\Omega_c, t\in\Omega_t) \tag{16}$$

$$0 \le P_{g,k,c,t}^G \le \sum_{p\in\Omega_p}^{(p-1)Y<t} \overline{P_k^G} y_{g,k,p}^G \qquad \forall(g\in\Omega_g, k\in\Omega_k^G, c\in\Omega_c, t\in\Omega_t) \tag{17}$$

$$-P_{g,k,c,t}^G\tan(\varphi_c^G) \le Q_{g,k,c,t}^G \le P_{g,k,c,t}^G\tan(\varphi_i^G) \qquad \forall(g\in\Omega_g, k\in\Omega_k^G, c\in\Omega_c, t\in\Omega_t) \tag{18}$$

$$\sum_{r\in\Omega_r}\sum_{k\in\Omega_K^W}P_{r,k,c,t}^{W}+\sum_{g\in\Omega_g}\sum_{k\in\Omega_K^G}P_{i,k,c,t}^{G}\leq\vartheta\sum_{i\in\Omega_n}f_c^D P_{i,t}^D \qquad \forall(c\in\Omega_c,t\in\Omega_t) \quad (19)$$

$$I_{l,a,c,t}^{sqrt}=\sqrt{2}(g_{ij,a}^2+(b_{ij,a}+\frac{b_{ij,a}^{sh}}{2})^2)\delta_{i,a,c,t}^{l}+\sqrt{2}(g_{ij,a}^2+b_{ij,a}^2)\delta_{j,a,c,t}^{l}-$$

$$2(g_{ij,c}^2+b_{ij,a}^2+\frac{b_{ij,a}b_{ij,a}^{sh}}{2})X_{ij,a,c,t}+g_{ij,a}b_{ij,a}^{sh}Y_{ij,a,c,t}\leq\overline{I}_a^2 \qquad \forall(l\in\Omega_l,a\in\Omega_a,c\in\Omega_c,t\in\Omega_t) \quad (20)$$

$$z_{l,a,t}^{L}\underline{V}_i^2\leq\sqrt{2}\delta_{i,a,c,t}^{l}\leq z_{l,a,t}^{L}\overline{V}_i^2 \qquad \forall(l\in\Omega_l,a\in\Omega_a,c\in\Omega_c,t\in\Omega_t) \quad (21)$$

$$z_{l,a,t}^{L}\underline{V}_j^2\leq\sqrt{2}\delta_{j,a,c,t}^{l}\leq z_{l,a,t}^{L}\overline{V}_j^2 \qquad \forall(l\in\Omega_l,a\in\Omega_a,c\in\Omega_c,t\in\Omega_t) \quad (22)$$

$$\sum_{a\in\Omega_a}\delta_{i,a,c,t}^{l}=\delta_{i,c,t} \qquad \forall(i\in\Omega_n,c\in\Omega_c,t\in\Omega_t) \quad (23)$$

$$\sum_{a\in\Omega_a}\delta_{j,a,c,t}^{l}=\delta_{j,c,t} \qquad \forall(j\in\Omega_n,c\in\Omega_c,t\in\Omega_t) \quad (24)$$

$$S_s^{o^2}+\sum_{p\in\Omega_p}^{(p-1)Y<t}\sum_{k\in\Omega_k^{SS}}y_{s,k,p}^{SS}(2S_s^oS_{s,k}^{'}+S_{s,k}^{'2})\geq(P_{c,t}^{SS})^2+(Q_{c,t}^{SS})^2 \qquad \forall(s\in\Omega_s,c\in\Omega_c,t\in\Omega_t) \quad (25)$$

$$2\delta_{i,a,c,t}^{l}\delta_{j,a,c,t}^{l}\geq X_{ij,a,c,t}^2+Y_{ij,a,c,t}^2 \qquad \forall(ij\in\Omega_l,a\in\Omega_a,c\in\Omega_c,t\in\Omega_t) \quad (26)$$

