Klinkhamer, F. R.; Volovik, G. E.

Tetrads and q-theory

Published in:
JETP Letters

DOI:
10.1134/S0021364019060031

Published: 01/03/2019

Document Version
Peer-reviewed accepted author manuscript, also known as Final accepted manuscript or Post-print

Please cite the original version:
Klinkhamer, F. R., & Volovik, G. E. (2019). Tetrads and q-theory. *JETP Letters*, *109*(6), 364-367.
https://doi.org/10.1134/S0021364019060031

JETP Lett. **109**, 364 (2019) arXiv:1812.07046

Tetrads and q-theory

F.R. Klinkhamer*

Institute for Theoretical Physics,
Karlsruhe Institute of Technology (KIT),
76128 Karlsruhe, Germany

G.E. Volovik†

Low Temperature Laboratory, Department of Applied Physics,
Aalto University, PO Box 15100, FI-00076 Aalto, Finland,
and
Landau Institute for Theoretical Physics, Russian Academy of Sciences,
Kosygina 2, 119334 Moscow, Russia

Abstract

As the microscopic structure of the deep relativistic quantum vacuum is unknown, a phenomenological approach (q-theory) has been proposed to describe the vacuum degrees of freedom and the dynamics of the vacuum energy after the Big Bang. The original q-theory was based on a four-form field strength from a three-form gauge potential. However, this realization of q-theory, just as others suggested so far, is rather artificial and does not take into account the fermionic nature of the vacuum. We now propose a more physical realization of the q-variable. In this approach, we assume that the vacuum has the properties of a plastic (malleable) fermionic crystalline medium. The new approach unites general relativity and fermionic microscopic (trans-Planckian) degrees of freedom, as the approach involves both the tetrad of standard gravity and the elasticity tetrad of the hypothetical vacuum crystal. This approach also allows for the description of possible topological phases of the quantum vacuum.

PACS numbers: 04.20.Cv, 95.36.+x, 98.80.Es

Keywords: general relativity, dark energy, cosmological constant

* frans.klinkhamer@kit.edu
† volovik@ltl.tkk.fi

I. INTRODUCTION

The q-theory framework [1, 2] provides a general phenomenological approach to the dynamics of vacuum energy, which may be useful for the resolution of problems related to the cosmological constant in the Einstein equation (a brief review of q-theory appears in Appendix A of Ref. [3]). The advantage of q-theory is that, at the classical level, the field equations of the theory essentially do not depend on the detailed microscopic (trans-Planckian) origin of the q-field. In the classical limit, the field equations of q-theory (i.e., the equation for the microscopic variable q describing the quantum vacuum and the modified Einstein equation for the metric) are universal.

The q-theory approach to the cosmological constant problem aims to describe the decay of the vacuum energy density from an initial Planck-scale value to the present value of the cosmological constant. However, the correct description of this decay requires the quantum version of q-theory, which may be different for different classes of realizations of the q-variable (see Sec. 1 in Ref. [4] for a general discussion of quantum-dissipative effects and Ref. [5] for a sample calculation).

Up till now, our discussions of q-theory have primarily used the nonlinear theory of a four-form field strength from a three-form gauge potential (the linear theory of the vacuum energy in terms of the four-form field strength has been considered by Hawking [6], in particular). However, the four-form field strength, though useful for the construction of the general phenomenological equations for the quantum vacuum, is rather abstract. The physical origin of such a field is not clear. Instead, we must look for another variable, which may have a direct relation to the underlying microscopic physics of the quantum vacuum.

Here, we propose a physical realization of the vacuum as a plastic (malleable) medium, which locally has the structure of a $(3+1)$-dimensional fermionic crystal. In this realization, the corresponding vacuum variable q is expressed in terms of both the gravitational tetrad and the elasticity tetrad of the underlying crystal. The new realization in terms of tetrad fields is more appropriate for the quantum theory of the fermionic vacuum of our Universe than the realization with a bosonic four-form field strength.

