Naderi, Ehsan; Pourakbari-Kasmaei, Mahdi; Lehtonen, Matti

Transmission expansion planning integrated with wind farms

Published in:
International Journal of Electrical Power and Energy Systems

DOI:
10.1016/j.ijepes.2019.105460

Published: 01/02/2020

Document Version
Peer reviewed version

Published under the following license:
CC BY-NC-ND

Please cite the original version:

This material is protected by copyright and other intellectual property rights, and duplication or sale of all or part of any of the repository collections is not permitted, except that material may be duplicated by you for your research use or educational purposes in electronic or print form. You must obtain permission for any other use. Electronic or print copies may not be offered, whether for sale or otherwise to anyone who is not an authorised user.
Transmission Expansion Planning Integrated with Wind Farms: A Review, Comparative Study, and a Novel Profound Search Approach

Ehsan Naderi¹, Mahdi Pourakbari-Kasmaei²*, and Matti Lehtonen²

¹ Electrical Engineering Department, Engineering Faculty, Razi University, Kermanshah, Iran.
² Department of Electrical Engineering and Automation, Alto University, 02150 Espoo, Finland.
E-mails: E.Naderi@stu.razi.ac.ir; Mahdi.Pourakbari@aalto.fi; Matti.Lehtonen@aalto.fi
* Corresponding Author, Mahdi Pourakbari-Kasmaei, Maarintie 8, 02150, Espoo, Finland.

Abstract—This paper develops a novel hybrid algorithm for solving transmission expansion planning (TEP) problems in electric power networks. Raising the awareness about immense contaminants produced by fossil fuels as well as depleting these resources, have pushed energy companies toward considering more renewable energy resources (RERs). The RESs are beneficial for the society and the power system utility, however, taking into account the uncertainties, which are inherent in RERs, increase the complexity of the optimization problems. In this work, a Monte-Carlo simulation (MCS) is used to address the intermittent nature of wind energy. To handle the resulted model, by modifying and combining three well-known evolutionary algorithms such as shuffled frog leaping algorithm (SFLA), particle swarm optimization (PSO), and teaching learning-based optimization (TLBO), a potent hybrid MSFLA-MPSO-MTLBO, namely combinatorial heuristic-based profound-search algorithm (CHPSA), is proposed. A self-adaptive probabilistic mutation operator (SAPMO) is employed to enhance the effectiveness and computational efficiency of the CHPSA. Ten commonly-used benchmark problems are introduced to corroborate the performance of the CHPSA, while the IEEE RTS 24-bus test system is used to validate the model. Results show that the proposed CHPSA is capable of obtaining better solutions than other algorithms, either implemented in this paper or borrowed from the literature.

Index Terms—Monte-Carlo simulation (MCS); Self-Adaptive Probabilistic Mutation Operator (SAPMO); Renewable Energy Resources (RERs); Transmission Expansion Planning (TEP)
Nomenclature

A. Index and sets

\( i \) and \( j \)  
Bus or node

\( L \)  
Set of all existing and also new corridors

\( p \)  
Particle in all of the optimization algorithms

\( s \)  
Sample in MCS

B. Parameters

\( \bar{A} V P^W \)  
Average value of drawn power from the wind power unit.

\( c \)  
Scale factor.

\( \bar{P}_d \)  
Vector of demand.

\( \bar{P}_g \)  
Vector of generation.

\( k \)  
Shape factor.

\( K_{Losses} \)  
Losses coefficient.

\( L_{ij} \)  
Length of the new line between buses \( i \) and \( j \) (Mile)

\( N_{ij}^{initial} \)  
Number of committed lines between buses \( i \) and \( j \)

\( N_{ij}^{max} \)  
Maximum number of candidate lines between buses \( i \) and \( j \)

\( P_{max} \)  
Maximum output power of wind turbine (MW)

\( P^{max}_{ij} \)  
Maximum active power flow for each line between buses \( i \) and \( j \)

\( P_g \)  
Generated power

\( P_{max}^g \)  
Maximum generated active power

\( R_{ij} \)  
Resistance of line between buses \( i \) and \( j \) (Ohm)

\( M_{LB}^T \)  
Transposed matrix of the network including line-bus components or the branch-node incidence matrix

\( S_N \)  
Maximum number of simulations

\( V_{cut-in} \)  
Cut-in velocity (m/s)

\( V_{cut-out} \)  
Cut-out velocity (m/s)

\( V_{rated} \)  
Rated velocity (m/s)

\( y_{ij} \)  
\( ij^{th} \) element in bus admittance matrix

\( \delta_i \)  
Voltage phase angle at bus \( i \)

\( \delta_j \)  
Voltage phase angle at bus \( j \)

C. Variables

\( C_{ij}^{investment} \)  
Investment costs of new lines constructed between buses \( i \) and \( j \) ($).

\( C_{ij}^{operation} \)  
Operating and maintenance costs of new lines constructed between buses \( i \) and \( j \) ($).

\( C_{Losses} \)  
Annual cost related to network losses ($).

\( C_{inc} \)  
Incremental cost of power ($/MWh).

\( F \)  
Objective function to be minimized ($).

\( I_{ij} \)  
Current flow between buses \( i \) and \( j \) (A).

\( N_{ij} \)  
An integer number of newly constructed lines between buses \( i \) and \( j \)

\( p(v) \)  
Weibull probability density function

\( \bar{P} \)  
Vector of active power flow through tie-lines

\( PF_{ij} \)  
Power flow through the lines in branches \( i \) to \( j \)

\( P_{losses} \)  
Network losses (MW)

\( P_{g}^{WT} \)  
Output power of wind turbine (MW)

\( v \)  
Wind velocity (m/s)

D. Acronyms

AC  
Alternative current
AC-OPF  AC optimal power flow
CHPSA  Combinatorial heuristic-based profound-search approach
DC  Direct current
DC-OPF  DC optimal power flow
DE  Differential Evolution
FAMPSO  Fuzzy adaptive modified particle swarm optimization
GA  Genetic algorithm
HMPSO-DE  Hybrid modified particle swarm optimization and differential evolution
ICA  Imperialist competitive algorithm
LB  Lower bound
MCS  Monte-Carlo simulation
MPSO  Modified particle swarm optimization
MSFLA  Modified shuffled frog leaping algorithm
MTLBO  Modified teaching learning-based optimization
MW  Megawatt
PPF  Probabilistic power flow
Proposed CHPSA  Hybrid modified shuffled frog leaping algorithm and modified particle swarm optimization and modified teaching learning-based optimization
PS  Population size
PSO  Particle swarm optimization
RER  Renewable energy resource
SAPMO  Self-adaptive probabilistic mutation operator
SF1  Schafer function 1
SFLA  Shuffled frog leaping algorithm
TEP  Transmission expansion planning
TLBO  Teaching learning-based optimization

1. Introduction

1.1. Background, definitions, and motivations

The transmission system has a vital role in power systems, especially in deregulated environments, where it intervenes to make an adoption between generation and distribution sections [1]. The main objective of transmission expansion planning (TEP) problem is to determine where, how many, and when to construct new lines while satisfying several techno-economic constraints, which allows the power system planners to spot the optimal facilities to be retrofitted in the transmission system aiming at enhancing the power transfer between the generation side and load points [2]. Since the electricity demand has a direct relationship with the development of countries, the faster a country is developed, the higher the demand becomes. However, more often than not, the transmission network cannot handle such demand growth, and this is where the impact of the TEP and other supporting tools comes under the spotlight. On the other hand, the electricity generation is experiencing some issues such as decreasing the fossil fuels by penetrating more renewable energy resources (RERs) or by making an appropriate trade-off between cost and emission [3]–[6]. The RESs are good alternatives and, among them, the wind is a promising source with an infinitesimal number of environmental hazards. Taking into account these resources, the model encounters different
types of uncertainties, which should be handled via appropriate strategies [7], [8]. In this regard, two main categories, random and non-random uncertainties, can be considered in TEP problems [9]. The most common objective function in TEP problems is minimizing the investment cost of constructing new lines, however, such objective does not take into account other issues in power systems such as operating and maintenance, and power losses, which are inseparable characteristics of each system [10]. To ameliorate these liabilities, the objective function of the TEP problem should be expanded to scrutinize different aspects. Accordingly, in this paper, unlike most of the studies in the literature and to have a more practical model, the objective function contains three terms such as operating cost, investment cost, and the cost of power losses while uncertainties related to the wind power and demands are taken into account.

1.2. Literature review

In order to handle the TEP problems, several classical and mathematically-based optimization methods have been proposed such as linear programming (LP) [11], nonlinear programming (NLP) [12], mixed-integer LP (MILP) [13], [14], [15], mixed-integer conic programming (MICP) [16], mixed-integer NLP (MINLP) [17], dynamic programming (DP) [18], quadratic programming (QP) and mixed-integer QP (MIQP) [19], Benders decomposition (BD) [20], [21], and branch and bound algorithm (B&B) [22], [23]. In addition, an adaptive zone division approach was introduced in [24] to find the optimal location of wind farms to be installed in a power system aiming at making full use of the reactive power of the installed wind farms. However, a new generation of optimizers has recently been introduced to solve a variety of optimization problems in different segments of power industry [25], [26], [27], [28]. In this connection, due to the large-scalability of TEP problems, heuristic-based approaches have been widely used to solve different TEP problems as well.

