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Partanen, Mikko; Tulkki, Jukka

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## Designing the measurement of the atomic mass density wave of a Gaussian mass-polariton pulse in optical fibers

Mikko Partanen  
Jukka Tulkki

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# Designing the measurement of the atomic mass density wave of a Gaussian mass-polariton pulse in optical fibers

Mikko Partanen\* and Jukka Tulkki

Aalto University, School of Science, Engineered Nanosystems Group, Aalto, Finland

**Abstract.** We have recently introduced the mass-polariton (MP) theory of light to describe the coupled dynamics of the field and matter when a light pulse propagates in a transparent medium. The theory is based on combining the electrodynamics of continuous media and continuum mechanics, which are both widely used standard theories in their fields of physics. The MP theory shows that a light pulse propagating in a transparent medium is accompanied by a mass density wave (MDW) of atoms set in motion by the optical force density of the light pulse. In the corresponding quantum picture, the covariant coupled state of the field and matter is described as the MP quasiparticle, which has coupled field and medium components. We study a schematic experimental setup that would enable measurements of the atomic displacements and the excess mass density related to the MDW of a Gaussian MP pulse propagating in an optical fiber made of fused silica. © 2019 Society of Photo-Optical Instrumentation Engineers (SPIE) [DOI: [10.1117/1.OE.58.3.036101](https://doi.org/10.1117/1.OE.58.3.036101)]

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## 1 Introduction

Conventionally, theories describing the propagation of light in a medium neglect the coupled dynamics of the field and matter by assuming that only the energy of the electromagnetic field propagates inside the medium, and the medium elements stand still. Thus, the response of the medium to the field is conventionally considered using a rigid body approximation. In contrast, we have recently shown that this assumption is not physical as it dismisses a physically substantial transport of medium mass by the mass density wave (MDW) of atoms set in motion by the optical force density.<sup>1,2</sup> We also note an early paper by Poynting,<sup>3</sup> where the existence of oscillatory “small longitudinal material waves accompanying light waves” is foretold. Thus, excluding the propagation of these material waves, he considered essentially the same phenomenon as the MDWs in our work.

In the classical continuum picture, the coupled dynamics of the field and matter is described by the optoelastic continuum dynamics (OCD), which combines the well-known optical force density and the elasticity theory with the Newtonian dynamics of the medium. In the quantum picture, this corresponds to describing light quanta in a medium as mass-polariton (MP) quasiparticles that have a nonzero transferred mass. In particular, we have shown that an electromagnetic pulse having field energy  $E_{\text{field}}$  propagating in a nondispersive dielectric transfers a mass equal to  $\delta M = (n^2 - 1)E_{\text{field}}/c^2$ , where  $n$  is the refractive index of the medium. The mass  $\delta M$  is transferred by the MDW, where the atoms are spaced more densely inside the light pulse as a result of the optical force field.

Another key observation of the MP theory of light is that in common semiconductors most of the momentum of light is transferred by semiconductor atoms moving under the influence of the optical force.<sup>1,2</sup> While the field's share of the total MP momentum is  $p_{\text{field}} = E/(nc)$ , the atoms coupled to

the field and moved with the MDW transfer momentum  $p_{\text{MDW}} = (n - 1/n)E/c$ . In the case of silicon at wavelength  $\lambda_0 = 1550$  nm, this corresponds to 92% of the total momentum of light. The sharing of the total MP momentum by the field and the MDW also provides a resolution to the centennial Abraham–Minkowski controversy of photon momentum in a medium,<sup>4–14</sup> which has also gained much experimental interest.<sup>15–26</sup> The MP theory of light has also very recently been applied to describe the angular momentum of light<sup>27</sup> and the light propagation in optical fibers.<sup>28</sup>

Without actually carrying out the simulations, it may be difficult to understand the atomic displacements in the optical fiber as a function of time. However, one can get a quick qualitative idea by considering the displacement of medium elements in two extreme cases: (1) if the fiber is free-standing, the medium elements in the core of the fiber move forward by an amount that is proportional to the total pulse energy per cross-sectional area. (2) If the fiber is mounted, the medium elements still move forward. However, because of the mounting, this leads to a strain that spreads from the interface between the fiber and the mounting in both directions. Consequently, the elastic forces quickly restore the original equilibrium positions of the medium elements. In both cases, the atomic-displacement-related strain will appear at the ends of the fiber and it will be relaxed by elastic forces. In our earlier works,<sup>1,2,28</sup> one can find a quantitative study of the atomic displacements and the relaxation of the medium in selected cases.

