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Flipping-Coin Experiment to Study Switching in Josephson Junctions and Superconducting Wires

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When probed with current pulses, Josephson junctions and superconducting wires exhibit stochastic switching from a superconducting to a stable nonzero-voltage state. The electrical current dependence of the switching probability (the so-called S curve) or the switching-current distribution is a fingerprint of the physics governing the escape process. This work addresses the criterion of the independent switching event, which is important for the credibility of the switching measurements of superconducting wires and various Josephson junctions involving superconductor-insulator-superconductor tunnel junctions, proximity junctions, and Dayem nanobridges. Treating the Josephson junction as an electrical coin with a current-tuned switching probability, we investigate the effect of correlation between switching events on the switching statistics. We show that such a correlation originates from the thermal dynamics of the superconducting wire.

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I. INTRODUCTION

The stochastic transition from the superconducting to the resistive state in Josephson junctions (JJs) and superconducting wires (SWs) offers a workbench for studying the decay of metastable states [1–5] and opens doors to the study of quantum phenomena. The JJ switching phenomenon is used for probing the state of superconducting quantum bits [6,7] and the development of superconducting quantum-information devices [8]. Hysteretic JJs and SWs are desired for the threshold detection of various physical signals. They have been employed [9] and considered [10] for on-chip current-noise measurements and study of the thermal dynamics of nanostructures [11] and proposed for single-photon counting [12].

It is common practice to measure the electrical current dependence of the switching probability (the so-called S curves) [13,14] or the switching-current distribution [15–18] for current-biased JJs and SWs. In the first case, the sample is probed with a train of N current pulses and a number of switchings n yields an estimate of the switching probability $p = n/N$. In the second case, the sample is tested with current ramps and for each ramp the switching current is recorded. The numbers of switchings in successive current intervals present the switching-current distribution. The shape of the measured S curves or distributions reveals information about a fundamental mechanism governing the transition from the superconducting to the normal state. The escape process is known to be driven by either thermal or quantum fluctuations [3,13,16,19]. In the former case, the fitting of an Arrhenius-like relation to the experimental data allows us to independently determine the temperature of the electromagnetic environment. In the latter case, the effective escape temperature is elevated above the bath temperature and indicates the regime of the macroscopic quantum tunneling (MQT). Importantly, the temperature dependence of the S-curve width (or the width of the switching current distribution) gives an insight into the physical process responsible for the switching. For the orthodox thermally driven escape, one observes a monotonic increase in the S-curve width with temperature increase. The basic model of thermal activation for tunnel junctions yields the width $\Delta I \sim T^{2/3}$ [2]. Intuitively, it is understood as a thermal broadening, which is larger for higher temperatures. There are also cases in which counterintuitive behavior is observed: reduction of the S-curve width when the temperature is increased. There are a few phenomena that are responsible for such anticorrelation. In the moderately damped Josephson junction, as the temperature is increased, the initial broadening of the switching threshold is followed by an apparent collapse of thermal activation [13,18,20]. Such a reentrant behavior is attributed to a retrapping process that sets the phase into diffusive motion and tends to keep the junction in the metastable state (the so-called phase diffusion regime). For the pure MQT, the escape is governed

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by the effective temperature, defined as $T_{\text{eff}} = \hbar\omega_p/2\pi k_B$, where ω_p characterizes the phase oscillations at the bottom of the confining potential. For junctions, w_p scales with the critical current. It corresponds to smaller widths of the S curve at higher temperatures for which I_C is reduced. Narrowing of the observable switching-current range with temperature has also been demonstrated for superconducting wires [16,21]. It was attributed to a multiphase-slip escape process, which is understood as follows. Each phase slip is a dissipative event, increasing the temperature in the wire. For low temperatures, a single phase slip is enough to heat the wire above T_C. At higher temperatures, a cascade of phase slips is required to exceed T_C, with each phase slip increasing both the temperature and the probability that the next phase slip will occur.

The probing of JJs and SWs with current pulses has relied on an unstated assumption: a JJ behaves like a coin for which a "heads and tails" experiment is performed. In the current work, we answer the following question: what makes a JJ a good coin? And what will happen if two successive "flips" are correlated? A good coin should exhibit stable values of the probabilities of two possible outcomes (here, we relax the requirement that the probability of each outcome should be 0.5). If this requirement is satisfied, the expected number of outcomes of each kind in a series of many identical experiments, each consisting of a fixed number of flips, is given by a binomial distribution.

