Parametric optimization of long-term multi-area heat and power production with power storage

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ABSTRACT

This paper develops a model and optimization method for multi-area heat and power production with power transmission and storage. The objective function of the model is to minimize the operating costs of the system. The model can be used both for planning optimal system operation, and for simulating the effects of extended production, transmission and storage capacity. The proposed parametric decomposition method is fast enough to solve problems with a large number of hourly models. The parametric decomposition method works in two phases. First, the problem is decomposed into hourly local energy production models without storages and transmission. Parametric linear programming analysis is applied to these models for determining the optimal marginal operating costs as a function of power production. In the second phase, the optimal marginal cost functions are encoded as a linear transshipment network model including storages and transmission network. The network model is solved using generic sparse linear programming software. The operation of each production plant is determined based on the network solution. The decomposition method was validated by comparing it against an integrated linear programming model. The decomposition method demonstrates good accuracy and solves yearly models up to 30 times faster than the integrated model.

Key words

Combined heat and power (CHP); optimization; power transmission; power storage; energy efficiency.

1. Introduction

The climate and energy framework targets adopted by the European Union (EU) are a 40% reduction in greenhouse gas emissions, a 27% increase in renewable energy, and a 27% enhancement in energy efficiency by 2030 [1]. Combined heat and power (CHP) production offers savings in energy of between 15-40% in comparison to separate heat and electricity production in conventional power plants and heat only boilers (HOB). CHP is a cost-efficient way to lower CO2 emissions to the environment. Currently, CHP generates 15% of heat and 10% of electricity in the EU, leading to saving of 200 million tonnes of CO2 emissions per
year. By 2030, CHP’s share of electricity production is estimated to be 20% and for heat 25% [2]. By 2016 in Finland, CHP had generated 73% of total heat and 31% of power. However, only 78% of Finnish power consumption was covered by national production while 22% was imported [3]. Improvements in CHP efficiency serves the targets of the EU by reducing production costs and lowering greenhouse gas emissions.

Optimization models can improve CHP efficiency. The CHP models determine heat and power production in the CHP system in order to minimize production costs. Determining the optimal operation of CHP plants along with separate heat and power production is complex due to the coupling of heat and power production. The complexity of CHP models increases when power transmission between different areas must be considered. Often it is possible to assume that the CHP system model is convex, which allows the use of linear programming (LP) models and specialized LP-related algorithms [4]. A specialized Simplex algorithm was formulated to optimize hourly CHP operation as an LP problem. A comparison results proved the improvement in solution time and the model was implemented for commercial use in industry. Considering the non-convex plant characteristics and the unit commitment (on/off states) of plants makes the problem non-convex and very difficult to solve. Such problems have been solved using heuristic techniques, mixed integer linear programming (MILP), and the Lagrangian relaxation technique. The authors of [5] developed a model for the unit commitment of multi-period CHP production. Initial solution in the model was improved, leading to an increasing in the accuracy of production cost. In [6], the unit commitment of power production was solved using a Lagrangian relaxation algorithm while considering ramp constraints. The results showed that the Lagrangian relaxation approach is preferable to MILP for large-scale instances, especially when fast solution time is needed. The ramp constraints limit the power production of the plants between two successive time steps. The impact of power ramp constraint on CHP production was investigated in [7]. A heuristic method was applied to optimize CHP operation with power ramp constraint. Solution time and optimality of the method were evaluated using numerical tests. A MILP model was applied to approximate ramping requirement in operations of a multi-timescale power system with solar power [8]. The authors considered costs of operation, start-up and ramping, and they formed piecewise linear costs curves for economic optimization. They showed that using optimal ramping gives economic benefits. In [9], both cost of energy production and cost of ramping were considered. It showed over 25% cost savings of the total operation cost, when there is 50% of renewable production. In [10], a MILP model was proposed for solving non-convex hourly CHP problems. In that study, a two phase method was applied to first generate single period subproblems, and then to merge them into a multi-period problem. The method was shown to be efficient for short and medium-term problems. In [11], the Lagrangian relaxation technique was applied to a non-convex CHP problem and tested using three benchmark CHP problems. The model optimizes each production unit locally to give maximum profit while satisfying the demand. In [12], a MILP optimizer was used to solve robust CHP commitment with heat storage. Uncertainty in the problem was due to the forecast for heat demand. The selected time horizon was 24 hours and results indicated that the method produces near-optimal solutions. In [13], a non-convex problem was converted into a convex problem to find the minimum average costs of two types of CHP systems. They
studied a conventional natural gas CHP system with a fixed heat-power ratio, and a CHP with renewable energy with an adjustable heat-power ratio. The results demonstrated lower average costs for CHP with renewable production than conventional CHP plant. In [14], a MILP model was applied for power system optimization. The model solved the problem of renewable power systems with different capacities for remote islands finding optimal capacities for renewable power systems. They compared a model with binary variables with an iterative solution process without binary variables. The iterative process solves the problem in a shorter time but near-optimally. MILP models have also been used for day-ahead power price optimization and demand response operation [15]. They reported results for a three-day time horizon for one plant. The model improved the solution time, profit for the plant, and reduced peak power demand.

Day-ahead market optimization simulates the power production and power transmission between regions based on hourly demand and supply curves created in the market for all areas. The deficit area with a higher price produces less power and imports from the supply area at a lower production price [16]. Day-ahead models apply to short-term problems to determine the power trade contracts for the next 24 hours in the market. The short-term operation of a power system with power transmission was modelled using a Lagrangian relaxation method in [17]. The problem was decomposed into sub-problems, and a network flow algorithm solved the economic dispatch problem with environmental and transmission constraints. Results showed that the approach was fast and efficient. A review for the economic optimization of short-term CHP problems was presented, and studies were classified based on different aspects [18].