A genetic algorithm (GA) was used in [29] to handle a coordination problem related to the transmission substations and sub-transmission networks expansion planning. A two-stage method was presented in [30] where, in the first step, a constructive heuristic algorithm (CHA) was used to obtain a reduced set of candidate lines, and in the second step, a particle swarm optimization (PSO) algorithm was applied to obtain the final expansion plan. A binary PSO (BPSO) algorithm was presented in [31] to find the optimal expansion plan while a multivariate interpolation routine was used to compute the operating cost. A modified gases Brownian motion optimization (MGBMO) algorithm was developed in [32] to solve the static TEP (STEP) problems considering the costs of wind energy. The authors in [33] presented an improved harmony search algorithm (HSA), namely IHSA, to solve an adequacy-security
constrained dynamic TEP problem in the restructured environment. A multi-operators evolutionary algorithm (MOEA) based on two sets of operators, evolutionary and specialized, was introduced in [34] and its potential was revealed by performing investigations on academic and practical test systems. In [35], a PSO algorithm was used in order to solve market-based TEP problems along with reactive power planning. Application of Firefly algorithm (FA) was introduced in [36] for solving multi-stage TEP problems in a restructured environment while considering investment cost, congestion cost, as well as reliability criterion. An improved PSO algorithm was presented in [37] for finding the optimal expansion plan of smart transmission networks in which a load shedding model and shunt compensation planning were considered in the problem. A hybrid algorithm for solving the TEP problem based on a combination of GA and PSO algorithm (HGAPSO) was proposed in [38]. A joint model to solve TEP and generation expansion planning (GEP) problems was introduced in [39] where the reserve and emission constraints were taken into account. In [40], a non-dominated sorting genetic algorithm-II (NSGA-II) was proposed to handle multi-objective TEP problem minimizing the wind curtailment. A mixed AC and DC power flow was proposed in [41] to address the TEP problems in the presence of a wind farm where an imperialist competitive algorithm (ICA) was used to solve it. A social spider algorithm (SSA) along with DC-power flow was presented in [42] to handle TEP problems. Authors in [43] proposed a static TEP (STEP) model to study the effects of plug-in electric vehicles (PEVs) and wind power integration under a demand response (DR) environment. In [44], a CHA algorithm was presented to solve the TEP problem using DC power flow. To address several issues such as investment cost, line overload capacity, and cost of network losses in the presence of wind power, a multi-objective model was presented in [45] where an epsilon multi-objective evolutionary algorithm (ε-MOEA) was used to handle this model. A comprehensive review of the TEP and GEP problems were presented in [46] and [47], respectively. In [46], the pertinent studies in the area of TEP problem have been classified from different aspects such as modeling, solving algorithms, reliability, distributed generation, electricity market, uncertainties, line congestion, and reactive power planning. In [47], previous works in the area of GEP problem have been categorized from different standpoints while reaching to a conclusion that the TEP problem is very crucial in the decision-making process regarding GEP problem in the upcoming years. Table 1 provides a quick overview of various existing works in this area of research.
Table 1. Outline and comprehensive study to solve different forms of TEP problem.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Solving Method</th>
<th>Objective Functions</th>
<th>Uncertainties</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>[2]</td>
<td>NSGA-II</td>
<td>Cost LMP, Security</td>
<td></td>
<td>1. FDM and PO were implemented.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Reliability, OC</td>
<td></td>
<td>2. Test system was IEEE RTS 24-bus.</td>
</tr>
<tr>
<td>[3],</td>
<td>NSGA-II</td>
<td>LMP, Security</td>
<td></td>
<td>1. FDM and PO were implemented.</td>
</tr>
<tr>
<td>[40]</td>
<td></td>
<td>Reliability, OC</td>
<td></td>
<td>2. EENS was considered.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>OC</td>
<td></td>
<td>3. The test system was IEEE RTS 24-bus.</td>
</tr>
<tr>
<td>[9]</td>
<td>BD&amp;MCS</td>
<td>LMP, Security</td>
<td></td>
<td>1. Test systems were Garver 6-bus and IEEE RTS 24-bus.</td>
</tr>
<tr>
<td>[11]</td>
<td>LP</td>
<td>LMP, Security</td>
<td></td>
<td>1. The test system was 6-bus RHTS.</td>
</tr>
<tr>
<td>[12]</td>
<td>NLP</td>
<td>LMP, Security</td>
<td></td>
<td>1. The test system was 28-bus Jordanian high voltage transmission network.</td>
</tr>
<tr>
<td>[13]</td>
<td>MILP</td>
<td>LMP, Security</td>
<td></td>
<td>1. Test systems were a simple 3-bus, Garver 6-bus, and IEEE RTS 24-bus.</td>
</tr>
<tr>
<td>[14]</td>
<td>MILP</td>
<td>LMP, Security</td>
<td></td>
<td>2. AC-PF and DC-PF were used.</td>
</tr>
<tr>
<td>[17]</td>
<td>MINLP</td>
<td>LMP, Security</td>
<td></td>
<td>1. DG units were considered.</td>
</tr>
<tr>
<td>[18]</td>
<td>DDP</td>
<td>LMP, Security</td>
<td></td>
<td>2. TEP and GEP problems were solved.</td>
</tr>
<tr>
<td>[19]</td>
<td>OP &amp; MOIP</td>
<td>LMP, Security</td>
<td></td>
<td>3. The test system was the Iranian power grid.</td>
</tr>
<tr>
<td>[20]</td>
<td>BD&amp;MCS</td>
<td>LMP, Security</td>
<td></td>
<td>1. A combination of deterministic search procedure of DDP, probabilistic search, and heuristic stopping criteria were considered.</td>
</tr>
<tr>
<td>[21]</td>
<td>B&amp;B</td>
<td>LMP, Security</td>
<td></td>
<td>1. DG allocation was solved.</td>
</tr>
<tr>
<td>[22]</td>
<td></td>
<td>LMP, Security</td>
<td></td>
<td>2. Problem was solved as a sub-transmission system expansion planning (SSEP).</td>
</tr>
<tr>
<td>[30]</td>
<td>PSO</td>
<td>LMP, Security</td>
<td></td>
<td>1. DC model for the transmission system was implemented.</td>
</tr>
<tr>
<td>[31]</td>
<td>BPSO</td>
<td>LMP, Security</td>
<td></td>
<td>2. Test systems are IEEE RTS 24-bus, Southern Brazil, and the Colombian systems.</td>
</tr>
<tr>
<td>[32]</td>
<td>MGBMO</td>
<td>LMP, Security</td>
<td></td>
<td>1. Test systems were Garver 6-bus and IEEE RTS 24-bus.</td>
</tr>
<tr>
<td>[33]</td>
<td>IHSOA</td>
<td>LMP, Security</td>
<td></td>
<td>2. STEP was solved by DC-PF.</td>
</tr>
<tr>
<td>[34]</td>
<td>MOEA</td>
<td>LMP, Security</td>
<td></td>
<td>2. Test systems were Garver 6-bus and IEEE RTS 24-bus.</td>
</tr>
<tr>
<td>[35]</td>
<td>PSO</td>
<td>LMP, Security</td>
<td></td>
<td>1. Test systems were IEEE RTS 24-bus and IEEE 118-bus.</td>
</tr>
<tr>
<td>[36]</td>
<td>FA</td>
<td>LMP, Security</td>
<td></td>
<td>1. DG allocation was solved.</td>
</tr>
<tr>
<td>[37]</td>
<td>Local PSO</td>
<td>LMP, Security</td>
<td></td>
<td>2. Test systems were Garver 6-bus and IEEE RTS 24-bus.</td>
</tr>
<tr>
<td>[38]</td>
<td>HGA PSO</td>
<td>LMP, Security</td>
<td></td>
<td>1. Multi-stage TEP was solved.</td>
</tr>
<tr>
<td>[41]</td>
<td>ICA</td>
<td>LMP, Security</td>
<td></td>
<td>2. Test systems were IEEE RTS 24-bus and IEEE 118-bus.</td>
</tr>
<tr>
<td>[42]</td>
<td>SSA</td>
<td>LMP, Security</td>
<td></td>
<td>1. Load shedding and shunt compensation planning were considered.</td>
</tr>
<tr>
<td>[43]</td>
<td>GABC</td>
<td>LMP, Security</td>
<td></td>
<td>2. Test systems were Garver 6-bus, IEEE RTS 24-bus, and Northeastern Brazilian 87-bus.</td>
</tr>
<tr>
<td>[44]</td>
<td>CHA</td>
<td>LMP, Security</td>
<td></td>
<td>1. Load shedding and shunt compensation planning were considered.</td>
</tr>
<tr>
<td>[45]</td>
<td>e-MOEAS</td>
<td>LMP, Security</td>
<td></td>
<td>2. Test systems were modified IEEE 24-bus, Brazilian 46-bus, and Colombian 93-bus.</td>
</tr>
<tr>
<td>[48]</td>
<td>PSO</td>
<td>LMP, Security</td>
<td></td>
<td>1. Coordinated GEP and TEP were solved.</td>
</tr>
</tbody>
</table>

Moreover, to illustrate the distribution by optimization algorithm of all relevant references for solving the different TEP problems, which have been reviewed in this study, the informative Fig. 1 is provided.

![Fig. 1. Distribution by optimization algorithm of reviewed papers for solving the TEP problem](image)

According to Fig. 1, it is incontrovertible that the heuristic-based approaches for solving the TEP problem are slightly more than the analytical algorithms. This verifies that the popularity and usefulness of the heuristic-based algorithms are a little higher than the analytical algorithms.

1.3. Contributions

The AC models are highly nonconvex and nonlinear that cause severe obstacles in finding a high-quality solution via commercial solvers. Utilizing the AC model in TEP problems results in nonconvex mixed-integer non-linear programming (MINLP) models, and the situation becomes even worse, while, on the other hand, the existing heuristic-based algorithms also show difficulties in convergence. Therefore, for the sake of simplicity and tractability, more often than not, a simplified model, namely the DC model, is used. It should be noted that in both the aforementioned models, optimal power flow (OPF) is solved with different constraints regarding each model. Although from the optimization standpoint, the DC model is beneficial, it may disregard some useful information, and consequently, the solution might be impractical. To this end, this paper presents an efficient framework in which the AC model is implemented in order to calculate the power losses of the system, and the DC model is used for achieving the best planning scheme. Consequently, the presented model by providing a proper tradeoff between the complexity and tractability of the AC and DC models
aims at enhancing the potential of the planning model. In order to handle the MINLP model of
the TEP problem and to obtain a cogent optimal solution, a robust and efficient optimization
approach is required.

