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# MULTICARRIER RADAR-COMMUNICATIONS WAVEFORM DESIGN FOR RF CONVERGENCE AND COEXISTENCE

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## ABSTRACT

RF convergence where the same transceiver is used for communications and sensing purposes is taking place. In this paper, a dual-use radar-communications multicarrier waveform is proposed, where different subcarriers are assigned to different subsystems. Two algorithms for subcarrier assignment and optimal power allocation for the radar and communications subsystems are developed. A compound mutual information (MI) based objective function is used for optimizing the power allocation of each subsystem in both design algorithms. The first proposed design algorithm assumes priority for the radar subsystem and it is called radar selfish design. The second proposed design algorithm is called cooperative design, in which both subsystems are jointly optimized by maximizing a compound MI based objective function.

**Index Terms**— RF convergence, mutual information, waveform optimization, coexistence

## 1. INTRODUCTION

The area of coexistence among radar and communications systems has received plenty of attention from the research community [1–9]. A recent review on different approaches to the problems of coexistence, spectrum sharing and RF convergence available in the literature is given in [10]. It is expected that in the future RF convergence will be a key enabling technology in radar-communications coexistence. Different approaches to RF convergence have been proposed in the literature. For example, the tasks of the two subsystems are performed sequentially [11] or simultaneously, by leveraging different degrees of freedom in frequency [2], antenna elements [12] or radiation patterns [13, 14]. Another approach is to embed communication symbols into the radar waveform as in [5–7, 15]. Multicarrier waveforms are particularly suitable for RF convergence since most modern communications systems use these and they also bring many desirable properties for radar systems [16, 17].

In this paper a multicarrier waveform is assumed, for which interleaved subcarriers or subsets of subcarriers can be assigned to the radar or the communications tasks. Each subsystem is assigned distinct subsets of subcarriers from the total available subcarriers, as illustrated in Fig. 1. In a similar fashion, a spectrally interleaved OFDM system concept, with applications in radar network or MIMO radar, is proposed in [18]. Each transmitter would be allocated a subset of subcarriers such that overall each transmitter still uses the full bandwidth of the system. It is claimed in [18] that the blank subcarriers in the signal spectrum do not affect the radar sensing performance. A dual-use radar-communications waveform is developed in this paper for RF convergence. Mutual information (MI) has been employed for radar waveform optimization in several works, for example in [19–22]. Commonly, it is employed for target characterization or classification tasks, which could benefit from maximized

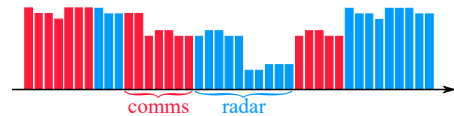


Fig. 1. Subcarrier assignment for radar or communications use.

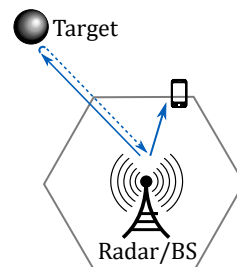


Fig. 2. Considered RF convergence system configuration.

MI between the received signal and target impulse response. In [23] based on [24] it is mentioned that, for a fixed probability of false alarm, MI maximization is related to the maximization of the probability of detection. Other optimization criteria such as probability of detection maximization [25] or CRB minimization [26] can be considered, however the optimization problems are no longer convex. In this paper a compound MI based objective function is formulated for subcarrier assignment and optimum power allocation to the radar and communications subsystems. Two algorithms for the design of the proposed dual-use waveform are also introduced, one called radar selfish design and one cooperative design. Also in [27] MI has been employed for a joint radar-communications system. However, therein all subcarriers are used for both tasks simultaneously and the objective function is formulated with respect to the maximized MI of the individual task.

**Notation:** A lower case bold letter  $\mathbf{x}$  denotes a column vector, while a capital bold letter  $\mathbf{X}$  denotes a matrix and  $\mathbf{X}^H$  its Hermitian transpose. The  $l$ th element in a vector is denoted  $\mathbf{x}[l]$ . By  $I(\cdot)$  we denote the MI between two random variables and by  $\|\mathbf{x}\|_0$  the  $\ell_0$  norm — the number of non-zero entries of the vector  $\mathbf{x}$ .