The climate and energy framework addresses medium or long-term challenges related to security of supply. Power production by renewables has high dependency on meteorological factors [19]. Medium and long-term planning models simulate interactions and behavior of the energy systems. The models also optimize operational planning and investment decisions to provide secure supply to consumers. One technique for medium and long-term models is to decompose them into a large number of hourly models. In [20], a market based multi-region heat and power system was modelled and used to examine the integration of intermittent renewable production into the power market. The model output power transmission between regions in the market, and the results were validated by historical data. The focus of the study was on the effect of Germany’s energy transition on power price in the market. However, the rapid solution of long-term hourly models is important [21]. In [22], a multi-site CHP system was formulated as an LP problem and solved by a two-stage network simplex algorithm. Yearly models (8760 hours) were tested numerically, and results showed that the proposed model was faster than commercial LP codes. However, dynamic constraints were not considered in the model.

Power storages can be used to balance the power production with fluctuating demand and to improve the flexibility of the power supply when it contains a significant fraction of intermittent resources such as solar or wind power. The power storage can store power when power demand and production costs are low, and discharge power from storage when the demand and production costs are high, resulting in reduced overall production costs. The benefit of storage has been studied by different approaches [23], [24], [25], [26], [27], [28].
The simulation of an energy system with 100% renewable power production shows that power storage is necessary to balance the power system [23]. The storage units also impose dynamic dependencies to the models of power systems, emphasizing the importance of the short solution time. Techno-economic analysis of a power storage was used to evaluate the economic feasibility of the storage and to improve its profitability in a power system [24]. By optimizing the storage size, the results implied that the benefit can be improved by increasing the storage size or applying waste heat in the system. Economic optimization of a residential solar system with battery storage was investigated in [25]. This provided a method for the evaluation and sizing of solar production with power storage. The main emphasis in the studies is on power systems; however, CHP plants can improve the flexibility of the power system by adjusting the heat production level.

The authors have earlier developed LP models of a CHP system with heat storage units [26] and [27], and validated the results with two energy optimizers EnergyPLAN [29] and EnergyPRO [30]. The efficiency of the LP model was proved for a smart hybrid renewable energy system for communities by testing the model over three typical weeks in different seasons [26]. The flexibility of the LP model for a CHP with heat storage and benefit of storage were demonstrated in the study [27]. To reduce the computational effort of CHP optimization with seasonal heat storage, a method using MILP model was derived based on aggregated typical operation periods [28]. The results showed a significant improvement in solution time and applicability of the method for long-term storage units.

A review study reported 37 computer tools for energy systems [31]. They categorized the tools based on their applications for different objectives. Different types of applications and analyses can include scenario analysis, planning optimization, and investment optimization. Many of the tools (such as EnergyPLAN, energyPRO, BALMOREL, COMPOSE, BCHP Screening, SIVAEL, etc.) include CHP analysis. They can be classified based on their scenario timeframe and scale of regions (ranging from single building to international).

Based on literature, very few studies proposed medium or long-term models for CHP systems considering both power and heat sectors with power transmission and storage units [20], [22], [32]. Due to increasing supply of intermittent renewables, load balancing is becoming more challenging, and there is a great need of long-term planning models that can handle power storages, transmission, and different production forms including CHP.

In this paper, we develop a novel two-phase parametric decomposition method for long-term planning problems to optimize a multi-area CHP system with power transmission and storage and optional ramp constraints for power production. Local hourly models are formulated, and following parametric analysis, they are encoded into a multi-period network model. The authors have previously proposed a model for a similar problem without power storage [32]. This paper is organized as follows. Section 2 explains the target energy system considered in this study. A generic CHP model with a convex operating region is presented in section 3, and an integrated general LP model is formulated for the multi-period and multi-area CHP system with power transmission and power storage. The decomposition method is developed in section 4. To evaluate the two-phase approach for accuracy and speed, the decomposition method is compared with the integrated model in section 5. Section 6 presents a conclusion, discussion, and suggestions for future research.
2. Target energy system

We consider an energy system consisting of multiple areas with heat and power production and demand. We assume that heat can be freely distributed within each area, but not between areas. On the other hand, power can be transmitted between areas. Figure 1 illustrates the structure of the energy system within each area. Each area can include any number of production units, including CHP plants, power-only plants and HOBs. At least one CHP plant should be present; otherwise it is not necessary to combine heat and power production planning. The power-only plants can include thermal power plants using different fuels, such as nuclear, fossil fuel, biomass, and also non-thermal production such as hydropower, photovoltaic or wind power. Heat production must match the heat demand locally. During periods of low power price or low heat demand, the CHP should be shut down, and then HOB plants can be used to produce heat. In Finland, volatile and low power price levels have led to a situation where some CHP plants will be replaced with new installations of HOBs by 2040 [33]. Power production must satisfy the power demand, the net amount of power transmitted out/from the area, and charge/discharge of the power storage. The area with the lowest marginal production cost produces more power and transfer to the area with higher marginal costs. Power storages can improve the efficiency and flexibility of the energy system by storing power during periods of low demand and discharging during periods of peak demand and high power price. Power storages can also lower the overall operating costs of the energy system.

Fig. 1. Schematic of the energy production problem in each area

Dependencies between subsystems require modelling and solving all components together. A CHP links heat and power production together, transmission lines link different areas together and storage links subsequent time steps (hours) together into a dynamic model. The objective is to minimize the variable production costs of the CHP system with power transmission and power storage over a given time horizon. The time horizon can be anything from short-term, i.e. several hours or days, to long-term, i.e. several months, a year or even longer.
3. Energy system modelling

3.1 Convex production unit model

In the following, we present a convex CHP plant model, which can also be applied for other production technologies. A similar extreme point based modelling technique was introduced in [4]. The characteristic operating region of a running CHP plant is a 3-dimensional surface \((p, q, c)\) where the \(p\)-coordinate is power production, \(q\) is heat production and \(c\) is (variable) operating costs corresponding to the power and heat production amount. The operating costs are mainly due to fuel consumption, but may also include variable operations and maintenance (O&M) costs. The 3D operating region can be represented in terms of a number of extreme characteristic operating points \((p_j, q_j, c_j)\). Assuming that the characteristic operating region is piecewise linear and convex, we can represent it using an LP model. Figure 2 illustrates such a convex CHP characteristic and its projection on the \((p, q)\) plane. Note that both the projection on the \((p, q)\) plane is convex and the operating cost is a convex function of \(p\) and \(q\). In practice, the characteristic of many kinds of CHP plants can be assumed to be convex. Convexity can also be assumed for non-convex CHP plants that can run in different modes for fractions of an hour. When the plant is not running, that corresponds to a characteristic point \((0, 0, 0)\), i.e. no heat or power is produced and no variable operating costs are incurred. Including the off-state into the characteristic can make the characteristic non-convex. However, in this paper we assume that the unit commitment (on-off states for each hour) of production plants is pre-determined, which means we can assume convex characteristics.