Accordingly, this paper develops a novel and efficient framework to obtain a more
reliable solution to TEP problems. The proposed algorithm takes the advantages and merits of
modified shuffled frog leaping algorithm (MSFLA), modified PSO (MPSO), and modified
teaching learning-based optimization (MTLBO) simultaneously while each of these algorithms
has been enhanced and modified by using a novel structure of self-adaptive probabilistic
mutation operator (SAPMO) [49]. Moreover, to have a comparative study, eleven different
optimization algorithms are employed to solve the benchmark functions as well as the TEP
problems. The Schaffer 1 problem (SF1) is introduced in detail to verify the powerful searching
ability of the proposed algorithm with respect to other alternatives. The accuracy of the
proposed algorithm is verified via the commonly used IEEE RTS 24-bus test system. The
obtained results are compared with the results of GA, PSO, MPSO, FAMPSO, SFLA, MSFLA,
TLBO, MTLBO, ICA, HMPSO-DE, and other existing algorithms in the literature. It is notable
that the proposed CHPSA is an effortless algorithm and has high computational efficiency.
Results demonstrate the stable performance of the proposed approach in solving such a large-
scale problem. Unlike the other heuristic- and meta-heuristic-based algorithms, the proposed
CHPSA has a flexible and well-balanced mechanism to explore the search space.

All in all, the main contributions of this paper are as follows.

- Providing evaluation and comparison among eleven swarm intelligence-based
  algorithms in the realm of solving a set of benchmark problems as well as the
  TEP problem.
- Modifying the original version of PSO, SFLA, and TLBO algorithms using a
  novel SAPMO strategy for diversifying their population and also improving their
  performances and searching capabilities.
- Introducing a novel CHPSA algorithm as a powerful optimization approach to
  solve the benchmark problems as well as the TEP problem, which is one of the
  most complicated problems in power systems.
- Considering three terms such as operating cost, investment cost, and the cost of
  power losses in the objective function to have a more practical model.
• Considering multi-level wind power penetration into the TEP problem to scrutinize the impact of wind power on the future plan; the penetration levels are considered to be 10%, 20%, and 30%.

• Considering uncertainties into the TEP problem, which are associating with wind power and annual demand growth.

1.4. Organization of the paper

The remainder of this paper is organized as follows. Section 2 presents the formulation of TEP problem. The proposed optimization algorithm is described in detail in section 3. Section 4 provides case studies and numerical results including optimizing of proposed benchmark functions as well as the TEP problem, which corroborates the powerful performance of the proposed algorithm compared to the other presented algorithms. Section 5 contains the concluding remarks.

2. Problem modeling and formulation

The TEP problem is a challenging issue in power system studies since transmission lines provide reliable power flows from the generation side to the demand points. Therefore, different formulations have been introduced to handle the problem appropriately. In general, the main goal of the TEP problem is minimizing the total investment costs while satisfying a set of techno-economic constraints, i.e., equality and inequality constraints. In this paper, the static or single-stage model of the TEP problem is taken into account. However, to achieve the most economical plan while satisfying all constraints and restrictions, some additional terms are considered in the conventional objective function. Solving the static TEP model only addresses the following primary concerns: (i) which facilities should be placed into the transmission network, and (ii) where should they be established. That is to say, in the single-stage model, only the optimal conditions of the transmission system is explored. In order to have a more precise and practical model, the conventional DC model is combined with the AC optimal power flow (ACOPF) to handle the problem appropriately. The main target of implementing AC model along with the DC model is to analyze the impact of power losses on the final optimal solution. This way, the annual cost of power losses is added as an additional term to the basic objective function. On the other hand, the existing uncertainties in demand and wind power production, which have a considerable impact on the TEP problem, are handled by the commonly probabilistic distribution functions including Weibull and normal distribution functions that are capable of modeling random behavior of the wind velocity and forthcoming demand.
The following subsections present the proposed objective function as well as the constraints associated with the TEP problem.

2.1. Objective function

Unlike most of the studies in the literature, in this paper, the proposed objective function has multiple terms such as investment costs, and operating and maintenance costs of new transmission lines, as well as the annual cost of power losses of the system. Therefore, the objective function is defined as (1)-(3) [41], [50].

\[
F = \min \left( \sum_{ij} \left( C_{ij}^{\text{investment}} \times N_{ij} \times L_{ij} + C_{ij}^{\text{operation}} \times N_{ij} \times L_{ij} \right) + C_{\text{losses}} \right) \tag{1}
\]

\[
C_{\text{losses}} = K_{\text{losses}} \times (P_{\text{losses}} \times C_{\text{inc}}) \times 8760 \tag{2}
\]

\[
P_{\text{losses}} = \sum_{ij} R_{ij} \times (I_{ij})^2 \tag{3}
\]

2.2. Constraints

The operational and technical constraints corresponding to the DC model are presented in the following sub-sections, while the details of the utilized AC model are available in [26].

2.2.1. Equality constraints

In order to guarantee the most effective economic and operational conditions, the generation and demand should be equal [42]. In this regard, (4) is related to the power balance at each bus, while (5) stands for the power flow of transmission lines, which is the foundation of the DC model.

\[
(\bar{M}_{LB} \times \bar{P}) + \bar{P}_a - \bar{P}_D = 0 \tag{4}
\]

\[
P_{Fij} - \left( y_{ij} \times (N_{ij}^{\text{initial}} + N_{ij}) \times (\delta_i - \delta_j) \right) = 0; \forall i, j = 1, 2, ..., L \tag{5}
\]

2.2.2. Inequality constraints

The inequality constraints are related to specific limits on electrical devices as well as system security such as (6)–(8) [44], [51]. The power flow limit on transmission lines is presented in (6), while (7) is the power generation limit, however, since in this study resizing of the units is not considered, the units’ output is set to the upper bound. Finally, (8) stands for the number of candidate lines in the predefined locations.

\[
|P_{Fij}| \leq P_{Fij}^{\text{max}} \times (N_{ij}^{\text{B}} + N_{ij}); \forall i, j = 1, 2, ..., L \tag{6}
\]

\[
0 \leq P_a \leq P_a^{\text{max}} \tag{7}
\]

\[
0 \leq N_{ij} \leq N_{ij}^{\text{max}}; \forall i, j = 1, 2, ..., L \tag{8}
\]
2.3. Wind generation model

Wind power has a pivotal role in the future of the power industry in many different parts of the world, however, in practice, the characteristics of wind turbines are different regarding the velocity of wind that they can handle, the blade shape, and other control characteristics. Consequently, according to the structure of wind turbines, the power output of each wind turbine unit has a non-linear relationship with wind velocity. This relationship can be illustrated via an ideal power-velocity curve such as Fig. 2 [45], [48].

\[
P_{\text{WT}}^{\text{net}} = \begin{cases} 
0 & 0 \leq v < V_{\text{cut-in}} \\
\frac{p_{\text{max}}}{v_{\text{rated}}} \times (A + Bv + Cv^2) & V_{\text{cut-in}} \leq v < V_{\text{rated}} \\
\frac{p_{\text{max}}}{v_{\text{rated}}} \times \left(\frac{V_{\text{cut-in}} + V_{\text{rated}}}{2 \times V_{\text{rated}}}\right)^3 & V_{\text{rated}} \leq v < V_{\text{cut-out}} \\
0 & v \geq V_{\text{cut-out}} 
\end{cases}
\] (9)

\[
A = \frac{1}{(V_{\text{cut-in}} - V_{\text{rated}})^2} \times \left\{ V_{\text{cut-in}} \times \left(\frac{V_{\text{cut-in}} + V_{\text{rated}}}{2 \times V_{\text{rated}}}\right)^3 \right\} (10)
\]

\[
B = \frac{1}{(V_{\text{cut-in}} - V_{\text{rated}})^2} \times \left\{ 4 \times V_{\text{cut-in}} \times V_{\text{rated}} \times \left(\frac{V_{\text{cut-in}} + V_{\text{rated}}}{2 \times V_{\text{rated}}}\right)^3 \right\} (11)
\]

\[
C = \frac{1}{(V_{\text{cut-in}} - V_{\text{rated}})^2} \times \left\{ 2 - 4 \times \left(\frac{V_{\text{cut-in}} + V_{\text{rated}}}{2 \times V_{\text{rated}}}\right)^3 \right\} (12)
\]
In this work, the commonly-used Weibull probabilistic distribution function, which is oriented toward two parameters, is utilized to model the wind speed variations. The Weibull probability density function can be written as (13).

\[ p(v) = \left( \frac{k}{c} \right) \left( \frac{v}{c} \right)^{k-1} \times EXP \left( -\left( \frac{v}{c} \right)^k \right) \] (13)

It is quite a tricky process to obtain \( c \) and \( k \) in terms of wind velocity, however, there are a limit number of approximations that can be implemented to do so. Interested readers are directed to [52] for more information. It should be noted that the aforementioned parameters of the Weibull function in this study are chosen based on Justus model in [52].

2.4. Power flow model

Since the uncertainties of wind power and load are considered in this study, the traditional power flow model shows some drawbacks to handle the power system operation integrated with RERs. To address this issue, Monte-Carlo simulation (MCS), which is a good candidate for representing the probabilistic power flow (PPF), is used. A significant merit of the MCS is the independent relationship between the required number of samples and the size of the problem to acquire an accurate trade-off. It is notable that, more often than not, the MCS method is very useful for large-scale nonlinear problems. In this paper, to implement the MCS for modeling the uncertainties of the proposed TEP problem following steps are considered [53], [54];

**Step 1:** Setting an initial state or an initial sample.

**Step 2:** Modify the parameter \( P_{WT} \) of the wind turbines.

**Step 3:** Modify the generation units, load points, and wind power units based on new numerical values.

**Step 4:** Evaluate the network conditions by applying power flow procedure.

**Step 5:** Terminate the algorithm if one of the termination criteria is satisfied, otherwise go to step 2.

It is worth mentioning that the uncertainties of wind power features are modeled as random variables. While the procedure is terminated, the average amount as well as the best estimation of variables, and power flows from wind units to the system, are determined by (14) [55].

\[ AVP^w = \frac{1}{S_N} \sum_{S=1}^{S_N} AVP^w \] (14)
3. Descriptions of the proposed approach

This section presents detail explanations on the optimization algorithms, which have been considered to solve the proposed TEP problem. The performances of the commonly used PSO, SFLA, and TLBO algorithms are enhanced and fortified by applying an appropriate configuration of SAPMO strategy. The proposed solution approach is explained in details as follows. It is noteworthy to mention that the details related to the aforementioned algorithms as well as the other studied ones are presented in Appendix A.

