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Deformation and fracture of cylindrical tubes under detonation loading: A review of numerical and experimental analyses

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Abstract

The dynamic load in tubes due to detonation has a number of applications, such as in oil pipeline systems and pulse detonation engines. Various experimental, analytical, and numerical investigations have been conducted to study the mechanical, thermo-mechanical, and fracture behavior of tubes under internal detonation loads. Regarding numerical analysis, different approaches such as interface cohesive element, mesh-free, and extended finite element methods have been used to model the propagation of crack(s) in a tube. This paper presents a review of relevant literature pertaining to numerical and experimental analyses of detonation-driven deformation and fracture, and of studies based on the analytical investigation of moving loads in detonation tubes. The corresponding findings are discussed in detail, and possible avenues for future research are highlighted.

Keywords: Cylindrical tube, Gaseous and explosive detonations, Numerical and experimental analyses, Deformation and fracture

1. Introduction

Presently, cylindrical tubes are being used in various applications, such as petroleum and aviation industries and energy production units. In some situations, cracks may occur in these tubes resulting in serious defects in the structure serviceability. Most of the currently available fracture analysis studies on tubes under internal moving loads address either quasi-static or fatigue loading [1]. The structural response of a tube to an internal dynamic load depends on several factors, such as the dimensions and material characteristics of the tube, and the magnitude and speed of the moving load [2]. Generally, three pressure levels can be defined for internal detonation loads, for which three different types of structural response can be expected. These pressures are designated as low, medium, and high with respect to the magnitude of resultant stresses, which can be less than, equal to, or higher than the dynamic ultimate tensile strength of the material. Detonation is an efficient means of combusting a fuel-oxidizer mixture and releasing its chemical energy content [3]. Gaseous detonation in pipes can be encountered in the nuclear [4] and transportation industries [5], and in technologies such as pulse detonation engines [6]. Detonation-driven fracture in tubes is distinguished from quasi-statically loaded tube fracture according to two main features: (1) Flexural waves caused by traveling loads can result in oscillatory strains whose amplitudes are dependent on the speed of the traveling load, and can be several times higher than those predicted by static formulae, and (2) dynamic fracture parameters may be different from equivalent static forms [7].

The purpose of this work is to review relevant studies on detonation-driven deformation and fracture from the numerical and experimental perspectives. This review focuses on high-speed moving pressures that can result in oscillating stresses/strains in metal tubes (such as aluminum or steel). While there are a number of studies on detonation and detonation-driven fracture analysis in detonation tubes (DTs), for example [8, 9, 10, 11], as well as a few review articles on detonation in tubes [3, 6, 12], the authors consider there is an absence of a detailed review of detonation and detonation-driven fracture analyses using numerical and experimental approaches. Such a study would be of value to the research community dealing with internal moving load analysis. Further, the key issues that need to be resolved in the future are addressed at the end of this review. The article is organized as follows: Section 2 presents a summary of typical detonation loads that can be treated as internal moving pressure, and experimental studies from literature are discussed in Section 3. Section 4 contains a discussion on analytical investigations of moving load analysis inside tubes, thermal analysis studies on pulse detonation (PD) engines, and numerical simulations on elastic and crack propagation modeling problems. Our concluding remarks are presented in the final section.
2. A Summary of Detonation Loads

Two types of detonation loads are discussed in this work: (1) gaseous detonation, and (2) explosive detonation. These two types of loads were previously used to conduct experimental and numerical analyses; hence, they should be defined.

2.1. Gaseous detonation

Gaseous detonation is a high-speed load that occurs with a velocity greater than that of sound. Two principal conditions are required for detonation onset [6]: (i) Formation of a shock wave with an intensity sufficient for an explosive mixture to auto-ignite, and (ii) increase in the local rate of energy release to a level sufficient for shock wave reproduction in the adjacent layer in the explosive mixture. A gaseous detonation wave consists of a tightly coupled shock wave and reaction zone [8]. An ideal gaseous mechanical shock has a velocity, the so-called Chapman-Jouguet velocity \( (V_{cj}) \), of 1500–3000 m/s, depending on the fuel-oxidizer combination. The maximum wave pressure caused by this load can be as high as 20–30 times the ambient pressure, and the gas temperature may exceed 2000 °C [13]. Figure 1(a) shows an example of the detonation propagation (pressure wave) in a tube with closed ends, while Figure 1(b) presents temperature distribution in the outer wall of a PD engine (PDE) under sequential detonation loads (frequency of 19 Hz, CJ speed of 1786 m/s, and maximum pressure of 1.9 MPa).

Idealized models of the gaseous detonation [14] predict the presence of a pressure peak, the so-called Von Neumann (VN) spike, at the front of the reaction zone \( (P_{VN} \approx [1.5–2] \times P_{cj}) \). This pressure spike is usually not resolved in experiments because of its localized nature and short duration [8].

The pressure history for this type of loading can be represented by an exponential approximation to the Taylor–Zeldovich model as follows [16]:

\[
P(t) = (P_1 - P_{atm}) + \left( P_2 - P_1 \right) + (P_2 - P_3) e^{-\frac{t}{T_{exp}}} \times \left[ 1 - H(x - V_{cj} t) \right]
\]

\[0 < x < L \]

\[P_3 = P_1 \left( \frac{C_p}{C_{cj}} \right)^{2\gamma (\gamma - 1)} \]

\[C_{cj} = \frac{\gamma + 1}{2} \gamma - 1 \]

where \( P_1, P_2, \) and \( P_3 \) are the initial, gas mixture, and peak pressures, respectively, \( T_{exp} \) is the exponential decay factor, \( P_{atm} \) is the atmospheric pressure, \( x \) is the distance variable, \( t \) is the time variable, \( H \) is the step function, \( \gamma \) is the ratio of specific heats, and \( C_{cj} \) is the isentropic sound speed. The reaction zone occurs in a short length compared to the typical DT length; therefore, its influence on the structural response is smaller than the main loading produced by the Taylor–Zeldovich pressure behind the detonation front (Figure 2) [8]. As a result, the influence of the VN pressure spike on the structural response was not considered in almost all numerical studies on gaseous detonation.

![Figure 1](image-url)
Table 1. Computed detonation parameters for stoichiometric mixtures at standard initial conditions (25 °C, 1 atm) [13]

<table>
<thead>
<tr>
<th>Mixture</th>
<th>Fuel</th>
<th>% (vol)</th>
<th>$P_{cj}$ (MPa)</th>
<th>$V_{cj}$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel-air</td>
<td>Hydrogen ($H_2$)</td>
<td>29.6</td>
<td>15.6</td>
<td>1971</td>
</tr>
<tr>
<td></td>
<td>Acetylene ($C_2H_2$)</td>
<td>7.75</td>
<td>19.1</td>
<td>1867</td>
</tr>
<tr>
<td></td>
<td>Ethylene ($C_2H_4$)</td>
<td>6.54</td>
<td>18.4</td>
<td>1825</td>
</tr>
<tr>
<td></td>
<td>Ethane ($C_2H_6$)</td>
<td>5.66</td>
<td>18.0</td>
<td>1825</td>
</tr>
<tr>
<td></td>
<td>Propane ($C_3H_8$)</td>
<td>4.03</td>
<td>18.3</td>
<td>1801</td>
</tr>
<tr>
<td></td>
<td>Methane ($CH_4$)</td>
<td>9.48</td>
<td>17.2</td>
<td>1804</td>
</tr>
<tr>
<td></td>
<td>Hydrogen ($H_2$)</td>
<td>66.7</td>
<td>19.0</td>
<td>2841</td>
</tr>
<tr>
<td></td>
<td>Acetylene ($C_2H_2$)</td>
<td>28.6</td>
<td>34.0</td>
<td>2425</td>
</tr>
<tr>
<td></td>
<td>Ethylene ($C_2H_4$)</td>
<td>25.0</td>
<td>33.7</td>
<td>2376</td>
</tr>
<tr>
<td></td>
<td>Ethane ($C_2H_6$)</td>
<td>22.2</td>
<td>34.3</td>
<td>2372</td>
</tr>
<tr>
<td></td>
<td>Propane ($C_3H_8$)</td>
<td>16.7</td>
<td>36.5</td>
<td>2360</td>
</tr>
<tr>
<td></td>
<td>Methane ($CH_4$)</td>
<td>33.3</td>
<td>29.6</td>
<td>2393</td>
</tr>
</tbody>
</table>