![A convex CHP plant operating region](image)

Fig. 2. A convex CHP plant operating region [32]

A convex CHP characteristic can be modelled efficiently as a convex combination of the characteristic extreme points \((p_j, q_j, c_j)\) in terms of linear constraints:
\[ C = \sum_{j \in J_u} c_j x_j \]
\[ P = \sum_{j \in J_u} p_j x_j \]
\[ Q = \sum_{j \in J_u} q_j x_j \]
\[ \sum_{j \in J_u} x_j = 1 \]
\[ x_j \geq 0 \]

Here the variables \( x_j \) are used to form the convex combination (weighted average with non-negative weights) and the index set \( J_u \) refers to the extreme characteristic points of CHP plant \( u \). The model formulation (1) allows the plant to run anywhere inside the 3D polyhedron spanned by extreme points. However, cost minimization always determines an optimal solution lying on the lower envelope surface. The lower envelope shown in Fig 2 corresponds to the minimal cost of the CHP operation for each combination of power and heat production, including part load operation. The maximum load operation happens at the characteristic point with maximal power and heat production. Part load operation happens when the plant operates at the other characteristic points or in the region between them.

Separate power and heat production units, such as heat-only boilers (HOB) and power-only (condensing power) plants can also be represented as special cases of this modelling technique. For HOB units defining each \( p_j = 0 \) can be applied. Similarly, for power-only units similarly, \( q_j = 0 \). This model also applies to heat pumps that consume power while producing heat (\( p_j \leq 0, q_j \geq 0 \)) and various demand side management components that adjust power or heat consumption up or down. In addition, the purchase and sale of heat or power outside of the system can be represented by this model. For example, sales of power to the market can be represented by a virtual unit with \( c_j \leq 0, p_j \leq 0 \) and \( q_j = 0 \).

### 3.2 Integrated LP model

We consider a long-term heat and power production problem including multiple areas, power transmission between areas and power storages within areas. For different kinds of production units we apply the convex production unit model presented in the previous section. The problem can be formulated as an integrated multi-period LP model as follows:

\[
\begin{align*}
\text{min} & \quad \sum_{t=1}^{T} \sum_{i \in N} \left( \sum_{u \in U_i} \sum_{j \in J_u} C_{u,j} x_{u,j}^t \right) + \sum_{k \in N \setminus \{i\}} \sum_{k \neq i} C_{ik} y_{lk}^t \\
\text{s.t.} & \quad \sum_{u \in U_i} \sum_{j \in J_u} q_{u,j} x_{u,j}^t = Q_i^t 
\end{align*}
\]
\[
\sum_{u \in U} \sum_{j \in J_u} p_{u,j} x_{u,j}^t + \sum_{k \neq i} y_{ki}^t - \sum_{i \neq k} y_{ik}^t + \eta_{\text{out},i} s_{\text{out},i}^t - s_{\text{in},i}^t = p_i^t
\]

(4)

\[
s_i^t = \eta_{s,i} s_i^{t-1} + \eta_{\text{in},i} s_{\text{in},i}^t - s_{\text{out},i}^t
\]

(5)

\[
\sum_{j \in J_u} x_{u,j}^t = 1 \quad (u \in U_i)
\]

(6)

\[
0 \leq x_{u,j}^t \quad (u \in U_i)
\]

(7)

\[
\begin{align*}
& s_i^0 = 0 \\
& 0 \leq s_i^t \leq S_i^{\text{max}} \\
& 0 \leq s_{\text{in},i}^t \leq S_{\text{in},i}^{\text{max}} \\
& 0 \leq s_{\text{out},i}^t \leq S_{\text{out},i}^{\text{max}} \\
& 0 \leq y_{ik}^t \leq Y_{ik}^t \quad (k \in N | k \neq i)
\end{align*}
\]

(8)

The objective function (2) minimizes the variable production costs \(C_{u,j} x_{u,j}^t\) of all production units \(u\) of each area and in each time step. Vector \(x_{u,j}^t\) contains the hourly decision variables for unit \(u\) and the corresponding objective function coefficients are in cost vector \(C_{u,j}^t\). The cost coefficients can be time-dependent, because some fuel prices may change over time. The amount of electricity transmitted from area \(i\) to \(k\) for each time step is represented by decision variables \(y_{ik}^t\) and the corresponding transmission unit costs are \(C_{ik}\).

In the constraints, index \(i\) is implicitly looped over the set of areas \(N\). Equation (3) is the heat balance constraint to satisfy heat demand \(Q_i^t\) locally in each area \(i\) and hour \(t\). The heat balance can also be written as an inequality constraint \((\geq)\) to allow the free disposal of excess heat. This is equivalent to subtracting a surplus variable without cost coefficient from the left hand side of the constraint. A cost coefficient can also be included for the surplus variable if extra heat can be disposed at a cost. Constraints (4) guarantee that in each area power demand is matched by production, transmission, power storage charge \(s_{\text{in},i}^t\) and discharge \(s_{\text{out},i}^t\). Discharge is subject to discharge efficiency \(\eta_{\text{out},i}\). The storage level constraints (5) compute the hourly storage level \(s_i^t\) using efficiency factors \(\eta_{s,i}\) between hours and \(\eta_{\text{in},i}\) for charging. Constraints (6) and (7) represent convexity constraints for all production units. Constraints (8) give bounds for decision variables. The initial storage level is set to zero. Limits for storage capacity, charge and discharge speed are defined as upper bounds for the storage variables. Transmission line capacities are similarly defined as upper bounds for transmission variables.