$$X_{ij,a,c,t}=X_{ji,a,c,t},\quad X_{ij,a,c,t}\geq 0 \qquad \forall(ij\in\Omega_l,a\in\Omega_a,c\in\Omega_c,t\in\Omega_t) \quad (27)$$

$$Y_{ij,a,c,t}=-Y_{ji,a,c,t} \qquad \forall(ij\in\Omega_l,a\in\Omega_a,c\in\Omega_c,t\in\Omega_t) \quad (28)$$

$$X_{ij,a,c,t}\leq\overline{V}_i\overline{V}_j z_{l,a,t}^{L} \qquad \forall(l/ij\in\Omega_l,a\in\Omega_a,c\in\Omega_c,t\in\Omega_t) \quad (29)$$

$$|Y_{ij,a,c,t}|\leq\overline{V}_i\overline{V}_j z_{l,a,t}^{L} \qquad \forall(l/ij\in\Omega_l,a\in\Omega_a,c\in\Omega_c,t\in\Omega_t) \quad (30)$$

$$\sum_{a\in\Omega_a}\sum_{p\in\Omega_p}y_{l,a,p}^{L}\leq 1 \qquad \forall(l\in\Omega_l) \quad (31)$$

$$\sum_{k\in\Omega_k^{SS}}\sum_{p\in\Omega_p}y_{s,k,p}^{SS}\leq 1 \qquad \forall(s\in\Omega_s) \quad (32)$$

$$\sum_{k\in\Omega_k^{W}}\sum_{p\in\Omega_p}y_{r,k,p}^{W}\leq 1 \qquad \forall(r\in\Omega_r) \quad (33)$$

$$\sum_{k\in\Omega_k^{G}}\sum_{p\in\Omega_p}y_{g,k,p}^{G}\leq 1 \qquad \forall(g\in\Omega_g) \quad (34)$$

$$z_{l,a,t}^{L}\leq\sum_{h=1}^{p}y_{l,a,h}^{L}+a_0^l \qquad \forall(l\in\Omega_l,a\in\Omega_a,p\in\Omega_p,t\in\Omega_t,|_{t\leq pY}) \quad (35)$$

$$\sum_{a\in\Omega_a}z_{l,a,t}^{L}=1 \qquad \forall(l\in\Omega_l,t\in\Omega_t) \quad (36)$$

$$\sum_{l\in\Omega_l}\sum_{a\in\Omega_a}y_{l,a,p}^{L}C_{a_0^l,a}^{I,L}l_l+\sum_{s\in\Omega_s}\sum_{k\in\Omega_k^{SS}}y_{s,k,p}^{SS}C_k^{I,SS}+$$

$$\sum_{r\in\Omega_r}\sum_{k\in\Omega_r^{W}}y_{r,k,p}^{W}C_k^{I,W}+\sum_{g\in\Omega_g}\sum_{k\in\Omega_k^{G}}y_{g,k,p}^{G}C^{I,G}\leq INV_p \qquad \forall(p\in\Omega_p) \quad (37)$$

In this presented planning scheme, the objective function (1) minimizes simultaneously several targets subject to (2)-(10). The amortized investment costs due to substation reinforcement by defining its new capacity is presented in (2), while (3) and (4)stand for the amortized costs of conductor replacement of overloaded circuits, and the wind- and gas-based DG units allocation, respectively. The operating costs of substations and DG units are represented by (5) and (6), respectively. Costs related to energy losses are defined by using the expression (7). The cost of the unserved energy by the EDS is consider in (8). Finally, costs of the carbon tax of the main grid and gas-based DG units are defined in (9) and (10).