According to Chao [1], stoichiometric hydrogen ($H_2$) and oxygen ($O_2$) mixtures can be used after dilution with various amounts with either nitrogen ($N_2$) or helium (HE) – $H_2 + 0.5O_2 + nN_2$ or $H_2 + 0.5O_2 + nHE$. These mixtures were chosen to vary the CJ velocity without significantly altering CJ pressure. Table 2 describes various types of mixtures and their corresponding detonation parameters.

Table 2. Gaseous detonation parameters for stoichiometric hydrogen and oxygen mixtures [1]

<table>
<thead>
<tr>
<th>Mixture</th>
<th>$n$</th>
<th>$P_{cj}$ (MPa)</th>
<th>$V_{cj}$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_2 + 0.5O_2 + nN_2$</td>
<td>1.0</td>
<td>1.72</td>
<td>2187</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
<td>1.77</td>
<td>2300</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>1.80</td>
<td>2398</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>1.85</td>
<td>2611</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>1.87</td>
<td>2712</td>
</tr>
<tr>
<td>$H_2 + 0.5O_2 + nHE$</td>
<td>0.0</td>
<td>1.90</td>
<td>2841</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>1.90</td>
<td>2907</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>1.90</td>
<td>2968</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>1.90</td>
<td>3125</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>1.88</td>
<td>3324</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>1.82</td>
<td>3576</td>
</tr>
</tbody>
</table>

According to Chao [1], fracture pattern in DTs is responsible for the variation in loading patterns. For a small change in the specific volume ($\Delta v$), the pressure drop ($\Delta P$) can be calculated as:
\[ \Delta P = -\frac{\Delta v}{K_s v} \]  
\[ K_s = \frac{v}{C_s^2} \]  

where \( v \) is the specific volume, \( C_s \) is the speed of sound in the detonation mixture, and \( K_s \) is isentropic compressibility. For example, venting only 0.5% of the initial volume of gaseous detonation products provides a \( \Delta P \) of approximately 36 kPa (with \( C_s \) of 1300 m/s, \( v \) of 0.24 m\(^3\)/kg, and \( K_s \) of \( 1.4 \times 10^{-7} \) m\(^2\)/N).

The calculation process for the heat dissipation of a single detonation is defined using the first law of thermodynamics as follows [18]:

\[
Q - W = m\left[(h_{pr} - h_{re}) - 1/2 \left(V_{pr}^2 - V_{re}^2\right) - g\left(z_{pr} - z_{re}\right)\right]
\]

where \( Q \) is the heat transferred, \( W \) is the work done, \( m \) is the mass, \( h_{pr} \) and \( h_{re} \) are the enthalpy of the product and reactant, respectively, \( V_{pr} \) and \( V_{re} \) are the speeds of the product and reactant, respectively, and \( g(z_{pr} - z_{re}) \) is the potential energy. Inlet and outlet terms are neglected during the detonation phase, and the following assumptions were made: There is no change in mass and the gas is at rest before and after detonation shock; hence \( V_{pr} = V_{re} = 0 \), and work done can be zero if no rotary component is used in the detonation tube. The energy equation according to these assumptions is as follows [19]:

\[
Q = m(h_{pr} - h_{re})
\]

Accordingly, the surface heat flux on the inner surface of the DT can be calculated as:

\[
q = \frac{Q}{A_{dt}}
\]

where \( A \) is the internal area of the DT and \( t_{dt} \) is the time taken by the detonation shock wave traveling at a pre-defined speed. This surface heat flux can be applied over the detonation tube during numerical simulations along with the mechanical load from Eq. (1) to analyze both mechanical and thermal effects of the detonation.

One of the main applications of gaseous detonation is in PDEs, in which a tube serves as the combustor [6], as shown in Figure 3. Here, the detonation wave rapidly traverses the combustor resulting in a constant-volume heat addition that produces high pressures in the combustor and provides thrust [6], as shown in Figure 3(a)–(e). The operation of multi-tube PDE configurations (Figure 3(f)) at high frequencies (\( \geq \)100 Hz) produces a near-constant thrust. The advantages of PDE for air-breathing propulsion include reduced fuel consumption and operability from a zero-approach stream velocity to high supersonic flight speeds.

![Figure 3](image-url)
In some fluid-structure interaction (FSI) detonation models, hydrostatic pressure is obtained using the equation-of-state of the gas based on its density ($\rho$) and specific internal energy ($e$). This approach was used by Cirak et al. [23], Deiterding [24, 25, 26], Deiterding et al. [27, 28, 29], and Du et al. [30] as follows:

$$P = (\gamma - 1) \rho e$$  
(7)
$$e = E - \frac{u^2}{2}$$  
(8)

According to [31], both the initial reactive and detonation products can be considered as polytropic gases in the case of gaseous detonation. Therefore, the connected chemical reaction can be coupled with fluid motion using the following equations [30]:

$$P = \lambda(\gamma - 1) \rho e$$  
(9)
$$\lambda = \begin{cases} 
0, & t \leq t_1 \\
\frac{t - t_1}{L_{\text{min}}/V_{cj}}, & t > t_1 
\end{cases}$$  
(10)

where $\lambda$ is a process variable that controls the release of chemical energy in the detonation model, $L_{\text{min}}$ is the minimum characteristic length of the grids, and $t_1$ is the lighting time. The lighting time is computed for each element by dividing the distance from the detonation point to the center of the element by the detonation velocity. If $\lambda$ exceeds 1, it must be reset to 1. Finally, Eq. (8) can be solved using equations of mass, momentum, and energy (first, second, and third equations), as follows:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$\frac{\partial (\rho \mathbf{u})}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) + \nabla P = 0$$

$$\frac{\partial (\rho E)}{\partial t} + \nabla \cdot (\rho \mathbf{u} E) = 0$$  
(11)

A similar FSI approach was used by Perotti et al. [32] to model inviscid fluid flow for loading investigations simpler than detonation loading. They utilized a 3D Euler equation in Cartesian coordinates written in the form of a conservation law to represent shock waves in a water-filled tube:

$$\frac{\partial}{\partial t} q(x, t) + \sum_{n=1}^{3} \frac{\partial}{\partial x_n} f_n(q(x, t)) = 0$$

$$q(x, t) = (\rho, \rho u_1, \rho u_2, \rho u_3, \rho E)^T$$

$$f_n(q) = (\rho u_n, \rho u_1 u_n + \delta_n P, \rho u_2 u_n + \delta_n P, \rho u_3 u_n + \delta_n P, \rho u_n (\rho E + P))^T$$

where $q$ is the vector-of-state, $f$ is a flux function, $u_n$ is the $n^{th}$ component of the vector $\mathbf{u}$, and $n = 1, 2, \text{ and } 3$. $\delta_n$ denotes the Kronecker delta and the hydrostatic pressure $P$ is given by the stiffened gas equation-of-state as

$$P = (\gamma - 1) \rho \left( E - \frac{1}{2} \mathbf{u}^T \mathbf{u} \right) - \gamma P \rho$$  
(13)

where $\mathbf{u}$ is the velocity vector, $E$ is the specific total energy, and $\rho$ represents fluid density. The role of $P_\rho$ is to enforce stiffening and thereby a liquid-like behavior, but note that the model consisting of governing equations and an equation-of-state is otherwise fully compressible. In particular, this includes the use of an energy equation leading to a velocity equal to that of the velocity of sound in the fluid.

$$C_s = \frac{\sqrt{\gamma P + P_\rho}}{\rho}$$  
(14)

This equation fully considers dynamic alterations in the fluid density and hydrodynamic pressure $P$ with respect to the flow field, which is a requirement for accurately simulating problems driven by a strong FSI [32].