In this work, we investigate possibilities of the experimental measurement of the mass  $\delta M$  transferred by the MDW atoms when a Gaussian MP pulse propagates in a single-mode fiber. In particular, we simulate the atomic displacements and the excess mass density of the MDW related to the propagation of a Gaussian MP pulse in an optical fiber made of fused silica, which is the most widely used material in common optical fibers. Our results show that, if the pulse

\*Address all correspondence to Mikko Partanen, E-mail: [mikko.p.partanen@aalto.fi](mailto:mikko.p.partanen@aalto.fi)

duration is increased to seconds, the MDW can lead to nanometer-size atomic displacements also at relatively low power levels below 1 W.

## 2 Mass-Polariton Theory of Light

### 2.1 Optoelastic Forces and Newton's Equation of Motion

As the electromagnetic field gives rise to an optical force density on the induced atomic dipoles in the medium, the dynamics of the field and matter become automatically coupled through Newton's equation of motion. In this OCD model, we denote the disturbed mass density of the medium as  $\rho_a(\mathbf{r}, t) = \rho_0 + \rho_{\text{MDW}}(\mathbf{r}, t)$ , where  $\rho_0$  is the equilibrium mass density and  $\rho_{\text{MDW}}(\mathbf{r}, t)$  is the mass density of the MDW. Since the atomic velocities in the MDW are nonrelativistic, Newton's equation of motion for the mass density of the medium is written as<sup>1</sup>

$$\rho_a(\mathbf{r}, t) \frac{d^2 \mathbf{r}_a(\mathbf{r}, t)}{dt^2} = \mathbf{f}_{\text{opt}}(\mathbf{r}, t) + \mathbf{f}_{\text{el}}(\mathbf{r}, t), \quad (1)$$

where  $\mathbf{r}_a(\mathbf{r}, t)$  is the atomic displacement field of the medium,  $\mathbf{f}_{\text{opt}}(\mathbf{r}, t)$  is the optical force density, and  $\mathbf{f}_{\text{el}}(\mathbf{r}, t)$  is the elastic force density. The elastic force density arises as a secondary effect trying to restore the mass equilibrium in the medium after it has been disturbed by the optical force density.

In the OCD model, we apply the well-known optical force density following from Maxwell's equations.<sup>29,30</sup> It is given for a dielectric medium as<sup>31</sup>

$$\mathbf{f}_{\text{opt}}(\mathbf{r}, t) = -\frac{\epsilon_0}{2} \mathbf{E}^2 \nabla n^2 + \frac{n^2 - 1}{c^2} \frac{\partial}{\partial t} \mathbf{E} \times \mathbf{H}, \quad (2)$$

where  $\mathbf{E}$  and  $\mathbf{H}$  are the electric and magnetic field vectors and  $\epsilon_0$  is the permittivity of vacuum. When solving Newton's equation of motion in Eq. (1), we use a perturbative approach in which we neglect the extremely small damping of the electromagnetic field due to the transfer of the field energy to the kinetic and elastic energies of atoms as a result from the optical force density. The accuracy of this approach has been studied in Ref. 1.