As distinct from the conventional gravitational tetrads, the elasticity tetrads have dimensions of inverse length or inverse time. This allows for the construction of topological Chern-Simons terms describing the mixed gravity-elasticity anomaly and the mixed gauge-elasticity anomaly of the fermionic vacuum.

Throughout, we use natural units with $c = \hbar = 1$ and take the metric signature $(-+++)$.

II. GRAVITY TETRAD

The tetrad formalism of torsion-less gravity is given by the following equations:

$$g_{\mu\nu} = \eta_{ab}\, e^a_\mu\, e^b_\nu\,, \tag{1a}$$

$$\nabla_\mu\, g^{\mu\nu} = 0\,, \tag{1b}$$

$$D_\mu\, e^a_\nu \equiv \nabla_\mu\, e^a_\nu + \omega^a_{\mu b}\, e^b_\nu = 0\,, \tag{1c}$$

where ∇_μ is the standard covariant derivative of general relativity and $\omega^a_{\mu b}$ the spin connection,

$$\omega^a_{\mu b} = e^a_\nu\, \nabla_\mu\, e^\nu_b\,. \tag{2}$$

Given the tetrad e^a_μ, the inverse tetrad e^μ_a is defined by

$$e^a_\mu\, e^\mu_b = \delta^b_a\,, \tag{3a}$$

or, equivalently,

$$e^a_\mu\, e^\nu_a = \delta^\nu_\mu\,. \tag{3b}$$

An introduction to the tetrad formalism of gravitation can be found in, e.g., Sec. 12.5 of Ref. [7].

III. ELASTICITY TETRAD

In the present article, we interpret the vacuum as a plastic (malleable) fermionic crystalline medium. The “vacuum crystal” could, in principle, also have bosonic “atoms,” but, for definiteness, we focus on the fermionic case. At each point of spacetime, we have a local system of four deformed crystallographic manifolds of constant phase $X^a(x) = 2\pi n^a$, for $n^a \in \mathbb{Z}$ with $a = 0,\, 1,\, 2,\, 3$. (For the usual atomic crystals in 3-space, the index a runs over 1, 2, 3. See Sec. 6 in Ref. [8] for a general discussion of the elasticity theory of crystals and Sec. 3.2 in Ref. [9] for a succinct summary of the role of the phase functions X^a.)

In addition to the conventional tetrad e^a_μ of gravity, we then introduce the following elasticity tetrad E^a_μ (cf. Refs. [8–10]):

$$E_\mu{}^a(x) = D_\mu\, X^a(x)\,, \tag{4}$$

where both indices a and μ take values from the set $\{0,\, 1,\, 2,\, 3\}$. Invariance under the local $SO(1,\, 3)$ group of rotations is implemented by defining

$$D_\mu\, X^a \equiv \nabla_\mu\, X^a + \omega^a_{\mu b}\, X^b = \partial_\mu\, X^a + \omega^a_{\mu b}\, X^b\,. \tag{5}$$

Note that the dimensionalities of the elasticity tetrads ${E_i}^a$ and ${E_0}^a$ are inverse length and inverse time, respectively.

IV. Q-FIELD REALIZATION BY TETRADS

Let us, now, consider the q-theory description of the quantum vacuum [1] by assuming that the vacuum energy density $\epsilon(q)$ in the action depends on the following type of q-field:

$$q(x) = \frac{1}{4}\, e^{\mu}_{a}(x)\, E^{a}_{\mu}(x)\,, \tag{6}$$

in terms of the inverse of the gravity tetrad e^a_μ from (1) and the elasticity tetrad E^a_μ from (4). The action in its simplest form is then given by

$$S = \int_{\mathbb{R}^4} d^4x\, e\, \left(\frac{R}{16\pi G_N} + \epsilon(q) \right)\,, \tag{7}$$

where R is the Ricci curvature scalar and e the tetrad determinant,

$$e \equiv \det e^a_\mu\,. \tag{8}$$

The pure-gravity part of the action (7) contains the integral

$$\int_{\mathbb{R}^4} d^4x\, e\, R = \int_{\mathbb{R}^4} d^4x\, e\, e^{\mu}_{a} e^{\nu}_{b}\, F^{ab}_{\mu\nu}\,, \tag{9}$$

with a curvature two-form given by

$$F^{ab} = d\,\omega^{ab} + \omega^{a}_{c} \wedge \omega^{bc}\,, \tag{10}$$

in terms of the spin connection (2).