On the other hand, if the probability of obtaining a "head" is affected by the result of previous flips, we talk about correlation. In the simplest case, the correlation involves only two adjacent trials, with the earlier one affecting the later one (nearest-neighbors correlation) but, as we show below, it may have a much more intricate character, with the result of each trial influenced by all previous outcomes. The switching of a superconducting weak link gives us a unique opportunity to realize and investigate both the noncorrelated and correlated switching scenarios.

We fabricate a superconducting nanobridge (width = 60 nm) interrupting a long nanowire (width = 600 nm), connected to large-area contact pads at both sides (Fig. 1). The structure is prepared by means of conventional one-step *e*-beam lithography, by the evaporation of 30-nm aluminum. We test the bridge with a train of N current pulses. In response to each pulse, depending on the probing-current amplitude and fluctuations, the bridge may remain in the superconducting state or transit to a normal state (switching). For low probing currents, the bridge never switches; for a high current, it always switches. In between, there is a current region in which the bridge switching is probabilistic, with the probability rendering the familiar S-shaped curve as the testing current increases (Fig. 1). For the detailed description of the method, see our earlier works [11,22]. The switching probability for a fixed testing current increases with temperature (see the vertical line in Fig. 1). Furthermore, we reserve the notion

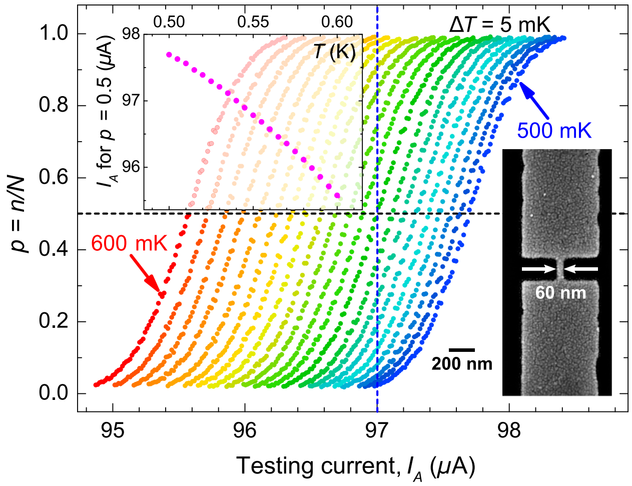

FIG. 1. S curves measured at different bath temperatures used to extract the switching-current dependence on the temperature (inset). The dashed lines correspond to $p = 0.5$ (horizontal) and to a constant testing current (vertical). The SEM image of the aluminum nanobridge is shown on the right-hand side.

of probability p for the independent events, while we generally talk about the switching number n/N, specifying the number of switching events n in the total number N of probing pulses.

To investigate the effect of correlation, we intentionally introduce a dependence between the probing pulses, using the time interval between the pulses, $\Delta\tau$, as a control knob for the strength of the correlation [Fig. 2(a)]. The current dependencies of the switching number $n(I_A)/N$ in the train of N pulses with different separation times $\Delta\tau$ are presented in Fig. 2(b). For sufficiently large $\Delta\tau > 50$ μs, the obtained S curves are the same, indicating that $\Delta\tau$ in this range does not influence the switching numbers. In such a case, we talk about independent switching events and we can associate the switching number $n(I_A)/N$ with the independent-switching probability p [23]. With reduction of the period of the probing pulses, switching in a single pulse starts to influence the result in subsequent pulses, leading to steepening of the S curve [Fig. 2(b)]. The further reduction of the period destroys the familiar picture of the S curve: the number of switchings $n(I_A)$ corresponding to a fixed probing-current amplitude seems to be completely random, as revealed by the scattered curves presented in Fig. 2(b). The observed correlation is of thermal origin, as we show in Sec. V, and appears when the interval between the probing pulses becomes shorter than the thermal relaxation time for the bridge that switches to the normal state, thus exceeding T_C, and is left to cool down. The bridge always returns to the superconducting state after single switching, with a retrapping time of approximately 10 ns [24], but it does not reach the base temperature prior to the arrival of the next testing pulse and the switching probability is enhanced, as shown in

