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On the current state of flotation modelling for process control

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Abstract: Despite significant effort in modelling and simulating flotation circuits, comprehensive model based control and optimisation implementations on industrial circuits remain scarce. In this paper, the factors preventing more widespread implementation of model-based control and optimisation applications are investigated by focussing on three aspects. Firstly, the critical variables required in a simplified flotation model are identified. Models that are currently used in control, optimisation and supervisory applications are thereafter analysed to determine to what extent the required variables are modelled. Finally, online instrumentation available to support these models are investigated, also including instrumentation that is still under development and not commonly available in commercial applications. Although models used in control applications tend to focus on subsections of the flotation process, there seem to be a good agreement between the required and modelled variables. Model fitting however often relies on extensive sampling campaigns that will need to be repeated regularly to maintain model accuracy. A number of online measurements of sufficient accuracy are still not available to support these models, compromising the long term reliable use of models in online applications. The fact that flotation processes are in many instances not extensively instrumented, constrains online maintenance and adaption of model based solutions further.

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1. INTRODUCTION

The origins of froth flotation can be traced back to the 1880s (Gaudin, 1957) while column flotation was patented in the early 1960s (Finch & Dobby, 1990). Froth flotation has been modelled extensively to include factors such as the chemical reactions (King, 1982), how different size classes participate in the process (Lynch, et al., 1981) and the physical processes such as particle-bubble collision (Finch & Dobby, 1990). Collaboration between industry and academia as part of programmes such as AMIRA P9 focussed efforts, and significant progress has been made during the past 25 years in understanding the intricacies of the froth flotation process. As a result, many of the principles described earlier have been integrated into a comprehensive simulator (Schwarz, et al., 2006).

Despite the rich modelling framework available, the number of successful industrial implementations of model based control and optimization strategies (other than basic level control as described by Schubert et al. (1995)) remain scarce (Shean & Cilliers, 2011). Reasons for this include a lack of instrumentation, lack of reliable dynamic models and inadequate regulatory control (Bergh & Yianatos, 2011). The issues of the lack of suitable dynamic models and insufficient instrumentation to interface with these models cannot be separated and need to be addressed simultaneously.

Model based implementations often fall into disuse after some time due to the models not being robust over a large range of operating conditions (Shean & Cilliers, 2011). The use of

empirical models fitted over a limited operating range has similar limitations, and require frequent recalibration when the operating points shift (Bouchard, et al., 2009). In the absence of online measurements to recalibrate these models, or at least alert the operator of the need for recalibration, these strategies are bound to fail.

The aim of this paper is hence to identify the key variables required in flotation control applications, determine if existing models take these variables into account, and consider to what extent online measurements required by these models are commonly available on froth flotation plants, or are being developed and can potentially be made available on industrial sites in future. The focus is on long term industrial implementations rather than short term pilot plant campaigns.

2. MODEL REQUIREMENTS

2.1 Structure

The motivation to develop many of the existing flotation models, is to model different circuit configurations and operating practices with high accuracy, in order to recommend changes that would improve operation. Examples of such activities are described by Schwarz et al. (2006). Under these circumstances a detailed sampling campaign can often be justified to provide the data to fit a comprehensive set of model parameters. Where models are to be used online, manual sampling campaigns are not viable on an ongoing basis, and online measurements would have to be used to maintain model integrity. A model for continuous control applications would

thus need to be structured in such a way that available online measurements provide the stimuli to the process and are also used to maintain model integrity by estimating parameters where possible.

Basic froth flotation models are typically extended with the aim of characterising some behaviour that cannot be explained by existing models. While more detailed modelling is essential in improving model accuracy and in advancing knowledge about the process, it does result in a significantly larger set of parameters to be fitted initially and updated regularly to maintain model consistency. For control purposes, some accuracy may be sacrificed in exchange for fewer parameters. The direction of change and relative magnitude is generally of more importance than the absolute value of a variable, as measurement feedback can correct for model inaccuracies. While the decision on which interactions to ignore are not trivial, a reliable model would have to be based on a significantly reduced parameter set, to ensure that model accuracy is not degraded by the use of estimates based on parameters that cannot be updated dynamically.