Variation of the action (7) over e^μ_a gives the Einstein equation [7],

$$R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\, R \;=\; 8\pi G_N\, \rho_V(q)\, g_{\mu\nu}\,, \tag{11}$$

where $\rho_V(q)$ will be discussed shortly, and variation over X^a gives the following differential equation for q (which is both a coordinate scalar and a Lorentz scalar):

$$\partial_\mu \left(\frac{d\epsilon(q)}{dq} \right) \;=\; 0\,, \tag{12}$$

where (1c) has been used. The vacuum energy density $\rho_V(q)$, which enters the Einstein equation (11) through a cosmological-constant-type term, is given by

$$\rho_V(q) \equiv \epsilon(q) - q\, \frac{d\epsilon(q)}{dq}\,, \tag{13}$$

with an extra term $-q\, d\epsilon/dq$ tracing back to the e^μ_a dependence of the q-realization (6).

The field equation (12) for q has the following general solution:

$$\frac{d\epsilon(q)}{dq} = \mu = \text{constant}\,, \tag{14}$$

where the arbitrary constant μ (interpreted as a "chemical potential" in Ref. [1]) is not to be confused with the spacetime index μ. The solution (14) allows us to rewrite the gravitating vacuum energy density (13) as $\rho_V(q) = \epsilon(q) - \mu\, q$.

The quantum vacuum in perfect equilibrium has a constant nonzero value of the q-field,

$$q(x) = q_0 = \text{constant}\,, \tag{15}$$

which gives a particular value μ_0 for the constant μ in (14),

$$\mu_0 = \left[\frac{d\epsilon(q)}{dq}\right]_{q=q_0}\,. \tag{16}$$

In addition, there are the following equilibrium conditions:

$$\rho_V(q_0) \;=\; 0\,, \tag{17a}$$

$$\left[\frac{d\,\rho_V(q)}{dq}\right]_{q=q_0} \;=\; 0\,, \tag{17b}$$

$$\left[\frac{d^2\,\rho_V(q)}{dq^2}\right]_{q=q_0} \;>\; 0\,. \tag{17c}$$

These conditions have been discussed for q-theory in general (see, e.g., Sec. II B in Ref. [1]). First, recall that the conditions (17a) and (17b) result from the self-adjustment of the conserved vacuum variable q [having, as mentioned before, the chemical potential $\mu = d\epsilon/dq$ and the thermodynamically active vacuum energy density $\rho_V(q) = \epsilon(q) - \mu\, q$], as follows from the Gibbs–Duhem relation for an isolated self-sustained system without external pressure. Second, recall that condition (17c) can be interpreted as having a positive isothermal compressibility. A further important consequence of the equilibrium conditions (17) will be discussed in the next section.

To summarize, we have obtained with (6) one further realization of the q-variable, in addition to the four-form realization [1] and the brane realization [2]. The advantage of this new realization is that it has a more direct physical origin.

V. DISCUSSION

The action (7) is special in the sense that the only dependence on the "vacuum crystal" is via the q-variable as defined by (6). Another approach is to assume that the action depends on the elasticity tetrad $E_\mu{}^a(x)$ in a general way, but still obeys all the gauge symmetries. One possible action density term would be, for example,

$$k\,\left[\nabla^\kappa(X^a e_a^\lambda)\right]\,\left[\nabla^\mu(X^b e_b^\nu)\right]\,g_{\kappa\mu}\,g_{\lambda\nu}\,, \tag{18}$$

with a constant k. Then, q only appears via the equilibrium solution of the field equation obtained by variation over X^a,

$$\Big[D_\mu X^a\Big]_{\text{equil. sol.}} = q\,e_\mu^a\,, \tag{19}$$

for constant q.

