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# A multi-ship following model for icebreaker convoy operations in ice-covered waters

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# ABSTRACT

Most vessels cannot safely sail in areas with sea ice. In the Northern Baltic Sea area, the presence of sea ice often requires icebreaking ships to open up ice-covered areas so merchant vessels can proceed to their destinations. In this environment there is an increasing number of ships. With growing transport volumes in sea ice environments, icebreaker operations such as convoys have become increasingly important to ensure navigation safety. This paper proposes a model of multi-ship following for icebreaker convoy operations in continuous icebreaking conditions, using modelling principles from earlier proposed car following models combined with considerations of safe distance and safe speed in ship convoy operations in ice conditions. The model parameters are calibrated with empirical data from actual icebreaker convoy operations, using data from the Automatic Identification System (AIS) and sea ice model data. The model determines ice resistance according to ice thickness, type, and bending strength, and the main dimensions of the icebreaker leading the convoy. The maximum safe navigable speed is used to assess the following ships' ability to sail in sea ice. The multi-ship following simulation results were found to be in good agreement with empirical data. The new proposed model can provide a theoretical reference for icebreaker convoy operations, can have practical use in ship simulators to improve training, and could be useful for traffic modelling and planning purposes.

KEYWORDS: Multi-ship following model; Ice condition; Convoy operations; Icebreaker; Maritime safety.

# 1. INTRODUCTION

Maritime transportation is an essential element of the global trade. According to statistics by the United Nations Conference on Trade and Development (UNCTAD), the volume of shipping trade has increased by 2.5 times over the last 40 years. It reached a staggering 9.1 billion tons in 2012, accounting for 70–90% of the total volume of the world's trade (UNCTAD, 2016). However, because many ships lack the ability to navigate independently in ice covered waters such as those found in the Arctic and in the Baltic Sea during winter, they require the assistance of an icebreaker-classed ship. Icebreaker operations to open up channels through ice-covered waters are essential to ensure smooth maritime transport flows. Despite the overall importance of icebreakers to improve navigational safety (Rosenblad, 2007), sailing in proximity to an icebreaker increases the risk of collisions (Kum and Sahin, 2015). Through field observations, Boström and Österman (2017) have indicated the importance of clear communication between icebreakers and escorted vessels to ensure the safety of operations. Zhang et al. (2019) have analysed accident reports of collisions under icebreaker assistance in Arctic conditions, finding that maneuvering failures, lack of situational awareness, and inability to maintain a safe speed and distance, are among the most significant contributing factors to these accident occurrences.

For the ice-covered waters of the Baltic Sea during winter, navigation in sea-ice environments is a common but complex operation, and icebreakers are essential for breaking the sea ice, to ensure smooth transportation flows. The Finnish-Swedish Winter Navigation System (FSWNS), a set of technical requirements and operational procedures to enhance winter navigation safety, succeeds to lower the overall risk of maritime transportation in winter conditions (FTSA, 2010; FTA, 2014). Nevertheless, a recent analysis of accident data indicates that most accidents in the area occur during icebreaker assistance operations. Collisions are the most common type of accident for general cargo ships navigating ice-covered waters, and such ships are most frequently involved in accidents (Goerlandt et al. 2017). The number of ships requiring assistance by one or more icebreakers depends on the extent, concentration, thickness, and strength of the sea ice (Valdez Banda et al. 2015). A risk analysis reported by Valdez Banda et al. (2016) furthermore indicates that training can help lower the accident risk, and be an effective way to reduce the risk of oil spills, which is a serious concern in the Baltic Sea area (HELCOM, 2018). Ship bridge simulators are important tools by which ships' crew can gain experience under different sailing conditions. Hence, developing realistic simulator environments is very important for strengthening training (Valdez Banda et al., 2016; Last et al., 2017). Models for assessing the performance of individual vessels in ice-breaking mode, such as those described by Lubbad and Løset (2011), as well as vessels kinematics in assisted operations, are key components of such simulators (Zhang et al., 2018).

Most models for ship performance in ice focus on continuous icebreaking in independent navigation, typically focusing on the assessment of ship resistance and speed, and ice loads on the hull (Li et al., 2018). Such models are mostly used for the design of the ship's structure (Su et al., 2011), but can also be applied for operational purposes such as route planning (Guinness et al., 2014).

Real-time simulation of ship models in sea ice conditions is very important for ensuring crew can train in realistic environments to ensure safe navigation. Lubbad and Løset (2011) proposed a method based on the PhysX modeling environment to track the ship-ice interaction process, for a ship in the continuous icebreaking mode. For escort operations, Tsoy (1983) developed a mathematical model for ships in ice-broken channels that did not consider the dynamics of, or interaction with, supporting icebreakers. Existing models for the coupled kinematics between icebreakers and assisted vessels only consider two ships, i.e. are limited to escort operations (Zhang et al., 2018). Also, a data-driven model for estimating the safe distance between icebreakers and assisted vessels only considered escort operations (Zhang et al., 2017). However, in complex ice conditions, especially in busy sea areas, many ships may need support, and convoy operations, where multiple vessels follow a single icebreaker, are common (Valdez Banda et al., 2015). Hence, there is a need to model the kinematic spatio-temporal interaction between multiple ships, but to the best of the author's knowledge, no such models are available. Developing such a model has practical significance for improving the realism of marine simulators, for enhancing ship crew training, and ultimately may contribute to the safe passage of vessel convoys in ice conditions (Valdez Banda et al., 2016).

The purpose of this paper is to develop a multi-ship following model for icebreaker convoy operations in ice-covered waters. Although the implementation is based on the data collected in the Baltic Sea region, the proposed modeling approach is expected to be applicable to other regions as well.

The remainder of this article is organized as follows. Section 2 outlines the problem, justifies the need for the development of a model for convoy operations, and defines the objectives. Section 3 outlines the context of icebreaker convoy operations, reviews existing models coupled kinematics of vehicles, and introduces basic data for developing the model. Section 4 presents the multi-ship following model in a step-wise manner, including the quantification of ice conditions and model calibration. Section 5 introduces the application of the model and validates it. Section 6 provides a discussion, addressing model limitations, uncertainties, and future work. Section 7 concludes.

# 2. PROBLEM DESCRIPTION AND MODEL OBJECTIVES

## 2.1 Icebreaker convoy operations for multiple ships

During cold winters, ships navigating the northern part of the Baltic Sea may face complex ice conditions. When a ship crosses an ice-covered area of the Baltic Sea in winter, the assistance of one or more icebreakers is often necessary. A common situation is that many ships follow a single icebreaker by forming a convoy, which increases the risk of collisions for these vessels, while leading to overall improvements to maritime transportation safety. More generally, five practical icebreaker assistance operations are usually distinguished (Rosenblad, 2007): escort, convoy, breaking loose, double convoy, and towing. When assisted by an icebreaker, ships must consider the rules of the road as stipulated in the Collision Regulations (SOLAS, 1974), while also considering the guidance and instructions by the icebreaker (Boström and Österman, 2017).

In a convoy operation, the icebreaker opens a channel through the sea ice, with the other vessels following it at a certain distance. This distance, as well as the speed under which the operations take place, depend on the ice conditions (Goerlandt et al., 2015). A safe distance needs to be maintained between the icebreaker and all following vessels, to avoid collision. Similarly, a safe speed needs to be adhered to.

The safe speed is the maximum speed at which hull and ship propulsion are not be damaged by ice when navigating in the ice-covered Baltic Sea. Even with an icebreaker opening a channel in the ice, sea ice can still cause damage to following vessels in the convoy. If the speed is too high, a ship's hull is vulnerable to damage, as higher speeds correspond to higher ice loads (Kotilainen et al., 2017). The thicker the ice, the smaller the distance between two ships and the lower their speed must be to ensure safety (Goerlandt et al. 2016; Zhang et al., 2017).

When multiple ships are following an icebreaker, the ships at the rear of the convoy are less affected by the ice, at least when navigating in sea areas without significant ice compression. However, the fluctuating ice resistance on the leading ship can cause speed variations to the icebreaker, which can have knock-on effects throughout the convoy. In case of multiple vessels following an icebreaker, each vessel must maintain a safe distance between its bow and the stern of the vessel ahead in the convoy, and its stern and the bow of the next vessel in the convoy. If this distance becomes too short, sudden speed fluctuations may result in collision as there is too little time to stop the vessel or break out of the channel (Goerlandt et al., 2017) However, if the distance becomes too long, the following vessel can encounter additional ice resistance and become beset in ice. This can occur especially in compressive ice conditions, as the channel opened up by the leading vessel(s) closes due to ice movement.

## 2.2 Model justification and objectives

Ice navigation training with a navigation simulator is considered to be an effective way to reduce the risks of winter navigation (Valdez Banda et al., 2016; Boström and Osterman 2017; Last et al. 2017). In the Northern Baltic Sea during winter conditions, most accidents occur during ship escort and convoy operations (Goerlandt et al., 2017), while these also represent the highest environmental pollution risks from winter navigation, due to the possibility of oil spill (Valdez Banda et al., 2016). Hence, reducing the probability of accidents through navigation training is desirable. On a systems level of the winter navigation system, it is important that there is a sufficient number of icebreakers available to assist merchant vessels, to ensure the smooth flow of cargo in international maritime trade. Maritime traffic simulation models can be used to estimate the number of icebreakers required to assist vessels in sea ice environments. Including the characteristics of different operations types is an important aspect of such a system-level analysis of the transportation system (Lindeberg et al., 2015). For both these aforementioned purposes, it is useful to develop mathematical models for common wintertime operations, which can be used in training or planning contexts.

Ship convoy operations are common icebreaker assistance types in the Baltic Sea area, and in other ice-covered waters (Rosenblad, 2007), which justifies the focus on developing a model for this operation type. The specific aim of this article is to develop a mathematical model for the coupled kinematics of multiple vessels following an icebreaker in a convoy. Such a model can be implemented in ship simulators, to enhance the realism of navigation training by mimicking the behavior of ships in convoys in sea ice environments. This can be done e.g. through generating objects in the simulation, i.e. other vessels with which the navigator in training should interact and learn how to safely proceed in an icebreaker-led convoy. The model presented in this article aims to automatically generate the coupled kinematics of vessels in a convoy, improving the realism of the navigation environment, reducing the workload of simulator operators to develop and implement specific scenarios, thus enhancing efficiency and cost-effectiveness. Thus, the model has similar objectives and use philosophy as the work by Last et al. (2017). It extends the work presented in Zhang et al. (2018), which presented a model for an icebreaker and a single following vessel (escort operations), to the multi-vessel following problem (convoy operations).

# 3. SELECTION OF MODELING APPROACH AND EVIDENCE FOR MODEL DEVELOPMENT

In order to select a suitable modeling approach for describing the kinematics of vessels in a convoy, it is useful to describe the convoy process as one of coupled kinematics subject to a number of constraints, similarly as done in Zhang et al. (2018) for escort operations. This is described qualitatively in Section 3.1. Furthermore, it is instrumental to identify modeling approaches used in other application areas where coupled kinematics are important for describing aspects of reality. In particular, inspiration for selecting a suitable modeling approach can be found in the car following literature, and in existing maritime traffic modeling literature. This literature is briefly presented in Section 3.2, to support the model selection in Section 4. Finally, it is important to introduce the data and evidence used for the actual model development, addressing the main features of the qualitative process of Section 3.1. These aspects are elaborated in Section 3.3 (influence of ice conditions), Section 3.4 (safe speed), and Section 3.5 (safe distance).

## 3.1 Icebreaker convoys as coupled kinematics processes

In an icebreaker convoy, the leading vessel is an icebreaker, which navigates in an unbroken icefield or in a previously broken ice channel (Rosenblad, 2007). This allows merchant vessels, which typically have less ice-going capability both in terms of propulsion power and hull strength than the icebreaker, to navigate through icefields to engage in trade. In practice, this process involves communication between the icebreaker and the following vessels, which attempt to keep up with possible speed changes of the icebreaker as it makes its way through the ice field, in order to stay close enough to the icebreaker to benefit from the cleared channel.