Some types of thermal units cannot be ramped up or down very fast. Power ramp constraints can easily be included in the model as

\[
-P_u^{\text{D Ramp}} \leq \sum_{j \in J_u} p_{u,j} x_{u,j}^t - \sum_{j \in J_u} p_{u,j} x_{u,j}^{t-1} \leq P_u^{\text{U Ramp}}
\]

(9)
4. Decomposition model formulation

In the following, we present the decomposition method for optimizing the energy production in multiple areas with power transmission and storage over multiple hours. The flowchart in Fig 3 outlines the decomposition modelling and solution technique. In the first phase, hourly local CHP optimization models are formulated. These models are formed from the integrated model by omitting storages and transmission, and considering a single area and hour at a time. The local hourly models are then analyzed in order to determine the optimal production cost as a function of power production for each area and each hour. In the second phase, the production cost functions for all areas and hours are composed together with transmission and storage constraints into a multi-period and multi-area network model for optimizing power production, transmission and storage. This model is power-only, because optimal heat production was already solved in the first phase. Based on the network solution and additional parametric analysis results, the optimal power and heat production of each plant is finally computed.

Fig. 3. Flowchart for the decomposition modelling approach

4.1. Hourly local CHP optimization model

An hourly local heat and power system model is similar to the single step problem presented earlier [32]. The model can include any number of CHP, power-only and HOB units. The following hourly model minimizes the overall costs of local production in a single area $i$ for time step $t$:

$$\min \sum_{u \in U_i} \sum_{j \in I_u} C_{u,j}^t x_{u,j}^t$$

s.t.
The objective function (10) minimizes the local production costs of all production units based on the cost coefficients $C_{u,j}^t$ of characteristic operating points of production units. Heat balance (11) makes sure that the heat demand $Q_{u,i}^t$ in the area is satisfied. As in the integrated model, the heat balance can also be written as an inequality constraint ($\geq$) or include a surplus variable with a cost coefficient for disposing of excess heat. Power balance (12) defines local power production $P_{prod,i}^t$ as a convex combination of the characteristic operating points of all production units. Constraints (13) and (14) introduce the convexity constraints for production units. The power and heat production of each unit $u$ in area $i$ can be expressed as

$$p_{u,i}^t = \sum_{j \in J_u} p_{u,j} x_{u,j}^t,$$

$$q_{u,i}^t = \sum_{j \in J_u} q_{u,j} x_{u,j}^t.$$

The power and heat production of each unit can be easily computed after solving the hourly local model; it is not necessary to include these as constraints in the model.

4.2. Parametric analysis of the local model

The model (10-14) is analyzed using parametric LP analysis for each time step and each area in order to determine the optimal production costs as a function of power production $P_{prod,i}^t$. Because the model is an LP model, the optimal costs form a piecewise linear convex function $C_{i}^{t} = C(p_{prod,i}^t)$. Parametric LP analysis determines this function in the form of a sequence of $L_{i}^t + 1$ points $[(C_{i}^{t,0}, P_{prod,i}^{t,0}), \ldots, (C_{i}^{t,L_{i}^t}, P_{prod,i}^{t,L_{i}^t})]$. Here $P_{prod,i}^{t,0}$ is the minimal and $P_{prod,i}^{t,L_{i}^t}$ is the maximal possible power generation. Some of the points may be duplicate ($P_{prod,i}^{t,L_{i}^t-1} = P_{prod,i}^{t,L_{i}^t}$) due to degenerate basic LP solutions. Duplicate points are removed from the parametric results. The line segments between corner points correspond to different basic LP solutions. The slope of each line segment is the marginal production cost within the corresponding power production range. The marginal production cost for line segment $l$ in area $i$ is

$$c_{i}^{t,l} = (C_{i}^{t,l} - C_{i}^{t,l-1}) / (P_{prod,i}^{t,l} - P_{prod,i}^{t,l-1}).$$
Figure 4 illustrates some piecewise linear optimal cost functions. The line segments determine the costs and capacities of production arcs in the network model described later. According to (4) in the integrated model, the local power production must cover local power demand, transfer and storage. Substituting (12) into (4) gives

\[ P_{\text{prod},i}^t = P_i^t - \left( \sum_{k \neq i} y_{ki}^t - \sum_{i \neq k} y_{ik}^t + \eta_{\text{out},i} s_{\text{out},i}^t - s_{\text{in},i}^t \right) \] (18)

This equation links the local production model to the network model described in the following.

In addition to the operating cost and power production at each point, parametric analysis also determines using (15) and (16) the corresponding power and heat production of each unit at each point \( l \): \( P_{u,tl}^t \) and \( Q_{u,tl}^t \). This information is necessary for computing the production of different units in the second phase after solving the encoded multi-period network model.

### 4.3. Generic network flow model

A network flow model is formulated in terms of a set of nodes \( N \) and a set of directed arcs \( A \) connecting pairs of nodes. The arcs allow non-negative flow of some commodity between nodes. Associated to each arc \( a \in A \) is the unit cost \( c_a \) per flow amount \( y_a \). The objective is to minimize the total cost of all flows subject to balance constraints at each node. The balance constraints state that the sum of outflows minus inflows to a node must equal the local supply \( d_i \) at the node. Local demand is represented as negative supply. Variants of the network flow problem can include capacity limits \( Y_a \) for the flow arcs, and losses or gains \( \eta_a \) for the flows, for example. The minimum cost generalized network flow model is formulated as [34]

\[
\text{min} \sum_{a \in A} c_a y_a \tag{19}
\]

s.t.

\[
\sum_{a \in A | \text{org}(a) = i} y_a^t - \sum_{a \in A | \text{dest}(a) = i} \eta_a y_a^t = d_i, i \in N
\]

\[
0 \leq y_a \leq Y_a, a \in A
\]

The above model is an LP model. In the special case without gains or losses for arcs, each \( \eta_a = 1 \) and the model simplifies into the standard transshipment flow model with capacity limits. Without losses or gains, the problem is also called a pure network flow problem. The advantage with the capacitated transshipment flow model is that very efficient algorithms exist for solving it such as the network simplex algorithm [34], [35]. Note that without losses and gains, it is necessary that \( \sum d_i = 0 \) to guarantee the existence of a feasible solution. This means that the balance constraints are linearly dependent, and the balance constraint for an arbitrary node can be eliminated from the model. If gains or losses are present, it is necessary
that at least one of the balance constraints is eliminated, i.e. the supply at corresponding nodes is not fixed beforehand.