In this approach with a general dependence of the action on the elasticity tetrad, we expect that the local Newtonian gravitational dynamics is ruined. The reason is that we have from (19) that

$$\big[q\big]_{\text{equil. sol.}} = \frac{1}{4}\,e_a^\mu\,D_\mu\,X^a = D_\mu\left(\frac{1}{4}\,e_a^\mu\,X^a\right) \equiv D_\mu\,\mathcal{A}^\mu = \nabla_\mu\,\mathcal{A}^\mu\,, \tag{20}$$

where the composite vector field $\mathcal{A}^\mu(x)$ plays a similar role as the fundamental vector field $A^\mu(x)$ in Dolgov's discussion of the cosmological constant problem [11, 12]. In fact, the action density term (18), rewritten in terms of $\mathcal{A}^\mu(x)$ from (20), corresponds precisely to Dolgov's term $[\nabla_\mu A_\nu]\,[\nabla^\mu A^\nu]$.

From (20), we now observe that, in a cosmological context, a constant q requires $\mathcal{A}_0 \propto t$. But precisely the behavior $\mathcal{A}_0 \propto t$ is the source of trouble for Newtonian gravity as shown by Rubakov and Tinyakov [13]. (It is possible to maintain Newtonian gravity by the introduction of further q-type fields, corresponding to different interpenetrating vacuum crystals, but the theory looks rather artificial [14–16].)

This disaster of ruining Newtonian gravity is not expected to occur in the approach of Sec. IV. The reason is two-fold. First, we observe that the corresponding field equations do not explicitly carry the composite vector field $\mathcal{A}^\mu(x)$ (as happens for the Dolgov theory; cf. (A1) with $\zeta = 1$ in Ref. [14]) but only contain the fields $g_{\mu\nu}(x)$ and $q(x)$, as shown by (11), (12), and (13). Second, we observe that the Einstein equation (11) with $\rho_V \sim (q-q_0)^2$, according to the equilibrium conditions (17), gives rise to the standard linearized Einstein equation for a small perturbation around flat Minkowski spacetime with $\rho_V(q_0) = 0$. Regarding the last observation, we refer to Ref. [17] for a discussion of the special case of four-form q-theory and to Sec. 2.2 in Ref. [18] for a general discussion.

In closing, we remark that our setup with vacuum elasticity resembles somewhat the setup of gravity as a gauge-theory-squared [19], where the metric is considered to be the product of two Yang–Mills fields. The elasticity tetrad plays a similar role as the Yang–Mills fields.

But, for us, the elasticity tetrad describes the vacuum as a (3+1)-dimensional fermionic crystalline medium, where dislocations correspond to torsion and disclinations to curvature [8]. Such a medium may also have different nontrivial topological phases, characterized by particular Chern–Simons-like terms with prefactors determined by momentum-space topological invariants [10].

Another interpretation of the vacuum crystal as described by an elasticity tetrad is that this crystal may provide a dynamic realization of the prior metric which enters a particular formulation of q-theory, where the q-variable essentially equals the determinant of the metric relative to the determinant of a fixed prior metric [18].

A further incentive for choosing the elasticity tetrad as a realization of the q-variable is based on the following observation. The quantum version of q-theory is sensitive to the particular realization of the q-field. Assuming q-theory to be relevant, the comparison with experiment may then provide information on the detailed structure of the fermionic quantum vacuum and, in particular, on the types of quantum anomalies. The fermionic crystalline model of the vacuum is one of the possible structures of the deep fermionic vacuum, distinct from a structure described by the abstract four-form field strength. This new fermionic structure gives, for example, rise to new types of quantum anomalies, where elasticity tetrads are mixed with gauge and spin-connection fields [10].