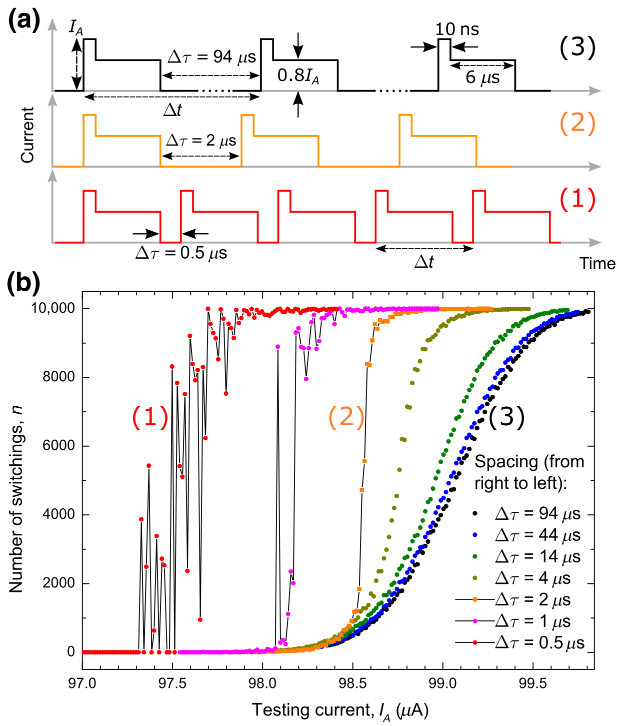

FIG. 2. (a) Three trains of N current pulses used to probe the bridge with different $\Delta\tau$, yielding dependencies (1), (2), and (3) as shown in (b). (b) The current dependencies of the switching number n for different spacings $\Delta\tau$ between the probing pulses.

Fig. 1 for a fixed testing-current amplitude. The strength of the correlation increases with the reduction of the time interval between testing pulses. In the current work, we focus on the distributions of the switching number for the fixed probing-current amplitude in three regimes: for (1) independent, (2) correlated, and (3) fully correlated switching events.

II. INDEPENDENT-SWITCHING REGIME: CRITERION FOR CLEAN-SWITCHING MEASUREMENT

We verify the assumption that the independent switching events are described by a binomial distribution:

$$P(n) = \binom{N}{n} p^n (1-p)^{N-n}, \tag{1}$$

where p is the independent-switching probability and $\binom{N}{n} = N!/[(N-n)!n!]$ is the number of different ways in which n switchings can be distributed among N trials. We send a train of $N = 10,000$ current pulses of fixed amplitude with a probing period of $\Delta t = 100\ \mu$s and measure the switching number $n(I_A)/N$. By repeating the same experiment many times, we reconstruct the switching distribution. Our procedure remains a fully analogous with flipping a coin (each pulse being a single toss) and indeed yields a binomial distribution (Fig. 3), thus confirming the independence of the switching events. The experimental distribution is a sensitive probe of the possible correlation between the testing pulses. However, the distribution would possibly be affected if there was a temperature instability in the cryostat or excessive current noise, resulting in variation of p and leading to premature switching or preventing the bridge from switching. In such a case, the testing pulses may be not correlated but the distribution can still be violated. Nevertheless, if it is binomial with the proper variance, it serves as a strong indication of the independent switching events and of the negligible influence of electrical noise and temperature instabilities on

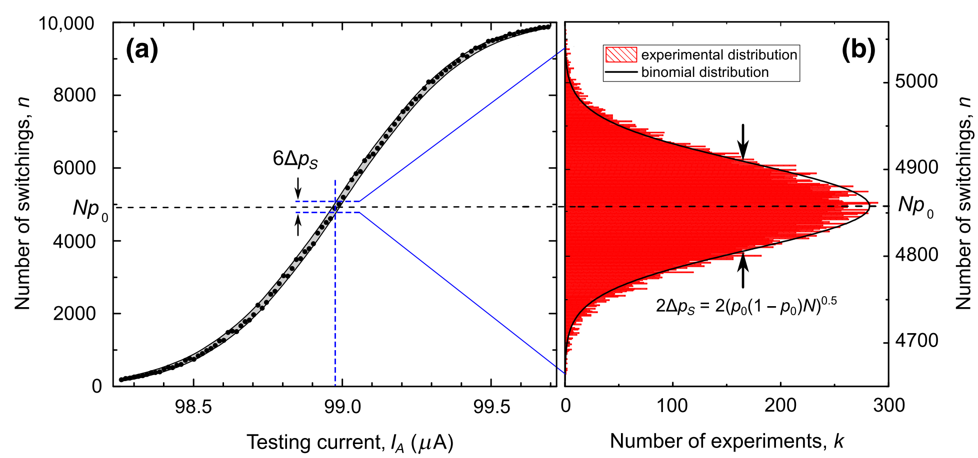

FIG. 3. The "flipping-coin" experiment for the independent events at $T = 300$ mK. (a) The S curve and its statistical broadening, imposed as a gray region. (b) The number of experiments (horizontal axis) resulting in the given number of switchings (vertical axis) for the constant current (98.97 μA), indicated with a dashed vertical line in (a). The single experiment consists of sending $N = 10,000$ pulses and measuring the number of switching events. The experiment is repeated 35,372 times. A binomial distribution is imposed, as shown by the black curve. $\Delta p_S \cong 50$ is the statistical broadening of the measurement at $p_0 \cong 0.486$.

the switching probability [25]. The compliance with the binomial distribution guarantees that the measurements are only limited statistically: the measured probability exhibits the binomial broadening characteristic of a finite number of trials. This broadening can be viewed as unavoidable stochastic noise.