Despite the requirement of minimising the set of parameters, the model needs to be able to estimate process dynamics required for control with sufficient accuracy. It must also be able to model non-linear phenomena such as peak air recovery (Hadler & Cilliers, 2009) and discrepancies between mass-pull and recovery (Hadler, et al., 2010) that currently receive research interest, with sufficient accuracy.

While laboratory and pilot-plant scale applications show some benefit of deriving empirical models (Bouchard et al., 2009), the long-term reliability of these models in the presence of changing operating conditions is a concern. Phenomenological models should thus take priority. Bouchard et al. (2009) however also commented that empirical models should not be dismissed completely, as all models as well as sensors require calibration.

2.2 Variables

A number of authors listed the key variables required in the control of froth flotation processes, for example Finch & Dobby (1990), Lynch et al. (1981), Bergh & Yianatos (2011) and Laurila et al. (2002). There is to a large extent agreement on the set of variables required, and a summary follows:

As inputs, or manipulated variables, the following variables can be used to drive the process in a desired direction:

- Reagent additions
- Pulp level setpoints
- Air flow rate setpoints
- Froth wash water rate (particularly in columns)

Lynch et al. (1981) also included addition points for reagents and collection points for concentrate, but these form part of circuit design parameters rather than online control parameters.

Depending on the circuit configuration, feed characteristics are either considered as disturbances or as manipulated variables,

as indicated by Lynch et al. (1981). Bergh & Yianatos (2011) considered slurry flow as a manipulated variable rather than a disturbance, which would also be the case in integrated grinding and flotation control and optimisation applications (Conradie et al., 2003). The following feed properties can be classified as manipulated variables or disturbances:

- Pulp density
- Volumetric flowrate
- Fineness of grind

The main outputs of the process relate to its economic performance, and are grade and recovery. Lynch, et al. (1981) also included concentrate density and flowrate in the outputs referred to as “performance variables”, as the total production also affects profitability.

A number of process states has a direct influence on the economic outputs of the process, and are typically affected by the manipulated variables and disturbances. The following states, also referred to as “intermediate variables” by Lynch, et al. (1981) are listed:

- Froth depth
- Gas holdup
- Bias superficial velocity (mostly columns)
- Air superficial velocity
- Feed, tailings and concentrate flow rates
- Mineral concentrations in all intermediate streams (grades)
- Densities of all streams

The largest discrepancy between the variables described by the authors, is in the variables considered disturbances. Bergh & Yianatos (2011) only included the first 3 disturbances listed below, while the full list is described by Laurila et al. (2002).

- Feed size distribution
- Feed grade (minerals concentration in feed)
- Feed density
- Feed mineralogy (fineness of crystallisation, minerals)
- Electrochemical potentials (Eh, pH)
- Particle properties (size distribution, shape, degree of liberation)
- Froth properties (speed, bubble size distribution, stability)

The fact that Bergh & Yianatos (2011) managed to explain 92% of variance with a reduced parameter set model, using only 6 latent variables obtained through principle component analysis, indicates that a model with a small parameter set may still provide sufficient accuracy for control purposes, but the complexity required is likely to be process dependent (Laurila et al., 2002). Shean & Cilliers (2011) confirmed that all these variables are not necessarily required to obtain good control performance, but that their impact needs to be considered. Simplifying assumptions, for example that the feed distribution and density would not vary significantly if the grinding circuit control is effective have been proposed (Wills & Napier-Munn, 2006) and can potentially be used to simplify models without degrading controller performance.

3. MODELS USED FOR CONTROL

The flotation process involves complex physical and chemical reactions with various contributing factors. Generally, flotation modelling approaches can be divided into two categories: Kinetic (first-principle) modelling and data-driven modelling. The kinetic modelling studies mainly include:

- Flotation rate modelling: The influence of operational parameters, such as pulp density, chemical reagents, aeration rate and, froth depth, on the flotation rate;
- Mass balance modelling: Mass and flow conservation in a flotation cell;
- Probability modelling: The probability of collision / adhesion / detachment between particles and bubbles, and the merging / bursting of bubbles.