This model is subject to steady-state operating conditions based on conic approach [32], subject to physical and operational constraints of each planning action and investment limits. In this regard, (11) and (12) stand for the active and reactive power balance, respectively. The active and reactive power flow through circuits are represented by (13) and (14). The operational limits of the active and reactive power by installed WT source are realized in (15) and (16). Analogously to WT model, the operational limits for active and reactive power by the installed gas turbines are shown in (17) and (18). The limit of DG penetration to the system is defined by (19). The current flow at circuit l is limited by (20), according to the installed conductor type. The upper and lower limits of voltage magnitude are presented by (21) and (22), respectively. In this expression, variables $\delta^l_{i,a,c,t}$ and $\delta^l_{j,a,c,t}$ are defined for each circuit l and for each conductor type a, in order to represent the voltage magnitudes of buses i and j for scenario c at year t. Equations (23) and (24) define the variable $u_{i,c,t}$ in terms of the selected conductor type at circuit l. The maximum capacity of the energy supplied by the substation is defined by (25).

The conic rotated constraints are represented in (26)-(28). These expressions relate the variables $\delta^l_{i,a,c}$, $\delta^l_{j,a,c}$, $X_{ij,a,c}$ and $Y_{ij,a,c}$ with power flow equations (11) and (12); $X_{ij,a,c}$ and $Y_{ij,a,c}$ are subject to the steady state operation of EDS, for which, the traditional power flow equation are represented and must be fulfilled for each circuit. Variables $X_{ij,a,c}$ and $Y_{ij,a,c}$ are bounded by (29) and (30), respectively. The basic concept of (26)-(30) has been derived from the second-order conic programming (SOCP) power flow formulation presented in [32], and has been adequately developed in our paper to consider alternatives such as conductor replacement, substation upgrading, optimal placement and sizing of DG units and uncertainty operational scenarios. In order to obtain an appropriate representation, the set of variables $\delta_i = V_i^2/\sqrt{2}; X_{ij} = V_i V_j \cos(\theta_i - \theta_j)$ and $Y_{ij} = V_i V_j \sin(\theta_i - \theta_j)$ are defined where V_i and θ_i are the voltage and angle at bus i, respectively. Therefore, the power flow equations can be expressed as $2\delta_i\delta_j = X_{ij}^2 + Y_{ij}^2$ as in (26) to represent a SOCP model. Finally, by considering radial networks, if the equality equation is attended, thereby, it implies that the optimal solution can be guaranteed [30].

To define that only one reinforcement, replacement, or placement is allowed for each system component

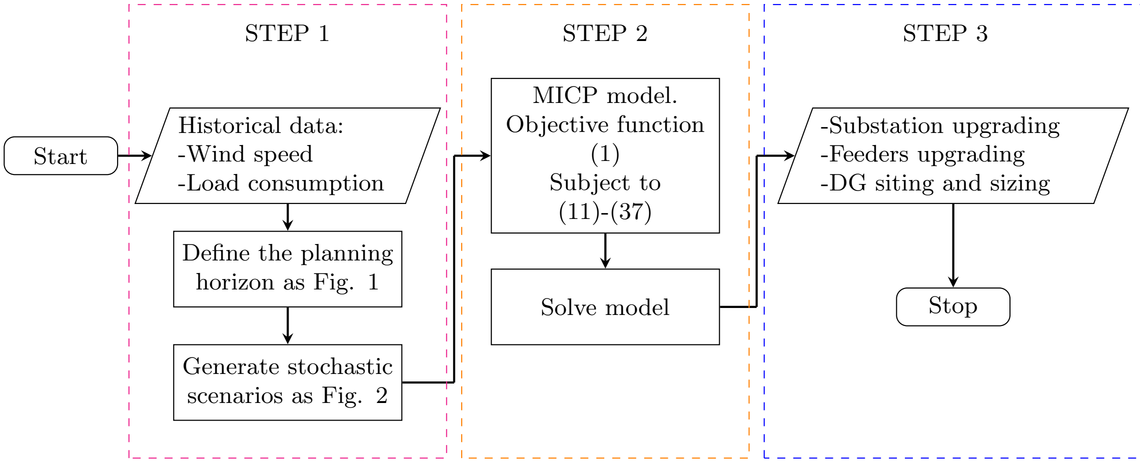

Fig. 3: Flowchart of the proposed approach.

along the planning horizon, (31)-(34) are taken into account. Equation (35) defines the conductor type used in each year considering the initially installed conductor type, while (36) defines that only one conductor should be selected for circuit l. Finally, an investment budget for each period is stated by (37).