3.1. Proposed CHPSA

According to Appendix A, all the three original versions of SFLA, PSO, and TLBO algorithms may be trapped into undesirable local optimal solutions, and this issue is inherent in these original algorithms. Despite this, these algorithms may have premature convergence under some circumstances. In order to address such drawbacks, first, the structure of each algorithm is modified to enhance their potential by applying the SAPMO technique. Then, all the three algorithms, i.e., MSFLA, MPSO, and MTLBO, are combined to introduce a novel hybrid configuration, namely CHPSA, for solving the proposed TEP problem. Toward this end, in the proposed hybrid algorithm, $3 \times PS$ initial population is generated randomly, where the $PS$ indicates the population size for each algorithm including MSFLA, MPSO, and MTLBO, that starts its own procedure. After each iteration, the best solution of MSFLA, MPSO, and MTLBO sub-algorithms are obtained, and among them, the best one is selected by comparison the results. The opted value is set as the best solution for all of the three algorithms in the next iteration. It is worth mentioning that among the three optimization algorithms, MSFLA shows higher potential in discovering an eligible optimal solution because this algorithm uses one extra local search with an independent number of iterations. Furthermore, by increasing the iteration number related to its local search process, the accuracy of results is increased. However, increasing the iteration number of local search results in low computational efficiency compared to the other optimization algorithms. Since the TEP problem is one of the long-term optimization problems in power system operation and planning, therefore, increasing the execution time in order to obtain a high-quality solution is justified. The flowchart of the hybridization process of MSFLA, MPSO, and MTLBO sub-algorithms in this paper is illustrated in Fig. 3. In this figure, the $PS$ stands for the population size.
3.2. Application of the optimization algorithms on the proposed TEP problem

This section presents the application of all the studied algorithms to solve the proposed TEP problem. To this end, the following steps are carried out for each one.

**Step 1:** Importing the required data such as system data, generator data, load data, branches data, wind power data, etc., to the algorithm.

**Step 2:** Tuning the algorithm parameters and generating the initial population randomly.

**Step 3:** Constructing new transmission lines out of the candidate ones in the test systems.

**Step 4:** Computing the total power losses of the system, including recently constructed candidate lines using the AC model.

**Step 5:** Computing the total cost to modify the system; the total cost is the summation of costs of new lines, power losses, and investment and maintenance cost.

**Step 6:** In this step, according to the features of the optimization algorithm, the evolutionary process is carried out for obtaining the least cost.

**Step 7:** DC model is applied for checking the obtained plan over the optimization algorithm.

**Step 8:** Termination criterion is checked. Therefore, if the algorithm reaches to the maximum iteration number or if other stopping criteria satisfied, the algorithm stops; otherwise, it goes to step three.

The flowchart of solving the proposed TEP problem for all optimization algorithms used/proposed in this paper is illustrated in Fig. 4.
4. Simulation, numerical results, and analysis

The main goal of this section is to show the robustness and potential of the proposed CHPSA to solve complex optimization problems. To this end, the stable performance of the proposed approach in solving complex optimization problems is verified by testing on a set of complex benchmark problems with qualitative differences, including unimodal and multimodal functions. The potential and potential of the proposed algorithm is verified by solving the TEP problem for the IEEE 24-bus test system. It is worth mentioning that to provide a fair comparison, the proposed algorithm and also other studied algorithms in this paper including GA, PSO, MPSO, FAMPSO, SFLA, MSFLA, TLBO, MTLBO, ICA, and HMPSO-DE have been implemented by the authors in MATLAB R2014A coding environment, and all
simulations have been carried out on a quad-core processor laptop machine with 1.6 GHz clock frequency and 4.0 GB of RAM.

4.1. Evaluation of the CHPSA on solving the benchmark functions

In this section, the proposed approach and other mentioned optimization algorithms are evaluated to solve a set of well-known benchmark problems. Toward this end, adequate descriptions of these problems are tabulated in Table 2 [56].

Table 2. Descriptions for all of the benchmark problems in this paper

<table>
<thead>
<tr>
<th>Functions</th>
<th>Problem Name</th>
<th>d</th>
<th>SD</th>
<th>GM</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_1(x) = -20 \cdot \exp \left( -0.02 \left( \frac{1}{n} \sum_{i=1}^{d} x_i^2 \right) \right) - \exp \left( \frac{1}{n} \sum_{i=1}^{d} \cos(2\pi x_i) \right) + 20 )</td>
<td>Ackley’s Problem (ACK)</td>
<td>10</td>
<td>-30 ≤ x_i ≤ 30</td>
<td>0</td>
</tr>
<tr>
<td>( f_2(x) = 1 + \frac{1}{4000} \sum_{i=1}^{d} x_i^2 - \frac{1}{e} \sum_{i=1}^{d} \cos \left( \frac{x_i}{\sqrt{i}} \right) )</td>
<td>Griewank Problem (GW)</td>
<td>10</td>
<td>-600 ≤ x_i ≤ 600</td>
<td>0</td>
</tr>
<tr>
<td>( f_3(x) = \sum_{i=1}^{d} \left[ \ln(x_i - 2) \right]^2 + \ln(10 - x_i) \right]^2 - \left( \frac{10}{\pi} x_i \right)^2 )</td>
<td>Paviani Problem (PP)</td>
<td>10</td>
<td>2 ≤ x_i ≤ 10</td>
<td>-45.778</td>
</tr>
<tr>
<td>( f_4(x) = \sum_{i=1}^{d} \left[ 100 \cdot (x_i - x_i^2)^2 + (x_i - 1)^2 \right] )</td>
<td>Rosenbrock Problem (RB)</td>
<td>10</td>
<td>-30 ≤ x_i ≤ 30</td>
<td>0</td>
</tr>
<tr>
<td>( f_5(x) = 0.5 + \frac{(\sin(\sqrt{x_1^2 + x_2^2}))^2}{(1 + 0.001 \cdot (x_1^2 + x_2^2))^2} - 0.5 )</td>
<td>Schaffer 1 Problem (SF1)</td>
<td>2</td>
<td>-100 ≤ x_i ≤ 100</td>
<td>0</td>
</tr>
<tr>
<td>( f_6(x) = \sum_{i=1}^{d} \left[ -x_i \cdot \sin \left( \sqrt{</td>
<td>x_i</td>
<td>} \right) \right] + 418.98280727243369 \cdot d )</td>
<td>Schwefel Problem 2.26 (SWF)</td>
<td>30</td>
</tr>
<tr>
<td>( f_7(x) = \sum_{i=1}^{d} \left[ x_i^2 - 10 \cos(2\pi x_i) + 10 \right] )</td>
<td>Rastrigin Problem (RG)</td>
<td>30</td>
<td>-5.12 ≤ x_i ≤ 5.12</td>
<td>0</td>
</tr>
<tr>
<td>( f_8(x) = \sum_{i=1}^{d} \left( \sum_{j=1}^{n} x_j^2 \right)^2 )</td>
<td>Schwefel Problem 1.2 (SWF)</td>
<td>30</td>
<td>-100 ≤ x_i ≤ 100</td>
<td>0</td>
</tr>
<tr>
<td>( f_9(x) = \sum_{i=1}^{d} \left[ (x_i + 0.5)^2 \right] )</td>
<td>Step 2 Function</td>
<td>30</td>
<td>-100 ≤ x_i ≤ 100</td>
<td>0</td>
</tr>
<tr>
<td>( f_{10}(x) = \sum_{i=1}^{d} \left[ 100(x_i - x_i^2)^2 + (x_i - 1)^2 \right] )</td>
<td>Rosenbrock Problem (RP)</td>
<td>10</td>
<td>-30 ≤ x_i ≤ 30</td>
<td>0</td>
</tr>
</tbody>
</table>

\( d \): the dimension of the problem, which is related to the number of decision variables; \( SD \): Search domain of the problem; and \( GM \), the global minimum value of the problem.

To solve the problems stated in Table 2, the population size and the maximum number of iterations for all different algorithms are set to 5 and 30, respectively. The obtained outcomes of all algorithms are presented in Table 3.