## 2. SYSTEM AND SIGNAL MODELS

A joint radar-communications system intended for RF convergence is considered in this paper. The assumed coexistence and spectrum sharing co-located configuration is presented in Fig. 2. A multicarrier waveform, OFDM in this paper, is used by the system for both purposes. The dual-use radar-communications waveform is reflected back from the target, for radar purpose, and received at a communications receiver, for data transmission.

Using the generalized multicarrier radar (GMR) model introduced in [28] and the special case for OFDM waveform with  $N$  subcarriers, the dual-use waveform proposed in this paper can be modeled as follows:

$$\mathbf{x} = \mathbf{F}^H [\mathbf{W}\mathbf{r} + (\mathbf{I} - \mathbf{W})\mathbf{c}], \quad (1)$$

where vector  $\mathbf{r}$  of size  $N$  contains the frequency domain transmitted radar symbols and vector  $\mathbf{c}$  of size  $N$  contains the frequency domain transmitted communication symbols. The diagonal matrix  $\mathbf{W}$  of size  $N \times N$  is a subcarrier selection matrix whose elements take the values  $\{0, 1\}$ , while matrix  $\mathbf{F}^H$  is the inverse discrete Fourier transform (IDFT) matrix.

The transmitted waveform passes through one radar channel  $h_r$ , which contains the target, and one communications channel  $h_c$ , between the joint transceiver and the communications receiver. The transmitted radar and communications symbols are assumed deterministic, while the radar and the communications channel impulse responses are assumed to be wide sense stationary (WSS) Gaussian processes with known second order statistics. These are similar assumptions as in [20, 29]. Also, at both subsystem receivers, additive zero mean white Gaussian noise is assumed. Consequently, the signals at the radar and the communications receiver respectively can be written using a matrix formulation as follows:

$$\begin{cases} \mathbf{y}_r = \mathbf{X}_r \mathbf{h}_r + \mathbf{X}_c \mathbf{h}_c + \mathbf{n} \\ \mathbf{y}_c = \mathbf{X}_r \mathbf{h}_c + \mathbf{X}_c \mathbf{h}_c + \mathbf{m}, \end{cases} \quad (2)$$

where  $\mathbf{X}_r$  and  $\mathbf{X}_c$  are Toeplitz matrices. These matrices can be approximated by circulant ones since their dimensions are sufficiently large [30]. Circulant matrices are diagonalized by unitary DFT matrices. The complex noise vectors  $\mathbf{n}$  and  $\mathbf{m}$ , the complex radar channel taps  $\mathbf{h}_r$  and communications channel taps  $\mathbf{h}_c$  are all assumed zero mean Gaussian random vectors with known covariance matrices:  $\sigma_n^2 \mathbf{I}$ ,  $\sigma_m^2 \mathbf{I}$ ,  $\mathbf{C}_{h_r}$  and  $\mathbf{C}_{h_c}$  respectively.

### 3. DUAL-USE SIGNAL DESIGN

The chosen criteria for subcarrier selection is based on mutual information (MI). This criteria has been employed for both communications [31–33] and radar [19, 20, 22] waveform design. The radar subsystem wishes to maximize the MI between the received target reflected signal and the impulse response of the target. The communications subsystem wishes to maximize the MI between the received signal at the communications receiver and the transmitted signal. Consequently, a compound objective function is considered in this paper. The MI based compound objective function is formulated as follows:

$$I(\mathbf{y}_r; \mathbf{h}_r) + I(\mathbf{y}_c; \mathbf{x}), \quad (3)$$

which may be written as shown in our earlier work [20, 29]:

$$\begin{aligned} & \frac{1}{2} \left[ \sum_{k=0}^{N-1} \log \left( 1 + \frac{\mathbf{w}^2[k] |\mathbf{r}[k]|^2 \sigma_{h_r}^2[k]}{\mathbf{u}^2[k] |\mathbf{c}[k]|^2 \sigma_{h_r}^2[k] + \sigma_n^2} \right) \right. \\ & \left. + \sum_{k=0}^{N-1} \log \left( 1 + \frac{\mathbf{u}^2[k] |\mathbf{c}[k]|^2 \sigma_{h_c}^2[k]}{\mathbf{w}^2[k] |\mathbf{r}[k]|^2 \sigma_{h_c}^2[k] + \sigma_m^2} \right) \right], \end{aligned} \quad (4)$$