### 2.3 Explosive detonation

The characteristics of explosive detonation can be described using a standard Jones–Wilkiss–Lee (JWL) equation-of-state, where the pressure is expressed as a function of volume and energy as follows [10, 33]:

$$P = C_1 \left[ 1 - \frac{\omega}{R \gamma} \right] e^{-R \gamma} + C_2 \left[ 1 - \frac{\omega}{R \gamma} \right] e^{-R \gamma} + \frac{\omega e_0}{\gamma}$$  
(15)
Here, $C_1, C_2, R_1, R_2,$ and $\omega$ are material-dependent constants; $P, v,$ and $e_0$ represent pressure, relative volume, and specific internal energy, respectively. The initial density $\rho_0,$ detonation velocity $D,$ and CJ pressure $P_{CJ}$ must be used to initialize the simulation. Table 3 shows the JWL parameters reported in two published works.

<table>
<thead>
<tr>
<th>Explosive Material</th>
<th>$C_1$ (GPa)</th>
<th>$C_2$ (GPa)</th>
<th>$R_1$</th>
<th>$R_2$</th>
<th>$\omega$</th>
<th>$e_0$ (J/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pentaerythritol Tetranitrate (PETN)</td>
<td>625.3</td>
<td>23.3</td>
<td>5.25</td>
<td>1.6</td>
<td>0.28</td>
<td>$8.56 \times 10^9$</td>
</tr>
<tr>
<td>Trinitrotoluene (TNT)</td>
<td>371.2</td>
<td>3.231</td>
<td>4.15</td>
<td>0.95</td>
<td>0.30</td>
<td>$2.6 \times 10^9$</td>
</tr>
</tbody>
</table>

3. Experimental Studies on Detonation Tubes

Several reports on different aspects of DTs from PDEs to explosive-driven DTs are available in literature. Table 4 presents a summary of the experimental studies on DTs focusing on structural analysis as well as some behavioral aspects of shock loading.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Year</th>
<th>Main aspects of the test</th>
<th>Main conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hinkey et al.</td>
<td>1997</td>
<td>- A rotary valve PDE combustor for high-frequency operation to analyze single/multiple combustors&lt;br&gt;- The measuring parameters were thrust, combustor wall pressure histories, oxidizer, and fuel mass flow rate.</td>
<td>The proposed approach for system operation successfully worked in a rotary valve multi-combustor PDE engine.</td>
</tr>
<tr>
<td>Cooper et al.</td>
<td>2001–02</td>
<td>- Impulse was measured using a ballistic pendulum for detonation and deflagration in a closed end tube. &lt;br&gt;- Studied the effect of internal obstacles on the transition from deflagration to detonation</td>
<td>Experimental results varied by 15% when compared to analytical results.</td>
</tr>
<tr>
<td>Kasahara et al.</td>
<td>2002</td>
<td>- Tested a PDE with two main tubes and nozzle to study pressure variation, thrust generation; and heat generation in 28 cycles of PDE operation. &lt;br&gt;- Measured tube wall temperature with a thermal sensor. &lt;br&gt;- Pipes with a nominal diameter of 300 mm were constructed from GRP or medium density polyethylene and subjected to transient internal pressure loads. &lt;br&gt;- Time-resolved strain measurements of the wall responses were obtained.</td>
<td>Temperature of the thermo-mechanical shock tube increased by 20 to 25 K during operation in 9 s.</td>
</tr>
<tr>
<td>Thomas</td>
<td>2002</td>
<td>- A high-speed valve injects a mixture at rates of up to 1000 Hz. &lt;br&gt;- Investigate the thrust and specific impulse</td>
<td>It highlighted the need to consider the behavior of all components over a wide range of timescales if the full range of potential failure modes are to be analyzed.</td>
</tr>
<tr>
<td>Allgood et al.</td>
<td>2006</td>
<td>- Measured the damped thrust of a multi-cycle engine with an exhaust nozzle &lt;br&gt;- Experiments on pulse detonation rocket engine model</td>
<td>A diverging nozzle increases performance with an increase in the fill fraction.</td>
</tr>
<tr>
<td>Li et al.</td>
<td>2007</td>
<td>- Mixture of kerosene, oxygen, and nitrogen &lt;br&gt;- Investigate the thrust and specific impulse</td>
<td>Thrust in this type of engines is nearly proportional to the detonation frequency.</td>
</tr>
<tr>
<td>Cutler</td>
<td>2008</td>
<td>- Experiments on high-speed PDEs &lt;br&gt;- A high-speed valve injects a mixture at rates of up to 1000 Hz.</td>
<td>Optimum fuel-air compositions and resonant frequencies are identified and design rules are postulated.</td>
</tr>
<tr>
<td>Panicker et al.</td>
<td>2009</td>
<td>- Studied techniques for DD transition (DDT). &lt;br&gt;- Considered Shchelkin spirals, grooves, converging-diverging nozzles, and orifice plates</td>
<td>They concluded that the Shchelkin spiral is the best performer for deflagration to detonation (DD) wave transition among other devices.</td>
</tr>
<tr>
<td>Inaba et al.</td>
<td>2010</td>
<td>- Studied the coupling of flexural waves in water-filled tubes with cavitation flow. &lt;br&gt;- A transparent polycarbonate tube was used to examine cavitation events.</td>
<td>Cavitation does not occur uniformly or simultaneously; it is observed near the bottom surface of the buffer, the middle of the tube, and at the bottom (closed) end.</td>
</tr>
<tr>
<td>Kasahara et al.</td>
<td>2009–11</td>
<td>- Fabrication of a single-tube PD rocket system that can slide on rails &lt;br&gt;- Tested engine systems at different operating conditions</td>
<td>System was operated for 13 cycles at the frequency of 6.7 Hz and the maximum produced thrust was greater than 70 N.</td>
</tr>
</tbody>
</table>
Li et al. [45] 2011
- Described the detonation initiation area in an open-ended detonation tube
- Tube consisted of thrust wall and ignition sections.
- Spiraling internal grooves such as semicircle, square, and inversed-triangle grooves were used.
- Performance analysis of PD engine with a bell-shaped convergent-divergent nozzle
- Mixture of kerosene, oxygen, and nitrogen
- Experiments on PD rocket engines with injectors and nozzles

Yan et al. [46, 47] 2011
- Performance analysis of PD engine with a bell-shaped convergent-divergent nozzle
- Mixture of kerosene, oxygen, and nitrogen
- Experiments on PD rocket engines with injectors and nozzles

Matsouka et al. [48] 2012
- Developing a rotary valve for a PDE
- Visualizing a multi-shot of a PDRE in a square cross-section combustor
- Operation frequency of 160 Hz

Chen et al. [49] 2012
- Investigated nozzle effects on the thrust and inlet pressure.
- Injectors were tested for atomization and mixing of two-phase reactants.