The speed of the icebreaker and following ships, and the distances between these, are subject to a number of restrictions. First, the assisted vessels must maintain a reasonable distance from the icebreaker and from one another. On the one hand, the distances between the ships should be large enough to prevent a collision if the preceding ship suddenly stops. On the other hand, the distances should be small enough to prevent following vessels from being blocked by ice during the voyage. This is especially important in dynamic ice conditions, particularly when compressive ice fields close the ship channels, leading to large added resistance forces causing following vessels to become beset in ice (Suominen and Kujala, 2012). Second, due to economic reasons, the speed should be as high as possible, but lower than the safe speed to prevent ice damage (Rosenblad, 2007).

## 3.2 Review of modeling approaches for coupled kinematics of vehicles

Considering the qualitative mechanism as described in Section 3.1, a coupled-kinematics following model is considered as a basis for the modeling of multiple ships under ice conditions, similarly as in Zhang et al. (2018) for escort operations. In order to justify the model selection and the main elements of its formulation, a brief overview of car following models is given in this section.

In road traffic, a driver is always inclined to follow the leading vehicle in a safe distance to avoid possible collisions in case of braking suddenly, which is similar to the icebreaker convoy problem. As collision avoidance is a major issue, it is desirable to maintain a constant convoy speed and constant distances between the vessels. However, unlike the car-following problem, the icebreaker needs to take into account the ice capability of the assisted vessels, as different ship designs have a different transit performance and safety level in different ice conditions (Trafi, 2017).

Traffic flow theory has been developed specifically for road traffic. This theory is used to describe car behavior on single-lane roads, i.e. roads without overtaking, and especially focus on how speed changes the first car will influence the speed of the car behind it. When ships are assisted by an icebreaker in a convoy operation, these cannot overtake the icebreaker, so the traffic conditions are similar to single-lane car following. Several car following models have been proposed, and although ships and cars are different e.g. in terms of kinematic behavior, the traffic conditions are qualitatively similar. Hence, modeling approaches developed for road traffic may be useful. In this context, it is interesting to note that apart from the icebreaker escort operations model presented in Zhang et al. (2018), other authors have also used modeling approaches from road traffic in maritime contexts. In particular where navigation conditions lead to similar following behavior as in road traffic, for instance in inland waterways and narrow straits, the analogies have been used as inspiration for model developments. Jun and Wei (2009) combined the car-following model into inland rivers study to calculate transportation capacity. Wu and Cheng (2012) considered the effects of following behavior when studying the U-turn behavior in a river. This illustrates the utility of proving some insights in the relevant road traffic literature for justifying the selection of the modeling approach.

Car following has been studied for many years, and a large number of papers have been published on the topic. The theory of vehicle following simply sees vehicular traffic as interacting particles (in cases where overtaking is not allowed), by studying the kinetic process of retreating from the front. It assumes that each vehicle must maintain a certain distance from other vehicles in order to avoid collisions, by considering the response of the driver to the vehicle's distance, speed, and other factors. The first such model was made by Pipes (1953), whereas Chandler et al., (1958) made an in-depth analysis of the California model (1958). In the 1950s, the General Motors (GM) laboratory conducted a large amount of theoretical research on vehicle tracking problems, which continues to the present day (Newell, 2002; Gazis, 2002).

In recent years, more advanced methods for describing the car following behavior have been proposed. In particular, the combination of data-driven solutions with model-driven approaches has become a new research direction in recent years (Papathanasopoulou and Antoniou, 2015). With larger amounts of data now available, more precise algorithms have been developed to estimate model parameters (Li et al., 2016). Meanwhile, the formulation of safety factors in the following behavior modelling still is a main concern in this research field (Przybyla et al., 2015). Considering these trends, taking a data-driven modeling approach, with particular attention to relevant and safety factors, is considered appropriate for the objectives of the icebreaker convoy modeling as described in Section 2.2.

The most important aspects of vehicle-following models are the vehicle's movement characteristics and the driver' behavior under following conditions (Jia et al., 2005). Generally, the response of a car driver can be summed up in the following three stages.

(1) Perception stage: The driver sees traffic and vehicle status information, including forward spacing, speed, and acceleration (Shao. 2008).

(2) Decision stage: The driver analyzes the information obtained and decides on a driving strategy (Liu et al., 2006).

(3) Control stage: The drivers control the vehicle according to their decisions (Chang, A. T. S.1994).

The vehicle-following model is a simplified abstraction of the stress-response class model (Wei et al., 2010) in the context of an in-depth analysis of driver response characteristics.

$$r_e = s_e s_t \tag{1}$$

here, $S_e$ represents the driver's ability to respond as a sensitivity factor to the stimulus, $s_t$ represents the role of the vehicle in front of the driver as a stimulus, and $r_e$ represents the driver's reactions to the behavior of the car in front.

The initial car-following model was proposed by Pipes (1953). Its equation of motion is as follows:

$$a_n = \frac{1}{T}\left(v_{n+1} - v_n\right) \tag{2}$$

where $v_n$ and $v_{n+1}$ are respectively expressed as the speeds of the leading vehicle and following vehicle, T is the vehicle response time, and $a_n$ is the acceleration of the nth vehicle. The left side of Equation (2) shows the difference in speeds. The basic idea of this model is that when the speed of leading vehicle is faster than the following vehicle, the following vehicle's driver accelerates to catch up and maintain a close car-following relationship. Likewise, the following driver decelerates to avoid a collision with the leading car when it slows down. Based on this model, Chandler et al., (1958) believe that the delayed effect of vehicle acceleration cannot be ignored; that is, acceleration is obtained by adjusting the following vehicle's speed difference between the front and following vehicles in the delay time, and the model is improved. The equation of motion is as follows:

$$a_n(t+\tau) = \lambda\left(v_{n+1} - v_n\right) \tag{3}$$

here, $\lambda$ is the driver's sensitivity coefficient, $\lambda = \frac{1}{T}$. $a_n(t+\tau)$ represents the acceleration of the *n*th vehicle at time $t+\tau$, here $\tau$ is the delay time, and $t$ is the current time. The other parameters are the same as for Equation (2).

In Section 4, a modeling approach for convoy operations is presented, based on the above described modeling formulations, combined with data-driven techniques for determining the model parameters, and with formulations for performance (icebreaker speed) safety considerations (safe distance and speed). The data used as a basis for deriving formulations for the latter aspects is introduced in the following sections.

## 3.3 Maritime traffic data and sea ice data

The SOLAS Convention (International Convention for the Safety of Life At Sea) requires that most ships above 300 GT (Gross ton) doing international voyages carry an Automatic Identification System (AIS) transceiver, whereby vessel movement information is transmitted between vessels and shore facilities (SOLAS, 1974). This system transmits data commonly referred to as AIS data. The proper use of AIS can help to improve the safety and efficiency of navigation (Zhang et al., 2016), which in turn strengthen the safety of lives at sea, while improving the protection of the marine environment. The primary role of AIS is to identify ships, assist in tracking targets, facilitate information exchange, and provide other ancillary information to avoid collisions. While originally developed for operational navigational support, AIS data has proven to be a very useful source of information for various other research purposes, as described in Lensu and Goerlandt (2018).

The ice data was obtained from the hindcasts performed with the HELMI multicategory sea-ice model, which can be found in Haapala et al., (2005) and Mårtensson et al., (2012). AIS Data from 1 to 12 March 2011 are used, the format of which is shown in Table 1. This period is in late winter in the Baltic Sea, during severe ice conditions. Since the main interested is in the convoy operations, which are carried out by icebreakers, the traffic data is organized per assisting icebreaker (SMHI, 2015). Table 2 lists some of the icebreaker's particulars and dimensions.

To analyze some of the characteristics of the escorted ship and the icebreaker, the parameters of the ice are needed, such as the bending strength of various types of ice, the ice compression rate, and ice concentration and thickness.

**Table 1. AIS data available for the proposed model**

| Data field | Unit | Explanation |
|---|---|---|
| MMSI number | - | A 9-digit code uniquely identifying a vessel |
| Timestamp | s | Time at which the message is recorded |
| Position | - | Longitude and latitude of transmitted message |
| Ship type | - | A 2-digit code identifying the type of vessel USCG (2012) |
| Ship length and width | m | Dimensions from bow to stern and side to side USCG (2012) |
| Ship speed | kn | Speed over ground |
| Ship course | ° | Course over ground, relative to north |
| Ship heading | ° | Direction of the ship, relative to north |

**Table 2. Main characteristics of the considered icebreakers**

| Name | MMSI | Ice class | Length (m) | Width (m) | Draft (m) |
|---|---|---|---|---|---|
| Fennica | 230245000 | POLAR-10 | 116 | 26.0 | 8.4 |
| Urho | 230290000 | IA Super | 104.6 | 23.8 | 8.3 |
| Zeus | 212409000 | IA Super | 45 | 14 | 8.2 |

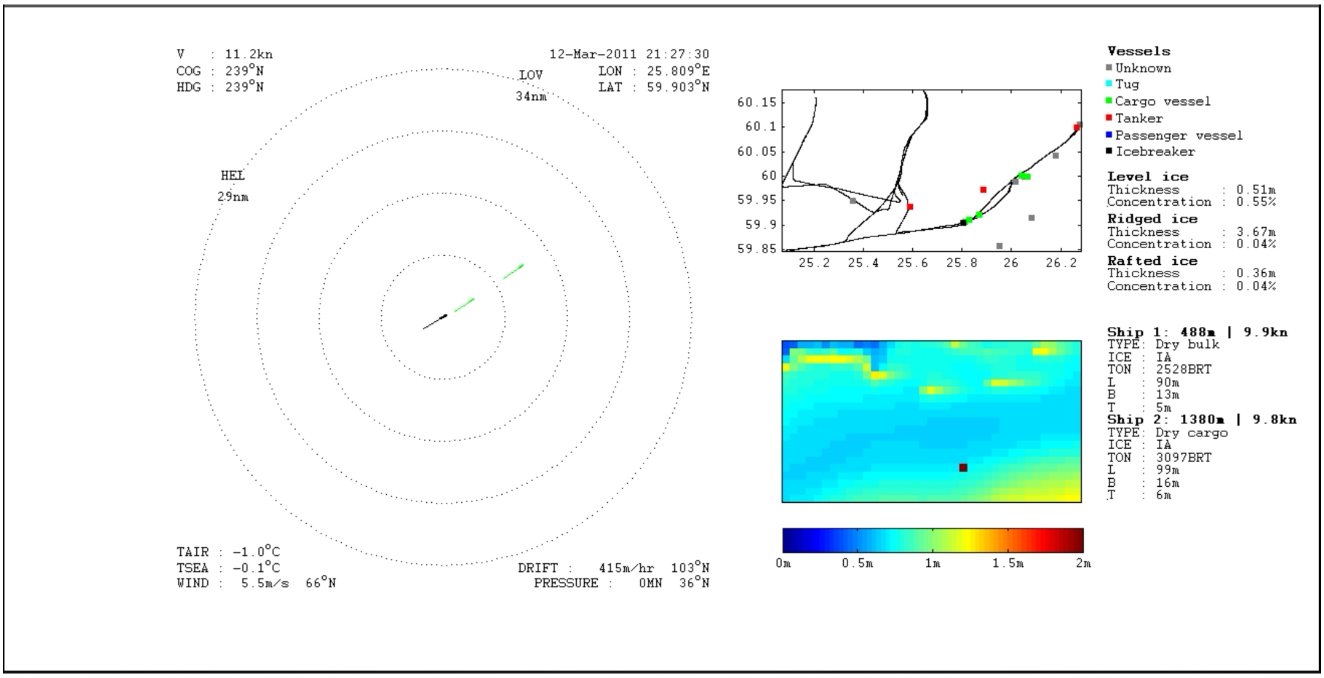



**Figure 1. Snapshot of video of icebreaker Fennica convoy operation**

For investigating the icebreaker operations, Goerlandt et al. (2016) developed a procedure to visualize AIS data along with spatio-temporal information about sea ice and atmospheric conditions, and to identify and classify icebreaker operations based on this integrated dataset. To facilitate this classification, videos of icebreaker operations were created, from which the different operation types introduced in Section 2.1, could be identified and further analysed. Such videos have also supported the development of models for ship performance in ice (Montewka et al., 2015), analysis of icebreaker escort and convoy operations (Goerlandt et al., 2016), analyses of winter navigation accidents (Goerlandt et al., 2017), and the development of a coupled kinematics model for icebreaker escort operations (Zhang et al., 2018).