where  $\sigma_{h_r}^2[k]$  and  $\sigma_{h_c}^2[k]$  are the gains of the corresponding radar and communications channels on  $k$ th subcarrier and  $\sigma_n^2$  and  $\sigma_m^2$  are the noise powers at the radar and communications receivers respectively. Also,  $\mathbf{w}[k]$  and  $\mathbf{u}[k]$  are the weights for the  $k$ th subcarrier of the radar and communications subsystem respectively, given by the diagonal elements of matrices  $\mathbf{W}$  and  $\mathbf{U} = \mathbf{I} - \mathbf{W}$ . These select the active or inactive subcarriers. As these take values only in  $\{0, 1\}$

and for any subcarrier  $k$  only  $\mathbf{w}[k]$  or  $\mathbf{u}[k]$  is non-zero, the objective can be simplified to:

$$\begin{aligned} & \frac{1}{2} \left[ \sum_{k=0}^{N-1} \log \left( 1 + \frac{\mathbf{w}[k] |\mathbf{r}[k]|^2 \sigma_{h_r}^2[k]}{\sigma_n^2} \right) \right. \\ & \left. + \sum_{k=0}^{N-1} \log \left( 1 + \frac{\mathbf{u}[k] |\mathbf{c}[k]|^2 \sigma_{h_c}^2[k]}{\sigma_m^2} \right) \right]. \end{aligned} \quad (5)$$

The objective function in (3) can be generalized and a weighting factor can be consider to each of the components.

Two dual-use waveform design strategies are proposed in the following. First, a radar selfish design is introduced, where the radar subsystem is designed first to achieve the best possible performance and may have access to all  $N$  subcarriers. The number of subcarriers for the radar subsystem is then minimized in a second step while allowing for a controlled MI decrease. Subcarriers with the lowest channel gain will be unassigned from the radar subsystem and more power would be allocated to the ones with higher channel gain. The communications subsystem is then allocated also the spared subcarriers from the radar subsystem and it is optimized as well. Next, a cooperative design is developed, where subcarriers are first assigned to either subsystem such that the compound objective function in (5) is maximized. After that, each subsystem is optimized. A communications selfish design can also be employed similarly to the radar selfish one. However, this is not explored in this paper.

#### 3.1. Radar selfish design

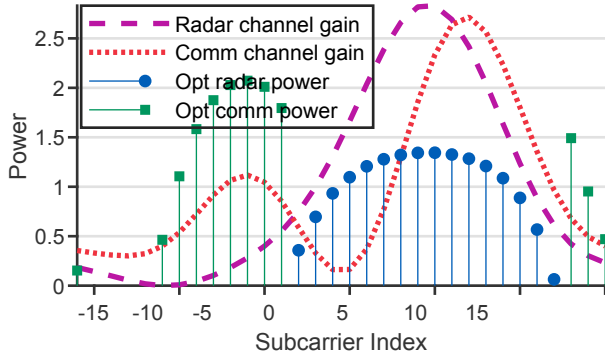
For this design strategy the radar waveform is first optimized based on MI maximization, similarly to [19–21, 29]. At this stage, it is assumed that the weights  $\mathbf{w}[k]$  are all 1 and the optimum power allocation for the radar subsystem is obtained based on MI maximization. As the optimization is performed for the radar subsystem only, the second sum in (5) can be ignored. Consequently, the optimization problem for the radar subsystem may be formulated as follows:

$$\begin{aligned} & \underset{\{|\mathbf{r}[k]|^2\}}{\text{maximize}} && \sum_{k=0}^{N-1} \log \left( 1 + \frac{\mathbf{w}[k] |\mathbf{r}[k]|^2 \sigma_{h_r}^2[k]}{\sigma_n^2} \right) \\ & \text{subject to} && \sum_{k=0}^{N-1} \mathbf{w}[k] |\mathbf{r}[k]|^2 \leq P_T. \end{aligned} \quad (6)$$