The other complementary way to judge the independence of the switching events is to calculate a pair-correlation function for measurements separated by n cycles in a long pulse train, e.g., with $N = 10^6$ pulses [22]. Such an approach should reveal coupling of the parasitic current modulation imposed on the testing pulses (e.g., 50-Hz power network coupling) but it may not be sensitive to slow drift or slow oscillations of the switching probability.

III. CORRELATED SWITCHING EVENTS

As we reduce the repetition time, the probing pulses become correlated. For $\Delta\tau = 2\ \mu$s [see Fig. 2(a)], when the S curve becomes very steep, the resulting switching distributions are no longer binomial (Fig. 4). It is difficult to describe them using a compact analytical distribution, since the correlations involved have a long-range character with a stochastic strength of correlation between pulses. The simplest numerical model, involving nearest-pulse correlations only, could assume two values of the switching probability: p for the case in which there was no switching in the previous pulse and q ($q > p$) if there was switching in the previous pulse. Such a model has a very limited range of validity, as it only works for the onset of correlations. Qualitatively, it is easy to observe that it accounts for the steepening of the S curve. The observed steepening (Figs. 2 and 4) can be viewed as reduction of the apparent effective escape temperature T_{eff}, where T_{eff} is dependent on the strength of the correlations. The case with $T_{\text{eff}} \approx 0$ is extremely sensitive to a change in the probing current. Inasmuch as S curves can be measured in a domain of current, temperature, or magnetic flux, an S curve with an engineered effective temperature would be a desirable building block for detectors, e.g., it could be employed for sensing magnetization reversals that produce tiny changes in magnetic flux.

IV. FULLY CORRELATED SWITCHING: PANIC DISTRIBUTION

We now move on to the case of fully correlated pulses. We reduce the probing period to $\Delta t = 6.5\ \mu$s [$\Delta\tau = 0.5\ \mu$s; see Fig. 2(a)]. Apparently, the switching number becomes completely unpredictable (Fig. 5). We enter a regime in which a single switching operation makes switching in the subsequent pulse certain, leading to a switching avalanche. Such a phenomenon is described using the following switching number $n(I_A)$ probability