2.3. Solution scheme

In summary, as illustrated in Fig. 3, the presented planning strategy is divided into three steps to determine the appropriate distribution network expansion plan. This figure shows the flowchart of the proposed approach where in the first step, the historical data related to wind speed and load consumption is used to generate scenarios via the respective distribution probability. Considering these scenarios (wind-based energy production and load consumption), the stochastic MICP model is developed and by using mature commercial solvers, the solution is carried out. Finally, the solution of this model is represented by investment decisions in order to maximize the efficiency of the network with the most appropriate plan. This information is shown in the third step where the substation and feeder upgrading, as well as the placement and sizing of DG units (WT and gas technologies), are the planning actions to be implemented.

3. Assumptions and Test Cases

The proposed stochastic conic model, described in subsection 2.2, is tested on a 34-node distribution system [33], and subsequently, to validate the scalability of the presented approach, a adapted 135 real distribution system is used [34]. This section presents technical and economic information, assumptions, and case studies to validate and reveal the potential of the proposed conic model.

3.1. Technical and financial information

3.1.1. The 34-node distribution system:

This system contains 33 load nodes and one substation with the supplying capacity of 5 MVA. The existing substation can be reinforced by adding a transformer of 5 MVA. The nominal voltage is 11kV, while the lower and upper limits are 0.95 and 1.05 p. u, respectively. The total conventional load is 5.4 MVA with an average power factor of 0.85. The initial system contains 33 circuits with different conductor types, 23 with $a1$ type, 3 with $a2$ type, 5 with $a3$ type, and 2 with $a4$ type.

Five conductor types are considered in the proposed analysis where the technical and financial specifications for each conductor type are presented in Table 2 and 3, respectively. In this regard, to replace the conductor of type ax with type ay, a replacement cost is associated. In this work, wind- and gas-based technologies for DG sources are considered. For each technology, two turbines are considered where the technical and financial information is presented in Table 1.

3.2. Assumptions for 34-node distribution system

In order to obtain a proper analysis and to validate the proposed mathematical representation of the long-term planning model, the following main assumptions are taken into account: a) planning horizon of 9 years is considered where it is divided into 3 short-term periods (3 years for each period), b) the annual load growth rate is 5% while the increase rate for energy price and carbon tax is 1% with the discount rate 8%, c) the maximum integration permissible of DG sources penetration is 25%; d) the investment budgetary for each short-term period is $500k; d) the cost of unserved energy is $150k per MWh; this value is used as penalty factor, e) the stochastic yearly information of load and wind power is divided into 4 time blocks and 3 levels of load and wind speed, for which an equiprobable combination of scenarios is developed [34], f) to upgrading the existing substation, an investment cost of $350k is considered, and g) the carbon tax rate is $10-ton considering an intensity of pollutant emission, at distribution level, of 0.92 kg CO_2/kWh [24].

3.2.1. The 135-node distribution system:

This system contains two substation S1 and S2, each with 15MVA capacity, for which the load of the system has been distributed in 135 load nodes. The nominal voltage is 13.8 kV, and the voltage level of the system is bounded between 0.95 and 1.05 p.u. In the initial stage, the active and reactive loads are 36.910 MW and 17.876 MVAr, respectively. On the other hand, the initial condition of the system has 2 types of conductors distributed in 134 circuits where 57 circuits are identified with $a6$ and 78 with $a7$. The reinforcement of substations considers two types of transformers of 7.5 and 15 MVA (TR1 and TR2, respectively). Two types of DG units such as gas-based DG units with the capacities of 1.0 and 1.5 MVA and renewable-based DG units (presented in Table 1) are considered. It is worth mentioning that different technical and financial characteristics of conductor types for this system are considered. Details related to the utilized data are available in [36].