The well-known elastic force density used in the OCD model follows from Hooke's law.<sup>32</sup> For an isotropic linear elastic medium, it can be given in terms of the material displacement field  $\mathbf{r}_a(\mathbf{r}, t)$  as<sup>33</sup>

$$\mathbf{f}_{\text{el}}(\mathbf{r}, t) = (\lambda_L + 2\mu_L) \nabla [\nabla \cdot \mathbf{r}_a(\mathbf{r}, t)] - \mu_L \nabla \times [\nabla \times \mathbf{r}_a(\mathbf{r}, t)], \quad (3)$$

where  $\lambda_L$  and  $\mu_L$  are the Lamé elastic constants of the medium. In place of the Lamé constants, one could also use any two independent elastic moduli, such as the bulk and shear moduli whose relation to the Lamé constants is described, e.g., in Ref. 34.

### 2.2 Transferred Mass, Energy, and Momentum of Mass Polaritons

Assuming a light pulse whose total electromagnetic energy is  $E_{\text{field}}$ , the total mass transferred by the MDW is given by

$$\delta M = \int \rho_{\text{MDW}} d^3 r = (n^2 - 1) E_{\text{field}} / c^2, \quad (4)$$

where the integration is performed over the light pulse. The excess mass density of the MDW  $\rho_{\text{MDW}}$  follows from the forward displacement of atoms resulting from the second term of the optical force density in Eq. (2).

The total energy of the coupled MP state of the field and matter  $E_{\text{MP}}$  and its shares  $E_{\text{field}}$  and  $E_{\text{MDW}}$  carried by the electromagnetic field and the atomic MDW are given by

$$E_{\text{field}} = \int \frac{1}{2} (\epsilon \mathbf{E}^2 + \mu \mathbf{H}^2) d^3 r, \quad (5)$$

$$E_{\text{MDW}} = \int \rho_{\text{MDW}} c^2 d^3 r = \delta M c^2 = (n^2 - 1) E_{\text{field}}, \quad (6)$$

$$\begin{aligned} E_{\text{MP}} &= \int \left[ \rho_{\text{MDW}} c^2 + \frac{1}{2} (\epsilon \mathbf{E}^2 + \mu \mathbf{H}^2) \right] d^3 r \\ &= \delta M c^2 + E_{\text{field}} = n^2 E_{\text{field}}, \end{aligned} \quad (7)$$

where  $\epsilon = n^2 \epsilon_0$  is the permittivity and  $\mu = \mu_0$  is the permeability of the dielectric medium. The right-hand side results can be derived using either the numerical simulations of the OCD model or the Lorentz transformation of the MP quasiparticle model.<sup>1</sup>

Correspondingly, the total momentum of the coupled MP state of the field and matter  $\mathbf{p}_{\text{MP}}$  and its shares  $\mathbf{p}_{\text{field}}$  and  $\mathbf{p}_{\text{MDW}}$  carried by the electromagnetic field and the atomic MDW are given by

$$\mathbf{p}_{\text{field}} = \int \frac{\mathbf{E} \times \mathbf{H}}{c^2} d^3 r = \frac{E_{\text{field}}}{nc} \hat{\mathbf{z}}, \quad (8)$$

$$\mathbf{p}_{\text{MDW}} = \int \rho_a \mathbf{v}_a d^3 r = \left( n - \frac{1}{n} \right) \frac{E_{\text{field}}}{c} \hat{\mathbf{z}}, \quad (9)$$

$$\mathbf{p}_{\text{MP}} = \int \left( \rho_a \mathbf{v}_a + \frac{\mathbf{E} \times \mathbf{H}}{c^2} \right) d^3 r = \frac{n E_{\text{field}}}{c} \hat{\mathbf{z}}, \quad (10)$$

where  $\mathbf{v}_a(\mathbf{r}, t) = \partial \mathbf{r}_a(\mathbf{r}, t) / \partial t$  is the atomic velocity field of the medium and  $\hat{\mathbf{z}}$  is the unit vector corresponding to the direction of propagation along the  $z$  axis. Again, the results on the right-hand side can be derived using either the numerical simulations of the OCD model or the Lorentz transformation of the MP quasiparticle model.<sup>1</sup>