Figure 1 shows a screenshot of such a video, depicting an icebreaker convoy operation with two assisted vessels. The following is an introduction to the data shown in this figure.

**A. Icebreaker.** Central in the 2 NM range of the display is icebreaker, shown by a black line at its current position.

**B. Assisted/nearby vessels.** Within the 2 NM range of the display, there are two vessels being assisted (green lines).

**C. Dynamic icebreaker data.** Current icebreaker data is displayed above the outer inspection domain circle, to the right and left sides. The icebreaker's speed, heading, course over ground, and geographic coordinates are provided. The date and time are shown as well.

**D. Data of assisted/nearby vessels.** The type, ice class, tonnage and main dimensions (length, width and draft) of the vessels in close vicinity to the icebreaker are shown, as well as their distance to the icebreaker and current speed.

**E. Distance to closest harbors.** Because no background map or sea chart is included in Figure 1, an indication of the position of the icebreaker relative to the two nearest harbors is given. Three-letter abbreviations of the port names are displayed at the edge of the inspection domain, in the direction of their location. The distance between the icebreaker and port is shown below the name.

**F. Environmental data.** Below the inspection domain circle, the air temperature, sea temperature, and wind speed and direction are provided for the time corresponding to the AIS timestamps.

**G. Sea ice data.** Below the inspection domain circle, the ice drift speed and direction and internal ice friction magnitude and direction are shown, for the time corresponding to the AIS timestamps.

**H. Sea ice data.** The level ice, ridged ice and rafted ice thicknesses and concentrations at the position of the icebreaker are shown at the right-hand side of the figure, for the time corresponding to the AIS timestamps.

**I. Ice chart.** An ice chart of the average weighted ice thickness (incorporating level, ridge and rafted ice) shows the instantaneous position of the icebreaker with a dark red square. It provides insight in the average ice conditions over the trajectory of the icebreaker.

**J. Dynamic traffic image.** The icebreaker's trajectory is recorded over a 24-h time period, and presented as a black line on the plot. The traffic image also shows the instantaneous positions of all other vessels within the icebreaker's range of travel. Each vessel is represented by a colored marker, indicating its vessel type according to the legend on the right.

In the course of a voyage, a ship needs to take full account of the impact of ice. Through the detailed analysis and treatment of ice conditions in the Baltic Sea area, the ice conditions can be better understood. One way to classify the safety level of vessels in ice environments is the Arctic Ice Regime Shipping System (AIRSS), on which the Polar Operational Limits Assessment Risk Indexing System (POLARIS) is based (Stoddard et al., 2016). AIRSS originates from Canada, and is intended to minimize the risk of pollution in Arctic waters due to damage of vessels by ice, to emphasize the responsibility of the shipowner and master for safety, and to provide a flexible framework for decision-making. AIRSS includes a four-step process, which consists of the following steps: i) characterizing the ice regime, ii) obtaining the vessel Class dependent Ice Multipliers, iii) determining the Ice Numeral, and iv) making route choice decision. The principles of the AIRSS process are generic, so it can also be applied to the analysis of the Baltic Sea. AIRSS can be used to determine the type of ice. The concentration of sea ice is directly considered as an ice type, and the ice thickness is classified using an ice multiplier (IM). Table 3 shows how ice multipliers are determined in AIRSS. Ice Multipliers are defined as weighting factors to represent the relative risk damage to a vessel by different ice types. The higher the value is, the lower is the risk of vessel damage due to ice conditions.

**Table 3. Ice multipliers used in the Arctic Ice Regime Shipping System (AIRSS)**

| Ship category | Ice type | | | | | | |
|---|---|---|---|---|---|---|---|
| | Open water | Gray ice | Gray white ice | Thin first-year ice 1st stage | Thin first-year ice 2nd stage | Medium first-year ice | Thick first-year ice |
| CAC3 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| CAC4 | 2 | 2 | 2 | 2 | 2 | 2 | 1 |
| Type A | 2 | 2 | 2 | 2 | 2 | 2 | -1 |
| Type B | 2 | 2 | 1 | 1 | 1 | -1 | -2 |
| Type C | 2 | 2 | 1 | 1 | -1 | -2 | -3 |
| Type D | 2 | 2 | 1 | -1 | -1 | -2 | -3 |
| Type E | 2 | 1 | -1 | -1 | -1 | -2 | -3 |

From Table 3, it can be seen that different ice types and ships with a different ice class correspond to specific ice multipliers (IM). These can be derived from Table 3 as follows:

$$IM = \begin{cases} 2, & 0 \le C<70 \\ 1, & 70 \le C<120 \\ -1, & 120 \le C<151 \\ -3, & 151 \le C<189 \end{cases} \tag{4}$$

here, C is the thickness of the ice, and the unit of C is centimeters(cm). In the AIRSS, six ice types with associated thickness ranges are identified as follows: Gray (10–15 cm), Gray white (15–30 cm), Thin first-year first stage (30–50 cm), Thin first-year second stage (50–70 cm), Medium first year (70 – 120 cm), and Thick first year ice (first-year ice over 120 cm).

## 3.4 Speed data analysis and processing

The safe speed is the maximum speed in an ice regime which is considered safe for a vessel of a given ice class (Zhang et al., 2017). The procedure is to first define the safety level, for which the AIRSS is used in the proposed model. Thus, the AIRSS is applied, using information of the sea ice conditions and the ship class. Based on this information, the maximum speed in the ice regime (i.e. the safe speed) is subsequently determined. In practical navigational contexts, there may be different judgments concerning what constitutes a safe speed, as this is not explicitly defined in the collision regulations (Hilgert and Baldauf, 1997). In sea ice conditions, it is commonly considered (e.g. in regulations) that this is affected by the ice conditions and the vessel's ice class (Stoddard et al., 2016; FTA, 2010; FTSA, 2014). The safety status can be modeled according to the thickness of the ice and vessel ice class. According to AIRSS, the safety of ship navigation in an ice field is given by the Ice Numeral (IN), which considers concentration and thickness of different types of ice, as follows:

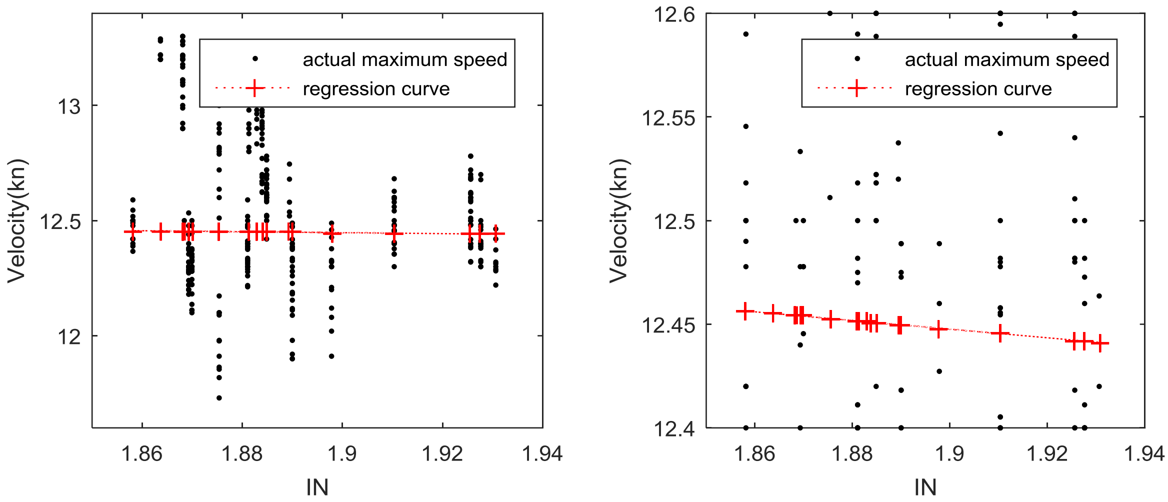
$$IN = C_a IM_a + C_b IM_b + ... + C_n IM_n \tag{5}$$

where $C_a$ represents the concentration of a type of ice, and $IM_a$ represents the ice multiplier of a given ice type. The empirical formula (6) is determined based on the dataset of ice conditions and speeds of convoy operations. In particular, the speed data is determined from the relevant AIS data fields, for three icebreakers and their following vessels, as described in Section 3.3. In total, 1032 data points are sampled to derive a regression formula linking the ice numerals with safe speeds.

Equation (6) is used to calculate the maximum safe speed of each ship including the icebreaker and the vessels following its lead in the convoy. This data mainly includes conditions with relatively high speeds, all of which are assumed to constitute safe speeds, as they are operational speeds selected by experienced icebreaker crew. The range of validity of this formula in terms of ice numerals is from -3 to 2. A polynomial regression model is used to obtain the parameters of Equation (6), and the relationship of the safe speed given by IN is as follows:

$$V_{max} = 0.1291 IN^3 - 0.6946 IN^2 + 1.0329 IN + 12.1071 \tag{6}$$

This formula is used to calculate the maximum allowable speed in the model proposed in Section 4, that is, the maximum speed at which a ship can safely proceed in the given ice conditions. Exceeding this speed increases the risk of hull damage. Figure 2 shows a graph of the polynomial regression according to Equation (6).




(a) Original figure (b) Partial enlarged figure

**Figure 2. Polynomial regression analysis of safe speed in relation to the Ice Numeral**

From Figure 2, it can be seen that while the historical ship speed data shows fluctuations for given ice numerals, the regression curve is very close to the mean value of the operational convoy speeds. The normr (Residual norm) value of fitting is 25.9133, which indicates high accuracy, as smaller values of normr indicate a more accurate fit. The mean IN is 1.8697 and the variance is 0.0283. Because only first year ice is present in the Baltic sea area, the ice thicknesses are relatively small, which leads to a rather small range of the Ice Numeral. Because the resulting regression line is very close to a straight horizontal line, it is possible to use a constant safe speed with a value of 12.5 knots in the model to represent the safe speed.

## 3.5 Safe distance data analysis

The safe distance between two ships is a factor that must be considered. Safety of navigation can only be guaranteed if the ships follow one another at a distance which, if the leading vessel stops, gives the following vessel sufficient time to avoid collision. At the same time, this distance should not become too large, because dynamic ice can close the broken channel, and impede the transit of the following vessel, which may become beset in ice. This can lead to issues with the smoothness of the entire convoy's transit through the ice field, while also presenting a hazard to vessels beset in ice.

The model for the safe distances requires data on ice conditions, ship ice class, and movement characteristics. Vessels also have technological equipment such as radars to support observation and decision making by navigators, and communication between vessels and the icebreaker can also be used to maintain a safe distance. However, studies on e.g. communication onboard icebreakers do not shed light how, quantitatively, this relates to a safe distance. Zhang et al., (2018) proposes a model for determining a safe distance between two ships, but this did not account for the coupled kinematics of both vessels.

In this paper, the following model is proposed to determine the safe distance. The thicker the ice, the smaller the safe distance is and the lower the speed is, which corresponds to empirical findings of actual vessel behavior in convoy operations (Goerlandt et al., 2016).

$$D_{safe-ice} = x_p(t_n+\tau) - x_f(t_n+\tau) - l_p - \Delta D \quad (7)$$

here, $x_p$ represents the position of the leading ship, $x_f$ represents the position of the following ship, $t_n$ represents the time when the leading ship begins to slow down, $\tau$ represents the deceleration time interval, $l_p$ represents the hull length of the leading ship, and $\Delta D$ represents the additional distance which vessels aim to maintain between their positions in the convoy, due to the ice conditions (Zhang et al., 2018).

$$x_p(t_n+\tau) = x_p(t_n) + \frac{(v_p(t_n))^2}{2a_p} \quad (8)$$

$$x_f(t_n+\tau) = x_f(t_n) + \frac{(v_f(t_n))^2}{2a_f} + \tau\frac{v_f(t_n)+v_f(t_n+\tau)}{2} + v_f(t_n+\tau)\theta \quad (9)$$

here, $v_p$ and $a_p$ represent the speed and acceleration of the leading ship, $v_f$ and $a_f$ represent the speed and acceleration of the following ship, and $\theta$ represents reaction time that means the reaction time of the following vessel to react to changes of the leading vessel. This reaction time both includes the technical constraints of the vessels (inertia effects), and the delays due to operator response times. The former is affected by the vessel design, whereas the latter has a relation to observation and decision making of the navigating officer on the following vessel, which in turn is affected by the communication between the following vessel and the icebreaker (Boström, 2018). Previous work by Zhang et al., (2018) has determined the mean value of the reaction time as $\theta$ = 84.45 seconds, which is used in this study.