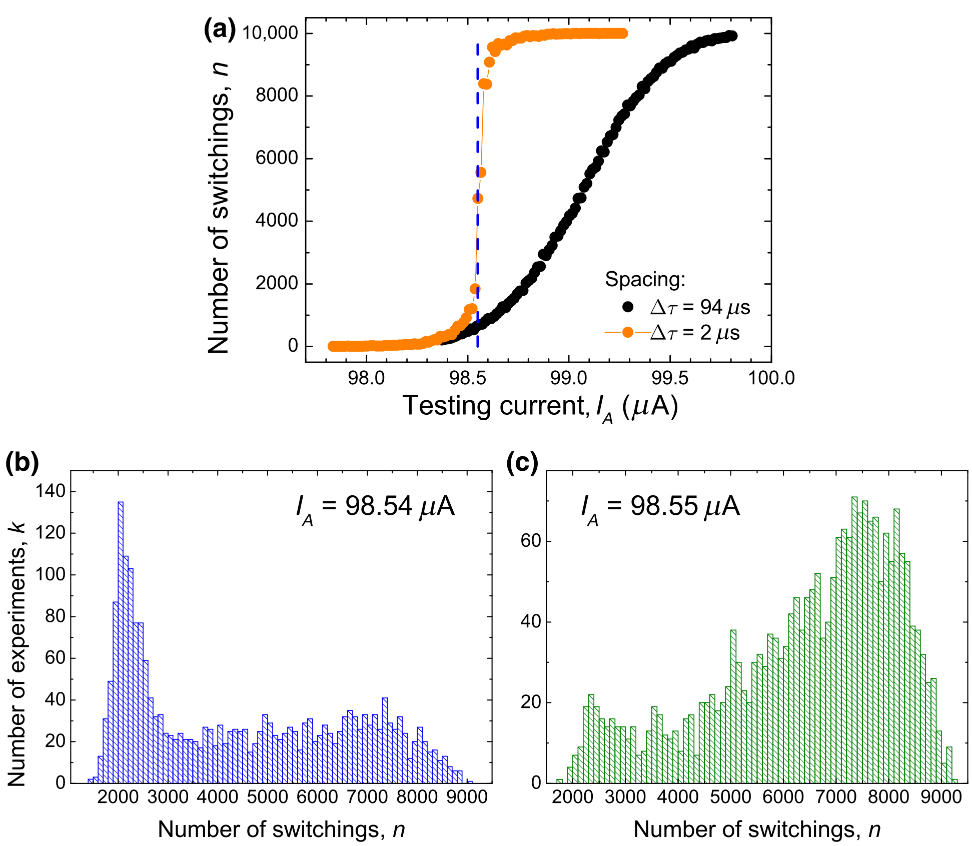

FIG. 4. The “flipping-coin” experiment for the correlated events at $T = 300$ mK. (a) Two S curves measured with pulse trains with different repetition rates (see Fig. 2). (b),(c) The number of experiments (vertical axis) resulting in the given number of switchings (horizontal axis) for the two slightly different current amplitudes indicated by the dashed vertical line in (a) (single line shown for two currents). The single experiment consists of sending $N = 10,000$ pulses and measuring the switching number. The experiment is repeated 2250 times.

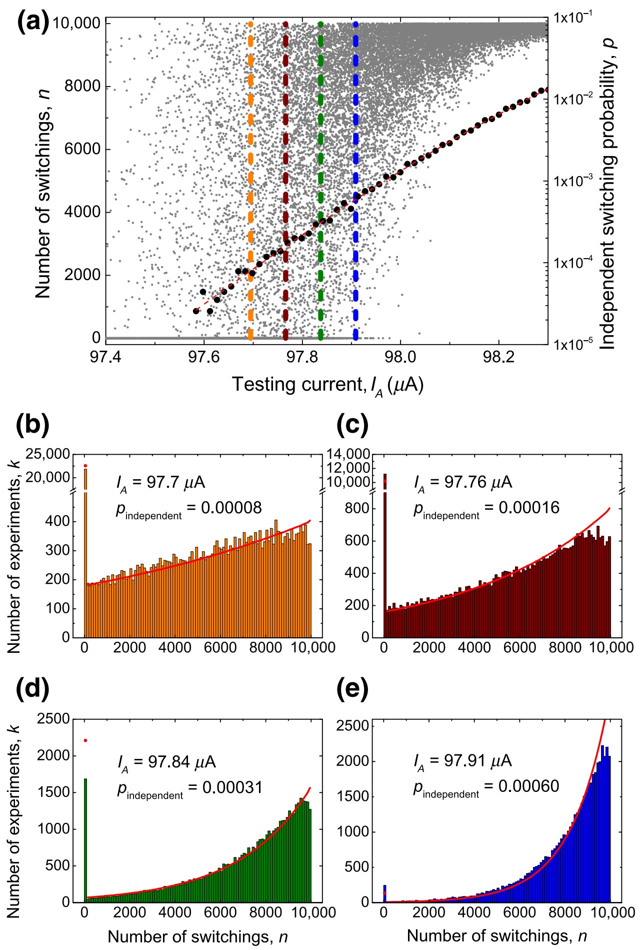

FIG. 5. The "flipping-coin" experiment for the fully correlated events at $T = 300$ mK. (a) The random cloud of the switching numbers (left-hand axis) for a very slowly swept current amplitude, with an imposed independent-switching probability measured in the limit of the independent-switching experiments (right-hand axis). Each single point is the result of a measurement with $N = 10,000$ pulses. (b)–(e) Experimental distributions (the number of experiments with corresponding numbers of switchings) recorded at constant current amplitudes indicated by the dashed vertical lines in (a). The single experiment consists of sending $N = 10,000$ pulses and measuring the switching number. The experiment is repeated 50,000 times for each distribution. The red lines and dots are predictions of the panic distributions with experimentally determined independent-switching probabilities p presented in (a).

distribution:

$$P[n(I_A)] = (1-p)^{N-n}p$$

for $n(I_A) \geq 1$ and

$$P[n(I_A) = 0] = (1-p)^N,$$

where N is the number of probing pulses. Since all of the pulses before the switching avalanche are not affected by the previous testing pulses, p is the independent-switching probability measured in the independent-switching regime [black dots in Fig. 5(a)].