After the optimal power allocation for the radar subsystem is obtained, the subcarriers which are allocated no power, due to very low channel quality, are assigned to the communications subsystem. Then, the optimization of the communications subsystem is formulated in a similar fashion, using the following optimization problem:

$$\begin{aligned} & \underset{\{|\mathbf{c}[k]|^2\}}{\text{maximize}} && \sum_{k=0}^{N-1} \log \left( 1 + \frac{\mathbf{u}[k] |\mathbf{c}[k]|^2 \sigma_{h_c}^2[k]}{\sigma_m^2} \right) \\ & \text{subject to} && \sum_{k=0}^{N-1} \mathbf{u}[k] |\mathbf{c}[k]|^2 \leq P_T. \end{aligned} \quad (7)$$

It is observed from (6) and (7) that the same total power constraint is imposed to both subsystems. Both (6) and (7) can be solved exactly, for example as in [28], using their Lagrangian form and the Karush-Kuhn-Tucker (KKT) conditions [34]. The solutions to (6) and (7) are water filling solutions, where more power is allocated to subcarriers with higher channel gain and lower noise and interference power. Due to lack of space the exact power allocation solutions are not analytically derived here. An example of optimal power allocation is illustrated in Fig. 3. In the next step, the goal is to minimize the number of subcarriers assigned to the radar subsystem while controlling the reduction from the maximal MI. This will likely improve



**Fig. 3.** Optimal power allocation for radar and communications subsystems. The radar system is optimized first and then the communications system using the spare subcarriers. Power is allocated to each subsystem to subcarriers with higher channel gain.

the performance of the communications subsystem. Consequently, the problem of minimizing the number of subcarriers assigned for the radar subsystem can be formulated as follows:

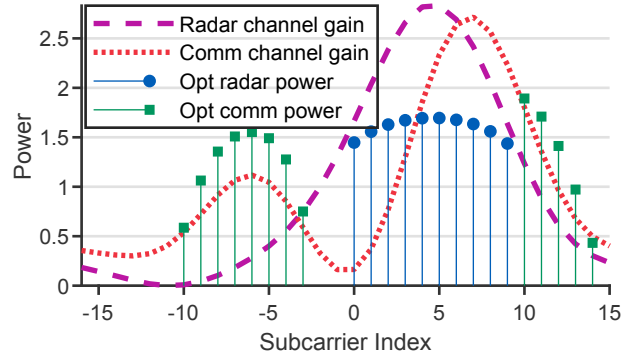
$$\begin{aligned} & \underset{\{\mathbf{w}[k]\}}{\text{minimize}} && \|\mathbf{w}\|_0 \\ & \text{subject to} && \sum_{k=0}^{N-1} \log \left( 1 + \frac{\mathbf{w}[k]|\mathbf{r}[k]|^2 \sigma_{h_r}^2[k]}{\sigma_n^2} \right) \geq t \\ & && \mathbf{w}[k] = \{0, 1\}, \forall k = 0 \dots N-1, \end{aligned} \quad (8)$$

where  $|\mathbf{r}[k]|^2$  are the optimal radar powers obtained in the previous step from (6) and  $t$  is a constraint on the minimum allowed radar MI. In this paper  $t$  is considered 5 – 25% smaller than the maximum MI initially obtained from solving (6) with  $\mathbf{w} = \mathbf{1}$ . This optimization problem is non-convex, however the best convex approximation can be used instead:

$$\begin{aligned} & \underset{\{\mathbf{w}[k]\}}{\text{minimize}} && \sum_{k=0}^{N-1} \mathbf{w}[k] \\ & \text{subject to} && \sum_{k=0}^{N-1} \log \left( 1 + \frac{\mathbf{w}[k]|\mathbf{r}[k]|^2 \sigma_{h_r}^2[k]}{\sigma_n^2} \right) \geq t \\ & && 0 \leq \mathbf{w}[k] \leq 1, \forall k = 0 \dots N-1, \end{aligned} \quad (9)$$

and the exact vector  $\mathbf{w}$  can be obtained by rounding the solution for (9) to one of the values  $\{0, 1\}$ . After the new vector  $\mathbf{w}$  is obtained, the optimization problems (6) and (9) are solved again. This procedure is repeated until vector  $\mathbf{w}$  does not change from one iteration to another. Next, the subcarriers assigned to the communications subsystems are obtained as  $\mathbf{u} = \mathbf{1} - \mathbf{w}$  and the power allocation for the communications subsystem is optimized using (7). Algorithm 1 summarizes the radar selfish design strategy for the proposed dual-use waveform. An example of the final power allocation for both subsystems is illustrated in Fig. 4.