## 3 Simulations of the Mass Transfer in an Optical Fiber

Next, we use the OCD model to simulate the mass transfer related to the propagation of a one-dimensional (1-D) Gaussian light pulse in fused silica, which is the standard material used in optical fibers. The mass density of fused silica is  $\rho_0 = 2200 \text{ kg/m}^3$ .<sup>35</sup> The phase and group refractive indices of fused silica are given by  $n_p = 1.44$  and  $n_g = 1.46$  for  $\lambda_0 = 1550 \text{ nm}$ , respectively.<sup>36</sup> As the dispersion is not very large, in this work, we neglect dispersion and use the phase refractive index only. However, the dispersion could be accounted for as explained in Ref. 2. The Lamé elastic constants for fused silica are  $\lambda_L = 15.4 \text{ GPa}$  and  $\mu_L = 31.3 \text{ GPa}$ .<sup>37</sup>

We assume a 1-D linearly polarized Gaussian light pulse of central wavelength  $\lambda_0 = 1550$  nm and a total electromagnetic energy of  $E_{\text{field}} = 0.1$  J,  $E_{\text{field}} = 0.5$  J, or  $E_{\text{field}} = 1$  J per cross-sectional area  $A = \pi(d/2)^2$ , where the diameter of the area is  $d = 2.5$   $\mu\text{m}$ . Large energy per cross-sectional area requires that the pulse duration is relatively long as described in more detail later. Even if it is not conventional for long pulse durations, we use the pulse energy to characterize the pulse since the atomic displacement in the fiber only depends on the pulse energy per cross-sectional area. For the 1-D Gaussian pulse, the electric and magnetic fields are obtained as exact solutions of Maxwell's equations as<sup>38</sup>

$$\begin{aligned} \mathbf{E}(\mathbf{r}, t) &= \text{Re} \left[ \int_{-\infty}^{\infty} \tilde{E}(k) e^{i[kz - \omega(k)t]} dk \right] \hat{\mathbf{x}} \\ &= \tilde{E}_0 \cos \left[ nk_0 \left( z - \frac{ct}{n} \right) \right] e^{-(n\Delta k_0)^2 (z - ct/n)^2 / 2} \hat{\mathbf{x}}, \quad (11) \end{aligned}$$

$$\begin{aligned} \mathbf{H}(\mathbf{r}, t) &= \text{Re} \left[ \int_{-\infty}^{\infty} \tilde{H}(k) e^{i[kz - \omega(k)t]} dk \right] \hat{\mathbf{y}} \\ &= \tilde{H}_0 \cos \left[ nk_0 \left( z - \frac{ct}{n} \right) \right] e^{-(n\Delta k_0)^2 (z - ct/n)^2 / 2} \hat{\mathbf{y}}, \quad (12) \end{aligned}$$

where  $\tilde{E}(k) = \tilde{E}_0 e^{-[(k - nk_0)/(n\Delta k_0)]^2 / 2} / (\sqrt{2\pi} n \Delta k_0)$  and  $\tilde{H}(k) = \tilde{H}_0 e^{-[(k - nk_0)/(n\Delta k_0)]^2 / 2} / (\sqrt{2\pi} n \Delta k_0)$  are the Gaussian Fourier components of the electric and magnetic fields with normalization factors  $\tilde{E}_0$  and  $\tilde{H}_0$ , respectively,  $k_0 = \omega_0/c$  is the wavenumber corresponding to the central angular frequency  $\omega_0 = 2\pi c/\lambda_0$  in vacuum,  $\Delta k_0$  is the standard deviation of the wavenumber in vacuum,  $\omega(k) = ck/n$  is the dispersion relation of the nondispersive medium, and  $\hat{\mathbf{x}}$  and  $\hat{\mathbf{y}}$  are the unit vectors with respect to  $x$  and  $y$  axes, respectively. The normalization factors  $\tilde{E}_0$  and  $\tilde{H}_0$  are not independent but related to each other by the wave impedance as  $\tilde{E}_0/\tilde{H}_0 = \sqrt{\mu/\epsilon}$ . In addition, the normalization factors become fixed by requiring that the integral of the energy density in Eq. (5) gives the energy  $E_{\text{field}}$  per cross-sectional area  $A$ , which gives  $\tilde{E}_0 = \sqrt{2\Delta k_0 E_{\text{field}} / (n\sqrt{\pi}\epsilon_0 A)}$  in the narrowband limit  $\Delta k_0 \ll k_0$ .