4.4. Encoding the hourly production and transmission problem

The hourly power transmission problem between multiple areas leads directly into model formulation (19). Each area in the power transmission problem is represented by a node with a given supply or demand for power. Each transmission line between areas is represented by an arc with a given transmission unit cost $c_a^t$, capacity $Y_a$, and possible loss. Observe that the model formulation allows zero or more transmission lines/arcs between each pair of areas. As a result, a network problem (19) is created for each hour $t$. While the topology of the transmission network remains the same for all hours, the flow variables and parameters depend on the hour and will therefore receive superscript $t$ as a time index.

Because the losses in power transmission are relatively small, they can be represented as transmission costs (cost of extra production to make up the transmission loss). Thus, the transmission problem results in the capacitated transshipment flow model.

Next, we combine the hourly transmission problem with local power production in each area. Local production is represented by the piecewise linear cost functions that were computed using parametric analysis. The nodes corresponding to each area are defined as sink nodes with $d^t_i = -P^t_i$, where $P^t_i$ is the local demand for power (at hour $t$ in area $i$). In addition, we create an artificial production node $i^0_t$ with supply $d^t_i = \sum_i P^t_i$. Then, we define production arcs from the production node to each area node with capacities equal to the lengths ($p^{t,0}_{prod, i} - p^{t,0}_{prod, i}$) and costs equal to the slopes ($c^{t,0}_i$) of the parametric line segments of the corresponding area. The constant terms of the parametric functions ($p^{t,0}_{prod, i}, c^{t,0}_i$) define the minimal power production and production cost in each area. The constant $p^{t,0}_{prod, i}$ is the least amount of power that can be produced in each area. The constant $c^{t,0}_i$ is the minimum power production cost in area $i$, which is the starting point for the piecewise line segments. It is the cost independent of the transmission amount for each area and time step. The hourly multi-area production and transmission model is formulated as (20)-(23).

$$\min \sum_{t=1}^{T} \sum_{i \in N} (C^{t,0}_i + \sum_{a \in A} c^{t}_a y_a^t) \quad (i \in N, t \in T, a \in A) \quad (20)$$

$$\sum_{a \in A | org(a) = i} y_a^t - \sum_{a \in A | dest(a) = i} y_a^t - \sum_{t=1}^{L_i} x_{i,t}^t = -P^t_i \quad (i \in N) \quad (21)$$

$$\sum_{i \in N} \sum_{t=1}^{L_i} x_{i,t}^t = \sum_{i \in N} P^t_i \quad (22)$$

(redundant)

$$0 \leq x_{i,t}^t \leq p^{t,0}_{prod, i} - p^{t,0}_{prod, i} \quad (a \in A) \quad (23)$$

$$0 \leq y_a^t \leq Y_a \quad (a \in A)$$
This model is a capacitated transshipment flow model. Because one of the balance constraints is redundant, we choose to remove constraint (22) for the production node from the model. Removing (22) would also be necessary and sufficient, if transmission losses were present.

4.5. Multi-period and multi-area model with storages

Without storages or other dynamic constraints, hourly models could be solved independently. With storages, it is necessary to solve hourly models together with storage constraints. Next, we extend the multi-area network model into a multi-period and multi-area model with power storages. To simplify notation, we include the production node \( i_0 \) for each time period into the set of nodes \( N \) and the production arcs into the set of arcs \( A \). This means that \( c_a^t \) includes unit costs both for transmission and production arcs, and \( Y_a^t \) includes similarly upper bounds for both transmission and production arcs.

Next, we combine the hourly network models into a single network model and combine the hourly models together with storage constraints. The formulation (24-27) shows the network model for the long-term hourly problem with power storage.

\[
\begin{align*}
\min & \sum_{t=1}^{T} \sum_{i \in N} \left( C_i^{t,0} + \sum_{a \in A} c_a^t y_a^t \right) \quad (i \in N, t \in T, a \in A) \\
\text{s.t.} \quad & \sum_{a \in A : \text{org}(a) = i} y_a^t - \sum_{a \in A : \text{dest}(a) = i} y_a^t + s_{i,n,i}^t - \eta_{out,i} s_{i,out,i}^t = P_{i,prod,i}^{t,0} - P_i^t \\
& s_i^t = \eta_{i,n,i} s_{i,n,i}^{t-1} + \eta_{i,in,i} s_{i,in,i}^t - s_{i,out,i}^t \\
& s_i^0 = 0 \\
& 0 \leq s_i^t \leq S_{i,\text{max}} \\
& 0 \leq s_{i,n,i}^t \leq S_{i,\text{max}} \\
& 0 \leq s_{i,out,i}^t \leq S_{i,\text{out,\text{max}}} \\
& 0 \leq y_a^t \leq Y_a^t
\end{align*}
\]

The objective of the multi-period network model (24) is to minimize the production and transmission costs over all time periods. The decision variables \( y_a^t \) represent power flow via transmission or production arcs with corresponding cost \( c_a^t \) and capacity \( Y_a^t \). The balance constraints (25) maintain the power balance in each area. The net power flow out of the node plus charge minus discharge of storage must equal the difference between \( P_{i,prod,i}^{t,0} \) and the local demand. Constraints (26) determine the power storage level at each time step. Constraints (27) set the initial storage level to zero, the limits for storage operation, and the upper bounds for power transmission and production flows. The efficiency factors \( \eta_{out,i}, \eta_{in,i} \) in (25) and (26) represent efficiency when discharging or charging the storage, and \( \eta_s \) similarly the efficiency for storing over time. If the storage losses are small, the approximate
value of 1 can be used for the efficiency ratios, and the model will again become a capacitated transshipment flow model.