3.3. Case Studies

The applicability and robustness of the proposed MICP model are validated by testing two distribution systems under three different perspectives. These cases are summarized as follows.

3.3.1. Case I - Without considering DG sources

This case aims at finding the optimal planning actions of substation reinforcement and conductor replacement. It is worth mentioning that, under this analysis, the uncertainty of demand is considered into 4 times blocks and 3 load levels that results in 12 scenarios.

3.3.2. Case II - Considering gas-based DG sources

In this case, planning actions are substation reinforcement, conductor replacement, and siting and sizing of gas-based DG sources.

3.3.3. Case III Considering all the investment alternatives

In this case, all the planning actions are considered in the optimization model. This case aims to show the robustness and effectiveness of the proposed model while several actions are simultaneously taken into account in order to find the best economic and environmental plan. In this case, the uncertainty is modeled into 4 blocks of times, 3 load levels, and 3 wind power factors, that is, in total 36 scenarios are taken into consideration.

4. Numerical Results

The simulations have been implemented on a Dell PowerEdge R910 with four Intel Xeon E7 4807 processors at 1.86 GHz and 125 GB of RAM using CPLEX 12.8.0 under OPL [35] while keeping the optimality gap below 1% as the stopping criterion.

4.1. 34-Node system

For the 34-distribution system and each case study, the economical results, environmental impact, and the proposed investment plan are presented in Tables 4, 5, and 6 respectively, while Fig. 4 shows the proposed DG siting and sizing.

Table 1: Data for DG alternatives

Type k	$\overline{P^{W;G}}$ [MW]	$\tan(\varphi_i^{W;G})$	$\tan(\varphi_c^{W;G})$	$C_k^{I,W;G}$ US$	$C_k^{W;G}$ US$
WT1	0.910	0.0	0.0	168350	7.10
WT2	2.050	0.0	0.0	379250	7.10
GT1	0.200	0.484	0.328	20000	185.00
GT2	0.400	0.484	0.328	40000	180.00

Table 2: Data for conductor alternatives

Type a	R (Ω/km)	X (Ω/km)	$\overline{I_a}$ A
$a1$	0.5240	0.0900	219.0
$a2$	0.3780	0.0860	304.0
$a3$	0.2990	0.0830	384.0
$a4$	0.1950	0.0800	590.0
$a5$	0.1740	0.0780	661.0

Table 3: Cost to replace branches ($/km)

	Type of conductor				
α_a	$a1$	$a2$	$a3$	$a4$	$a5$
$a1$	0	7500	13500	21500	29500
$a2$	-	0	11000	18500	25500
$a3$	-	-	0	14000	22000
$a4$	-	-	-	0	17500

4.1.1. Case I: Without considering DG sources:

This case aims at evaluating the proposed planning framework disregarding the DG sources allocation. The obtained solution for this case shows a total cost of $25.859M. The investment decisions for the first and second stages show the costs of $499.450k and $20.400, respectively. This cost contains $1.356M regarding the carbon taxation, $405.791k of energy losses costs, $639.57k of planning investment costs, and $23.45M regarding the energy production of EDS. This solution proposes the following plan for the planning horizon. For the first stage, the obtained plan is to reinforce the substation with a total cost of $350k, conductor replacement of 13 circuits with an investment cost of $149.45k. For the second stage, the obtained plan is replacing the conductor of three circuits with an investment cost of $20.4k. It is worth mentioning that, the optimal solution of this case reveals that no investment actions are required for stage 3.

4.1.2. Case II: Considering gas-based DG sources:

The solution of this case shows that the costs to obtain the optimal plan is about $25.336M. The investment plan for the first and third stages have the costs of $174.225k and $370k, respectively. This cost is divided into $499.579k of the investment costs, $307.453k of the energy losses costs, $1.302M of carbon taxation and

$23.227M regarding the energy production costs. The investment cost of each stage can be summarized as follows. In the first stage, the investment cost of allocation of gas-based DG sources with the power capacity of 1200kW is $120k; costs of conductor replacement of 5 circuits is $54.225k. For the second stage, no investment actions are required. In the third stage, the investment cost of siting and sizing of gas-based DG sources with the installed capacity of 100kW are $20k while the substation reinforcement cost is $350k.