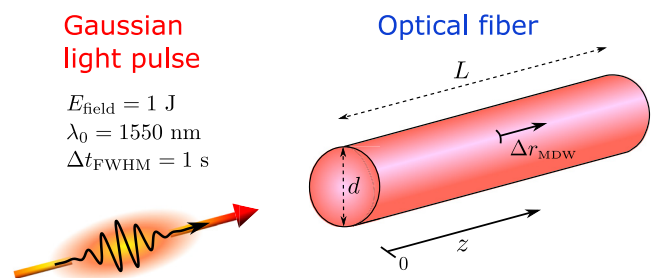
In our example, the temporal FWHM of the pulse is  $\Delta t_{\text{FWHM}} = 1$  s, which corresponds to a relative spectral width of  $\Delta\omega/\omega_0 = \Delta k_0/k_0 = 1.37 \times 10^{-15}$ . The pulse used here has a very small spectral width, which follows from its relation to the temporal FWHM of the Gaussian pulse, which is relatively long in the time domain. However, in practical measurements of the atomic displacement resulting from the MDW, one could use, e.g., a top-hat pulse of broadband radiation. Small spectral width is not required as the magnitude of the total atomic displacement due to the MDW studied in this work is proportional to the total energy of the light pulse per cross-sectional area, and therefore, it is not sensitive to the spectral width. For the three pulse energies given above, the peak intensities of the Gaussian pulse averaged over the harmonic cycle and the cross section of the fiber are  $I_{\text{peak}} = 1.91 \times 10^6$  W/cm<sup>2</sup>,  $I_{\text{peak}} = 9.55 \times 10^6$  W/cm<sup>2</sup>, and  $I_{\text{peak}} = 1.91 \times 10^7$  W/cm<sup>2</sup>, which correspond to the peak powers  $P_{\text{peak}} = 94$  mW,  $P_{\text{peak}} = 470$  mW, and  $P_{\text{peak}} = 939$  mW, respectively. For comparison, the corresponding peak powers for a top-hat pulse are  $P_{\text{peak}} = 100$  mW,  $P_{\text{peak}} = 500$  mW, and  $P_{\text{peak}} = 1000$  mW. Transmitting large

powers through a single-mode fiber is expected to require the use of broadband radiation to reduce nonlinear effects, such as the stimulated Brillouin and Raman scattering.

We simulate the time dependence of the atomic displacement resulting from the MDW. The simulated atomic displacement corresponds to that measured in the middle of the optical fiber in a setup in which the pulse energy is propagating through an optical fiber as illustrated in Fig. 1. Due to the interface effects, the physical cross-sectional area of the fiber is not directly comparable with the cross-sectional area used in our simulations. The possible cladding layer, waveguide dispersion, and other factors that affect the spreading of the pulse energy in the lateral direction should also be taken into account in more accurate modeling of the possible experiments. All these factors can be easily included in the OCD simulations. The main goal of the simulations presented here is to give order of magnitude estimates for the atomic displacement resulting from the transfer of pulse energy through an optical fiber. In this work, we neglect the interface recoil effects taking place at the ends of the fiber although the relaxation of these effects propagates at the velocity of sound and can contribute to the atomic displacement in the middle of the fiber in the relatively long time scale used in the simulation. These relaxation effects have been studied in Ref. 1. Investigating possible ways to reduce the influence of the interface effects at long time scales is a topic of further work.