Perotti et al. [32] 2013
- Investigated the response of filament-wound composite tubes subjected to axial shock wave loading in water.
- Two-phase dual-tube engine
- Analyzed valve-less multi-tube characteristics

Peng et al. [50] 2013
- Analysis of spiral configurations in PDE
- Provided the design data for deflagration to detonation transition in a curved detonation chamber.

Wang et al. [51] 2013
- Many experiments were carried out in the PDRE system using ethylene as the fuel and oxygen-enriched air as the oxidizer.
- Tests for high frequencies

Lu et al. [52] 2016
- Fully developed detonation waves were obtained and a steady operating frequency of 200 Hz was achieved. Higher frequencies can be expected when the supplying flow rate is increased or a smaller tube is utilized.

As can be seen from Table 4, experimental investigations on structural and deformation analysis mainly dealt with the internal components of the tube, number of engine tubes, and types of mixture, in addition to performance analyses of the produced thrust. One interesting study was conducted by Kasahara et al. [37], in which they analyzed the history of pressure at the thrust end wall of the PDE tubes as well as the net outgoing momentum and heat transfer from detonation gases to the tube walls. The experimental tube is schematically shown in Figure 4. Almost all experimental setups for structural and deformation analyses of PDE tubes are similar to that shown in Figure 4; the test setup included detonation, test, and extension tubes. The only differences between them are the gas mixture type, tube material, and analysis type (i.e., deformation, fracture, or both).
Conversely, there are only a few experimental investigations on the deformation and fracture analysis of tubes subjected to detonation loading. Table 5 presents a summary of experimental investigations on the fracture analysis of DTs under shock and detonation loading.

Table 5. Summary of experimental works on DTs dealing with fracture analysis

<table>
<thead>
<tr>
<th>Reference</th>
<th>Year</th>
<th>Main aspects of the test</th>
<th>Main conclusion</th>
</tr>
</thead>
</table>
| Chao et al. [1, 7, 9] | 2004–05| - Observe the fracture behavior of thin-walled aluminum tubes during internal gaseous detonation loading  
- The load speed was equal to 2.4 km/s and the maximum pressure magnitude was in the range of 2 to 6 MPa.  
- Initial flaws were machined as external surface notches. | Experiments and analyses demonstrated that the fracture mechanics approach can be useful in studying detonation-driven fracture in aluminum tubes. |
| Shepherd et al. [17] | 2008  | - To isolate and simplify phenomena involved in detonation-driven fracture that had been observed in previous works  
- Two sets of experiments were conducted to validate numerical results in literature. | The detonation loading process is repeatable. Using pre-cut flaps with reinforcement on the tube retains some couplings between structural and fluid mechanics. |
| Ma et al. [53] | 2010  | - Studied the dynamic response of explosion containment vessels (ECVs) to impulsive loading.  
- Shear failure mode of ECVs was studied for the first time, including potential initial flaws at the meso-scale in the vessel. | Failure analysis indicated that the rate-dependent failure criterion governs the damage mode of the vessel; however, the initial flaw mainly ignites the shear band, which has a minor effect on the final failure mode of the vessel. |
| Schildberg et al. [54] | 2013  | - Initial pressures were high enough to produce detonation pressures that caused significant bulging in the pipe walls.  
- Eight different pressure scenarios that can be distinguished for gas phase detonation in pipes were used. | The proposed approach helped establish reliable static equivalent pressures for different pressure scenarios for gas phase detonation in a pipe. |
Mirzaei et al. [10] 2015

- Presented dynamic ductile rupture of steel pipes subjected to high-speed internal moving pressures.
- Experiments included the detonation of tiny explosive cords inside small segments of ordinary gas pipes.

The results showed that self-similar propagation of initial cracks in the pipe followed incremental cyclic growth governed by structural waves.

A series of experimental tests was conducted by Chao et al. [1, 7, 9] in which they prepared tubes of different lengths and flaw sizes. Figure 5 shows a picture of the experimental setup used by Chao [1] along with a ruptured tube with an initial flaw size of 12.7 mm, $P_{cj} = 6.1$ MPa, and $V_{cj} = 2404$ m/s. Their results indicated that the fracture mechanics approach was useful in studying detonation-driven fracture in aluminum 6061-T6 tubes. Crack flaps, crack fronts, detonation wave front, and shock waves can be captured with transparent specimens and advanced optical techniques [1].

![Figure 5. Experimental setup used by Chao [1]: (a) Tube assembly, and (b) Ruptured tube with an initial notch of 12.7mm.](image)

Mirzaei et al. [10] reported experimental and numerical analyses of the dynamic rupture of steel pipes subjected to high-speed internal moving pressures due to detonations in tiny explosive cords inside small segments of the pipes, as shown in Figure 6(a). Experimental and numerical analyses were conducted using ANSYS LS-DYNA, as shown in Figure 6(b), for a tube with a crack length of 3 mm and a 16 gr/m explosive cord. The results showed that the dynamic rupture caused by this loading has its own characteristics, including incremental propagation in axial cracks by detonation-induced flexural waves, and crack curving and branching adjacent to the bulged area.

![Figure 6. Experimental setup used by Mirzaei et al. [10]: (a) Experimental setup, (b) Experimental and numerical results for a crack of 3 mm length](image)
Figure 7 describes the materials used for DT tubes employed in experimental studies. It can be seen from the two charts in Figure 7 that stainless steel is the first choice for both deformation and fracture-type experiments. However, there have been fewer studies on aluminum tubes. Moreover, non-metallic materials (such as epoxy-fiber, polycarbonate, and polyethylene), when used as base materials for detonation tubes, were restricted to deformation analysis.

![Figure 7. DT tube material types. (a) Deformation study (18 experiments) and (b) fracture study (5 experiments) ](image)

### 4. Numerical Simulations of Detonation Tubes

Although experimental tests are required for real observations on high-speed moving loads such as detonation, numerical simulations are equally essential to identify the main reasons behind them and the reasons for the phenomena occurring during experimental or actual accidents. The first step in this context is to investigate the nature of loading, using either analytical or numerical analysis. There have been several studies on the structural response of thin-walled tubes to internal shock/detonation loading. Tang [55] and Reismann [18] developed comprehensive approaches for calculating the elastic response of a tube under a moving load. The Tang model predicted the response of a thin shell to internal shock loading, while Reismann’s model took into consideration the pre-stress effect on the structural response. De Malherbe et al. [56] compared the results of detonation-loading motion in a tube with experimental values. Shepherd [57] utilized the cross-sectional model to predict the response of a tube to internal detonation loading.