Data driven modelling mainly involve:

- Performance evaluation: The relationship between flotation performance and froth features;
- Grade / recovery prediction: Predict the concentrate grade / recovery using inlet conditions and operational variables;
- Soft sensing: Estimate key process variables, e.g. pH, pulp level, when the default instrument is not available or out of order.

Bascur (2005) developed a detailed phenomenological flotation model. The model linked together the particle / bubble and water transport mechanisms, as well as the hydrodynamic characteristic of a flotation cell, and is able to describe the behaviour of particles with different mineralogical composition and particle sizes under a wide range of steady state and dynamic operating conditions. This model provides detailed understanding of flotation operations at the expense of increasing complexity.

The simplest models used in froth flotation control have been derived to stabilise pulp levels in interacting cells in the presence of variations in feed rate. Despite their simplicity, flotation level control systems, such as Mintek's FloatStar (Schubert et al., 1995) can significantly improve the economic performance of a flotation plant (Craig and Henning, 2000; Craig and Koch, 2003).

In Jämsä-Jounela (1992), a simplified flotation model based on both mass and volume balances was developed for the control of a rougher flotation bank. The model parameters were obtained through an industrial experimental campaign.

The level (y_i) response of a cell can be calculated from the difference between the slurry inflow from an upstream (Q_{i-1}) cell and the slurry outflow from the cell (Q_i), as shown in (1), assuming a constant cross section area (A_i) over the cell height.

$$\frac{\partial y_i}{\partial t} = \frac{Q_{i-1} - Q_i}{A_i} \quad (1)$$

Jämsä-Jounela et al. (2003) improved on this model by taking the structure of flotation cells and valve sizing into account. Various control strategies for pulp level control, including PI control, feed-forward control, decoupling control and multivariable control, have been tested and compared using this model (Kämpjärvi & Jämsä-Jounela, 2003).

When the froth phase is considered in addition to the slurry phase as part of a model, a mass balance forms the core of the model to track mass flows between the feed, tailings and concentrate streams. Depending on the complexity of the model and data available for fitting, the mass balance may be performed on specific components in a stream, for example PGMs, chromite and gangue (Du Preez et al. 2013) or further divided into floatability classes per component based on granulometry (typically size classes) (Putz & Cipriano, 2015).

Maldonado, et al. (2009) implemented a Model Predictive Control (MPC) strategy to control a flotation column pilot plant. Gas holdup in the collection zone was measured using a conductivity probe, while bias rate was calculated from a difference in conductivities between the wash-water and feed water streams in a two-phase system. A 2x2 model was implemented in an MPC controller, with the outputs defined as gas holdup and bias rate, and the inputs as wash water feed rate and aeration rate. The models were however empirical models fitted from operational data. Upper and lower limits for both gas holdup and bias rate were included in the controller, based on a desired operating range and expected constraints that would ensure such operation.

Bergh & Yianatos (2013) developed a simulator for rougher flotation banks and calibrated it using experimental data. The simulator was initially used in parallel with the existing control system to evaluate the effect of changes in operating conditions. It was later redesigned to act as an expert system, but this functionality has only been tested in a simulated environment. Other industrial implementations of expert systems (Kewe, et al., 2014; Kewe, Moffat and Schaffer, 2014) and optimisation strategies (Baas, et al., 2007) are often rule based, with limited detail provided on the underlying models.

Bergh & Yianatos (2013) modelled mass transfer between the pulp and froth phase by defining a global cell recovery (R_G) based on collection zone recovery (R_C) and froth recovery (R_F), as defined in (2). Yianatos et al. (2008) used a similar approach. Although (2) can be simplified by lumping the froth- and collection zone recoveries together, Du Preez et al. (2013) noted that a more detailed model is likely to improve their results that was based on a single parameter model.

$$R_G = \frac{R_C \cdot R_F}{1 - R_C(1 - R_F)} \quad (2)$$

The complexity of the equations describing collection zone and froth recoveries shows some variation between models. Bergh & Yianatos (2013) modelled R_C as a function of pulp residence time, flotation rate constants and maximum achievable recovery. R_F was modelled using two equations. The first is a function of bubble load, superficial gas rate, concentrate mass flow, cell dimensions and concentrate- and bubble load grade. The other is a function of a froth stability parameter, froth depth, gas holdup, superficial gas velocity and maximum froth recovery. For both R_C and R_F empirical equations were fitted to model the degradation in flotation rate distribution and the reduction of froth stability when moving down a bank of flotation cells. Gangue recovery is modelled based on water recovery (a function of superficial gas rate, froth depth and froth stability). The model fit requires an

extensive sampling campaign, including grade and mass data per size class on all streams, density (per stream), and also air holdup, bubble loading and grade, and aeration rates for each cell.