The outcomes of this case demonstrate that by allocating DG sources, several benefits can be obtained (see Table 4). Comparing with the first case, the installation of gas turbines represents a reduction of 4.09% in the pollutant emission (see Table 5). It is worth noting that, this reduction in pollutant emission represents an economic saving of $55.00k due to the carbon tax. On the other hand, the cost regarding the energy losses in case II is about 24.34% less than that in case I.

4.1.3. Case III: Considering all the investment alternatives

This case aims at showing the effectiveness of the proposed planning scheme considering all the investment decisions. The solution of this case reveals a total cost of $22.002M, where the investment plan for the first and second stages have the costs of $496.475k and $423.300k, respectively. The investment costs are summarized as follows. In the first stage, the cost of allocating DG sources based on gas with the 600 kW capacity and wind turbine with 2050 kW capacity is $439.25k; investment cost of conductor replacement of 6 circuits with new conductor types is $57.225k. In the second stage, the proposed planning determines that the investment cost of allocating wind-based DG sources with the capacity of 2050 kW and gas-based DG sources with the capacity of 400 kW are $419.250k; investment cost of conductor replacement of one circuit with a new conductor type is $4.050k. Finally, in stage 3 the substation was reinforced, this action has a cost of $350.000k.

By comparing case III with cases I and II, it can be concluded that the allocation of wind-based DG sources increases the total investment cost. However, this technology provides economic and environmental benefits. These environmental benefits resulted in a reduction about 19.89% in pollutant emission comparing the result of case I, while comparing with case II, it shows a reduction about 16.47%, this information can be seen in Table 5. These environmental results represent economic savings of $268k and $213k, compared to the carbon tax of case I and case II, respectively. In addition, compared with case I and case II, the cost of energy losses had a reduction of 27.58% and 4.23%, respectively.

As expected, the proposed planning scheme provides appropriate solutions for all case studies. For the test system under different instances correspond to each case, the carbon tax had an impact of 5.36%, 5.23%, and 5.23% in the total planning cost, respectively. The numerical results and comparison of cases show that to apply this environmental policy with planning actions simultaneously, specifically DG sources allocation,