We term the presented distribution the *panic distribution*. In Figs. 5(b)–5(e), we present four experimentally measured distributions with a direct comparison to the postulated panic distribution. We use p values measured in the independent-switching regime [black dots in Fig. 5(a)]. The remarkable agreement suggests that by performing measurements in the fully correlated regime, one may measure the independent-switching probability when this probability is vanishingly small.

Interestingly, the panic distribution may describe social and economic variables in situations when a single person affects the behavior of all other people within a certain group. It may also find application in situations when cascades of failures lead to the breakdown of a system; for example, electrical blackouts frequently result from a cascade of failures between interdependent networks [26].

V. DYNAMICS OF TEMPERATURE IN THE SWITCHING EXPERIMENT

In order to understand the physical origin of the correlations, we intentionally heat the bridge with the set of M prepulses preceding the actual testing pulse and vary the time between the last prepulse and the test pulse [Fig. 6(a)]. The current amplitude for the prepulses is set constant, at a level exceeding the switching threshold, yielding forced switching in each prepulse. After a single prepulse ($M = 1$), the electron temperature of the nanobridge exceeds T_C and relaxes to the bath temperature on a time scale of a few microseconds [Fig. 6(c)]. This relaxation is governed by electron-phonon coupling and by hot-electron diffusion [11]. Both energy-relaxation channels bring electrons into thermal equilibrium with phonons. As we increase the number of prepulses M, the whole substrate becomes overheated. This overheating relaxes at an approximately 1000 times slower rate than it takes for the hot electrons to achieve the local phonon temperature [Fig. 6(b)]. Since the two processes exhibit such different dynamics, they can be described in the linear regime using the sum of two exponential decays. It is also instructive to measure the increase in temperature of the substrate with the number of prepulses after the fast electron-phonon relaxation is over and the slow phonon-bath relaxation has not yet started. This can be accomplished by adjusting the relaxation time to 10 μs [see point A in Fig. 6(b)] and the result is presented in Fig. 6(d). Recalculation of the switching current in terms of temperature with the aid of a calibration curve (see the inset of Fig. 1) yields a $T_{\mathrm{sub}}(N)$ dependence, showing a slow but monotonous increase with a tendency for saturation [Fig. 6(e)]. The local substrate (phonon) temperature does not relax to the bath temperature before the arrival of the next testing pulse. Each switching deposits a bit of