# 4. SHIP FOLLOWING MODELING IN ICE CONDITIONS

## 4.1 Ship following model: process and icebreaking resistance

The most important aspects of establishing a multi-ship following model is the safe distance between the icebreaker and following ships, and their safe speeds. In the practice of convoy navigation, vessel speed and distance between vessels affect one another. When the distance between the two ships shortens, the following ships need to slow down. Conversely, they accelerate when the inter-vessel distance increases. There will be an optimal speed for traffic stability based on the distance between the ships. Based on the above, it is clear that the distance between two ships should be large enough to prevent collisions, but the distance could not so large that the following ship becomes strongly affected by the sea ice.

Compared with the original car-following model proposed by Pipes (1953), ship navigation in ice is more complex, due to the influence of the ice conditions on the vessel dynamics: the presence of ice affects the ship's speed and acceleration, where thicker ice reduces the ships' speed more than thinner (Li et al., 2018). Ice also poses a hazard to the ship hull, where higher speeds correspond to higher ice loads (Kotilainen et al., 2018). In order to reduce damage to the hull, the speed cannot be too high, and the following vessels need to adjust their speed and distance. The acquisition and processing of information for gaining appropriate situational awareness is also highly demanding for the officers on watch, so icebreaker convoy operations are relatively difficult to perform. These processes are illustrated in Figure 3, where the start point of the algorithm is printed in bold. No end point is shown in the algorithm, which is due to the modeling scope, which is limited to ongoing convoy operations. In practice, if in simulator environments the algorithm needs to be terminated, stopping the icebreaker vessel will also lead to the following vessels to decelerate to a halt.

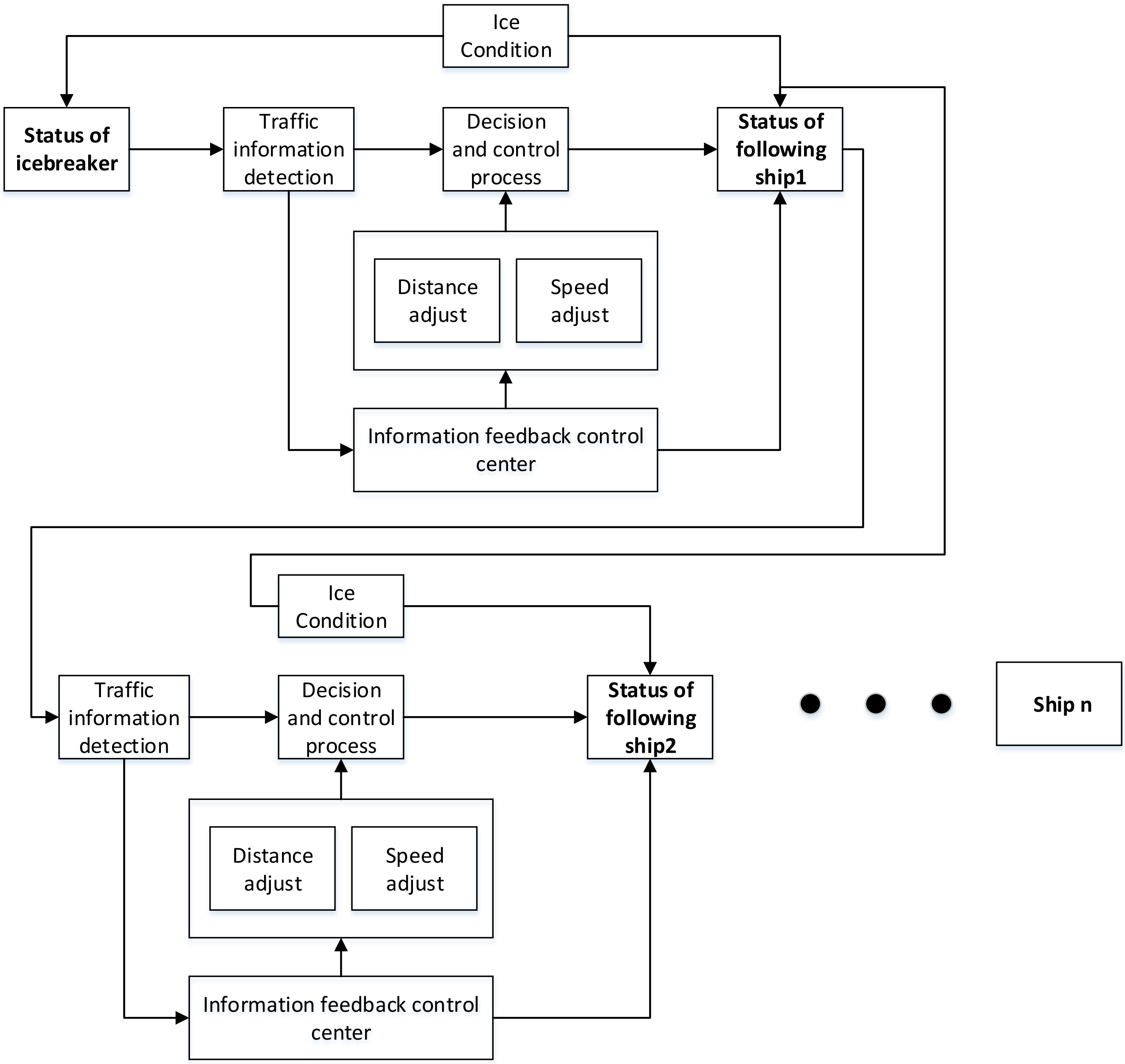



**Figure 3. Schematic model of the multi-ship following behavior in ice conditions**

In the proposed model, it is assumed that the operating speed of the icebreaker is affected the following ships by considering the fleet sailing in safe speed, which is based on operational practices where icebreakers accounts for the ice-going capabilities of the weakest vessel in the convoy. The following ship's behavior is determined by the leading ship in the sense that its speed is taken as a reference value which the following vessels aim to match, while also keeping a safe inter-vessel distance.

As the leading vessel, the icebreaker must create a channel to facilitate the transit of the vessels in convoy. Breaking this ice is associated with a high icebreaking resistance. There are many studies of ice resistance of ships (Li et al., 2018). In this study, a relatively simple formula is applied for calculating the ice resistance (Cho et al., 2015):

$$R_{BR} = 1.8\sigma_f^{0.853} h^{1.656} v^{0.259} M^{0.147} B^{-0.099} \tag{10}$$

where $\sigma_f$ represents the bending strength of ice, $h$ represents the thickness of the ice, $v$ represents the speed of the ship, $M$ represents the mass of the ship, and $B$ represents the ship's beam (width). Some of the required values are described in detail in Table 2, whereas the bending strength of ice is taken as 500 kPa in the Baltic Sea (Wang, 2010). The ship's length $L$, Width $W$ and draft $D$ are used to determine the ship's mass, where $\rho_w$ represents the density of water, $V_d$ represents the volume of the ship. $C_B$ represents block coefficient (Bertram and Schneekluth, 1998; Watson, 1998). $V_d$ is calculated as the Lightweight Displacement, which is the difference between gross registered tonnage and the gross deadweight tonnage. The following formula is used to solve the ship's mass (Scharrer et al., 2002):

$$M * g = \rho_w * g * V_d = 1\times 10^3 g \times L \times W \times D \times C_B \tag{11}$$

It may be noted here that the selected ice resistance model of Equation (10) is rather simplistic. Other regression type models exist, see e.g. Lindqvist (1989) and Riska et al. (1997), but these have no significant advantages over the selected model as these types of models are predicated on similar assumptions. Detailed numerical models exist for ship performance in ice, see Li et al. (2018), but these are computationally too slow to implement in real-time simulators. The only publicly described model for real-time icebreaking simulation, by Lubbad and Løset (2011) was not available for this study.

## 4.2 Ship following model: mathematical formulation

In this paper, the development of the ship following model of convoy operations in ice conditions consists of three steps. First, the model structure is established, which is described in detail in Section 4.2.1 to 4.2.3. The model introduction begins with a two-ship formulation which does not consider ice conditions in Section 4.2.1, and culminates in a multi-ship formulation where sea ice conditions are includes, in Section 4.2.3. Then, the parameters in the model are calibrated in Section 4.3. Finally, the model's performance is evaluated through a set of case studies, shown in Section 5.

Based on the original formula of Pipes (1953) and the CM (coupled map) following model, a mathematical multi-ship model is established, also considering the ice conditions in the Baltic Sea area. In the following, the model is introduced in a step-by-step process, from two to multiple ships and from ice-free conditions to operations in ice-covered waters. In Section 4.2.1, the initial two-ship model is introduced, which describes the kinematic aspects of the ship and is very similar to the original model of Pipes and the CM following model, which is also based on traffic flow theory. In Section 4.2.2, the model is extended to multiple ships and their kinematic interactions. Finally, in Section 4.2.3, the model is extended further to account for the impact of ice on the vessel behaviors.

### 4.2.1 Initial two-ship model formulation

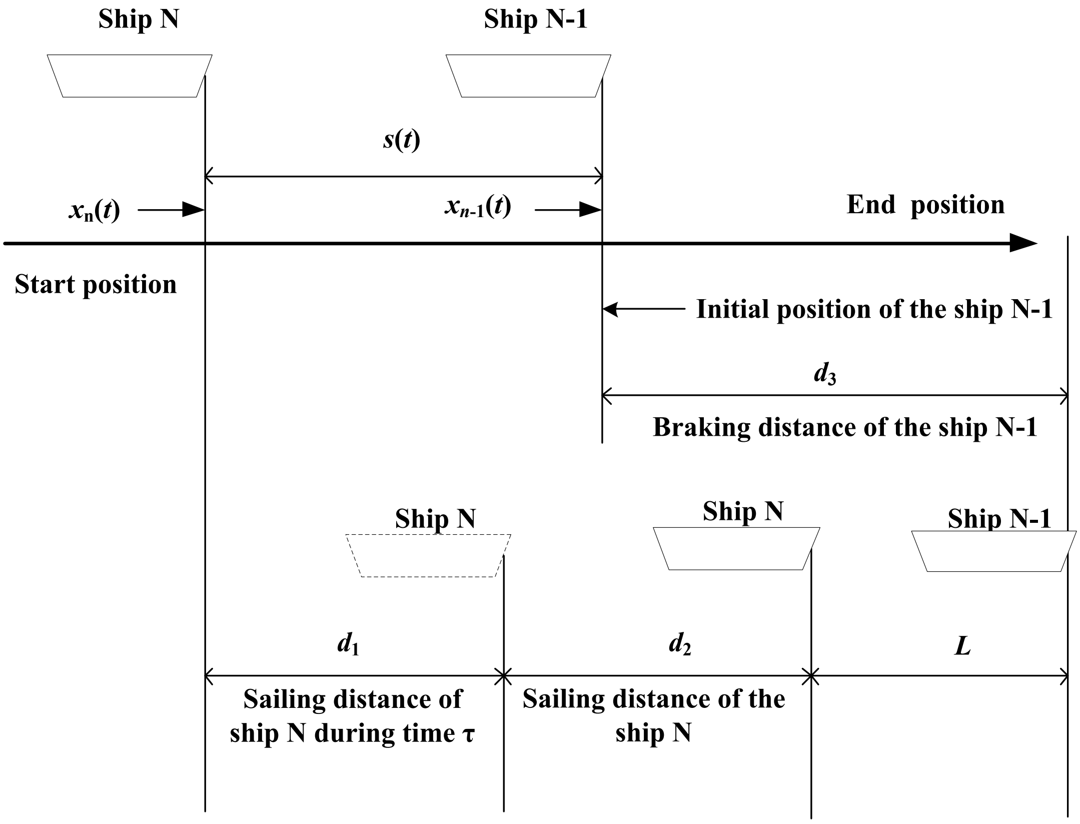



**Figure 4. Schematic diagram of the linear ship-following model**

Figure 4 illustrates the kinematics of the two vessels. The leading ship N-1 begins to slow down at the time instance *t*, after which the following ship N begins to slow down after a certain delay period. Ship N-1 sails and slows down a distance $d_3$, then finally stops. The reaction of ship N is delayed a time period $\tau$, and in this time interval proceeds a distance $d_1$. From the diagram, the equation (12) and (13) can be deduced, where $x_{N-1}(t)$ represents initial position of ship N-1, $x_N(t)$ represents the initial position of the ship *N*, $d_1$ represents the sailing distance of ship *N* during time $\tau$, $d_2$ represents the sailing distance of ship *N*, and L represents the distance from bow to bow between the leading ship and the following ship.