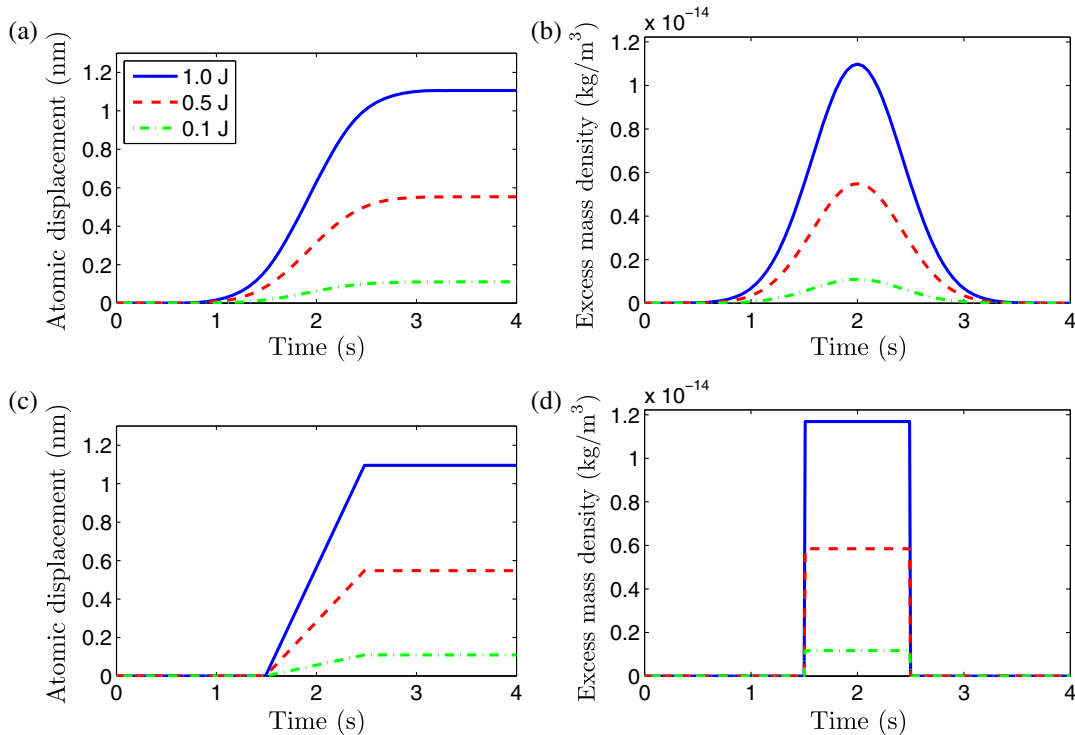
Figure 2(a) shows the simulated atomic displacement due to a Gaussian pulse in the fiber as a function of time. At time  $t = 0$  s, the atomic displacement is zero as the light pulse has not yet reached the reference plane, where the atomic displacement is studied. The front end of the pulse reaches the reference plane approximately at  $t = 1$  s. After this, the atomic displacement starts to increase monotonically until the tail of the pulse has passed the reference plane approximately at  $t = 3$  s. The atomic displacement after the light pulse has gone is  $\Delta r_{\text{MDW}} = \delta M / (\rho_0 A) \approx 1.1$  nm. This shows that, if the pulse duration is increased, nanometer-size atomic displacements can be obtained even for lower field intensities than originally studied in Ref. 1. In addition, instead of fused silica, the previous studies were carried out for silicon, which is a more favorable material for the studies of the MDW due to its higher refractive index and larger atomic displacements.

Figure 2(b) depicts the excess mass density  $\rho_{\text{MDW}}$  of the MDW in the fiber as a function of time. It is observed to follow the Gaussian shape of the light pulse. The excess mass



**Fig. 1** Schematic illustration of a Gaussian light pulse and an optical fiber with a core diameter  $d$  and length  $L$ . The mass transfer of the MDW can be deduced from the measurement of the longitudinal atomic displacement  $\Delta r_{\text{MDW}}$  on the fiber surface as a function of time.