Additional investigations include the response analysis of a tube to internal dust detonation with non-rotatory symmetric loading by Van de Ven et al. [58]; measuring strains produced in a thick-wall tube by detonation due to acetylene decomposition (Sperber et al.) [59] and extending this process to the analysis of thick-wall tubes [60]; analyzing the structural response of a thin-walled shell to internal shock loading using experimental, analytical, and numerical studies (Beltman et al.) [8]. Moreover, a modified Tang’s model was proposed by Mirzaei et al. [61, 62] to develop a new transient analytical model for finite tubes. Their approach included all the essential terms in the governing equation. Mirzaei [63] described the development of an enhanced form and analytical solutions for transverse shear strain. Hu et al. [64] presented analytical solutions for cylindrical shells under detonation loading; their solutions included static and transient states. A modification of the analytical formulation proposed by Mirzaei [63] was subsequently proposed [65], which included sequential detonation loadings. More recently, the accuracy of the analytical model reported in [65] was enhanced using a modified loading function [20], and the formulations were extended to include a description of orthotropic material behavior. Figure 8 shows analytical predictions of the vibrational response of a tube to four sequential detonations [20]. In this analysis, the time interval between detonations was set at 10.135 ms, and a very good agreement was observed between analytical and finite element (FE) solutions.
Figure 8. Analytical predictions of the vibrational response of a tube to four sequential detonations with loading profiles of $P_2 = 1.7$ MPa, $P_1 = P_3 = 0.1$ MPa, $V_{cj} = 1699.7$ m/s, and $T = 4.34 \times 10^{-4}$ s [20]

In the case of the analytical results shown in Figure 8, the boundary conditions were treated as simple supports [61]. Analytical solutions were obtained for a duration that included initial detonation loading and the subsequent reflections of flexural waves at the flanges.

4.1. Simulation under thermal load

Strain measured on the outer surface of a DT is not only due to the high pressure inside the tube, but also due to thermal stresses [66]. Hot combustion products transfer heat to the inner tube wall, which creates a thin layer of heated material inside the tube, as shown in Figure 9(a). The outer layer does not expand thermally, but experiences strain from the expanding hot inner thin layer. This effect (thermal stress), contributes further to hoop strain on the outer surface [66]. According to Pintgen et al. [66], a characteristic rise time ($t_c$) of the thermal loading signal can be used to calculate the penetration depth ($h$) of the heat into the inner tube layer, as shown in Figure 9(b). According to the experimental observations presented by Pintgen et al. [66], this rise time is approximately 50 ms. Therefore, the penetration depth can be defined as $h = \sqrt{\kappa t_c}$ using a one-dimensional heat equation, where $\kappa$ is the thermal diffusivity of the tube. For example, for a DT tube made of steel, $\kappa$ is $3.5 \times 10^{-6}$ m$^2$/s and the penetration depth is approximately 0.4 mm. Depending on tube thickness, this depth can represent either a thin or thick heat transfer layer; hence, its effect on the total strain/stress of the tube can be estimated, meaning whether the thermal strain/stress is large or small.

Figure 9. (a) Thermal penetration depth into the inner DT surface and (b) square-wave approximation of the temperature profile on the inner tube surface [17]

In high-frequency detonation tubes, thermal stresses are important due to their longtime thrust functionality. In this case, although the tube wall withstands mechanical stresses, thermal stresses can possibly cause fatigue and may lead to tube failure. Therefore, this kind of problem can be treated by coupled dynamic analysis of thermo-mechanical problems. Several studies employed the elasticity approach to analyze the thermal shock problem; in most cases, either an infinite elastic body with a spherical cavity (Stemberg and Chakravorty [67]) or hollow spherical shells (Tsui and Kraus [68], Zaker [69], and Hata [70]) were used. Wang and Gong [71] presented an effective analytical method to analyze thermo-mechanical stress distribution in a thick hollow cylinder subjected to shock loading. Subsequently, Wang [72] presented an effective method for gaining the histories and distribution of dynamic thermo-elastic stress in a hollow cylinder subjected to rapid arbitrary heating. A review of analytical methods to solve thermo-elastic problems for non-homogeneous materials was presented by Tanigawa [73]. Cho and Kardomates [74] developed a closed form elasto-dynamic solution for the stress field in a thick cylindrical shell.
under thermal shock loading. Further, a number of investigations were conducted on the thermal stress analysis of functionally graded cylinders. For example, a study on the thermo-elastic analysis of functionally graded cylindrical shells subjected to transient thermal shock loading was carried out by Santos et al. [75]. Details can be found in [76, 77] on cylinders made of functionally graded materials. Studies on the mechanical and thermal response of sandwich tubes subjected to an internal dynamic load were conducted by Zhou et al. [78] and Zhang [79]. Ghandikota [80] proposed a two-step method to thermally analyze PDEs. The first step involved in the thermal analysis of a PDE is to determine the heat dissipated from the engine using unsteady state analysis. The second step is to develop a preliminary heat exchanger design. Thermal analysis of the effect of sensible heat release on the walls of a shock chamber for different mixtures was described by Kalidindi [81]. Nagarajan and Lu [15] concluded that the high heat released, even at low operating frequencies, results in extremely hot walls. Further, they indicated that the temperature continued to increase, although at a slower rate, even after 12000 cycles (Figure 10). Zhou et al. [82] presented a model to study the transient thermal response in thick orthotropic hollow cylinders with a finite length, using a higher order shell theory. They applied this method for thermal stress analysis of a PDE as the case study to examine the accuracy of the extracted relations in their model. Malekan et al. [83] studied thermo-mechanical stresses and strains in a tube subjected to detonation load. Their numerical results consisted of simulations on single and multiple detonation cycles.

4.2. Simulation under mechanical load

There are three main approaches to the numerical modeling of thin-shell structures: shell theory, degenerated continuum theory, and a direct 3D continuum approach. The first two approaches require complicated numerical methods from an implementation perspective [84]. However, the 3D continuum approach often deploys multiple nodes in the shell thickness to obtain reasonable gradient fields. This results in a degradation of both the conditioning of the discrete system and the accuracy of the numerical solution. This section addresses numerical simulations that cover either elastic analysis only or fracture analysis, conducted using in-house codes as well as commercial software (ABAQUS®, ANSYS LS-DYNA®, COMSOL®, and WARP3D®).

4.2.1. Deformation Simulation

Deformation analyses include elastic, elasto-plastic, and fluid-structure interactions (FSI) constructed for either metallic or composite structures. Some studies involving metallic structures are as follows:

- Bakhshandeh et al. [85] studied the effect of thickness ratio on the response of a cylinder tube subjected to an internal dynamic pressure using the FE method.
- Zhou et al. [86, 87] studied the structural response of a prismatic sandwich tube under internal moving pressure loads. The sandwich tube was modeled using a multi-layer sandwich theory, taking into account the effects of transverse shear deformation and compressibility of the core. Figure 11 shows numerical schematics of two of their models – the effective (with an effective homogenized core) and complete models. Zhou et al.’s results [86] showed that the transient analytical and steady-state models can predict the critical velocity of the effective
model. Moreover, the structural response obtained using the complete model is almost similar to that obtained using other models when the load moves at critical velocities. It has also been demonstrated that a rectangular core is preferred among three core topologies (rectangular, triangular, and Kagome) [87].

- Karnesky et al. [88, 89] investigated the elasto-plastic [88] and plastic [89] deformation of tubes due to detonation loadings and their reflected waves. Their computational and experimental results were in good agreement when both the strain rate and hardening effects were considered in the computational model. Figure 12 shows the hoop stress and strain curves for mild steel as determined by a single degree of freedom (DOF) model for detonation with an initial pressure of 2 bar; here, the axial location for each case was chosen such that the reflected shock wave arrives half-way between the third peak and trough of the oscillation.

- Mirzaei [65] presented analytical solutions for the structural response of thin-walled tubes subjected to sequential internal moving pressures.