Solving the model determines the optimal overall power production in each area, optimal power transmission between areas, and optimal charge and discharge of the storages. Parametric analysis provided the production of power and heat for different production units \( p_{u,i}^{l,t} \) and \( q_{u,i}^{l,t} \) at different points. Using this information, the production of power and heat at different plants can be expressed as linear functions of decision variables \( x_{i,t}^{l} \).

\[
P_{u,i}^{t} = p_{u,i}^{t,0} + \sum_{l=1}^{L_{k}^{l}} x_{i,t}^{l} \frac{p_{u,i}^{t,l} - p_{u,i}^{t,l-1}}{p_{prod,i}^{t,l} - p_{prod,i}^{t,l-1}}
\]

\[
Q_{u,i}^{t} = q_{u,i}^{t,0} + \sum_{l=1}^{L_{k}^{l}} x_{i,t}^{l} \frac{q_{u,i}^{t,l} - q_{u,i}^{t,l-1}}{q_{prod,i}^{t,l} - q_{prod,i}^{t,l-1}}
\]

Using (28), ramp constraints for power production can be included as linear constraints in the network model as

\[
-P_{u}^{dRamp} \leq P_{u,i}^{t} - P_{u,i}^{t-1} \leq P_{u}^{uRamp}
\]

The ramp constraints do not conform to the network formulation. To solve the model with ramp constraints, either a general LP solver, or a network solver that can handle (30) as side-constraints must be used.

5. Results

In the following, we first illustrate the decomposition method using a small example. Then we describe model validation. After that, we evaluate the performance of the method using differently sized multi-period models with time horizons extending from one week (168 h) up to one year (8760 h).

5.1. Sample model

We consider four areas, each containing three CHP units, one power-only unit and one HOB. For simplicity, the production units in each area are identical. The properties of the production units are summarized in Table 1. The first CHP unit has three characteristic operating points, which means that it could be a CHP plant with a bleeder turbine where the intermediate extraction of heat can be closed to reduce heat output a little in proportion to power production. The two other CHP plants have two characteristic points each, which means that they are simple backpressure plants. The power-only unit is a typical modern condensing power plant with 40% efficiency ratio and 150 MW capacity. The HOB has an 89% efficiency ratio and 2700 MW capacity. The capacities of the power-only units and HOBs are large enough to satisfy the local demand in all operating situations. When both heat
and power are needed, the production costs per produced energy for CHP1 are cheapest, followed by CHP2, CHP3, HOB and then the power-only plant. However, when there is a large mismatch (non-coincidence) in heat and power demand, it may be beneficial to run the CHP units at a lower production level and produce more using separate heat or power units. The optimal operation of the CHP system in different demand and fuel price conditions is quite complex, and can be best found by using an optimization model.

Table 1. Input data of the plants for each area

<table>
<thead>
<tr>
<th>Plant</th>
<th>CHP1</th>
<th>CHP2</th>
<th>CHP3</th>
</tr>
</thead>
<tbody>
<tr>
<td>p (MWh)</td>
<td>3</td>
<td>9.4</td>
<td>12.5</td>
</tr>
<tr>
<td>q (MWh)</td>
<td>10</td>
<td>24.2</td>
<td>38</td>
</tr>
<tr>
<td>c (€)</td>
<td>315</td>
<td>753.9</td>
<td>1092</td>
</tr>
</tbody>
</table>

Table 2 shows sample heat and power demand data for different areas. Power transmission capacities between all areas are 10 MW and transmission costs are 1 €/MWh.

Table 2. Technical input data

<table>
<thead>
<tr>
<th>Area</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power demand (MWh)</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>Heat demand (MWh)</td>
<td>50</td>
<td>60</td>
<td>70</td>
<td>80</td>
</tr>
<tr>
<td>Transmission capacity (MW)</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Transmission cost (€/MWh)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

First, we perform a parametric analysis of the local CHP problems (10-14) for each area. Figure 4 shows the results of parametric analysis. The piecewise linear curves show the optimal production costs in each area as a function of power production. Because the production units are identical in all areas, the parametric curves are of similar shapes, but they are on different levels due to different heat demand in the areas. The curves with lower heat demand have lower levels of production cost due to lower fuel consumption. It is interesting to observe that the marginal costs are negative at low levels of power production for each area (the slope of the line segment is negative). This happens because when the power production is low, CHP production is insufficient to satisfy heat demand, and HOB operates to produce heat, which is more expensive than CHP. In Fig 4, CHP plants at point A in area 4 produce 8 MWh power. To cover all heat demand, HOB produces 51 MWh of heat. By increasing power production at point B, CHP plants can satisfy all heat demand, and HOB and power-only are not needed. Then, the production cost of point B will be lower than point A. At point C, where total power production is 175.5 MWh, the more expensive power-only
plant produces 150 MWh, while the production of CHP plants are 25.5 MWh. Therefore, the production costs increase from point B to point C.

![Graph showing optimal production cost curves as a function of power production in each area.](image)

Fig. 4. Optimal production cost curves as a function of power production in each area.

The minimal power production in each area $P_{\text{prod},i}^t$ is 8 MWh. According to (25), the difference between $P_{\text{prod},i}^t$ and the local power demand in each area (5, 10, 15, and 20, correspondingly) will be the adjusted demand/supply at each area node. Figure 5 shows the resulting network flow problem with the adjusted demands at the area nodes, and the transmission and production arcs with corresponding unit costs and capacities. The network model is solved for the new demand of the area showing in Fig 5, and then $(P_{\text{prod},i}^t)$ values will be added to the results of power production in each area to get the final solution.

![Network flow problem for a single hour.](image)

Fig. 5. Network flow problem for a single hour
Figure 6 shows the optimized flows of the network model. The transmission between areas appears on the transmission arcs. To obtain the power production in each area, the minimal production amounts $p_{\text{prod},i}^0$ must be added to the production arc flows. The optimal objective function value (24) is 10102.31 €.

![Diagram showing network flows and production](image)

Fig. 6. Solution to the network flow problem

5.2 Model validation

We validated the parametric decomposition method for accuracy by solving different models and comparing the solutions with the integrated LP model (2-8). In principle, the decomposition method is equivalent to the integrated model. However, numerical inaccuracies will be cumulated in both phases of the decomposition method, and therefore the solution could be less accurate than that of the integrated LP model. First, we validated individual hourly models without storage. Then, we formed multi-period models with storages, optional ramp constraints, and different time horizons extending from one week (168 h) up to a year (8760 h) and validated them.