**Fig. 2** Simulation of the mass transfer due to a 1-D Gaussian and top-hat light pulses in a silica fiber as a function of time: (a) atomic displacement of the Gaussian pulse, (b) excess mass density of the Gaussian pulse, (c) atomic displacement of the top-hat pulse, and (d) excess mass density of the top-hat pulse. The assumed electromagnetic pulse energies are 0.1, 0.5, and 1.0 J, corresponding to peak powers 94, 470, and 939 mW in the case of the Gaussian pulse and peak powers 100, 500, and 1000 mW in the case of the top-hat pulse, respectively. The powers have been averaged over the harmonic cycle and the cross section of the fiber. The wavelength is  $\lambda_0 = 1550$  nm and the duration of the pulse is 1 s. The diameter of the cylindrical fiber core is  $d = 2.5 \mu\text{m}$ .

density of the MDW has its origin in the atomic displacement as shown in Fig. 2(a). Since the atoms have been shifted forward after the light pulse and atoms in front of the pulse are still at their original equilibrium positions, the atoms are more densely spaced at the position of the light pulse. For the Gaussian pulse studied in this work, the MDW mass density is seen to be also Gaussian. Correspondingly, e.g., for a top-hat pulse, the excess mass density of the MDW would also have a top-hat shape.

For comparison, Figs. 2(c) and 2(d) show the atomic displacement and the excess mass density of the MDW resulting from top-hat pulses with total energies corresponding to those of the Gaussian pulses in Figs. 2(a) and 2(b). In the course of time, the atomic displacements in Fig. 2(c) approach the same values as those in Fig. 2(a). This follows from the fact that the total atomic displacements only depend on the transmitted pulse energies per cross-sectional area, which are the same for the Gaussian and top-hat pulses in our example. The excess mass densities of the Gaussian and top-hat pulses in Figs. 2(b) and 2(d) compare to each other so that the integrals of these curves are equal. This means that the Gaussian and top-hat pulses in our example transfer equal net MDW masses.

The atomic displacement of nanometer size is expected to be feasible to measure. For example, recent measurements of picometer-size atomic displacements related to the elastic waves induced by light reflection from a highly reflective mirror by Požar et al.<sup>39,40</sup> are very promising. It might be

possible to apply the piezoelectric sensor technique used in these measurements together with a D-shaped optical fiber studied by Cui et al.<sup>41</sup> However, at relatively large time scales, it may be difficult to separate the effect of the MDW from other nonlinear and thermal effects, which also contribute to the total atomic displacement. One possibility to separate the MDW effect would be to study the atomic displacement in the fiber as a function of time using a top-hat pulse for which the increase of the atomic displacement predicted by the MP theory is linear when the pulse is propagating in the fiber. For other nonlinear and thermal effects, the atomic displacement is expected to increase with an accelerating rate once the field has been switched on.

#### 4 Conclusions

In conclusion, we have simulated the atomic displacement and the excess mass density of the atomic MDW of a Gaussian light pulse propagating in a silica fiber. The total energy and momentum of the light pulse are shared between the electromagnetic field and the medium atoms in the MDW. Our results show that the MDW can lead to nanometer-size atomic displacements also at relatively low power levels if the pulse duration is increased so that it is of the order of seconds. In our simulations, we have neglected the interface effects and waveguide dispersion, but these factors can also be easily accounted for in the OCD simulations to allow more detailed modeling of the possible experiments. The optical shock wave property of the MDW, which

propagates with the velocity of light instead of the velocity of sound, prompts for engineering of device concepts, such as very high-frequency mechanical oscillators not limited by the acoustic cutoff frequency.

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**Mikko Partanen** received his BS and MS degrees in engineering physics and his PhD in computational science from the School of Science, Aalto University, Finland, in 2011, 2013, and 2017. Currently, he is a postdoctoral researcher at the Aalto University and his research interests include the quantization of the electromagnetic field and the description of optical energy and momentum transfer in dielectric structures in nonequilibrium conditions.

**Jukka Tulkki** is a professor of computational engineering at the Aalto University. His recent research has focused in theoretical and computational studies of photonic transport and thermal properties of nanoscale and atomistic materials and devices. He has more than 150 peer-reviewed publications that cover topics in theoretical and computational photonics, simulations of LEDs and thermophotonic devices, and modeling of light-matter interaction at wavelengths ranging from x-rays to wavelengths in semiconductor optics.