Table 3. Comparison of minimum values obtained by all different algorithms

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>( f_1(x) )</th>
<th>( f_2(x) )</th>
<th>( f_3(x) )</th>
<th>( f_4(x) )</th>
<th>( f_5(x) )</th>
<th>( f_6(x) )</th>
<th>( f_7(x) )</th>
<th>( f_8(x) )</th>
<th>( f_9(x) )</th>
<th>( f_{10}(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSO</td>
<td>5.1505</td>
<td>9.0128</td>
<td>-22.000</td>
<td>0.1194</td>
<td>4.0344</td>
<td>18.006</td>
<td>9.5554</td>
<td>45.149</td>
<td>11.337</td>
<td>19.000</td>
</tr>
<tr>
<td>MPSO</td>
<td>2.1903</td>
<td>5.0316</td>
<td>-37.019</td>
<td>0.0088</td>
<td>3.0219</td>
<td>11.001</td>
<td>9.4077</td>
<td>36.609</td>
<td>10.900</td>
<td>13.662</td>
</tr>
<tr>
<td>FAMPSO</td>
<td>2.1539</td>
<td>4.8999</td>
<td>-37.685</td>
<td>0.0081</td>
<td>3.0177</td>
<td>11.001</td>
<td>8.0117</td>
<td>35.009</td>
<td>10.000</td>
<td>13.008</td>
</tr>
<tr>
<td>SFLA</td>
<td>0.8722</td>
<td>2.0156</td>
<td>-40.200</td>
<td>0.0035</td>
<td>0.7210</td>
<td>4.5863</td>
<td>3.2919</td>
<td>11.995</td>
<td>3.4554</td>
<td>8.3107</td>
</tr>
<tr>
<td>MSFLA</td>
<td>0.0085</td>
<td>0.5211</td>
<td>-43.665</td>
<td>0.0009</td>
<td>0.0498</td>
<td>0.9108</td>
<td>0.5541</td>
<td>1.0005</td>
<td>0.3395</td>
<td>0.6122</td>
</tr>
<tr>
<td>MTLBO</td>
<td>2.2161</td>
<td>4.0713</td>
<td>-39.029</td>
<td>0.0774</td>
<td>3.3868</td>
<td>15.000</td>
<td>6.7369</td>
<td>41.731</td>
<td>10.012</td>
<td>17.777</td>
</tr>
<tr>
<td>HMPDO-DE</td>
<td>1.1e-10</td>
<td>1.1e-8</td>
<td>-16.982</td>
<td>4.0e-5</td>
<td>0.0005</td>
<td>0.0001</td>
<td>0.0008</td>
<td>6.5e-5</td>
<td>1.0e-4</td>
<td>4.5e-6</td>
</tr>
<tr>
<td>Proposed CHPSA</td>
<td>1.0e-19</td>
<td>1.9e-11</td>
<td>-45.031</td>
<td>1.0e-6</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>2.23e-9</td>
<td>0.0000</td>
<td>3.6e-7</td>
</tr>
</tbody>
</table>

16
As can be seen from Table 3, the proposed algorithm shows better and stronger performance compared to the other heuristic-based approaches.

To have a profound analysis of the searching ability of the proposed algorithm, a detailed analysis of the most complex function in Table 2, 5th function, namely Schaffer 1 problem (SF1) is performed. This problem is very complex and challenges the optimization approaches to find the best optimal solution. As the results show, most of the algorithms trap into local minima and are unable to find the global optima of this function. To this end, detail information of searching ability and stability performance of the proposed hybrid approach is scrutinized for minimizing the Schaffer’ function with two decision variables including $x_1$ and $x_2$ via a 3-D and contour graphs of Schaffer’ function in Fig. 5 and Fig. 6, respectively.

In Fig. 5 and Fig. 6, different areas, from the worst to the best, are demonstrated by the following colors: dark red, light red, dark yellow, light yellow, dark blue, and light blue, respectively. The globally optimal solution of the SF1 is equal to 0, which is occurred in $(0, 0)$. The obtained results from incipient and final iterations are displayed in Fig. 7 for the proposed approach and also some of the studied algorithms. It is necessary to note that the obtained results of the proposed algorithm, original SFLA, original PSO, and original TLBO after the first iteration are illustrated in subfigures $a_1$ to $a_4$, respectively, while subfigures $b_1$ to $b_4$ stand for the obtained results after 30 iterations of the aforementioned approaches, respectively.
By considering Fig. 7, it is revealed that unlike the presented original algorithms that trapped into local optimal points, the proposed algorithm finds the optimal global solution of
the Schaffer’s problem. This is where the impact of the proposed algorithm, by addressing the drawbacks of the original algorithms, comes under the spotlight. Thanks to the proposed SAPMO strategy as well as the hybridization process, the proposed algorithm presents such powerful performance in finding the global optimal.

4.2. Investigation of the proposed CHPSA in solving the TEP problem

The proposed algorithm and the other studied algorithms are investigated on the IEEE RTS 24-bus, which is a commonly-used test system in the area of TEP problems. The single-line diagram of this system is shown in Fig. 8, and the detail data can be extracted from [57].

Fig. 8. Single-line diagram of IEEE RTS 24-bus test system including two wind power units

The IEEE RTS 24-bus system has 41 transmission lines, including 34 available lines and 7 prospective corridors, each of which has three possible parallel lines to be installed. In addition, two wind power plants with a capacity of 300 MW, which are located far from the network and close to buses #3 and #4, are added to the system with transmission lines of 175 MVA [58]. In addition, the data including different generation plans, G1 to G4, has been presented in Appendix B. It is noteworthy to mention that among these plans, G4 is chosen to compare the performance of the proposed approach with other methods in the literature. The basic network demand and the total generations are 2850 MW and 3405 MW, respectively, where bus #13 is considered as the slack bus. The data related to the circuits and demands are available in [44].
A series of assumptions on IEEE RTS 24-bus test system should be taken into consideration to handle the proposed TEP problem over a 10-year planning horizon. As a quick reference, these assumptions are tabulated in Table 4 [34]. As mentioned before, a commonly probabilistic distribution function is used for modeling the random behaviors of the future system demand.

<table>
<thead>
<tr>
<th>Description</th>
<th>Related value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expansion planning horizon</td>
<td>10 years</td>
</tr>
<tr>
<td>Load of the system</td>
<td>3 × basic demand</td>
</tr>
<tr>
<td>Active power generation of system</td>
<td>3 × basic generation</td>
</tr>
<tr>
<td>The annual growth rate of system load</td>
<td>11.6% per year</td>
</tr>
</tbody>
</table>

In heuristic-based algorithms, several parameters and intervals should be initially set to handle a specific problem efficiently. Table 5 presents the settings related to each algorithm. In this paper, $PS$ and $K_{max}$ stand for the population size and the maximum number of iterations for all optimization algorithms; $K_{LS, max}$ is the maximum number of iteration for improving the worst solution in SFLA related to its local search process; $T_{SAPMO}$ is the maximum number of iterations in SAPMO strategy for enhancing the obtained results; $G_{max}$ and $C_{CO}$ respectively stand for the maximum number of generation and crossover constant in GA and DE algorithms; $P_M$ is the probability mutation in GA; $w$ is the inertia coefficient of particles in PSO algorithm, while $\Delta w$ is an acceptable boundary for $w$ variation in PSO algorithm; $c_1$ and $c_2$ are cognitive and social learning factors in PSO algorithm, respectively; $N_m$ is the number of memeplexes in SFLA, and $N_f$ is the number of frogs in each memeplex; $T_F$ is the teaching factor in TLBO algorithm; $\beta$ is a positive value which conducts the colonies towards their imperialists in ICA, while $\xi$ is a positive value which indicates the portion of the colonies in the power of an empire in ICA; and $R_r$ is the revolution rate in ICA.
4.3. Case studies and computational results

In the area of the TEP studies, the lower bound (LB) criteria, provides important information about the required costs, total costs, and marginal costs of a network to satisfy the forecasted demand, as well as environmental goals, which plays a key role in the decision-making process, [59]. To obtain an acceptable LB, the basic TEP problem is solved via the DC-OPF with 2000 iterations where the power losses and the possibility of little investments are neglected.

Accordingly, to clarify the solving ability of the proposed algorithm in handling the TEP problem, five different case studies are considered. Table 6 presents the conditions of these cases.

<table>
<thead>
<tr>
<th>Case Studies</th>
<th>Basic TEP</th>
<th>AC Model</th>
<th>Wind Power Penetration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I</td>
<td>✔️</td>
<td>–</td>
<td>✔️ – – –</td>
</tr>
<tr>
<td>Case II</td>
<td>✔️</td>
<td>✔️</td>
<td>✔️ – – –</td>
</tr>
<tr>
<td>Case III</td>
<td>✔️</td>
<td>✔️</td>
<td>– ✔️ – –</td>
</tr>
<tr>
<td>Case IV</td>
<td>✔️</td>
<td>✔️</td>
<td>– – – ✔️</td>
</tr>
<tr>
<td>Case V</td>
<td>✔️</td>
<td>✔️</td>
<td>– – – ✔️</td>
</tr>
</tbody>
</table>

The first case is used to obtain the LB, and in this regards, the basic TEP problem is solved via a DC-OPF while the power losses and the possibility of fractional investments are neglected. In this regard, interested readers are directed to [60] for more information. For this experiment, the maximum number of iteration is set to 2000. The convergence trend for this case is illustrated in Fig. 9.

![Fig. 9. The LB convergence plot obtained from solving Case I](image)

After obtaining the approximated value of LB for the TEP problem, the simulations and numerical results are used to evaluate other case studies. The obtained optimal solution by the
proposed algorithm, including candidate lines, total cost, and execution time for case II, case III, case IV, and case V are presented in Tables 7-10, respectively. It is worth mentioning that, due to the lack of sufficient room and also for the sake of brevity, only the results of case II is presented in detail in Table 7, while Tables 8-10 only present the total number of lines, total cost, and execution time.

**Table 7. Obtained results from solving the TEP problem on Case II of IEEE RTS 24-bus test system**

<table>
<thead>
<tr>
<th>Lines</th>
<th>GA</th>
<th>PSO</th>
<th>MPSO</th>
<th>FAMPSO</th>
<th>SFLA</th>
<th>MSFLA</th>
<th>TLBO</th>
<th>MTLBO</th>
<th>ICA</th>
<th>HMPSO-DE</th>
<th>Proposed CHPSA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>From</td>
<td>To</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>8</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>9</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>11</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>12</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>11</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>12</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>13</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>11</td>
<td>14</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>13</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>23</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>14</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>23</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>16</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>23</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>15</td>
<td>16</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>15</td>
<td>21</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>15</td>
<td>24</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>16</td>
<td>17</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>16</td>
<td>19</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>16</td>
<td>23</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>17</td>
<td>18</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>18</td>
<td>21</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>19</td>
<td>20</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>19</td>
<td>22</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>19</td>
<td>23</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td>23</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>21</td>
<td>22</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

| Total Lines | 29 | 26 | 22 | 22 | 29 | 18 | 24 | 17 | 22 | 17 | 11 |
| Cost (10^6 $) | 1304 | 1256 | 1077 | 927 | 1096 | 666 | 1009 | 779 | 891 | 620 | 431 |
| Time (s) | 19.55 | 28.42 | 22.13 | 31.08 | 35.18 | 40.00 | 10.01 | 16.21 |
According to Tables 7-10, it is incontrovertible that the proposed CHPSA, compared to the other algorithms, converges to a lower cost by constructing fewer lines. This approves the strength of the proposed algorithm in solving the TEP problem, which is a very complex planning problem in power systems. Thanks to the usage of the proposed SAPMO strategy as well as the hybridization process, the proposed algorithm could have much better performance in finding the most economical plan.