The compartment model described by Savassi (2005) include similar elements as described by Bergh & Yianatos (2013), but all the factors contributing to overall recovery is included in a single equation. In (3), collection zone recovery is described by k_{cz} (flotation rate for the collection zone) and τ_{cz} (residence time in the collection zone). Similar to (2), R_F refers to froth recovery while ENT and R_w refers to degree of entrainment and water recovery to the concentrate stream, respectively. Dos Santos et al. (2014) showed how phenomenological models can be used as part of (3) to model entrainment, water recovery and froth recovery.

$$R_G = \frac{k_{cz} \cdot \tau_{cz} \cdot R_F \cdot (1 - R_w) + ENT \cdot R_w}{(1 + k_{cz} \cdot \tau_{cz} \cdot R_F) \cdot (1 - R_w) + ENT \cdot R_w} \quad (3)$$

Putz & Cipriano (2015) used a hybrid Model Predictive Control (HMPC) strategy on a simulator to control the final tailings grade of a flotation circuit subject to level constraints. The hybrid functionality was used to include scenarios where pulp overflows and when froth flow is zero, in addition to normal operation. The core of the model is a mass balance performed on a number of defined granulometries (size classes) for each species considered. Pulp levels in cells are calculated using a similar approach as described in (1), including valve dynamics and the effect of relative heights of interacting cells. Collection rates (per granulometry class) defines mass transfer between the pulp and froth phase, while a drainage rates determines mass transfer between the froth and pulp phase. Although the model includes parameters such as air hold-up (in the level calculation) and reagent addition, no attempt was made to manipulate aeration rate or reagent addition for control purposes.

Operating flotation cells at their peak air recoveries have been shown to maximise both grade and recovery (Smith, et al., 2010). A theoretical model has been developed to calculate air recovery, based on froth film characteristics (Neethling & Cilliers, 2008). A controller implemented to operate a pilot scale flotation cell at its peak air recovery point however used a peak-seeking strategy without including any model (Shean, et al., 2017).

Maldonado, et al. (2007) used phenomological models in a dynamic programming application, with the aim of optimising the froth level profile for a bank of cells. Seguel, et al. (2015) used the same model, but with a different cost function (maximising overall Cu recovery compared to minimising the sum of squared Cu grades in tailings flows). In both cases the model used a single flotation rate constant per species. The flotation rate constant is modelled as a function of froth depth, residence time and slurry grade, and was fitted using industrial data. Concentrate flow is calculated as a function of froth depth, and the rest of the model is based on mass balances. Although the model was not used in an online control application, it could potentially be used as a simulator in parallel to the plant.

While this section covers a small portion of the available flotation models used in control, it provides an overview of the modelling techniques commonly employed and highlights the scarcity of model based controllers in flotation circuits. The MPC strategy described above was only implemented on a pilot plant, and the HMPC strategy on a simulator, highlighting the scarcity of model based controllers in industrial applications. The key variables listed in 2.2 are mostly modelled, with the exception being the large set of disturbance variables.

4. INSTRUMENTATION

Reliable online measurements are essential in ensuring reliable long-term use of model based control strategies (Hodouin, 2011). Wills & Napier-Munn (2006) stated that the key to effective flotation control is online chemical analysis. In all the flotation models described above, the mineral compositions of process streams form a core part of the models, supporting this statement. On-line X-ray fluorescence (XRF) analysers can provide assays on several elements as well as solids content, but the sampling delay varies between 15 seconds and one minute, and, depending on the number of samples analysed, cycle time can vary between 5 and 15 minutes (Laurila, et al., 2002). Visual and near-infrared reflectance spectroscopic analysis can complement XRF devices and provide grade analyses at a much higher frequency (Shean & Cilliers, 2011).