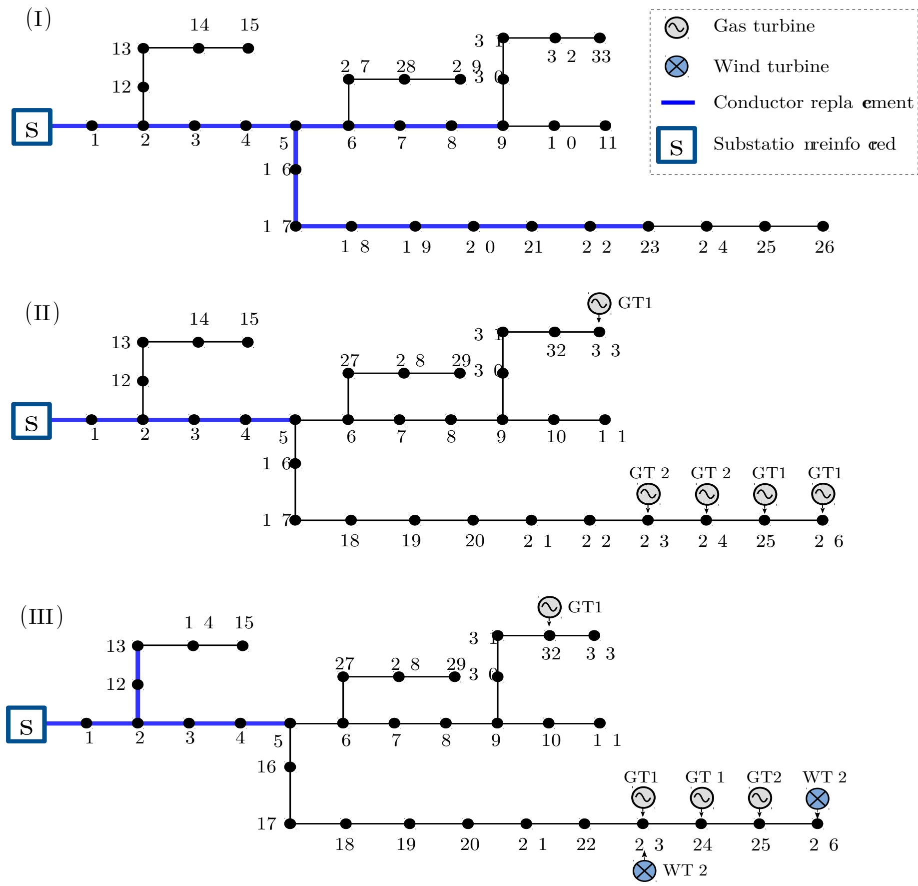

Fig. 4: Solutions of cases I, II, and III for 34-node distribution test system.

the pollutant emission at distribution level can be reduced effectively. In this regard, to fulfill the technical and operational requirements, and to obtain a low-carbon 34-node distribution system, a set of planning actions was obtained for each case. This information has been summarized in Fig. 4, Tables 4, 5 and 6. In Fig. 4, the planning alternatives are illustrated by representing the location of the DG units to be installed, the reconductoring of circuits and the substation reinforcement. On the other hand, in Table 4, the economic information related to the total cost, investment, and production cost, cost of energy losses, cost of unserved energy and carbon tax for each case is presented. The pollutant emission from the main grid, DG sources based on gas, and the total emission at distribution level are presented in Table 5. Finally, in Table 6, investment plans for each case are presented in three stages that represents the divisions of the planning horizon. This table identifies the planning actions made in each stage where substation reinforces are identified by ✓, the conductor replacement at circuit (ij) is identified by the new conductor type ax, and the type of allocated DG source at node (i), wind or/and gas unit.

Table 4: 34-Node system-Present value of investment and operational cost ($\$10^6$)

	Study cases		
Costs	I	II	III
OF	25.859	25.336	22.002
Investment	0.639	0.499	1.325
Production	23.450	23.227	20.783
Losses	0.406	0.307	0.294
Unserved energy	0.008	0.000	0.000
CO_2 tax	1.356	1.301	1.088

Table 5: 34-Node system-Expected Emission (kTon CO_2)

	Study cases		
In	I	II	III
DG	0.000	15.026	3.094
Main grid	178.540	156.208	140.033
Total	178.540	171.234	143.027

4.2. 135-Node system

This system is used to validate the applicability and robustness of the proposed approach on larger networks. Results of the 135-node distribution system are reported in Tables 7-9. In Table 7, the financial information for each case is summarized. It is worth noting that by considering the conditions of the case I, a high unserved energy cost of \$330.475M has been obtained. As can be seen for cases II and III, this value is reduced with the investment of DG alternatives. On the other hand, reductions of 5.62% and 5.55% in the CO_2 emission by the main grid and gas-based DG have been obtained by the investment in wind-based DG units, as can be seen in the Table 8. Finally, Table 9 identifies the planning actions for each stage where the substations upgrading is represented by type TRx in each substation (S1 and S2). Besides, the conductor reinforcement is presented by the new conductor type ax at the circuit ij, the allocation of gas-based DG units are described by the types (GT1 and GT2) with capacity at node i, respectively. Analogously, the placement of renewable-based DG units is identified by the type (WT1 and WT2) at the node i.