The convergence epochs of different algorithms in solving the second case study are illustrated in Fig. 10.

![Convergence curve of cost for different algorithms on Case II of IEEE RTS 24-bus test system](image)

**Fig. 10.** Convergence curve of cost for different algorithms on Case II of IEEE RTS 24-bus test system

By comparing Figs. 9 and 10, it is clear that the LB for solving the TEP problem in Case I is about 49.5% below the obtained optimal solution by the proposed algorithm for solving Case II. That is to say, the comparison of the obtained optimal costs of Case I and Case II confirms that the solution of the first case performs an LB for the proposed TEP model. As
mentioned earlier, the LB, by providing useful information, plays an important role in facilitating the search process for the planning problems.

In this paper, the proposed algorithm is run 30 times, and the obtained results are presented in Table 11. This table includes the best, mediocre, and worst values of the objective function as well as the standard deviation for Case II to Case V.

Table 11. Obtained results by the proposed algorithm for all main cases and in 30 independent runs

<table>
<thead>
<tr>
<th>Case Studies</th>
<th>Best solution</th>
<th>Mediocre solution</th>
<th>Worst solution</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case II</td>
<td>431.00</td>
<td>431.00</td>
<td>431.00</td>
<td>0</td>
</tr>
<tr>
<td>Case III</td>
<td>400.55</td>
<td>400.55</td>
<td>400.55</td>
<td>0</td>
</tr>
<tr>
<td>Case IV</td>
<td>410.00</td>
<td>410.00</td>
<td>410.00</td>
<td>0</td>
</tr>
<tr>
<td>Case V</td>
<td>422.90</td>
<td>422.90</td>
<td>422.90</td>
<td>0</td>
</tr>
</tbody>
</table>

According to Table 11, all of the best, mediocre, and worst values of the proposed approach are equal to each other. As a result, the standard deviation is equal to zero; therefore, the obtained results verify the high performance and accuracy of the proposed hybrid algorithm in solving the TEP problem from different aspects. However, to better understand the powerful performance of the proposed algorithm, the distribution of the obtained values of the total cost related to Case V by the proposed approach and original GA, ICA, and PSO algorithms for 30 independent trials are illustrated in Fig. 11.

Fig. 11. Distribution of obtained results related to the total cost of Case V

Fig. 11 reveals that the obtained results by the proposed CHPSA through all the 30 independent trials are similar, which approves the powerful performance of the proposed algorithm. In other words, the proposed algorithm could recognize the minimum optimal solution in each iteration and converged to $423 million, which confirms the stable
performance of the proposed algorithm in solving the TEP problem. This means that its performance remains stable regardless of the complications of the problem since the standard deviation of the proposed algorithm is always equal to zero.

In addition, Fig. 12 provides a comparison between the proposed algorithm and original SFLA, PSO, and TLBO algorithms in order to obtain the best, mediocre, and worst values of the objective function for solving Case III.

![Comparison of algorithms](image)

Fig. 12. The best, mediocre, and worst obtained values of the objective function for solving Case III in 30 independent runs

The ability and supremacy of the proposed algorithm are presented in Fig. 16. From this figure, it can be seen that unlike the algorithms such as original SFLA, PSO, and TLBO that show different values for the best, mediocre, and worst results, the proposed algorithm obtains the same value for them.

In order to provide a quick reference, the obtained results of the proposed approach and the other algorithms, implemented by the authors, for solving Cases II-V are presented in Table 12. This table reveals that the execution times of HPSO-DE and proposed algorithm are less than the other mentioned algorithms. Considering Case IV reveals a higher computational efficiency of the HPSO-DE than the proposed approach, with about 8 seconds faster process, although the solution quality of the proposed approach is much higher. This lower computational efficiency is mainly because of the profound searchability of the proposed algorithm using three different sub-algorithms that one of them is SFLA, while the SFLA uses one extra local search sub-routine, which is repeated for a specific number of iterations to improve the worst frogs. Moreover, the TEP problem is one of the most complicated optimization problems in power system optimization, and finding a high-quality solution for that is one of the significant existing challenges. Therefore, increasing the execution time to solve the problem is the price of obtaining a high-quality solution and an effective plan.
Table 12. Comparison of the costs (10^6 $) and execution times (s) from different algorithms for Cases II to V

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>Case II Cost</th>
<th>Time (s)</th>
<th>Case III Cost</th>
<th>Time (s)</th>
<th>Case IV Cost</th>
<th>Time (s)</th>
<th>Case V Cost</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSO</td>
<td>1256</td>
<td>28.4219</td>
<td>1177</td>
<td>28.9719</td>
<td>1205</td>
<td>29.4544</td>
<td>1226</td>
<td>31.6225</td>
</tr>
<tr>
<td>MPSO</td>
<td>1077</td>
<td>22.1341</td>
<td>1009</td>
<td>21.2924</td>
<td>1034</td>
<td>22.7121</td>
<td>1051</td>
<td>25.1719</td>
</tr>
<tr>
<td>FAMPSO</td>
<td>927</td>
<td>31.0825</td>
<td>869</td>
<td>31.6643</td>
<td>889</td>
<td>32.6032</td>
<td>905</td>
<td>33.0347</td>
</tr>
<tr>
<td>SFLA</td>
<td>1096</td>
<td>79.7744</td>
<td>1026</td>
<td>78.0510</td>
<td>1050</td>
<td>80.0022</td>
<td>1072</td>
<td>81.8724</td>
</tr>
<tr>
<td>MSFLA</td>
<td>666</td>
<td>85.9530</td>
<td>625</td>
<td>86.1822</td>
<td>637</td>
<td>87.0615</td>
<td>651</td>
<td>87.2239</td>
</tr>
<tr>
<td>TLBO</td>
<td>1009</td>
<td>44.5439</td>
<td>945</td>
<td>45.1229</td>
<td>968</td>
<td>44.9928</td>
<td>983</td>
<td>47.1244</td>
</tr>
<tr>
<td>MTLBO</td>
<td>779</td>
<td>40.0033</td>
<td>731</td>
<td>42.2249</td>
<td>747</td>
<td>41.2118</td>
<td>760</td>
<td>43.5442</td>
</tr>
<tr>
<td>ICA</td>
<td>891</td>
<td>35.1802</td>
<td>836</td>
<td>41.4534</td>
<td>855</td>
<td>43.5543</td>
<td>869</td>
<td>44.3011</td>
</tr>
<tr>
<td>HMPSO-DE</td>
<td>620</td>
<td>10.0122</td>
<td>582</td>
<td>11.5415</td>
<td>591</td>
<td>11.8745</td>
<td>602</td>
<td>12.9605</td>
</tr>
<tr>
<td>Proposed CHPSA</td>
<td>431</td>
<td>16.2128</td>
<td>400</td>
<td>19.6544</td>
<td>410</td>
<td>19.9537</td>
<td>423</td>
<td>17.7541</td>
</tr>
</tbody>
</table>

According to Table 7 to Table 12, the proposed hybrid algorithm obtains much higher-quality solutions that confirm its potential and effectiveness in handling such complex TEP problems.

In order to provide a better view, Table 13 presents a fair comparison between the best-obtained results by the proposed algorithm and those available in the literature such as GA [41], ICA [41], market-based model (MBM) in [61], CHA and interior point method (CHA-IPM) [62], and CHA with SNOPT solver (CHA-SNOPT) [62] for fixed generation plan G4, whose dispatch data is provided in Appendix B. It is noteworthy to note that for this comparison, only the first part of Eq. (1), i.e., \( C_{ij}^{\text{investment}} \times N_{ij} \), is considered.

Table 13. Comparison of the best costs (10^6 $) and execution times (s) of different approaches

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>Minimum Cost (10^6 $)</th>
<th>Time (s)</th>
<th>Number of Iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>GA [41]</td>
<td>350.0</td>
<td>108.6</td>
<td>300</td>
</tr>
<tr>
<td>ICA [41]</td>
<td>342.0</td>
<td>27.6</td>
<td>300</td>
</tr>
<tr>
<td>MBM [61]</td>
<td>376.0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>CHA-IPM [62]</td>
<td>376.0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>CHA-SNOPT [62]</td>
<td>376.0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Proposed CHPSA</td>
<td>342.0</td>
<td>14.4</td>
<td>1000</td>
</tr>
</tbody>
</table>

As can be seen from Table 13, among the approaches in the literature, the ICA [41] obtained the best result so far, $342 million, which has been obtained in 27.6s within 300 iterations; however, the proposed algorithm by a fusion structure proposes the same plan in 14.4s considering 1000 iterations. That is to say, each iteration took, on average, only 14.4 ms, which is equivalent to 48% improvement in the execution time with respect to ICA [41], thanks to the profound search capability of the proposed approach in the exploration and exploitation processes.
4.4. Discussion

According to Table 7, which represent the results of TEP problem without considering wind farms at buses 3 and 4, it can be seen that the obtained costs by the proposed algorithm and HMPSO-DE are $431 million and $620 million, respectively. These results are better than the other studied algorithms in this work. The optimal plan obtained by the CHPSA proposes the construction of 11 new transmissions lines that results in at least $189 million lower cost than the best results of other studied algorithms, which is equal to 30% saving.