- Smeulers et al. [90] investigated the occurrence of mode coupling in tubes by considering the effect of limited plasticity. Mode coupling refers to the excitation of circumferential bending modes, coupled with the initial hoop stress, which may lead to an increased local maximum stress. Their results showed that mode coupling can be reduced by considering the plasticity effect.

- Bitter et al. [91] analyzed the capability of shell models for predicting stresses and strains in thick-walled tubes under detonation loads.

- Bitter et al. [92] proposed a simplified model for forecasting axial displacement as well as stress and strain in tubes subjected to internal shock loading. The effects of the radial and rotary inertia of the pipe were neglected in their model owing to which the predictions represented only spatially averaged responses of the tube.

- Gwak et al. [93] used hybrid particle level-sets that modeled both the tube structure and detonating gas to track interface motion between the gas and deforming wall for a tube subjected to detonation load.

Figure 11. Schematic of computational models and finite element meshes corresponding to the (a) effective model and (b) complete model [86, 87]
Figure 12. Hoop stress and strain curves exhibiting the response of mild steel as determined by a single DOF model; the study conditions included stoichiometric ethylene-oxygen detonation at an initial pressure of 2 bar. In each case, detonation arrives at A and the reflected shock wave arrives at C. B marks the maximum stress/strain achieved before the reflected shock arrives and D marks the overall stress/strain maximum [89].

In addition, investigations were conducted on composite structures.

- Perotti et al. [32] studied the response of filament-wound composite tubes subjected to axial shock wave loading in water. Their study was focused on the FSI between pressures waves in the fluid coupled with flexural waves in the tube inner wall.
- Mirzaei et al. [20] presented a set of analytical solutions for the transient response of orthotropic thin tubes subjected to sequential moving loads. Their results demonstrated that a realistic analysis of the overall vibrational behavior requires the simulation of hundreds of sequential detonations.

Table 6 includes details of gaseous detonation properties used in numerical analysis studies on the deformation of DTs. The tube length in these studies ranged from 35 mm to 5 m. The detonation properties shown in this table refer to different gaseous mixtures described previously in Tables 1 to 3.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Maximum pressure (MPa)</th>
<th>Velocity (m/s)</th>
<th>Tube length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zhou et al. [86, 87]</td>
<td>5.0</td>
<td>600 – 1600</td>
<td>1.0</td>
</tr>
<tr>
<td>Karnesky et al. [88, 89]</td>
<td>1.64 – 10.54</td>
<td>2343 – 2430</td>
<td>1.076</td>
</tr>
<tr>
<td>Mirzaei [65]</td>
<td>1.35 – 1.70</td>
<td>1478.8 – 1699.7</td>
<td>2.38</td>
</tr>
<tr>
<td>Smeulers et al. [90]</td>
<td>6.7</td>
<td>400 – 1500</td>
<td>5.0</td>
</tr>
<tr>
<td>Bitter et al. [91]</td>
<td>1.5 – 2.5</td>
<td>1800 – 2324</td>
<td>2.0</td>
</tr>
<tr>
<td>Bitter et al. [92]</td>
<td>2.52</td>
<td>2088</td>
<td>2.0</td>
</tr>
<tr>
<td>Gwak et al. [93]</td>
<td>1.773 – 3.26</td>
<td>2343 – 2845</td>
<td>0.35</td>
</tr>
<tr>
<td>Mirzaei et al. [20]</td>
<td>1.35 – 1.7</td>
<td>1478.8 – 1699.7</td>
<td>2.38</td>
</tr>
<tr>
<td>Perotti et al. [32]</td>
<td>15.0</td>
<td>5.3</td>
<td>0.8</td>
</tr>
<tr>
<td>Bakhshandeh et al. [85]</td>
<td>1.3</td>
<td>12 – 30</td>
<td>0.035</td>
</tr>
</tbody>
</table>

4.2.2. Fracture Simulation

Besides mechanical and thermo-mechanical numerical simulations, there are several reports on the fracture analysis of tubes subjected to detonation loads. These investigations can be divided into two types: coupled fluid-structure interactions, and decoupled mechanical simulations. In a coupled simulation, the loading process (detonation) is simulated at the same time as structural fracture. Fracture due to detonation can be predicted more realistically considering the interaction between the blast wave and tube structure. On the other hand, the decoupled simulation initially calculates the detonation load and then applies the calculated load to the structure to be failed. In addition, continuous (including nonlocal constitutive, gradient enhanced damage, smeared crack, and emerged phase field models) or discrete (including zero-thickness cohesive interface elements, elements with embedded strong discontinuities, crack-tip opening angle (CTOA), extended finite element method (XFEM), phantom node method, and mesh-free methods) mathematical formulations can be used to develop FE formulations for problems dealing
with fracture. The main aspect of continuous approaches is that failure is defined by material degradation; hence, the incorporation of a length scale is adopted at the constitutive level. Moreover, discontinuous approaches use explicit crack demonstrations, and therefore allow material separation to be reproduced as a geometrical discontinuity.

4.2.2.1 Computational techniques for crack propagation modeling

A few computational approaches are described here to evaluate the fracture behavior of tubes subjected to detonation loading. Ductile fracture is always accompanied by a significant amount of plastic deformation, while brittle fracture is characterized by very little plastic deformation. If the size of the damage zone is small, the concept of linear elastic fracture mechanics can be applied to the problem [2]. The assumption of linear elasticity is not correct for large damage zone conditions; hence, non-linear formulations must be used. In circumstances where crack-tip plasticity is widespread, crack-tip opening displacement and angle (CTOD and CTOA) are the appropriate parameters to model fracture. CTOA can be defined by nodal displacements normal to the crack plane [94] as follows:

\[
CTOA = 2\tan^{-1}(\frac{CTOD}{2x}) 
\]

In this scenario, \(x\) is the distance from the crack front where CTOA and CTOD are calculated. This approach operates by advancing the crack front by a prescribed distance when the CTOA reaches a critical value. The critical CTOA is a measure of fracture toughness. Further information on using CTOA for crack propagation can be found in [2, 94].

Single-parameter simulation of ductile crack growth may yield inaccurate results when there occurs widespread yielding [95]. A well-established solution to this problem is to use the cohesive zone (CZ) model, which needs two fracture mechanics parameters. This model has been shown to be useful when incorporated within interface elements owing to its computational efficiency. The cohesive damage law is represented by the traction-separation law; the cohesive interface element has three parameters of cohesive fracture energy \(\Gamma_c\), cohesive strength \(\sigma_c\), and critical separation \(\delta_c\). The last two parameters are independent, and cohesive fracture energy can be extracted from them as

\[
\Gamma_c = \exp(10) \sigma_c \delta_c 
\]

In general, cohesive interface elements consist of two surfaces connecting the faces of two adjacent solid elements. Crack propagation occurs when the energy release rate reaches a critical value \(G_c\). Cohesive strength was selected as \(\sigma_c = 2\sigma_{ys}\) using the procedure outlined by Li & Siegmund [96], where \(\sigma_{ys}\) is the yield strength of the bulk material. To obtain accurate results on crack modeling using the CZ model, a minimum number of elements is required to accurately simulate variations within the CZ [97]. More information on the implementation of cohesive elements in commercial software packages can be found in [98, 99].