The trade-off between reduced maximum MI for radar and increased maximum MI for communications is evaluated next using simulations. For the communications subsystem the maximum MI is directly related to the capacity [35]. On the other hand, it may be difficult to quantify the performance loss in a radar task caused by a reduced maximum MI. Nevertheless, MI maximization is connected to minimum mean square error (MMSE), see [22, 36]. Also, it was shown in [25] that a slight reduced detection performance is expected when waveforms and power allocation optimized based on MI are employed in a NP detector. It is shown in Fig. 5 how the maximum MI of each subsystem has changed from the first to the



**Fig. 4.** Optimal power allocation for both subsystems after reducing the number of subcarriers for the radar subsystem.

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#### Algorithm 1: Radar selfish design

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**Result:** Optimal power allocation ( $\mathbf{p}_r$  and  $\mathbf{p}_c$ ) and subcarrier selection ( $\mathbf{w}$  and  $\mathbf{u}$ )

Assume  $\mathbf{w} = \mathbf{1}$  and solve (6) for initial optimum radar power allocation  $\mathbf{p}_r$ ;

Based on  $\mathbf{p}_r$  find new  $\mathbf{w}$ ;

Compute  $t$  using  $\mathbf{w}$  and  $\mathbf{p}_r$ ;

**while** no change in  $\mathbf{w}$  **do**

    Solve (9) and round to  $\{0, 1\}$  for new  $\mathbf{w}$ ;

    Solve (6) for new  $\mathbf{p}_r$ ;

**end**

Find  $\mathbf{u} = \mathbf{1} - \mathbf{w}$ ;

Using solve (7) for optimum communications power allocation  $\mathbf{p}_c$ ;

---

last step of Algorithm 1. The average maximum MI change is shown in Fig. 5 for 500 different channel realizations and a number of subcarriers  $N = 32, 64, 128$ . It is observed that small decrease in radar maximum MI allows for a larger increase in communications maximum MI. Thus it can be concluded that it is worth taking a small loss in radar performance for a potential much larger gain for the communications subsystem. This result is consistent for different number of available subcarriers.

### 3.2. Cooperative design

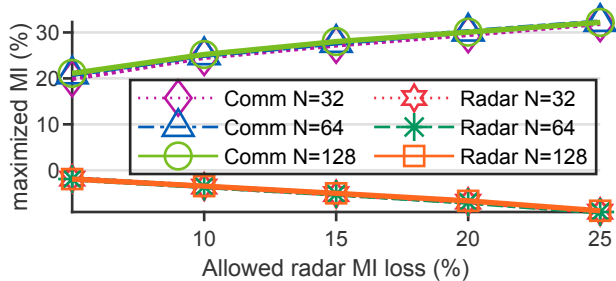
For this design strategy the available subcarriers are assigned for the radar or the communications subsystem based on maximizing the compound objective in (5). Considering uniform power over the subcarriers for both radar and communications subsystems and the fact that for any given  $k$ th subcarrier either  $\mathbf{w}[k]$  or  $\mathbf{u}[k]$  is non-zero, the compound objective function in (5) can be simplified to:

$$\frac{1}{2} \left[ \sum_{k=0}^{N-1} \log \left( 1 + \frac{\mathbf{w}[k] \sigma_{h_r}^2[k]}{\sigma_n^2} + \frac{\mathbf{u}[k] \sigma_{h_c}^2[k]}{\sigma_n^2} \right) \right]. \quad (10)$$

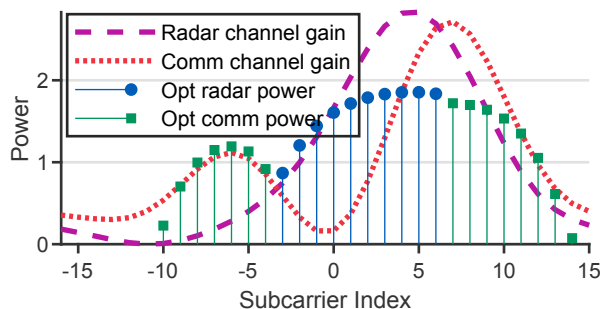
It can be found that the optimum  $\mathbf{w}[k]$  and  $\mathbf{u}[k]$  which maximize the objective in (10) are given by:

$$\begin{cases} \text{If } \frac{\sigma_{h_c}^2[k]}{\sigma_m^2} > \frac{\sigma_{h_r}^2[k]}{\sigma_n^2} & \text{then } \mathbf{w}[k] = 0, \mathbf{u}[k] = 1 \\ \text{If } \frac{\sigma_{h_c}^2[k]}{\sigma_m^2} \leq \frac{\sigma_{h_r}^2[k]}{\sigma_n^2} & \text{then } \mathbf{w}[k] = 1, \mathbf{u}[k] = 0. \end{cases} \quad (11)$$

After the vectors  $\mathbf{w}$  and  $\mathbf{u}$  are obtained, the optimal power allocation based on MI maximization is found for both the radar and the communications subsystems. An example of optimal power allocation



**Fig. 5.** The change in maximum MI for each subsystem. It pays off to allow a small decrease in radar maximum MI for a larger communications maximum MI – which translates to a higher capacity. This is valid for waveforms with  $N = 32, 64, 128$  subcarriers.



**Fig. 6.** Optimal power allocation for radar and communications subsystem when the cooperative design is employed.

for both radar and communications subsystems using this cooperative design is shown in Fig. 6.

### 3.3. Radar selfish versus cooperative design

We can quantify how much each subsystem gains in the cooperative design versus the radar selfish design. The final maximum MI for both subsystems obtained using the radar selfish and the cooperative design strategies is shown in Fig. 7. As expected, both subsystems achieve similar maximum MI using the cooperative design. It can be concluded from the results in Fig. 7 that the cooperative design is better for the communications subsystem given the choice of 5 – 25% loss in maximum radar MI. As expected, for the radar subsystem the radar selfish design is more favorable as long as there is not too much loss allowed for its maximum MI. For the simulation results presented in Fig. 7 this point is around 22% loss.

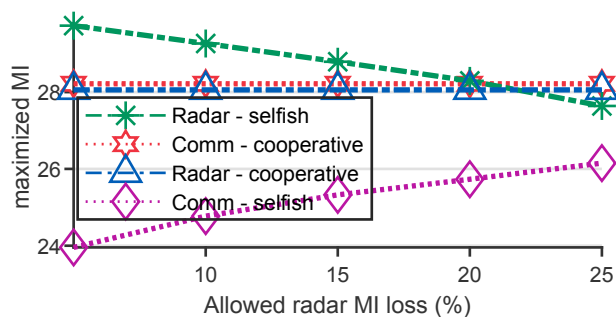
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#### Algorithm 2: Cooperative design

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**Result:** Optimal power allocation and subcarrier selection  
**Steps**

- Assume  $|\mathbf{r}[k]|^2 = |\mathbf{c}[k]|^2 = 1$  and apply (11) for optimum  $\mathbf{w}$  and  $\mathbf{u}$ ;
  - Based on the obtained  $\mathbf{w}$  and  $\mathbf{u}$  solve (6) and (7) for the optimum radar and communications power allocation respectively;
- 



**Fig. 7.** Comparison between the proposed designs for waveforms with  $N = 128$  subcarriers. The cooperative design is better for the communications subsystem, while the radar selfish design is better for the radar subsystem as long as there is not too much loss allowed for its maximum MI.

## 4. CONCLUSIONS

A dual-function radar-communications OFDM waveform is proposed in this paper. Two design strategies for subcarrier assignment and optimal power allocation using MI based criteria are also proposed. The first design strategy is radar selfish, which means that the MI optimal performance for the radar is ensured first. After that, by allowing certain maximal MI loss in the radar subsystem, the number of subcarriers for the radar subsystem is minimized and the spare subcarriers are reassigned to the communications subsystem. The purpose of this is to allow for higher communications data rates. The second design strategy is cooperative, which means that subcarriers are assigned to either subsystem based on the channel gain experienced by the corresponding subcarriers. After that, the optimal power allocation for both subsystems is obtained using MI based criteria. It is shown using simulation results that the cooperative design is better for the communication subsystem, while the radar selfish design is better for the radar one as long as the loss in maximum MI is not too large.

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