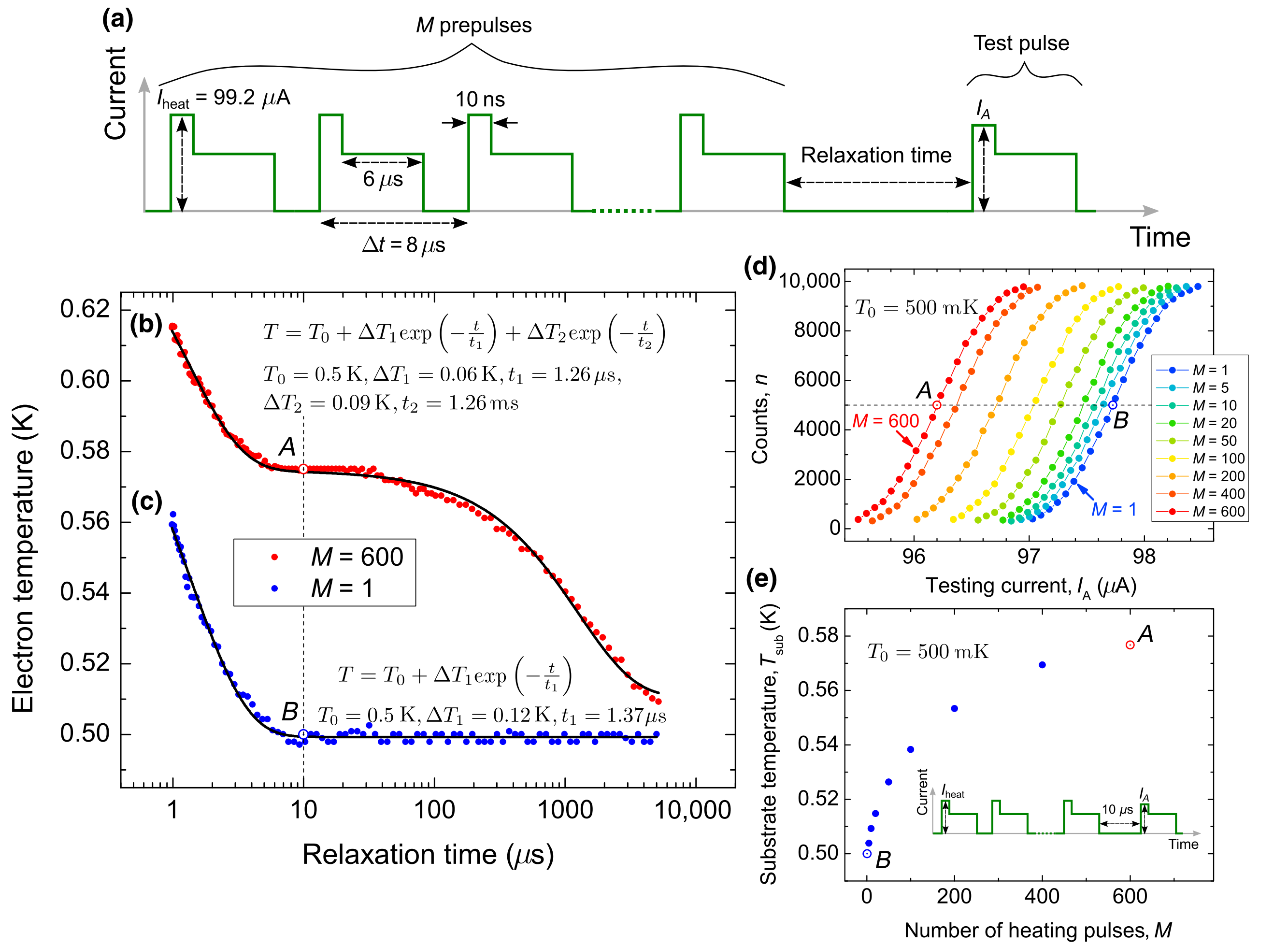

FIG. 6. The thermal dynamics of the nanobridge presented in the text. (a) The definition of the testing sequence. The single sequence consists of M heating prepulses followed by the testing pulse. The sequence is repeated N times to measure the switching number n. The repetition period is 10 ms, to allow for proper equilibration after each sequence. (b) Relaxation of the nanobridge after $M = 600$ forced switchings: two relaxation mechanisms are visible—the fast process is the same as in (c) and the slow one is approximately 1000 times slower and is attributed to relaxation of the local phonon temperature (substrate) toward the bath temperature. (c) Relaxation of the excess hot-electron energy toward equilibrium with local phonons after a single forced switching ($M = 1$): the local phonons are at bath temperature. (d) S curves measured for different numbers of heating prepulses 10 μs after the end of the last prepulse, when the electrons are already thermalized at the local phonon temperature. (e) The temperature rise of the local phonons as extracted from the S curves presented in (d) along the dotted line $n = 5000$, with the aid of the calibration curve (see inset of Fig. 1).

energy in the substrate, increasing the local phonon temperature at the nanobridge. The substrate overheating is the main cause of the correlation even between distant pulses. The fast relaxation process, which equalizes the electron and phonon temperatures, is only responsible for correlation between nearest pulses. It governs the behavior of the ensemble in the fully correlated case.

The relaxation time in the linear-response regime reads $\tau = C_p/G_{\mathrm{exp}}$, where C_p is the heat capacity of volume Ω ($C_p = \gamma\Omega T_e$, with $\gamma \sim 100$ J K^{-2} m^{-3} in the free electron model [11,27]) and G_{exp} is the dominant thermal conductance that is responsible for the relaxation. In general, G is governed by both hot-electron diffusion and electron-phonon coupling. For the electron-phonon relaxation channel, the linearized electron-phonon conductivity in the normal state is $G_{e\text{-ph},n} = 5\Sigma\Omega T_e^4$, where $\Sigma \cong 2\times 10^9$ W/m^{3}K^5 is the aluminum electron-phonon coupling constant, and we obtain a relaxation time $\tau_{e\text{-ph},n} = \gamma/(5\Sigma T_e^3) \sim 100$ ns. It must be noted that in a superconductor, quasiparticle-phonon coupling is significantly weaker than in the normal state [28]. The power flow in the normal state is given by a well-known formula, $P_{e\text{-ph}} = \Sigma(T_e^5 - T_{\mathrm{ph}}^5)$, while the power transferred in the superconducting state requires more involved calculation [11,29]. In particular, at 0.55 K it is reduced by 1 order of magnitude, thus increasing the relaxation by factor of 10: $\tau_{e\text{-ph},s} \cong 10\tau_{e\text{-ph},n}$, in agreement with the relaxation time observed in our experiment. Considering the long-wire limit, we may