For individual hourly models, the parametric decomposition method gave the same objective function values to an accuracy of six decimal places. For example, for the sample model of the previous section, the optimal solution (2) of the integrated model is 10102.39 € (compared to 10102.31 € for the decomposition model). The production amounts in different areas and power transmission between areas can sometimes be a little different because the model may have multiple optima.

To validate the multi-period model with storage, we considered three-area models with the same plant characteristics mentioned in Table 1. To guarantee a feasible solution for each hour, we included a surplus variable with high cost coefficient for the heat balance (3) in the integrated model and (11) in the parametric model. In area 2, we included a 1000 MWh power storage with 500 MW charge and discharge limit. The capacity is large enough to cover 2-3 day demand in area 2. In addition, we considered optional hourly power ramp constraints for one plant in area 2 with up and down ramp rate equal to 10% of nominal capacity. To obtain realistic power and heat demand data, we modified real data to match the production capacity of different areas (Figs 7 and 8). In particular, we generated the heat
demand data using the heating degree-days method on historical temperature data for different locations [20]. Table 3 lists other input data.

![Power demand in each area](image1)

**Fig. 7. Power demand in each area**

![Heat demand in each area](image2)

**Fig. 8. Heat demand in each area**

<table>
<thead>
<tr>
<th>Area</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Storage capacity (MWh)</td>
<td>0</td>
<td>1000</td>
<td>0</td>
</tr>
<tr>
<td>Charge and discharge rate (MW)</td>
<td>0</td>
<td>500</td>
<td>0</td>
</tr>
<tr>
<td>Transmission capacity (MWh)</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Transmission cost (€/MWh)</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Power ramp limit (MWh)</td>
<td>-</td>
<td>±10%</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 3. Technical input data
We formed two versions of the models with storage: versions without and with power ramp constraints. For both versions, we obtained almost identical objective function values between the decomposition method and the integrated model. Table 4 reports the objective function values between the two models with power ramps for different time horizons. The results are the same to an accuracy of seven decimal places, i.e. the relative accuracy is even better than in the individual hourly models.

Table 4. Objective function values of the two models with ramp constraints

<table>
<thead>
<tr>
<th>Time horizon (h)</th>
<th>Decomposition (€)</th>
<th>Integrated (€)</th>
</tr>
</thead>
<tbody>
<tr>
<td>168</td>
<td>2155486.8</td>
<td>2155486.9</td>
</tr>
<tr>
<td>336</td>
<td>4506361.1</td>
<td>4506361.3</td>
</tr>
<tr>
<td>720</td>
<td>10351210</td>
<td>10351211</td>
</tr>
<tr>
<td>1440</td>
<td>20225954</td>
<td>20225954</td>
</tr>
<tr>
<td>2000</td>
<td>27691058</td>
<td>27691059</td>
</tr>
<tr>
<td>2500</td>
<td>34468317</td>
<td>34468318</td>
</tr>
<tr>
<td>3600</td>
<td>50561894</td>
<td>50561896</td>
</tr>
<tr>
<td>3880</td>
<td>53693839</td>
<td>53693841</td>
</tr>
<tr>
<td>4000</td>
<td>56843384</td>
<td>56843387</td>
</tr>
<tr>
<td>5000</td>
<td>72570429</td>
<td>72570432</td>
</tr>
<tr>
<td>6000</td>
<td>88621238</td>
<td>88621241</td>
</tr>
<tr>
<td>7000</td>
<td>102981420</td>
<td>102981430</td>
</tr>
<tr>
<td>8760</td>
<td>126451890</td>
<td>126451900</td>
</tr>
</tbody>
</table>

5.3. Results for different units

In the following, we present optimization results from the decomposition model with ramp constraints. Results are based on solving the full-year models, but only the first week of January is displayed in the figures.

Figure 9 shows optimal power production of units in all areas. CHP2 and CHP3 plants operate in all areas at their minimum power during the week. In areas 1 and 3, CHP1 operates at variable part-load, following the local heat demand that fluctuates significantly. In area 2, CHP1 operates constantly at minimum load because heat demand is very low. The rest of the demand is satisfied by power-only plants in all areas. In area 2, we observe that the power-only unit follows the hourly 10% ramp rate constraint.

Figure 10 shows optimal storage operation in area 2. The storage is charged when demand is low and power can be produced cheaply. Storage is discharged when demand and production price high.
Fig. 9. Power production by different units in areas 1, 2, and 3
5.4. Solution time for long-term problems

The decomposition method was developed to speed up the optimization. We have solved the decomposition model and compared the solution time with the integrated LP model. The three-area models were solved with different time horizons from one week (168 h) up to one year (8760 h). Two variants of the models were tested: with and without ramp constraints. For the decomposition method, we added together the solution times of the local parametric models and the network model. All models were solved using LP2, which is an efficient solver for large sparse LP problems using the product form of inverse and allowing upper bounds for variables [4]. The yearly integrated model without ramp constraints has 166440 constraints and 367920 structural variables plus one slack variable per constraint. The corresponding network model has 61320 constraints and 249328 variables. The ramp constraints add 8760 double-sided constraints to each model. The tests were run by a PC running the 64-bit Windows 10 operating system with 2.5 GHz Intel® Core(TM)-7300U CPU @ 2.60GHz 2.71GHz processor, and 8.00GB installed RAM (7.41 GB usable memory during the analysis). The test runs were repeated twice to assess the size and reduce the effect of random variation in CPU time.

Figure 11 shows the solution times for different size models. The solid lines are the CPU times for models without ramp constraints and the dashed lines for models with ramp constraints. The difference in the solution time between the models with and without ramp constraints is quite small. However, it is interesting to see that the integrated models with ramp constraints solve a little faster than the corresponding no-ramp models, for almost all time horizons. The network model with ramp constraints is slower than the version without ramp constraints.