Another approach that can be used to analyze material failure under impulsive loading is the competitive interaction between the effects of strain rate hardening, strain hardening, and thermal softening [100]. Considering the high strain rate behavior of detonation loading, a Johnson–Cook type constitutive relation can be used to define the mechanical behavior of the tube [101]:

\[
\sigma = (\sigma_0 + E\dot{\varepsilon}) \left( 1 + g \ln \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right) \left( 1 - \alpha \frac{T - T_0}{T_0} \right)
\]

Here, \(\sigma\) and \(\dot{\varepsilon}\) are the effective stress and strain rate, respectively, \(T\) is the material temperature, \(\sigma_0\) is the reference stress, \(\dot{\varepsilon}_0\) is the reference strain rate, and \(T_0\) is material temperature. Terms \(E\), \(g\), and \(\alpha\) are material constants characterizing strain hardening, strain rate hardening, and temperature softening, respectively.

Metals often fail in high strain rate conditions by adiabatic shear band formation. Based on this fact, a rate-dependent failure criterion can be obtained from the instability analysis of Eq. (18) as follows:

\[
\left( \frac{\sigma_0}{E} + \varepsilon \right) \left( 1 + g \ln \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right) \left( A - \frac{\beta E}{\rho C} \varepsilon \right) = 1
\]

where \(\rho\) is density, \(C\) is the specific heat capacity of the material, \(\beta\) is the transformation coefficient of heat energy, and constant \(A\) characterizes the state of adiabatic shear.
In the cracking particle method (CPM), a discrete crack can be introduced by adding enrichment functions to the model. In this case, a pre-defined cracking criterion can be met at the particle level [102]. The crack or discontinuity length depends on the domain size of the particle enrichment function. Once a particle is cracked, tractions over discontinuity surfaces depend on the relative displacement between them. Therefore, a cohesive traction can be considered as a function of relative displacement of the crack in the normal and tangential directions. Further information on this approach can be found in [102].

Many reports can be found in the literature on other computational approaches for detonation-driven fracture modeling, such as smoothed particle hydrodynamics [103, 104], the extended finite element method [105, 106, 107, 108], and the mesh-free method [109]. Table 7 describes the advantages and disadvantages of different computational techniques that have been used for detonation-driven fractures.

**Table 7.** Computational techniques for detonation-driven fracture modeling – advantages and disadvantages

<table>
<thead>
<tr>
<th>Method</th>
<th>Advantage</th>
<th>Disadvantage</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTOA [110]</td>
<td>It requires only a single parameter (critical CTOA) to model the fracture process.</td>
<td>Only initially flawed structures can be analyzed using this method.</td>
</tr>
<tr>
<td>CZ model [111]</td>
<td>It leads to very good results for structures with different size and constraint conditions.</td>
<td>Input parameters should be calibrated due to deviation between the theoretical and actual material properties of the adhesive.</td>
</tr>
<tr>
<td>Strain rate hardening [100]</td>
<td>The two parameters of the cohesive model are assumed to be constant and thus can easily be determined by parameter fitting.</td>
<td>Increased computational time due to time-consuming non-linear analysis.</td>
</tr>
<tr>
<td>CPM [112]</td>
<td>The presence of an initial crack is not essential for this model.</td>
<td>Analytical approaches for elements of Eq. (13) are inflexible and fit more or less only specific classes of materials.</td>
</tr>
<tr>
<td>XFEM [113]</td>
<td>The model offers removal of elements from the mesh as a result of tearing or ripping of the structure.</td>
<td>This method uses a Lagrangian description of motion, which means that the points follow material motion. Therefore, it might be a disadvantage if material motion contains steep gradients or discontinuities.</td>
</tr>
</tbody>
</table>

### 4.2.2.2 Simulations

Coupled simulations addressing crack propagation modeling in tubes subjected to detonation loading are described in this section. Cirak et al. [23] and Deiterding et al. [27] performed large-scale FSI simulation of deformation and fracture in experimental tubes using the methodology described in [7]; their results show some inconsistencies with the experiment results, mainly in terms of crack growth magnitude, speed, and premature cracking of flap edges [114]. Rabczuk et al. [115] modeled detonation-driven fracture of tubes using the FSI approach in which both the fluid and structure were modeled by a mesh-free formulation. Du et al. [30, 33, 101] presented a coupled smoothed particle hydrodynamics-FE method to simulate fracture in thin-walled tubes subjected to internal detonation. Wang et al. [116] used computational fluid dynamics along with either element deletion or an extended finite element method to model dynamic crack propagation in experimental tubes described in [7]. Du et al. [117] presented a coupled FSI method to investigate the dynamic fracture of elbow pipes subjected to internal hydrogen-oxygen detonation. The initial flaws were through-thickness cracks located at three different locations.
called extrados, crown, and intrados. More recently, Du et al. [118] investigated pipe fracture under gaseous detonation using a coupled fluid-structure-fracture approach.

Figure 13 shows the fracture comparison in [101] for a load of 745 g of trinitrotoluene. It was found that the crack does not extend for very long after its bifurcation in decoupled simulations; however, the coupled SPH-FEM simulation resulted in a fracture morphology, which agreed well with the experimental results. For details on the FSI concept, its applications, and different approaches and formulations, please refer to [119, 120, 121].

Figure 13. Fracture comparison between simulations and experiment [101]: (a) decoupled simulation, (b) coupled SPH-FEM simulation, and (c) experiment [53].

Several studies have also been conducted on decoupled simulations. Chao [1] simulated crack propagation in a tube under detonation load; however, his results exhibited some inconsistency with experimental results, especially in terms of crack growth speed. Mirzaei & Karimi performed FE simulations on detonation-driven fracture of a thin tube using the crack tip opening angle [122], see Eq. (16). In another study, Mirzaei et al. [95] carried out FE simulations on an exploded gas cylinder; their results showed that the self-similar growth of the initial axial crack in the cylinder was a fatigue-type incremental growth governed by structural waves. A mesh-free shell formulation was developed for finite strains as well as plasticity and crack growth modeling by Ranczuk et al. [123]. Song & Belytschko developed a method to predict dynamic crack propagation in shells with explicit FE methods [124] while Gato studied detonation-driven dynamic fracture in thin-walled shells using a mesh-free method [84]. Liu [125] analyzed dynamic fracture in thin-walled structures upon impact loading using a meshless smoothed particle hydrodynamic shell formulation based on the Mindlin-Reissner theory. Larsson et al. [126] numerically modeled dynamic fracture in a cylinder under TNT-induced blast loading using the XFEM approach; numerical simulations of fracture in a detonation tube using cohesive elements were undertaken by Malekan et al. [127]. Mirzaei et al. simulated catastrophic failure in a CNG fuel tank [128] due to a shock-like moving load and Mostofizadeh et al. [129] presented an XFEM model for dynamic fracture in cylindrical tubes under TNT-induced blast loading. Ma et al. [130] proposed a rate-dependent failure criterion as well as FE analysis to describe adiabatic shear-band propagation in a tube subjected to explosive loads. Rouzegar & Mirzaei [131] studied dynamic crack propagation in a tube under internal moving load using an extended finite element method. Song et al. [132] presented an extended FE method for dynamic fracture in thin-walled structures subjected to detonation load; they implemented the proposed method on discrete Kirchhoff triangular shell elements. Shie [133] used mesh-free formulations for dynamic fracture modeling in thin tubes subjected to detonation loading while Pagani & Perego [134] proposed a computational explicit dynamics FE approach for fracture simulation in thin shells using directional cohesive interface elements, in which solid-shell elements were used for discretization. Malekan [135] investigated crack propagation in a tube under internal detonation using cohesive elements and compared their experimental results with literature values. Mirzaei et al. [136] investigated the role of stress waves in the ductile fracture of cylindrical tubes using a combination of computational and experimental analyses. Meanwhile, Malekan & Cimini [137] numerically modeled the effect of composite patches made of either glass or boron/epoxy on the response of a tube fractured under an internal dynamic moving load and more recently, Eskandari [138] described the linear elastic response of a cracked tube coated with a functionally graded material (FGM) and subjected to a detonation load. Table 8 summarizes these numerical investigations and describes the software/codes as well techniques used in these studies.