In conclusion, the integer solution obtained from the proposed stochastic model depends on the quality of the selected scenarios to represent the uncertainty information. Therefore, to evaluate the quality of this solution, for each case, all the decision variables (reinforcement plan such as: location, the capacity of the DG units, the new capacity of the substation, and new conductor types) have been fixed. In this way, the information was used to evaluate 8760 scenarios represented by time resolution of 1 hour through of a conventional power flow. The solution obtained from the conventional power flow determines that more than 99.00% of the integer solutions found by the proposed approach were feasible. This fact validates the

Table 6: 34-Node system-Investment proposed plan

Case	Stage	SE	Conductor replacement	Gas unit	Wind unit
	1	✓	$a5$(S-1, 1-2, 2-3, 3-4, 4-5, 5-16, 16-17, 17-18)	x	x
I		-	$a4$(5-6, 18-19, 19-20, 20-21, 21-22) $a2$(6-7)		
	2	-	$a4$(22-23) $a2$(7-8, 8-9)	x	x
	3	-	-	x	x
	1	-	$a5$(S-1, 1-2, 2-3, 3-4, 4-5)	GT1(26, 33) GT2(23, 24)	x
II	2	-	-	-	x
	3	✓	-	GT1(25)	x
	1	-	$a5$(S-1, 1-2, 2-3, 3-4, 4-5) $a2$(2-12)	GT1(23) GT2(25)	WT2(26)
III	2	-	$a2$(12-13)	GT1(24, 32)	WT2(23)
	3	✓	-	-	-

Table 7: 135-Node system-Present value of investment and operational cost ($\$10^6$)

	Study cases		
Costs	I	II	III
OF	531.883	201.745	193.639
Investment	1.678	1.761	2.670
Production	174.256	174.505	166.864
Losses	1.693	1.700	1.497
Unserved energy	330.475	0.000	0.000
CO_2 tax	23.779	23.779	22.608

acceptable quality of the solution found by the proposed approach to determine the best long-term plan.

5. Conclusion

In this paper, an uncertainty-based mixed-integer conic programming model for solving the distribution system planning has been proposed. The solution of this model determines an effective economic-environmental strategy that aims to maximize the performance of an electrical distribution system. In order to incorporate the uncertainties of electricity load and renewable output power, a two-stage stochastic programming has been used. This approach, in which the electricity demand and the variability of renewable energy-based DG units were estimated via multiples scenarios and duly considered in the planning process, is an appropriate way to represent uncertainties of a system.

Numerical results obtained for all cases validate the proposed formulation as a robust tool, which can be useful for the system operator in a decision making process. According to these results, a set of proper planning actions have obtained that besides fulfilling the operational and financial criteria on a long-term horizon, low carbon-emitting EDS has been guaranteed. It can be seen that applying carbon policies and considering the integration of renewable-based DG units, specifically wind turbines, a significant reduction of pollutant emission at distribution level can be obtained.

Table 8: 135-Node system-Expected Emission (kTon CO_2)

	Study cases		
In	I	II	III
DG	0.00	1.955	0.884
Main grid	3222.904	3220.678	3041.674
Total	3222.904	3222.633	3042.558

Table 9: 135-Node system-Investment proposed plan

Case	Stage	SE	Conductor replacement	Gas unit	Wind unit
I	1	TR2(S1)	$a8$ (S1-135, 35-36, 36-38, 38-39) $a7$ (67-68, 137-74)	x	x
	2	TR1(S2)	$a8$ (17-18, 60-61, 137-86)	x	x
	3	-	$a8$ (74-75)	x	x
II	1	TR1(S1)	$a8$(S1-17, 18-19, S1-35, 35-36, 36-38, 38-39, S2-74), $a2$(75-76)	GT2(88)	x
	2	TR1(S2)	$a8$(S1-1)	GT1(59, 105)	x
	3	-	$a8$(60-61)	GT1(16) GT2(15, 56)	x
III	1	TR1(S1)	$a8$(S1-35, 35-36, 36-38, 38-39)	GT2(59, 88)	WT1(85)
	2	TR1(S1)	-	GT1(105)	WT2(122)
	3	-	-	GT1(15, 56)	WT2(52, 58)

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