On the other hand, knowing the value of lower bound is very important for planning problems as it provides useful information such as total system costs for satisfying the prognosticated demand in the future. To this end, by comparing Fig. 9 and Fig. 10, it is obvious that the lower bound obtained by solving Case I, is about 49.5% lower than the obtained optimal result by the proposed algorithm in solving Case II, which is a good approximated bound served as the lower bound of the TEP problem. To obtain this lower bound, a linear formulation of the problem with the DC model and without considering power losses and the possibility of fractional investment has been utilized. In addition, similar results have been obtained for solving the TEP problem considering different levels of wind power penetration into the system.

Moreover, according to Tables 8-10, it is clear that the obtained costs by the proposed algorithm and also HMPSO-DE are better than the other alternatives, thanks to the usage of the proposed hybrid mechanism, as well as the novel SAPMO strategy, the proposed algorithm provides much better performance in recognizing the optimal cost. From these tables, it can be seen that the number of new transmission lines added to the system in Cases IV and V are more than the other case studies.

In addition, based on Table 11, Fig. 11, and Fig. 12, the stable and powerful performance of the proposed algorithm in order to solve the TEP problem from different aspects is profoundly obvious. In this connection, all of the best, mediocre, and worst obtained values by the proposed algorithm are equal to each other; therefore, the standard deviation is equal to zero. This is a very important achievement since, in general, power system operators in different segments of the power industry should make as precise as possible decisions, therefore, the proposed algorithm with its profound search mechanism can be an appropriate optimization tool in this regard.

To show the computational efficiency of the proposed algorithm for solving the TEP problem expeditiously, Table 12 corroborates that the obtained execution times of the proposed algorithm and HMPSO-DE are less than the other alternatives to solve Cases II-V. In this connection, for instance, the execution time obtained by the proposed algorithm and HMPSO-
DE in handling the TEP problem in Case III are equal to 19.6s and 11.5s, respectively, which are quicker than the other alternatives. The execution time obtained by the proposed algorithm for solving Case IV is approximately 8 seconds more than the HMPSO-DE algorithm. Evidently, this is the effect of using three various sub-algorithms simultaneously, while the SFLA has only one auxiliary local search sub-routine. However, the TEP problem is not the real-time problem to be solved in a fraction of seconds or even in a few minutes; the result may be used for the projects in upcoming months or years. Therefore, the little increase in execution time is negligible as far as a better solution is obtained. In addition, the proposed approach managed to improve the execution time in solving the TEP problem by 48% with respect to the other available alternatives in the literature. It is, therefore, inferable that the proposed algorithm is capable of handling the TEP problem while not only obtaining a lower cost but also with higher computational efficiency that definitely makes it a suitable candidate tool for power system operators to cope with different optimization problems in power industry.

According to above discussion, it can be deduced that the proposed hybrid algorithm has fascinating merits among all of the studied algorithms in this paper, and also other available approaches in the literature by obtaining the best plan with high computational efficiency.

5. Conclusions

The TEP problem considering the penetration of renewable energy resources is one of the most challenging and prominent optimization problems in power system planning. In order to have a more practical model, the uncertainties of wind power plants and electricity demands have been taken into account that complicates the problem. To handle this problem, which has numerous local optima, a novel hybrid configuration of several heuristic algorithms has been developed to obtain a hybrid MSFLA-MPSO-MLBO, namely combinatorial heuristic-based profound-search algorithm (CHPSA). The potential of CHPSA has been verified on several commonly used benchmark problems. Besides, this study has provided a comprehensive comparison between the proposed approach and several other heuristic-based algorithms to solve the TEP problem considering wind power plants. Since the original version of SFLA, PSO, and TLBO algorithms often converge to locally optimal solutions in most of the circumstances, in this paper, before hybridization of these algorithms, a useful modification has been applied to them in order to enhance the performance of the algorithms and avoid from trapping to local optimal points and premature convergence as well. This modification is called SAPMO technique, and it is based on a mutant process for all of the components in the population according to their probabilities. The outcomes of the proposed CHPSA algorithm
on TEP problem considering different case studies have been compared with the results of several implemented algorithms such as PSO, MPSO, FAMPSO, SFLA, MSFLA, TLBO, MTLBO, ICA, and HMPSO-DE, and other existing approaches in the literature. Results reveal the performance of the proposed algorithm by studying the IEEE RTS 24-bus test system where the probability and capability of the CHPSA to converge to a near-global optimal is much more than other mentioned algorithms. All in all, the proposed approach managed to propose an appropriate expansion plan by constructing fewer transmission lines. Since in almost all of the works in TEP problems, the candidate lines and the associated buses are predefined, and this might negatively affect the planning process, then in the future work, the authors aim at finding the most appropriate pair of buses to construct new lines.

6. Appendix A

This section provides detail information related to the heuristic algorithms used to propose the novel profound search algorithm and a comparative study on the TEP problem as well. As pointed out at the beginning of section 2, the static or single-stage model of the TEP problem is considered in this paper, which can be graphically demonstrated in Fig. A1. In addition, Fig. A1 illustrates a comparison between the concepts behind both the single-stage model and the multi-stage model (dynamic model) as well. According to Fig. A1, the TEP problem is typically considered for a long-term planning horizon (the blue curly bracket), which may span even more than thirty years. In this connection, in the single-stage TEP model, the expansion planning decisions are made at the beginning of the horizon (yellow circle), whereas the target of the multi-stage TEP is to establish the expansion decisions at various time intervals of the planning horizon (see Fig. A1 (b)).
In the following subsections, first, the proposed SAPMO strategy, as a powerful technique to improve the heuristic-based algorithms, is introduced, and then, all of the studied optimization algorithms are introduced briefly from the TEP problem-solving standpoint. It is worth mentioning that three of these algorithms, i.e., PSO, SFLA, and TLBO, which are the sub-algorithms of the proposed hybrid approach, are modified using the SAPMO technique.

6.1. SAPMO strategy

The commonly used PSO, SFLA, and TLBO algorithms often have some drawbacks like immature convergence to undesirable local optimal points in most of the circumstances. To address this issue, a novel and powerful structure of SAPMO strategy [49] is used in this paper. Therefore, in the aforementioned algorithms after producing a fortuitous initial population, three mutant vectors are randomly chosen based on different mutant rules to enhance the impact of this strategy. All of the members in the population can be mutated based on their acquisition probabilities. To this end, an appropriate model for acquisitive probability can function according to the fitness of every component in these algorithms such as particles, birds, frogs, etc. It is worth mentioning that at the beginning of this process each algorithm has a probability equal to 0.25. In addition, a new parameter, namely repository or reservoir, is implemented; the initial value of this parameter is zero. During the iterative process, Eq. (A1) is applied and the population is stored and retained in a repository at each iteration [63].

\[ WC_p = \frac{\log(PS - p + 1)}{\log(1) + \log(1) + \cdots + \log(PS)}; \forall p = 1, 2, \ldots, PS \]  \hspace{1cm} (A1)

where, \( p \) is related to the particles and it can take values from one up to \( PS \) for the best and worst particles, respectively; and \( WC \) is the weight coefficient that assigned to each particle.

After computing the \( WC \) for stored particles in a repository, the repository of every algorithm can be updated by Eq. (A2).

\[ R_k^M = R_k^M + \frac{WC_p}{N_k^M} + \cdots + N_k^M; k = 1, 2, 3 \] \hspace{1cm} (A2)

where \( R_k^M \) is the repository of \( k^{th} \) algorithm where \( k \) is equal to one, two, or three, which correspond to the PSO, SFLA, and TLBO algorithms, respectively.

The probability value of each algorithm is calculated by Eq. (A3).

\[ Prob_k^M = Prob_k^M \times (1 - \psi) + \left( \psi \times \frac{R_k^M}{K_{max}} \right); k = 1, 2, 3 \] \hspace{1cm} (A3)

where \( \psi \) is a predefined parameter that stands for the velocity of exchanging data between individuals or particles in every algorithm; \( Prob_k^M \) is the probability of \( k^{th} \) algorithm in the
SAPMO technique; index $k$ is related to the optimization algorithms, and $K_{max}$ is the maximum number of iterations of each algorithm.

At the end of this process, it is necessary to normalize the obtained values of $Prob_k$.

In this paper, the normalized probability (NP) is obtained by Eq. (A4).

$$NP = \frac{Prob_k}{\sum_{k=1}^{i} Prob_k}; \forall k = 1, 2, 3$$

### 6.2. Optimization algorithms

A brief and informative description for all of the optimization algorithms for solving the proposed TEP problem is presented in the following sub-sections. It should be noted that each particle in the following algorithms indicates a valid solution that entails an acceptable number of transmission lines to be presented in each of the corridors in which the solution is taken into account with integers considering the maximum number of transmission lines allowed in a specific corridor. As an illustration, the IEEE 24-bus test system is considered.

In the IEEE 24-bus test system, the set of new candidate lines has seven members, which includes (1-8), (2-8), (6-7), (13-14), (14-23), (16-23), and (19-23) and can create new connections between different buses. Amongst these, the operator may decide on the number of transmission lines to be installed into the network. For instance, in the IEEE 24-bus test system, the maximum number of new transmission lines allowed in a specific corridor is equal to three. Consequently, a possible solution, which is a particle in the heuristic-based algorithms, is a set of numbers that all of them are equal or less than three. For example, Table A1 tabulates the possible solutions.