ACKNOWLEDGMENTS

The work of GEV has been supported by the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (Grant Agreement No. 694248).

[1] F.R. Klinkhamer and G.E. Volovik, "Self-tuning vacuum variable and cosmological constant," Phys. Rev. D **77**, 085015 (2008), arXiv:0711.3170.

[2] F.R. Klinkhamer and G.E. Volovik, "Brane realization of q-theory and the cosmological constant problem," JETP Lett. **103**, 627 (2016), arXiv:1604.06060.

[3] F.R. Klinkhamer and G.E. Volovik, "Dynamics of the quantum vacuum: Cosmology as relaxation to the equilibrium state," J. Phys. Conf. Ser. **314**, 012004 (2011), arXiv:1102.3152.

[4] F.R. Klinkhamer, M.Savelainen, and G.E. Volovik, "Relaxation of vacuum energy in q-theory," JETP **125**, 268 (2017), arXiv:1601.04676.

[5] F.R. Klinkhamer and G.E. Volovik, "Dynamic cancellation of a cosmological constant and approach to the Minkowski vacuum," Mod. Phys. Lett. A **31**, 1650160 (2016), arXiv:1601.00601.

[6] S.W. Hawking, "The cosmological constant is probably zero," Phys. Lett. B **134**, 403 (1984).

[7] S. Weinberg, *Gravitation and Cosmology : Principles and Applications of the General Theory of Relativity* (John Wiley and Sons, New York, 1972).

[8] I.E. Dzyaloshinskii, and G.E. Volovick, "Poisson brackets in condensed matter," Ann. Phys. **125**, 67 (1980).

[9] J. Nissinen and G.E. Volovik, "Tetrads in solids: from elasticity theory to topological quantum Hall systems and Weyl fermions," JETP **127**, 948 (2018), arXiv:1803.09234.

[10] J. Nissinen and G.E. Volovik, "3d QHE and elasticity tetrads," arXiv:1812.03175.

[11] A.D. Dolgov, "Field model with a dynamic cancellation of the cosmological constant," JETP Lett. **41**, 345 (1985).

[12] A.D. Dolgov, "Higher spin fields and the problem of cosmological constant," Phys. Rev. D **55**, 5881 (1997), arXiv:astro-ph/9608175.

[13] V.A. Rubakov and P.G. Tinyakov, "Ruling out a higher spin field solution to the cosmological constant problem," Phys. Rev. D **61**, 087503 (2000), arXiv:hep-ph/9906239.

[14] V. Emelyanov and F.R. Klinkhamer, "Reconsidering a higher-spin-field solution to the main cosmological constant problem," Phys. Rev. D **85**, 063522 (2012), arXiv:1107.0961.

[15] V. Emelyanov and F.R. Klinkhamer, "Possible solution to the main cosmological constant problem," Phys. Rev. D **85**, 103508 (2012), arXiv:1109.4915.

[16] O. Santillan and M. Scornavacche, "The Newton constant and gravitational waves in some vector field adjusting mechanisms," JCAP **1710**, 048 (2017), arXiv:1705.04897.

[17] F.R. Klinkhamer, "Brief Report: Linearized Einstein gravity from four-form q-theory at equilibrium," preprint KA–TP–25–2016 [available from `https://www.itp.kit.edu/publications/preprints/2016`].

[18] F.R. Klinkhamer, "A generalization of unimodular gravity with vacuum-matter energy exchange," Int. J. Mod. Phys. D **26**, 1750006 (2016), arXiv:1604.03065.

[19] A. Anastasiou, L. Borsten, M.J. Duff, S. Nagy, and M. Zoccali, "Gravity as gauge theory squared: A ghost story," Phys. Rev. Lett. **121**, 211601 (2018), arXiv:1807.02486.