The solution time as a function of number of hours grows much slower for the decomposition method compared to the integrated model. For the problems until 720 hours (one month), the integrated model solves a little faster. However, for larger models, the
decomposition method increasingly outperforms the integrated model. Without ramp constraints, the decomposition method is 2 times faster for 1440 hours (2 months), 11 times faster for 4000 hours, and 30 times faster for 8760 hours. With ramp constraints, the speed ratios change to 1.5, 6, and 15 times faster for 1440, 4000 and 8760 hours respectively. This demonstrates the advantage of the decomposition method for long-term problems with thousands of hours.

Fig. 11. CPU time for solving different models

6. Conclusion and discussion

The purpose of this study was to speed up the optimization of long-term models for multi-area heat and power production with power transmission and power storage. We introduced a method decomposing the problem into two models: local hourly models, and a multi-period network model. Parametric analysis on the local models were used to determine the optimal production costs as function of power production. These parametric functions were encoded together with power transmission and storage constraints into a network model minimizing the overall production costs and determining the power storage operation and hourly power transmission between areas. The decomposition method was validated by comparing it with an integrated linear programming model.

For long-term models with thousands of hours, the decomposition method solves much faster than the integrated model. For example, with 8760 hours, the decomposition method solves 30 times faster than the integrated model. The method is therefore particularly suitable for long-term models, with planning horizons from months up to a year and even longer.

Power ramp constraints can be optionally included as side-constraints (30) at cost of somewhat longer solution times. If only part of the plants require ramp constraints, the decomposition method is still much faster than the integrated model. Many types of modern thermal units are capable of ramping about 1-2% of nominal power per minute [36], which means 60-120% per hour. When typical minimum power of a thermal unit is 30-50% of
nominal power, this means that ramp constraints for such plants can be omitted in an hourly multi-period model. Modern light water nuclear reactors are (by design) capable of ramp rates of ±5% per minute, which means they do not require ramp constraints in an hourly model [37].

In this study, we used an efficient general purpose sparse linear programming algorithm for solving the different problems. In further research, the decomposition method can be made even faster by applying specialized solvers. Parametric analysis of the hourly local models can be implemented efficiently by the Power Simplex algorithm [4]. The network model can be solved efficiently by the Network Simplex algorithm in the absence of losses, and by the Generalized Network Simplex algorithm when losses are present [34]. If ramp constraints are present as side-constraints, general linear programming is needed to solve the network model.

**Acknowledgments**

This research was funded by STEEM - Sustainable Transition of European Energy Market, which is a project aiming at novel solutions with improved energy efficiency as one of the Aalto Energy Efficiency Research Programs and STORE – Stochastic Optimization of Renewable Energy in large polygeneration systems, a project funded by the Academy of Finland, grant number 298317.
<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>$d_i$, supply in node $i$ (MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abbreviations</td>
<td>$P^0_{\text{prod}}$, minimal power production (MWh)</td>
</tr>
<tr>
<td>CHP, combined heat and power</td>
<td>$C^0$, cost of minimal power production (€)</td>
</tr>
<tr>
<td>HOB, heat only boiler</td>
<td>$x_{i,t}$, production in area $i$ and line segment $l$ (MWh)</td>
</tr>
<tr>
<td>LP, linear programming</td>
<td>Index sets</td>
</tr>
<tr>
<td>MILP, mixed integer linear programming</td>
<td>$N$, set of areas or nodes in network</td>
</tr>
<tr>
<td>Symbols</td>
<td>$U_i$, index set for production unit at area $i$</td>
</tr>
<tr>
<td>$C$, production cost (€)</td>
<td>$J_u$, extreme characteristic points of production unit $u$</td>
</tr>
<tr>
<td>$P$, power demand (MWh)</td>
<td>$A$, set of arcs in network</td>
</tr>
<tr>
<td>$Q$, heat demand (MWh)</td>
<td>dest($a$), set of arcs into node</td>
</tr>
<tr>
<td>$c_j, p_j, q_j$, production price, power generation, and heat production at characteristic point $j$ (MWh)</td>
<td>org($a$), set of arcs out from node</td>
</tr>
<tr>
<td>$x_j$, variables used to encode convex combination of operating region</td>
<td>Superscripts and subscripts</td>
</tr>
<tr>
<td>$C_{ik}$, power transmission price from area $i$ to $k$ (€/MWh)</td>
<td>$t$, time</td>
</tr>
<tr>
<td>$y_{ik}$, electricity transmitted from area $i$ to $k$ (MWh)</td>
<td>$i, k$, areas $i$ and $k$</td>
</tr>
<tr>
<td>$\eta$, efficiency factor</td>
<td>$\text{DRamp}$ and $\text{URamp}$, maximum down and up ramp rate</td>
</tr>
<tr>
<td>$s_{in}$, power charge to storage (MWh)</td>
<td>$\text{in}$, charge into storage</td>
</tr>
<tr>
<td>$s_{out}$, power discharge out of storage (MWh)</td>
<td>$\text{out}$, discharge out of storage</td>
</tr>
<tr>
<td>$s$, storage level (MWh)</td>
<td>$\text{max}$, maximum</td>
</tr>
<tr>
<td>$c_a$, transmission price (€/MWh)</td>
<td>$\text{min}$, minimum</td>
</tr>
<tr>
<td>$y_a$, power flow in arc $a$ (MWh)</td>
<td>$s$, storage</td>
</tr>
<tr>
<td>$Y_a$, capacity limit for flow in arc (MWh)</td>
<td>$\text{prod}$, production</td>
</tr>
<tr>
<td>$C^*$, optimal costs of power production $P_{\text{prod}}$ (€)</td>
<td>$i_0$, production node</td>
</tr>
<tr>
<td>$L_i$, number of line segments in production area $i$</td>
<td>$l$, point or line segment of piecewise linear curves</td>
</tr>
<tr>
<td>--------------------------------------------------</td>
<td>--------------------------------------------------</td>
</tr>
<tr>
<td>$c^l$, marginal production costs (slope of line segment $l$) (€/MWh)</td>
<td></td>
</tr>
</tbody>
</table>