Table 8. Summary of numerical investigations conducted on fractured tubes

<table>
<thead>
<tr>
<th>Reference</th>
<th>Year</th>
<th>Software/Code</th>
<th>Technique</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chao [1]</td>
<td>2004</td>
<td>LS-DYNA</td>
<td>Elements were set to fail at a failure strain of 12% and failure stress of 310 MPa</td>
</tr>
</tbody>
</table>
Figure 14(a) shows the type of software/codes used by researchers for fracture modeling of detonation tubes. The usage of commercial codes has increased in recent years compared to in-house codes. This is mainly due to commercial code development either by adding new features or user-written subroutine development. In addition, the type of simulations (coupled and decoupled) for two periods has been shown. It can be seen in Figure 14(b) that the number of FSI simulations increased by more than 200%, while decoupled simulations increased by 185%. This is due to developments in FSI formulations as well as their implementation either in in-house or commercial codes.
Figure 14. Distribution of investigations for two periods. (a) Type of software/code utilized and (b) type of simulations

Table 9 presents the loading properties that have been used in the numerical analyses studies detailed in Table 8, including loading velocity and peak pressure. It can be inferred from this table that the majority of simulations considered a loading velocity of 2000 to 3000 m/s and peak pressure of 2 to 7 MPa. However, there are some cases with higher velocities and peak pressures corresponding to the use of trinitrotoluene. These detonation properties refer to different gaseous or explosive mixtures described in Tables 1 to 3. In addition, there are some investigations with small velocities, which are related to a simpler but not detonation-based loading type.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Maximum Pressure (MPa)</th>
<th>Velocity (m/s)</th>
<th>Tube Length (mm)</th>
<th>Crack Length (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chao [1]</td>
<td>3.3 – 6.1</td>
<td>2376 – 2404</td>
<td>610 – 914</td>
<td>21.9 – 85.4</td>
</tr>
<tr>
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<tr>
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<td>220 – 410</td>
<td>7040</td>
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</table>
Table 10 presents different numerical simulations of experimental tube detonation studied by Chao [1] for different initial flaw sizes. Regardless of some expected differences in the crack path, the numerical simulation results were in good agreement with the experimental results. One reason for the mismatch between numerical and experimental results is that the mechanical properties of each experimental tube cannot be completely uniform, whereas simulations consider tubes to be perfect materials.

Table 10. Numerical simulations and comparison with experimental results

<table>
<thead>
<tr>
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<tr>
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<td>Shie [133]</td>
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<tr>
<td>Du et al. [30]</td>
<td></td>
</tr>
</tbody>
</table>
Figure 15 shows a number of actual propagation characteristics including flap bulging in a detonation tube, where propagation at the front tip (f) is more than that at the back tip (b). This difference is due to a pressure difference between the two crack sides; therefore, the front tip experienced a higher pressure most of the time. However, there is a major inconsistency between the FE simulation results in Figure 15 and the actual crack growth reported in literature; the crack propagation speed observed in simulations was two times greater than the actual average propagation speed for self-similar growth. This is because simulations were conducted without considering the retardation effects of cyclic crack growth [95].

An important aspect of detonation-driven fracture is the cyclic bulging of crack flaps [1], which occurs due to the vibrational motion of crack surface points. Detonation load produces both transverse and longitudinal structural waves, with transverse waves being stronger. These waves remain in the tube even after the detonation front leaves the tube. Further, they produce radial and vibrational motion that results in a vibrational strain in the circumferential direction and increases the bulge area. As a result of bulge-area extension, large tensile stresses can occur in this region in the longitudinal direction of the tube. Because cracks usually have a tendency to propagate perpendicular to the maximum value of principal stress, the initial self-similar crack propagation changes into circumferential crack propagation by curving around the bulged region. Due to symmetry, if the energy release rate of the crack is high enough to support two crack fronts, then branching at this point is also possible. This effect was presented with both experimental and numerical results by Mirzaei et al. [136] (Figure 16).
Figure 16. First principal stress distribution (MPa) along with crack growth for a steel DT with $V_{ij} = 7040$ m/s, $P_1 = 0$, $P_{ij} = 410$ MPa, $P_3 = 70$ MPa, and $T = 0.003$ ms. The tube length and initial crack were 500 mm and 20 mm, respectively.

5. Conclusions

This paper presents a comprehensive review of the deformation and fracture of tubes subjected to detonation loading. Experimental, numerical, and relevant analytical investigations were described in detail. Both experimental and numerical investigations analyzed tube detonation from the deformation and fracture perspectives, while analytical studies are related to detonation tubes without cracks. The first experimental study on tube structural response was conducted in 1997 (Hinkey et al. [34]), and it was only seven years later that experimental investigations were conducted on the deformation and fracture of pre-flawed tubes subjected to detonation loading [1, 7, 9]. Subsequently, many experimental and numerical studies have been published on the deformation and fracture of detonation tubes. Aside from numerical analysis of the deformation and fracture of DTs, there are various analytical solutions and numerical thermal analyses of tubes under detonation loading.

Generally, initial cracks in detonation tubes (and similar structures) are formed by excessive local shear stresses. Subsequently, dynamic internal pressure due to detonation loads induces stresses that might cause either crack formation or propagation. This crack propagation can result in partial fragmentation of the tube, as reported in both experimental and numerical works. Currently, commercial software packages cannot directly model both crack initiation and propagation in detonation tubes, and mostly they require implementation of specific techniques in the modeling process and/or additional coding efforts for loading simulation. Therefore, many in-house codes have been developed to model these kinds of problems with ease (similar to those presented in section 4.2.2.2).

Current experimental studies on detonation fracture are limited due to their high cost and the risk of explosions, and only a few studies concentrated on crack growth rate rather than crack characteristics. Therefore, further experimental studies are needed to fill this gap. In addition, various experimental investigations were conducted on metal tubes to study deformation or fracture behavior under detonation, but there are no experimental works available on orthotropic materials. Therefore, tubes made of composites can be used as test tubes, with and without initial cracks, subjected to detonation loads. Another interesting topic for experimental analysis is the effect of repair patches in metal/composite materials on tube fracture. Various composite patches (made of high-strength materials) can be used to study the effectiveness and characteristics of patches with respect to crack-growth resistance. This can...
also be accomplished under single and multiple detonation loadings to obtain patch responses under different loading conditions.

From the numerical perspective, although different works modeled detonation-driven fracture using the FSI approach [23-30, 114-118], the majority of these results do not match well with the experimental results. However, coupled investigations delivered results closer to experimental studies when compared to decoupled investigations. Further, all these studies only considered single detonation. Therefore, fluid-structure interaction modeling of tubes with and without initial crack under sequential detonation loading is another topic that needs to be investigated. This can be accompanied by experimental investigations under similar conditions to validate and provide reliable numerical models for DTs under sequential detonation loads.

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References


Highlights

- A comprehensive and thorough review of detonation-driven deformation and fracture analysis in tubes
- Quantitative investigation of experimental and numerical studies
- Comparative study between investigations in terms of methodology, loading characteristics, and geometry specifications
- Pointing out the research gaps and potential future research directions