$$s(t) = x_{N-1}(t) - x_N(t) = d_1 + d_2 + L - d_3 \tag{12}$$

$$d_1 = \tau \cdot v_N(t) = \tau \cdot v_N(t+\tau) = \tau \cdot v_N(t+\tau) \tag{13}$$

If the leading ship is not affected by ice, the stopping distance of the ship *N* and ship *N*-1 is same, that is, $d_2 = d_3$.

$$s(t) = x_{N-1}(t) - x_N(t) = d_1 + L \tag{14}$$

Combining Equation (13) and (14), the following is obtained:

$$x_{N-1}(t) - x_N(t) = \tau \cdot v_N(t+\tau) + L \tag{15}$$

Taking the derivative with respect to time $t$, the following is obtained:

$$v_{N-1}(t) - v_N(t) = \tau \cdot a_N(t+\tau) \tag{16}$$

Equation (16) can also be written as follows:

$$a_N(t+\tau) = \lambda\left(v_{N+1} - v_N\right) \tag{17}$$

where $\lambda = 1/\tau$ denotes the sensitivity coefficient of the following ship, which indicates the response coefficient of the following ship depending on following situations.

### 4.2.2 Initial multi-ship model formulation

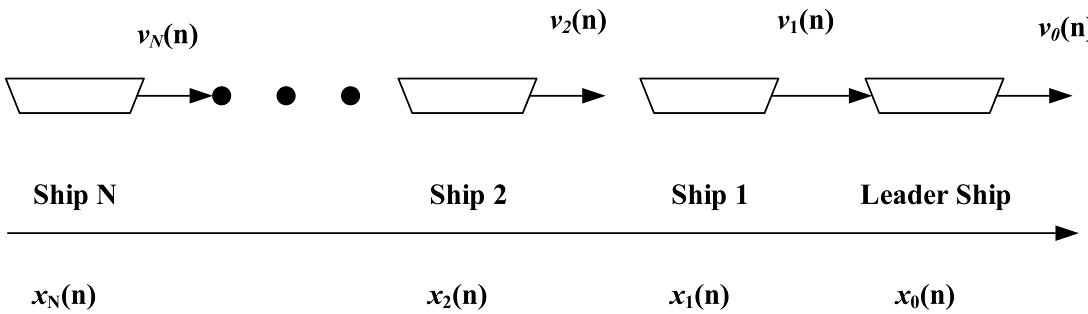



**Figure 5. Schematic diagram of multi-ship following model**

Fig. 5 demonstrates the location $x$ and speed $v$ of all ships in the fleet at time $n$. The kinematic equation of the leading ship (the icebreaker) can be written as:

$$x_0(n+1) = v_0 T + x_0(n) \tag{18}$$

where $x_0(n) > 0$ represents the location of the leading ship at moment $t = nT$, which serves as reference point in the model description, $v_0$ represents the speed of the leading ship, and $T > 0$ represents the sampling period.

Assuming that the motion of the leading ship is not affected by other ships, the kinematic equation of the following ships can be expressed as follows:

$$\mathrm{x}_i(n+1) = v_i(n) \cdot T + x_i(n) \tag{19}$$

where $x_i(n) > 0$ represents the location of ship $i$, and $v_i(n) > 0$ represents its speed. $N$ represents the total number of ships in the fleet, including the leading ship and its following ships. The formula for the speed of the following ships can be expressed as follows:

$$v_i(n+1)=\left[\lambda_i\left(v(y_i(n))-v_i(n)\right)\right]T+v_i(n) \tag{20}$$

where $\lambda_i=1/\tau$ indicates the sensitivity coefficient of the following ship, and $v(y_i(n))$ is the optimal velocity function that the assisted ships try to keep stable during navigation. The change of optimal velocity is only related to the distance between ships.

$$y_i(n)=x_{i-1}(n)-x_i(n) \tag{21}$$

where $y_i(n)$ represents the distance between the two ships. The formula for the speed can be written as follows:

$$v(y_i(n))=\frac{v_{\max}}{2}\left[H_{sat}\left(2\frac{y_i(n)-h_i}{\xi_i}\right)+1\right] \tag{22}$$

where $v_{\max}$ represents the safe speed, $h_i$ indicates the safe distance, and $\xi_i$ represents the safe distance parameter of ship *i*. During operations, the ship will try to reduce the relative speed between the two ships. At the same time, it will follow the previous ship at a safe distance. The specific limitations are as follows:

$$\rho=2\frac{y_i(n)-h_i}{\xi_i} \tag{23}$$

$$H_{sat}(\rho)=\begin{cases}+1, & \rho>+1\\ \rho, & -1\le\rho\le+1\\ -1, & \rho<-1\end{cases} \tag{24}$$

Here, $y_i(n)$ can be longer or shorter than the safe distance $h_i$, which means that $\rho$ can be positive or negative accordingly, and that $H_{sat}(\rho)\in[-1,1]$.

While the ship is moving, the following ship tries to reduce its speed difference relative to the ship ahead. At the same time, it tends to follow that ship at a safe distance, with the crew adjusting the speed to keep the distance between the vessels at a safe reference level. If there is a significant difference between the speed of ships, it is assumed that the crews accelerate or decelerate. Therefore, a function $H_{sat}(\rho)$, as shown in Equation (24), is constructed. This is a kinematic function used to represents the crew navigation control mechanism which has the intention to keep the ship safe.

## 4.2.3 Updated model formulation including effect of ice conditions

The models in Sections 4.2.1 and 4.2.2 do not consider the effect of ice on the safe distance. However, an analysis by Goerlandt et al. (2017) clearly shows that ice conditions have a large influence: as the ice thickness increases, the distance between vessels in a convoy decreases. Next, the proposed following model is further extended by considering the effect of ice. Studies by Seong-Rak Cho et al. (2015) show that ice can have a hindering effect on a ship and, thus, affect its speed and acceleration. According to Newton's second law, the ratio of ice resistance to ship mass gives the acceleration value. So, it is reasonable to utilize this information in the ship's kinematic equation. The notation $\Delta D$ is used to indicate the difference in distance between a vessel pair, in situations without sea ice, and situations where sea ice is present.

Due to the influence of ice, the leading ship has a shorter stopping distance than the following ships. Hence: $d_2 > d_3$, and Equation (14) in Section 4.3.1 is further developed as follows:

$$d_2 = d_3 + \Delta D \tag{25}$$

$$s(t) = x_{n-1}(t) - x_n(t) = d_1 + L + \Delta D \tag{26}$$

Combining Equation (13) and Equation (26), the following is obtained:

$$x_{n-1}(t) - x_n(t) = \tau \cdot v_n(t+\tau) + L + \Delta D \tag{27}$$

Taking the derivative with respect to time *t*, it follows that:

$$a_n(t+\tau) = \lambda_i \left( v_{n-1}(t) - v_n(t) - \Delta D^{'} \right) \tag{28}$$

where $\lambda_i = 1/\tau$ indicates the sensitivity coefficient of the crew on the following ship, $a_i = \lambda_i \Delta D^{'}$. The other parameters are as per Section 4.2.2.

$$a_n(t+\tau) = \lambda_i \left( v_{n-1}(t) - v_n(t) \right) \text{-} a_i \tag{29}$$

So, when there are multiple ships, and considering the ice situation, Equation (20) is modified to:

$$v_i(n+1) = \left[ \lambda_i \left( v(y_i(n)) - v_i(n) \right) - a_i \right] T + v_i(n) \tag{30}$$

where $a_i$ represents the acceleration of the icebreaker when it breaks the ice. The expression for optimal velocity $v(y_i(n))$ in the formula is as shown in Equation (20).

According to Newton's second law, the acceleration of icebreakers is described as follows: $a_i = \dfrac{R_{BR}}{M}$. The method for determining the mass of the icebreaker and the resistance of the icebreaker is detailed in Section 4.1.

## 4.3 Quantification of ice condition

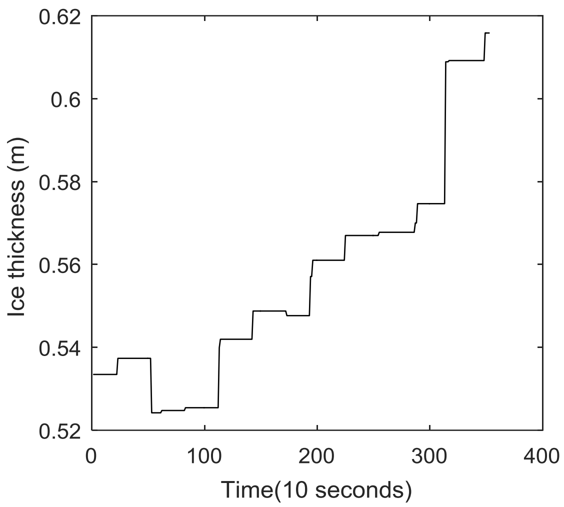
Navigation in sea ice is a highly hazardous operation, and strongly relates to environmental conditions such as ice type and thickness (Goerlandt et al., 2017). In addition, the vessel safety has a direct relation to the ship's hull structure, sailing speed, and engine performance (Trafi, 2017).

In this paper, the data concerning the ice conditions is obtained from hindcast runs using the HELMI sea ice model, as outlined in Section 3.3. The Finnish-Swedish ice class rules establish six ice classes assigned to operating in first-year ice in the Baltic Sea on the basis of requirements for hull structural design, engine output and performance in ice (FTSA 2010). The six ice classes are IA Super, IA, IB, IC, II and III. Currently, an analytic relationship between the effect of ice conditions on the coupled vessel dynamics in focus in the ship following problem has not yet been established. Therefore, in this study, the average ice thickness is used to represent ice condition, and assume the effect to the coupled vessel dynamics is linearly proportional to ice average thickness. This HELMI data is utilized in this paper to describe the ice conditions.