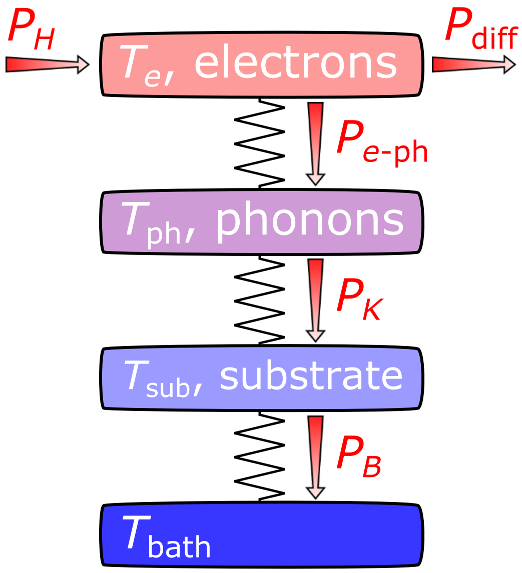

FIG. 7. The thermal model of a nanowire on a substrate thermally coupled to the bath temperature. P_H denotes the Joule heating, P_{diff} describes the hot-electron diffusion, $P_{e\text{-ph}}$ is the heat transferred from electrons to phonons, P_K is the Kapitza heat flow through the interface between the aluminum nanostructure and the substrate, and P_B is heat sink from the substrate to the bath.

neglect hot-electron diffusion in the middle of the wire, where the temperature-sensitive nanobridge is placed. To identify the origin of the slow relaxation process, we compare the linearized Kapitza conductance $G_k = A_k T^3$ with the electron-phonon conductance in the superconducting state, $G_{e\text{-ph},s}$ (Fig. 7). With a typical value of A_k observed for common metal-to-dielectric interfaces [30,31] ranging from 100 to 1000 $\text{W m}^{-2}\ \text{K}^{-4}$, we obtain $G_k/G_{e\text{-ph},s} = P_k/P_{e\text{-ph},s} \sim 8 - 80$, suggesting that local phonons in the wire are not overheated with respect to substrate phonons. Thus we conclude that $T_{\text{ph}} = T_{\text{sub}}$. We associate the slow relaxation process with local overheating of the substrate with respect to the sample-holder temperature (the bath temperature).

VI. CONCLUSION

We propose the "flipping-coin" experiment as a tool for studying correlation in switching experiments. We measure switching distributions for fixed current amplitudes in three regimes: for independent, correlated, and fully correlated switching events. We use the time interval between pulses as a control knob for the correlations. Our experiment provides an interesting approach not only to the study but also, perhaps more importantly, to the engineering of stochastic processes. We show that an independent regime is manifested by the binomial distribution with the proper variance. We demonstrate tuning of the apparent effective escape temperature in the correlated regime. We find that in the fully correlated case, the switching statistics are described by the "panic distribution," which exhibits high sensitivity to the independent-switching probability.

Our study gives an insight into the thermal dynamics of the switching experiment for a superconducting nanowire after it has switched to a normal state. It identifies two relaxation mechanisms in the switching measurements of superconducting bridges: the fast mechanism involves thermalization of electrons with substrate phonons and the slow one thermalization of substrate phonons with the thermal environment. The first mechanism is always present and proceeds through electron-phonon coupling and electron diffusion. The second one appears in measurements for which the testing pulses are too close to each other, allowing for a slow build-up of the phonon temperature. Its dynamics depend on the coupling strength between the substrate and the thermal bath.

It is a well-known practice to measure the magnetic field dependence of the switching current (i.e., the Fraunhofer pattern) to prove junction homogeneity prior to more advanced studies. Similarly, we propose to perform the "flipping coin" experiment to strengthen the quality of measured S curves and switching-current distributions and, consequently, the credibility of the escape mechanism that defines their shape.

ACKNOWLEDGMENTS

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[23] In fact, what is measured in the independent-switching regime is an estimator for the probability, since measurement of the probability itself would need to involve an infinite number of probing pulses.

[24] See the Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevApplied.11.054070 for the experimental estimation of the retrapping time.

[25] For symmetric noise and/or instability ($\langle \Delta I_A \rangle = 0$, $\langle \Delta T \rangle = 0$) of sufficiently low amplitude and probing at an average probability $p_{AV} = 0.5$, fluctuations would not be revealed in the experimental distribution because the number of pulses with suppressed p values would be equal to the number of pulses with enhanced p values (the S curve can be linearized around $p = 0.5$, $\Delta p \sim \Delta I_A$, $\Delta p \sim \Delta T$). The same argument does *not* hold for the distribution collected at $p_{AV} = 0.9$, which should be sensitive to symmetric noise.

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