Remes et al. (2005) developed a dynamic flotation model to study the influence of measurement accuracy and sampling frequency of online XRF analysers on the economical performance of the flotation process. It was shown that, in order to reduce the error caused by the measurement delay, fast basic measurement and control is necessary to complement process analysers, and to keep the process stable until the next assay arrives.

Due to the close relationship between visual froth surface features and flotation performance, a lot of research has been done on the use of flotation cameras, as a soft sensor for grades, to provide measurements at a faster rate than what can be provided by XRF analysers. Significant progress has been made in understanding froth behaviour, and to quantify the impact of manipulated and disturbance variables on froth characteristics, for example the effect of reagent dosage on bubble size (Zhu, et al., 2014). He et al. (2013) utilized a probability density function (PDF) of the froth colour texture unit number to characterise froths based on colour and texture. A nonparametric estimation method based on the fixed normal kernel basis was proposed to describe this distribution. Xu et al. (2015) proposed a complex network-based texture extraction and classification method for froth imaging to extract the distinctive froth texture features in different production states.

Liu & MacGregor (2008) developed a control strategy to achieve desired froth image properties that are related to froth stability, by manipulating reagent addition rates. In Zhu, et al. (2016), a B-spline estimator is used to describe the bubble size PDF, in order to classify bubble sizes with non-Gaussian features. A multi-output least square support vector machine (MLS-SVM) is then applied to establish a dynamical

relationship between the weights of the B-spline estimator and the reagent dosage. Based on this structure, a reagent addition control strategy was implemented to track a desired bubble size PDF.

Aldrich, et al. (2010) however concluded that, despite several advances in machine vision on flotation froths, conflicting results were obtained on linking image features to froth grade, and that no long term fully automated control system based on machine vision have been developed to date.

There may however be scope in using froth image properties in combination with a flotation model to calculate grade, rather than searching for a direct link. Other soft-sensing applications also exist using froth images. Xu et al. (2016) proposed a multi-model soft measurement method to estimate the froth layer thickness based on the visual features. The froth layer thickness was established by the kernel extreme learning machine (KELM) models under different working conditions.

An experimental device to measure froth recovery is described by Rahman, et al. (2013). Although still in an early stage of development, it could potentially be used in future to measure the froth recovery, which is used in several models.

Yianatos, et al. (2008) describes a manual gas-holdup sensor and bubble-load sensor. Both however require manual interaction and cannot be used as online measurements. The bubble load sensor described by Moys, et al. (2010) shows good accuracy, but also requires manual interaction. A gas holdup sensor described by Vinnett, et al. (2016) uses a combination of images and superficial gas velocity, and has the potential to provide an online gas holdup measurement. There is also ongoing research on bubble size measurement, as detailed in Bhondayi & Moys (2014).

Laurila, et al. (2002) describes several other measurements that may be available on a flotation plant, including their limitations.

- Slurry flow measurements are typically performed using magnetic flow meters, but these are generally considered problematic due to solid particles and suspended air bubbles causing inaccuracies.
- If other techniques are not viable, concentrate flow rates in open channels with known dimensions can be calculated using ultrasonic level transmitters, but are not accurate.
- Density measurements may be provided by some XRF devices, or alternatively, nuclear density meters may be used. Installation is critical in preventing inaccuracies caused by bubbles.
- Slurry level measurements using a float with a target plate and ultrasonic level transmitter are commonly used, while measurements based on hydrostatic pressure or direct ultrasonic measurements are often troublesome or sensitive to variations in density.
- A number of airflow measurement techniques are available and are generally considered accurate.
- pH measurements are often problematic due to contamination of the electrode. In some cases, conductivity probes can however be used as substitute.

The fact that measurement devices are available however does not imply that every process stream would be instrumented. Flow rates and on-stream analyses would typically only be available on critical streams, and pH measurements only in the conditioning tank (Laurila, et al., 2002).

5. CONCLUSIONS

Several good dynamic froth flotation models have been developed that can potentially be used in control and optimisation applications. The long-term reliability of these models however depends on the availability of accurate, fast and reliable online measurements, which still seem to be lagging.

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