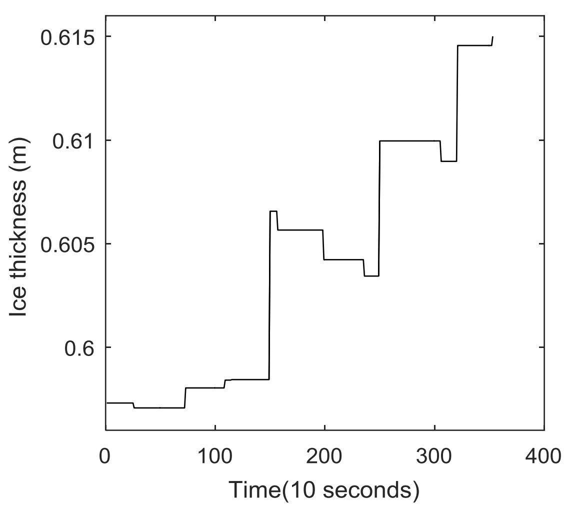
The ice thickness is calculated as follows:

$$h_{ave} = \frac{\sum_{i=1}^{n} h_i}{\sum_{i=1}^{n} A_i} \tag{31}$$

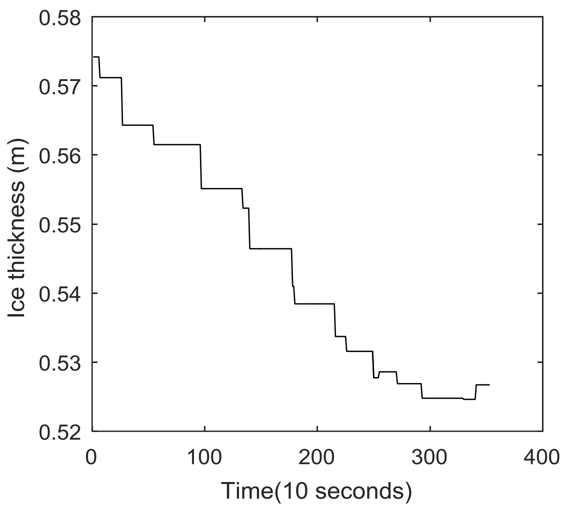
where $h_i (i = 1,2,3,4,5,6,7)$ represents the thickness of seven level ice categories (5 separated level ice, averaged ridged ice, and averaged rafted ice), and $A_i$ represents the concentration corresponding to each ice category. Figure 7 shows the average thickness of ice obtained at different escorting times and routes using Equation (31). Case 1 (icebreaker Fennica) lasts from 16:49:20 to 17:48:20 on March 1, 2011. Case 2 (icebreaker Urho) lasts from 21:40:50 to 22:39:50 on March 1, 2011. Case 3 (icebreaker Zeus) lasts from 14:03:50 to 15:02:50, on March 6, 2011.




**(a) Ice thickness of ship Fennica**




**(b) Ice thickness of ship Urho**




**(c) Ice thickness of ship Zeus**

**Figure 6. Average ice thickness change over time**

According to the ice resistance model introduced in Section 4.1, the relationship between ice thickness and ship resistance can be plotted using the calculations of Equation (10), using icebreaker parameters shown in Table 2. It is clear that when the ship speed is constant, the resistance of the ice will increase as its thickness increases. When the ice thickness is constant, the resistance will increase as the ship speed increases. There is also an association between ice thickness and ship speed whereby the speed of the ship is constrained by the ice thickness.

Figures 7 and 8 below show the relation between ice thickness and resistance, and ship speed and resistance, for three different ships. Figure 7 indicates that for increasing ice thickness, the icebreaking resistance increases as well. Figure 8 shows the relationship of ship speed to resistance, indicating that the faster the ship speed is, the greater the corresponding resistance is. The resistance values for the three icebreakers are different, but the overall trends are the same.

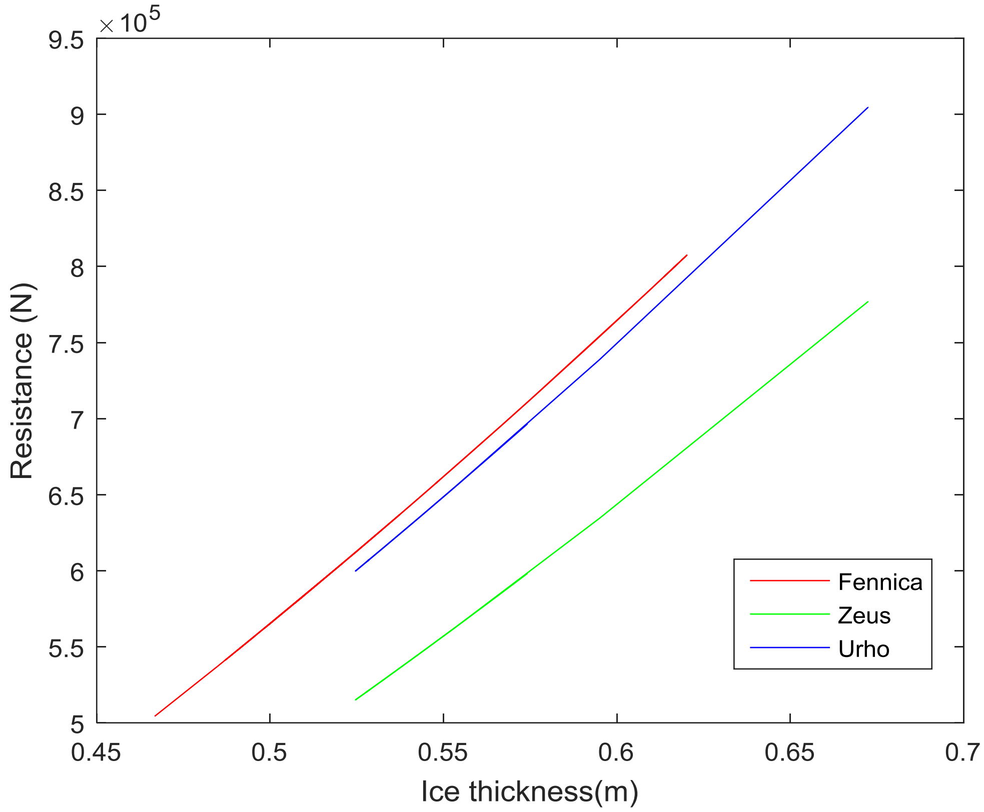



**Figure 7. Resistance vs ice thickness in icebreaking condition**

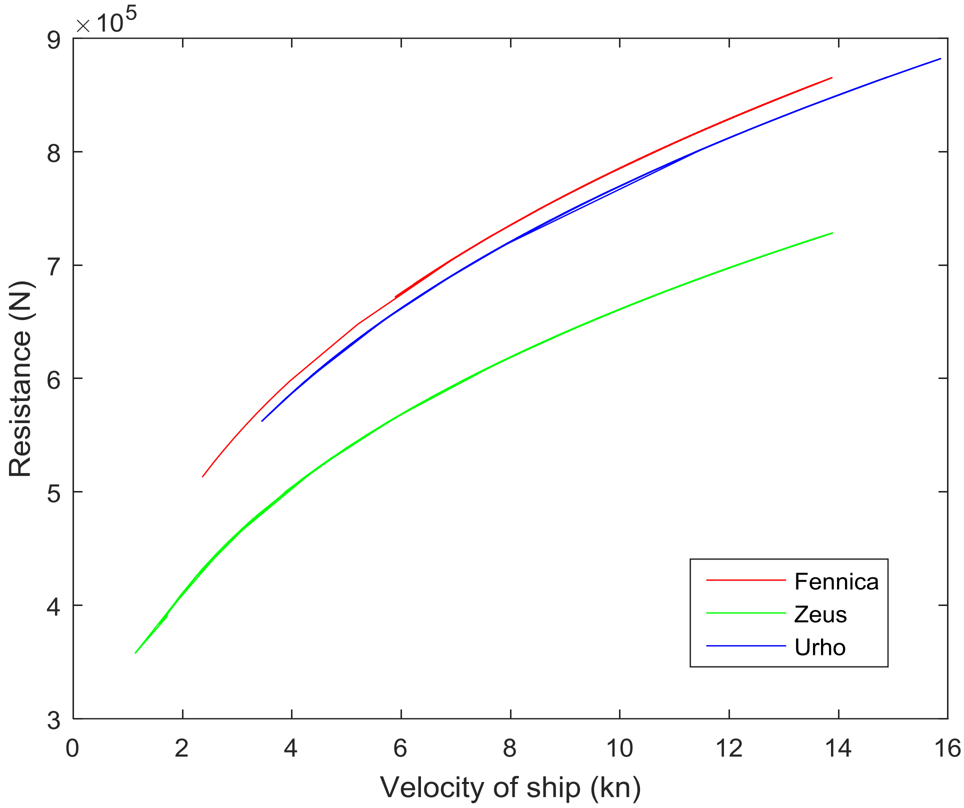



**Figure 8. Resistance vs ship speed in icebreaking condition**

## 4.4 Parameter selection and calibration

Parametric calibration refers to the process of applying a computational method to determine the value of a specific parameter based on the structure of the model in which the parameter is included, and the available relevant information, in particular known input and output variables. Inputs variables are speed and acceleration of icebreakers and following ships in different time instances, and corresponding ice thicknesses. The outputs are speeds of following ships in different time instances. In this summary, the maximum speed parameters $v_{\max}$, the sensitivity coefficient $\lambda_i$ and the distance parameters $\xi_i$ are calibrated. Finally, the sensitivity coefficient $\lambda_i$ of Equation (32) is solved. The sensitivity coefficient is dependent on the input and output variables, such as speed and acceleration. It is not directly reflected in the model structure, but has an impact on it.

### 4.4.1 Safe speed

The safe speed is the maximum speed allowed during convoy operations, and depends on the specific ice conditions and the vessel ice class. According to the description of the safe speed in Section 3.4, the maximum speed ranges for the three considered icebreakers and assisted vessels are shown in Figure 9, using box plots. The range of safe speeds is obtained from the empirical formula given in Equation (6), as introduced in Section 3.4.

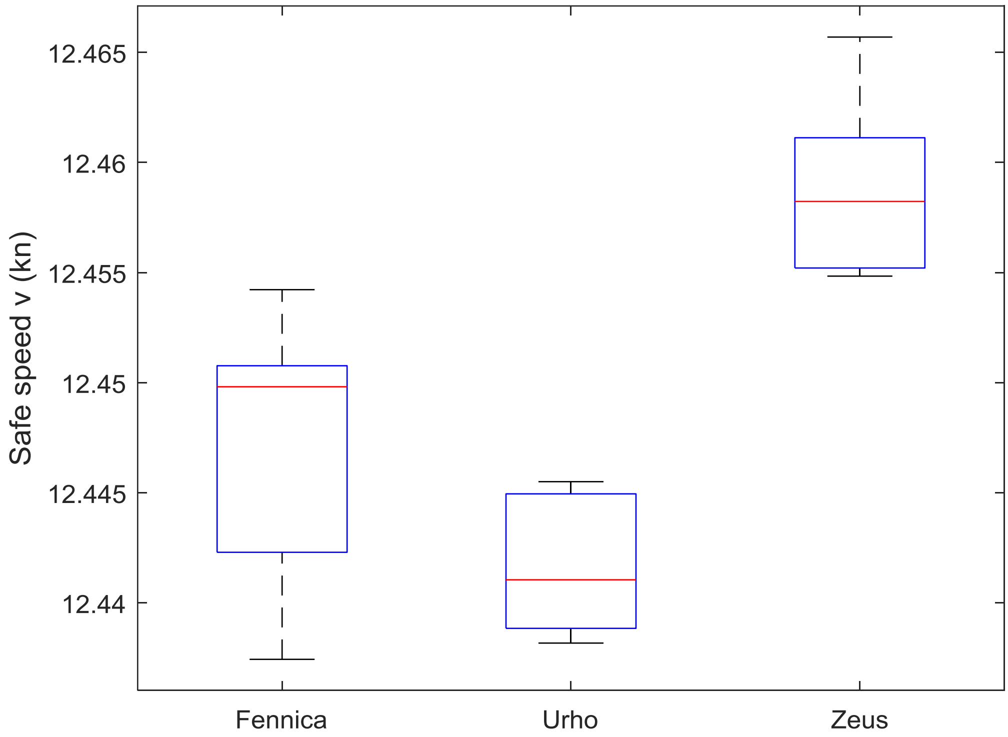



**Figure 9. Boxplots of Safe speed for the three icebreakers and their assisted vessels**

In this paper, the safe speed information is used to make a conservative judgment about the maximum convoy speeds. In the model formulation of Section 4.2.3, the maximum convoy speed is set at 12.5 knots.

## 4.4.2 Sensitivity coefficient and safe distance

Calibration of the sensitivity coefficient and safe distance can be explored from the sampled data and models. According to known input variables $a_1$, $v_i(n)$, $T$, $h_i$, $v_{max}$ and output variables $v_i(n+1)$, the undetermined parameter $\lambda_i$ and $\xi_i$ are considered to be the solution variable. Then, the model is validated by comparing the results obtained when utilizing the estimated parameter with empirical data. This parameter estimation is performed using a genetic algorithm for solving nonlinear equations proposed by Tadaki et al. (1997), to which is referred for the specific calculation process. In the calculation process, the crossover rate is set to 0.87 and the mutation rate is set to 0.03. Roulette selection method is applied together with a uniform crossover method, and an admissible convergence error of 1.0E-9. Table 4 shows the corresponding parameter calibration results. Equation (32) and (33) are the optimized formula.

$$v_i(n+1)=\left[\lambda_i\left(v(y_i(n))-v_i(n)\right)-a_1\right]T+v_i(n) \quad (32)$$

$$v(y_i(n))=\frac{v_{\max}}{2}\left[H_{sat}\left(2\frac{y_i(n)-h_i}{\xi_i}\right)+1\right] \quad (33)$$

**Table 4. Calibration of multi-ship following model**

| Name | Number of samples | Sensitivity coefficient | Distance parameter | Correlation coefficient |
|---|---|---|---|---|
| Case1(Fennica) | 487 | 36.42 | 190.90 | 0.87 |
| Case2(Urho) | 326 | 37.21 | 110.12 | 089 |
| Case 3(Zeus) | 436 | 35.78 | 114.23 | 0.91 |

From the values shown in Table 4, it is seen that a strong relationship exists between the distance coefficient and the safe distance. The table shows that there are most samples for Fennica (487) and Zeus (436), with least for Urho (326). The correlation coefficient measures the degree of correlation between the model output and the measured value, when the calibration parameters shown in Table 4 are used in the model as formulated in Section 4.2.3 and the outputs compared with the empirical data. The higher the value is, the stronger the correlation is. Correlation coefficients of 0.87 to 0.91 can be considered high.

# 5. MODEL VALIDATION

In this section, the proposed mathematical model of Equation (30) is validated by comparing historical data and simulated data. First, a case is considered with only one following ship, then a case with two following ships. The simulated speeds of the two following ships are compared with the actual sailing speeds. The empirical data and the ice data were described in Section 3.3. Three metrics are applied to evaluate the performance of the simulation results: mean absolute error (MAE), mean absolute percentage error (MAPE), and root mean square error (RMSE):

$$MAE=\frac{\sum_{i=1}^{n}\left|v_{model,i}-v_{obs,i}\right|}{n} \quad (34)$$

$$MAPE=\frac{1}{n}\sum_{i=1}^{n}\frac{\left|v_{model,i}-v_{obs,i}\right|}{v_{obs,i}}\times 100\% \quad (35)$$

$$RMSE=\sqrt{\frac{\sum_{i=1}^{n}\left(v_{model,i}-v_{obs,i}\right)^2}{n-1}} \quad (36)$$

Here, $n$ denotes the number of sample data points, while $v_{obs,i}$ denotes the actual ship's speed at time $i$, and $v_{model,i}$ denotes the speed at which the simulated ship sails at time $i$. Figure 10 shows a comparison of the historical speed and the simulated speed of three different ships in their ice regimes. The first ship is the icebreaker Fennica, from 14:49 to 20:14 on March 1, 2011. The second is the Urho icebreaker, from 21:21 to 23:05 on March 1, 2011. The third is the icebreaker Zeus, from 14:03 to 15:49

on March 6, 2011. The figure shows Tukey box plots for simulated model speeds and empirical data, where the red lines shows the median values, the blue boxes the first and third quartile values, and the red crosses indicate outlier values (Tukey, 1977).

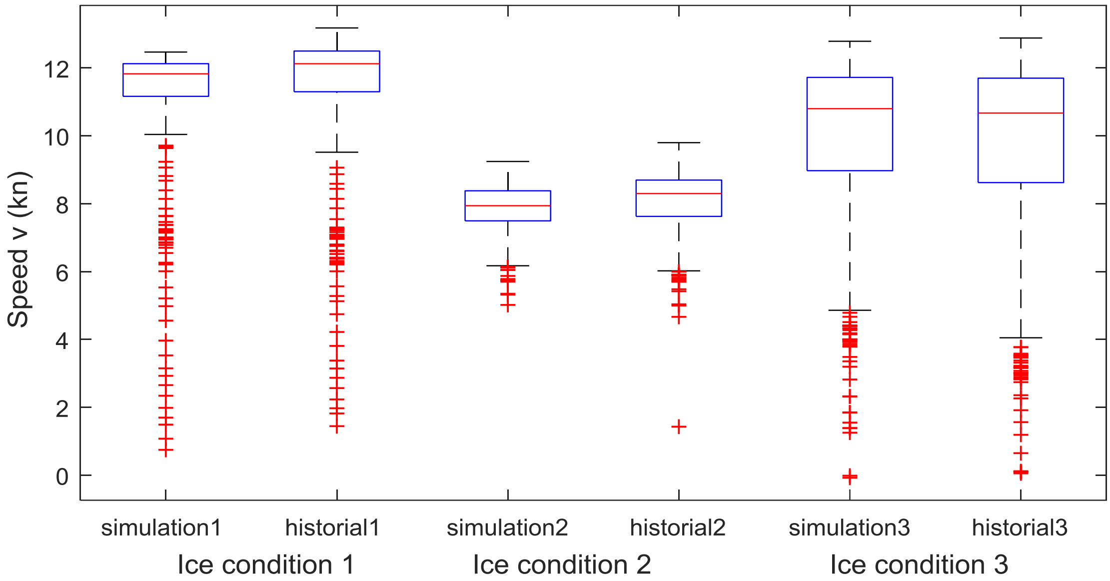
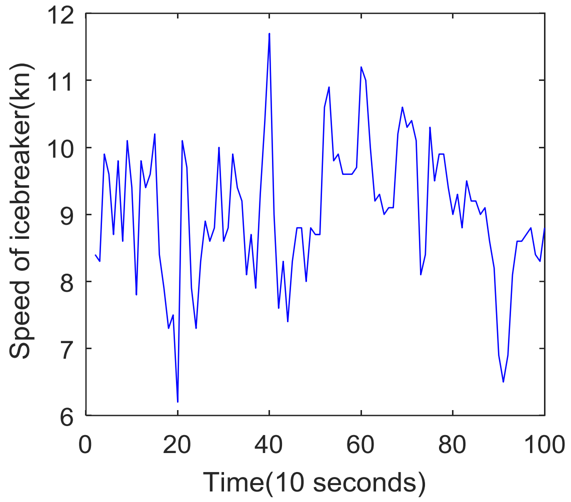
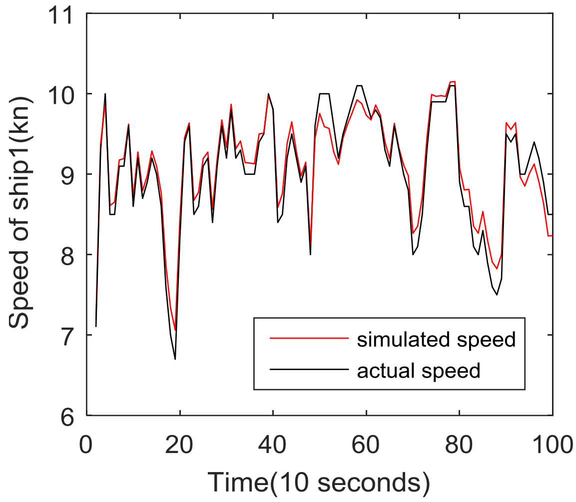



**Figure 10. Speeds in different ice conditions: comparison of simulation and empirical historical data**

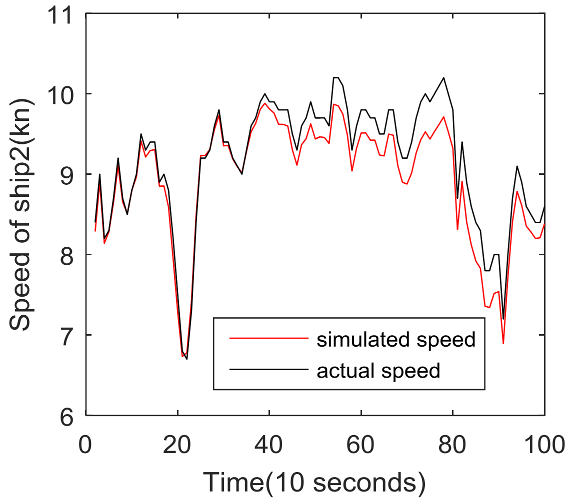
Figure 11(a) shows the actual sailing speed of the icebreaker Fennica on March 20, 2011, from 20:52:30 to 22:22:00. The time in Figure 11(b) and (c) are consistent with that of the icebreaker. Figure 11(b) shows the actual and simulated speeds of the first following ship (ship 1), and Figure 11(c) shows the actual and simulated speeds of the second following vessel (ship 2). The real distance and the simulated distance between the first following ship and the second following ship, are shown in Figure 11(d). The real distance and the simulated distance between icebreaker and the first following ship, are shown in Figure 11(e). When the distance decreases, the corresponding speed decreases, and when the distance increases, the speed increases as well. Overall, a good qualitative and quantitative correspondence between the model and the empirical data is found, although there are some deviations, e.g. in Figure 11(d), from around 650s onwards. Due to the complex interactions between the dynamic variables in the coupled kinematics equations, it is however not straightforward to assign specific causes to this behavior. The extent to which such deviations are important for the intended model purposes, outlined in Section 2.2, should be considered before using the model in practice, see also Section 6.



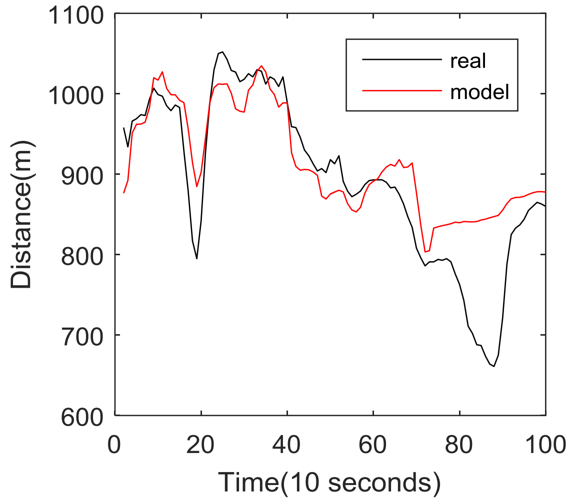

(a) Speed of icebreaker



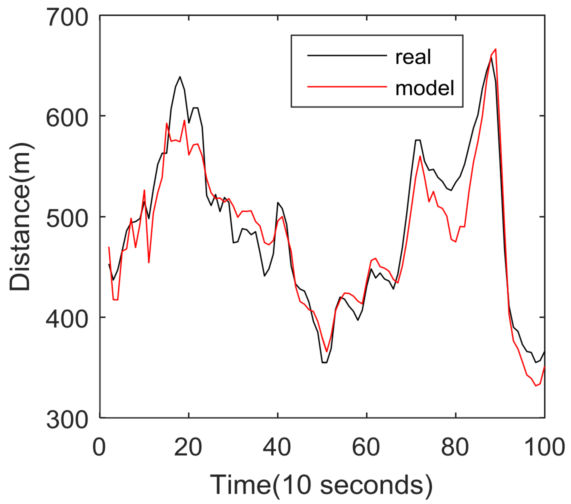

(b) Speed of following ship 1




(c) Speed of following ship 2




(d) Distance between following ships




(e) Distance between icebreaker and ship 1

**Figure 11. Speeds of and distances between different ships: comparison between model results and empirical data**

Next, the errors between the empirical and simulated speeds are determined, using Equations (34), (35) and (36). The results are shown in Table 5 and 6.

**Table 5. Evaluation of simulated speed for the three case studies**

| | MAE (kn) | MAPE (%) | RMSE (kn) |
|---|---|---|---|
| Case 1 (*Fennica*) | 0.3116 | 2.74 | 0.3853 |
| Case 2 (*Urho*) | 0.3126 | 4.55 | 0.5196 |
| Case 2 (*Zeus*) | 0.2957 | 12.98 | 0.4685 |

**Table 6. Evaluation of simulated speed for the two following ships**

| *Fennica* | MAE (kn) | MAPE (%) | RMSE (kn) |
|---|---|---|---|
| Ship 1 | 0.1495 | 1.71 | 0.1780 |
| Ship 2 | 0.2277 | 2.49 | 0.2757 |

**Table 7. Evaluation of simulated distance for the three ships**

| *Icebreaker(Fennica)* | MAE (m) | MAPE [%] | RMSE(m) |
|---|---|---|---|
| Ship 1 and icebreaker | 20.8838 | 4.18 | 24.9483 |
| Ship 1 and ship 2 | 42.8499 | 5.35 | 60.6539 |

From the results in Table 5 and 6, it can be seen that the simulation results are consistent with the actual speeds, and the overall range of the MAPE is less than 13%, with in most cases significantly smaller errors. The result of Table 7 indicates that all values of MAPE are less than 6%. This deviation can be considered small, and confirms the visual impressions from Figure 11 that the model is in good agreement with the empirical observations. Hence, in the authors' view, the model can provide reasonable estimates of the speed and distance of the assisted vessels under different ice conditions. From direct observation of Figure 11(b) and (c), it can be clearly seen that the simulated and actual speeds are generally consistent, and it is also possible to reasonably simulate multiple ships. However, the authors believe that better understanding the potential inaccuracies and shortcomings of the modeling approach for a larger set of test cases is important before implementing the model in ship simulators, see also Section 6.

# 6. DISCUSSION

So far, this paper has introduced the need for a model for icebreaker convoy operations, has shown evidence for developing the model, provided an overview of the model structure and parameterization, and has presented a validation of the model output in comparison with empirical data. It is concluded that, in the authors' view, the model performs reasonably well in terms of matching the model outputs with empirical data.

Nevertheless, due to the complexity of ice conditions and shipping operations, there are a number of factors that will affect the accuracy of the model. These include the determination of safe speeds and distances, the ice resistance of the leading icebreaker, and the sensitivity coefficients of following ships. In addition, the model in its current implementation has a number of limitations in scope, which to some extent limit the cases for which the model can be used. Finally, there are various aspects of validation and testing of the developed model, which need further consideration before implementing the proposed model in bridge simulator environments to facilitate navigator training.

These issues are addressed in the following sections, by critically assessing the model as shown, while providing guidance towards follow-up research. Section 6.1 focuses on the determination of safe speed, Section 6.2 addresses issues related to the ice resistance of vessels in the convoy and its relation to the safe distance. Section 6.3 discusses a number of other model limitations and related uncertainties, and Section 6.4 focuses on the issue of validation and testing of the model for its intended purpose.

## 6.1 Safe speed

The Arctic Ice Regime Shipping System (AIRSS), introduced in Section 3.3, distinguishes eight types of ice in assessing the safety of navigation for a given vessel in a given ice regime. The ice multipliers (IM) are based on the ice type and the vessel ice class. A negative IM means that there is a major danger in the ice situation, which should therefore be avoided. The ice multiples are shown in Table 3, and are used to determine the Ice Numeral (IN), as introduced in Equation (5). This IN is further used to determine the maximum safe ship speed, using the empirical regression formula of Equation (6).

This formula describes the relationship between the ice conditions in the Baltic Sea, the vessels' ice class and the maximum safe navigation speed. This safe speed is thus predicated on the conditions in the Baltic Sea, both in terms of ice conditions, vessel types, and operational practices. If it is to be used in other sea areas, care should be taken with Equation (6) and the maximum safe speed of 12.5 kn used in the currently proposed model, as ice conditions and operational practices may be different in other sea areas. Hence, it is recommended to apply the same procedure with data for other sea areas, or use other methods to ensure that the safe speed is meaningful for those areas.

## 6.2 Ice resistance and relation to safe distance

Section 4.1 introduced how the icebreaker's ice resistance is modeled in the proposed convoy following modeling. In particular, the resistance formula is shown in Equation (10), which requires an estimate of the vessel mass, for which Equation (11) is used. This regression formulation is a rather simplified approach to estimate the ice resistance, which makes an abstraction many influencing factors. It is known that factors such as the local hull geometry and waterline angles can have significant effects on the ice resistance (Li et al., 2018). Also, complex ice conditions such as ridges and compressive ice have a significant influence on the resistance (Montewka et al., 2015).

Regression formulae for ice resistance, such as those by Lindqvist (1989) and Riska et al. (1997) are commonly used for route planning purposes (Kotovirta et al., 2009; Guinness et al., 2014). As shown in Section 5, the approach taken to model ice resistance leads to reasonable results for the multi-ship following problem. Nevertheless, it is meaningful to further develop the model so it can account for more complex ice conditions, in particular ice ridges and compressive ice, as it is especially in such harsh conditions that convoys operations are required to ensure safe navigation. However, most existing models for ship performance in ice in more complex ice conditions are computationally too intensive for real-time simulation (Li et al., 2018). Existing approaches for real-time icebreaking simulation, e.g. by Lubbad and Løset (2011) also account only for rather simplified ice conditions. Hence, further research can be directed to developing fast-time simulation models for more complex ice conditions.

The safe distance as introduced in Section 3.5 and in particular Equation (7) also accounts for the effect of sea ice through the variable $\Delta D$. This is necessary, as an empirical study by Goerlandt et al., (2017) shows that the relationship between ice conditions and safe distance is complicated and varies significantly with the ice conditions. Furthermore, it is plausible that also the meteorological conditions, the experience with ice navigation of the officers on watch, and the performance of the assisted vessels in ice conditions, affect the safe distance.

**Table 8. Safe distances for different icebreakers at different ice conditions**

| *Icebreaker* | Ice thickness (m) | Safe distance (m) |
|---|---|---|
| Fennica | 0.5335 | 197.584 |
| Zeus | 0.5742 | 124.3 |
| Urho | 0.5973 | 121.967 |

Table 8 shows an example of safe distances for different icebreakers at different ice conditions. The ice conditions are obtained from start position of each icebreaker in Fig. 6, as well speeds data of each icebreaker accordingly. Then the safe distances are calculated according to Equation (7). Table 8 reveals a preliminary reverse trend between safe distance and ice thickness. However, accurate relationships between safe distance and ice condition require more data and analysis because more variables such as speed and acceleration of leading vessels and following vessels are involved.

An important issue with respect to the safe distance is the specific effects of compressive ice on the added resistance of following vessels in a convoy. In compressive ice conditions, the channel opened up by the icebreaker closes due to the movement of the ice sheet (Montewka et al., 2015). Consequently, the hull of the following vessels come in contact with ice along the waterline, which leads to significant added resistance to these vessels (Kaups, 2012). In the currently proposed model, it is assumed that only the leading icebreaker breaks the ice, and that the following vessels do not need to overcome significant ice resistances. It follows from this that the model as presented is limited to convoy operations in non-compressive ice conditions. As convoys are used in practice also in compressive ice conditions (Goerlandt et al., 2017), it is a worthwhile avenue for future research to develop the model to account for such conditions by implementing the added resistance on the following vessels.

## 6.3 Other model limitations and related uncertainties

In the process of building the coupled kinematics model for icebreaker convoy operations, a number of simplifying assumptions have been made. Aspects of this have been addressed in Section 6.1 and 6.2, focusing on safe speed, icebreaking resistance, and safe distance. In this section, a number of model limitations and related uncertainties are briefly discussed.

First, it is assumed that the assisted vessels in the convoy follow an icebreaker which breaks the ice continuously. This presupposes for instance that all vessels navigate in a contiguous, stationary ice field, i.e. possible patches of open sea are not considered, and neither are effects of dynamic ice.

Second, it is assumed that all ships have the same sensitivity coefficient $\lambda$, keep the same target safe distance h, have the same safe distance parameter $\xi$ and same safe speed $v_{max}$ for the same ice conditions. Such deterministic assumptions simplify the model formulation, and judging from the validation presented in Section 5, the approach appears to work reasonably well. However, it is clear that not all vessels have the same performance in ice, due to different hull shapes and installed propulsion power as required by the ice class rules (Trafi, 2017). Hence, it is a possible avenue for future research to explicitly consider parameter uncertainties and stochastic effects, as these may have some influence on the results. This also applies to the ice conditions: in the current model, the ice conditions are utilized in a rather simplified manner through considering the average ice thickness. It would be an avenue for research and development to explicitly account for the stochastic nature of the ice conditions.

Third, while the overall model performance is considered good based on the analysis of Section 5, Figure 11 (d) shows a deviation between the modeled and empirically determined distance between the following vessels, especially in the time interval from 800 s to 900 s. Due to the complex interdependencies in the model formulation of Section 4.2.3, and the possible effects of parameter uncertainties, it is not possible to explain why this behavior occurs. This points to the need for future validation tests, as described in Section 6.4. Another approach to understand the significance of parameter choices to the model performance when the causal dependencies within a model are complex is sensitivity analysis. By systematically varying parameters about which there is uncertainty, and evaluating the model outputs in light of empirical data, it is possible to gain understanding of the range of possible deviations due to different parameterizations. This is especially important when it is assumed that the same parameters ($\lambda$, $\xi$, h, $v_{max}$) can be used for all ships, as discussed above.

A final model limitation concerns the types of propulsion controls applied by the vessels in the convoy. Fixed pitch propellers cannot freely adjust the propeller pitch, and hence rely on changes in engine speed to effect vessel speed changes through changes in propulsive force. In contrast, variable pitch propellers can adjust the pitch freely, and can adjust the propulsive force and hence the vessel speed with constance engine speed. In convoy operations as modeled in this paper, frequent speed adjustments are required to keep a safe distance between the vessels. Such an approach is possible for variable pitch propellers as this does not require constant adjustments to engine speed, but is more problematic for fixed propellers. Hence, the model application is possibly restricted to simulating the performance in convoys of vessels with a variable pitch propeller.

## 6.4 Model validation and testing

In Section 5, a model validation has been presented, where the model outputs were compared both qualitatively and quantitatively. While overall a good agreement is found between model and empirical data, it is important to test the model for a wider range of scenarios than those presented in this paper.

Moreover, as the model is primarily intended to be used for navigator training in ship simulator environments to enhance the realism of how convoy operations are represented, as outlined in Section 2.2, it is important to test the model in the actual intended environment. More specifically, it should be assessed whether the model's behavior is considered realistic for the users of a ship simulator, and if the simulated vessels' kinematics in the simulator provide a good environment to facilitate ice navigation training. An appropriate test procedure should be developed and executed to this effect. It is beyond the intended scope of this paper to elaborate what exactly such a test procedure would look like. What is clear however is that apart from technical aspects, elaborated e.g. in Gavrilin (2018), also procedural and educational aspects of training with should be considered and analyzed (Sellberg, 2017).

# 7. CONCLUSIONS

This paper proposes a multi-ship following model under safety constraints for icebreaker convoy operations, which considers safe navigation in sea ice conditions. The core of the method is a set of coupled kinematic equations, supplemented with information on ice resistance, safe speed, and safe distance, derived from various sources. The ultimate purpose of the model is to be implemented in ship simulators, providing a more realistic environment for navigational training in ice covered waters. Because single-lane road traffic and icebreaker convoy operations have similarities in terms of following behavior, well-known road traffic modeling approaches are adapted to consider the sea ice conditions and vessel performance, from which a new model has been developed. The model can be used to calculate the speed of following ships, and the distance between these, under sea ice conditions. While the proposed modeling approach is general, data from the Baltic Sea has been applied for model calibration and testing. This specifically concerns the formulation of the ice resistance and the safe speed.

After developing the model, the degree of agreement between simulated and actual data has been determined. Based on a set of case studies, where the speed and distance of the vessels in the convoy have been assessed qualitatively and using a number of statistical performance measures, the proposed model has been tested against empirical data from real world operations. The results indicate that the model performance is relatively good, despite the assumptions and simplifications made, especially considering that no earlier work is known where models for icebreaker convoy operations are proposed. Future research can focus on developing more accurate formulations for ship safe speed and safe distance, incorporating ship resistance models which can better reflect the effects of local ice conditions such as ridged, rafted, and compressive ice. Especially the latter is likely important, because ice compression can lead to closing of the ice channel, which will lead to an added resistance to the following vessels, which is not considered in the current model formulation. Finally, future research should focus on testing the developed model (or an improved version thereof) in ship simulator settings for navigation training purposes.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