<table>
<thead>
<tr>
<th>Particles</th>
<th>Corridors</th>
<th>Particles</th>
<th>Corridors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1-8)</td>
<td>(2-8)</td>
<td>(6-7)</td>
</tr>
<tr>
<td>p_1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>p_2</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>p_3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

According to Table A1, the dimension of a sample particle, which varies algorithm to algorithm based on their operator, e.g., bird, fish, and frog, etc., is seven, corresponding to seven corridors (it is necessary to note that the IEEE 24-bus test system has entirely 41 possible paths; nevertheless, this is only an example to capture the concept behind heuristic-based algorithms while generating their initial population). In addition, since the maximum number of transmission lines to be added into the system is equal to three, each particle is considered as a string of seven number, each of which is either zero, one, two, or three. As a case in point, based on Table A1, p_2 is equal to [1 0 1 2 0 3 2], which means that between buses #1 and #8, one new line must be added; between buses #2 and #8, the system does not need any new
transmission lines, however, between buses #16 and #23, three new lines must be added, and the interpretations for other corridors is the same as the aforementioned ones. Moreover, to have a better view, if the size of the population is ten, for example, then the randomly generated matrix of the population can be as follows in which each row indicates a particle or a member of the population with dimension seven.

\[
\begin{bmatrix}
1 & 0 & 1 & 2 & 1 & 3 & 0 \\
0 & 2 & 1 & 1 & 3 & 0 & 2 \\
1 & 1 & 0 & 0 & 2 & 2 & 1 \\
3 & 2 & 1 & 0 & 1 & 1 & 1 \\
2 & 1 & 0 & 3 & 3 & 1 & 2 \\
1 & 1 & 1 & 1 & 1 & 3 & 1 \\
2 & 1 & 1 & 3 & 0 & 1 & 0 \\
3 & 0 & 2 & 1 & 1 & 2 & 1 \\
0 & 2 & 1 & 0 & 1 & 3 & 1 \\
1 & 1 & 0 & 3 & 1 & 2 & 1
\end{bmatrix}_{10 \times 7}
\]

- Initial Population

After producing the initial population, each of its members must be progressed toward the optimal solution in the search space. This process is done by applying a set of laws and formula that varies algorithm to algorithm according to their qualitative discrepancies, which are elaborated upon in the following sub-sections.

6.2.1. GA algorithm

The GA-based optimization algorithms often work according to the improvements in the fitness value through a progressive process like biological systems. In GAs, a sample solution of any given problem is considered in a string form, which is called chromosome and includes a set of components namely genes. In any optimization problem, these genes contain numerical values of each control variable [64]. The procedure of GAs almost always starts with an initial population, which is generated randomly. This population represents chromosomes that can be the solutions to the problem. The fitness of every chromosome can be recognized in the evaluating process of the objective function. To this end, the data are exchanged between the best chromosomes, parents, through mutation or crossover operators in order to generate the new offspring ones. Then, the offspring chromosomes are investigated, and according to this fact that the new solutions must be better than the weak ones in the previous population, a new population is generated. This process is promoted for a specific number of generation to obtain a high-quality solution. The data exchanging between both parents in the crossover process is illustrated in Fig. A2.
Fig. A2. Operation of crossover in order to produce the new child in offspring process

It should be noted that the GAs have some important parameters that directly affect the performance of the algorithm. These essential parameters include the size of the population, number of generations, the rate of mutation operator, as well as the rate of crossover operator. In this paper, a steady-state version of GA is implemented. The steady-state operation of GA works such that during the optimization process a specific offspring can replace the worst chromosome if it is better than the previous one.

6.2.2. PSO, MPSO, and FAMPSO algorithms

a) PSO algorithm

The PSO algorithm is one of the most well-known swarm intelligence-based optimization algorithms which has been inspired by the social manner of bird’s migration. The PSO algorithm works based on the probability laws [65]. In the PSO algorithm, at first, a random population is produced then it promotes toward optimality by updating its information in the search space. In this algorithm, each individual represents a feasible solution of the optimization problem where a sample solution is one particle in the search space, which has two important characteristics including position and velocity. During the simulation, the position and velocity of every particle are updated by Eqs. (A5)–(A7).

\[
X_p^{t+1} = X_p^t + V_p^{t+1}, \forall p
\]

\[
V_p^{t+1} = \left[ w \times V_p^t \right] + \left[ c_1 \times rand \times \left[ P_{best,p}^t - X^t_p \right] \right] + \left[ c_2 \times rand \times \left[ G_{best}^t - X^t_p \right] \right], \forall p
\]

\[
w = w_{max} - \left( \frac{w_{max} - w_{min}}{K_{max}} \right) \times K
\]

where \(X_p^{t+1}\) and \(V_p^{t+1}\) are the position and velocity of particle \(p\) at iteration \((t + 1)\), respectively; \(X_p^t\) and \(V_p^t\) are the position and velocity of particle \(p\) at iteration \(t\), respectively; \(c_1\) and \(c_2\) are two positive parameters, cognitive and social learning coefficients, respectively, which are bounded in range \([1, 2]\); \(rand\) is a random number in the range \([0, 1]\); \(P_{best,p}^t\) is the best personal fitness value of particle \(p\) at iteration \(t\); \(G_{best}^t\) is the best value among all \(P_{best,p}^t\) at iteration \(t\); \(w\), \(w_{min}\), and \(w_{max}\) are the inertia coefficient, minimum, and maximum values of inertia coefficient, respectively; \(K\) and \(K_{max}\) are the current iteration and the maximum number of
iterations, respectively; $P_S$ is the population size; and $d$ is the dimension of problem or number of control variables.

b) MPSO algorithm

The proposed structure of the MPSO algorithm is designed according to the SAPMO strategy; therefore, the mutant vector for this algorithm can be written as Eq. (A8).

\[ x_{PSO}^M = x_i^1 + rand_d \times (G_{best}^d - x_i^1) + rand_d \times (x_i^3 - x_i^1) \]  

(A8)

where, $x_i^1$, $x_i^2$, $x_i^3$, and $X_i^4$ are the mutant vectors that are randomly chosen from initial population; and $X_i^M$ is the mutant vector of the PSO algorithm.

c) FAMPSO algorithm

The MPSO algorithm has one important parameter, inertia coefficient $w$, which extremely influences the algorithm’s performance. Therefore, a Fuzzy Interface System (FIS) is designed to obtain the best inertia coefficient based on fuzzy rules during the iterative process. The input variables include current best performance and current inertia coefficient, while the output variable includes updated and desirable inertia coefficients. The sample structure of proposed FIS can be written as Eqs. (A9) and (A10), where the fuzzy rules are presented in Table A2 [66].

\[ NFV = \frac{[FV - FV_{min}]}{[FV_{max} - FV_{min}]} \]  

(A9)

\[ w_d = w_c + \Delta w \]  

(A10)

where $w_c$ and $w_d$ are the current and desirable inertia coefficients, respectively; $\Delta w$ is an eligible boundary for variation of $w$; $NFV$ is the normalized fitness value; and $FV_{max}$ is a big value.

Table A2. List of fuzzy rules to acquire the best inertia coefficient through the process

<table>
<thead>
<tr>
<th>$Delta w$</th>
<th>$w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>Small</td>
</tr>
<tr>
<td>Medium</td>
<td>Zero</td>
</tr>
<tr>
<td>Large</td>
<td>Positive</td>
</tr>
</tbody>
</table>

It should be noted that by applying the proposed FIS structure into MPSO algorithm, the FAMPSO algorithm results. Interested readers are directed to [67] for more information about intelligent scheduling techniques, notably fuzzy logic.

6.2.3. SFLA and MSFLA algorithms

a) SFLA algorithm

The SFLA is a stochastic optimization algorithm that almost always works by an initial population of frogs which search their lake for finding their food. This algorithm is a fusion model of GA and PSO algorithms. In addition, these frogs are known as memes that are in fact
the control variables of any given problem [68]. In the SFLA, the initial population of frogs, which is generated randomly, are split into some discrete categories, called memeplexes, and each memeplex searches for its own food. If the number of memeplexes is equal to \( N_m \) and each of them contains \( N_f \) frogs, then the population size of frogs can be calculated as \( PS = N_m \times N_f \). It is worth mentioning that the process of filling the memeplexes is continuous such that the first frog goes to the first memeplex, the second one goes to the second memeplex, and so on, \( (N_f)^{th} \) frog goes to \( (N_m)^{th} \) memeplex, and this procedure is repeated until all frogs filled all memeplexes. Afterward, in each memeplex, the best and worst frogs are identified. The best frog in all memeplexes is also recognized, which belongs to the population and not to one memeplex. The SFLA algorithm tries to improve the positions of the worst frogs by Eqs. (A11) and (A12). The geometrical representation of this process is illustrated in Fig. A3 [69].

\[
\begin{align*}
D_p &= \{rand_d \times [X_b - X_w]\} + \{rand_d \times [X_G - X_w]\}; \forall p \quad \text{(A11)}
X_w^n &= X_w^p + D_p; \forall p \quad \text{(A12)}
-D_{\max} \leq D_p \leq D_{\max}; \forall p \quad \text{(A13)}
\end{align*}
\]

where, \( D_p \) is the displacement of \( p^{th} \) frog; \( rand \) is a random number in the range \([0, 1]\); \( X_w \) and \( X_b \) are the worst and best frog in a memeplex, respectively, while \( X_G \) is the best frog among all memeplexes; \( D_{\min} \) and \( D_{\max} \) are the minimum and maximum displacement in frog’s position, respectively; and \( X_w^0 \) and \( X_w^n \) are the old and new position of frogs, respectively.

![Fig. A3. Progress of the worst frogs toward the best ones in the SFLA](https://example.com/sfla_diagram.png)

**b) MSFLA algorithm**

The proposed structure of MSFLA is designed according to the SAPMO strategy; therefore, the mutant vector for this algorithm can be written as Eq. (A14).

\[
X_{SFLA}^M = X_c^0 + rand_d \times (X_b^1 - X_c^2)
\]

where \( X_c^1 \) and \( X_c^2 \) are mutant vectors, which are randomly chosen from the initial population; and \( X_{SFLA}^M \) is a mutant vector of SFLA algorithm.
6.2.4. TLBO and MTLBO algorithms

a) TLBO algorithm

The TLBO algorithm works based on simulation of the teaching process in a classroom, and it is clear that the teacher has a key role in the learning process of students. The main procedure of TLBO algorithm is split into two major phases including teaching phase (teacher) and learning phase (student), which are elaborated upon hereunder [70].

**Teaching Phase:** An experienced teacher is one who can enhance the knowledge of the students. However, in the real world, the level of student’s knowledge increases only near to the level of the teacher and not beyond that if the teacher is the only source of information. This mentioned phase can mathematically be modeled as follows [70].

\[
\begin{align*}
X_{\text{diff}}^t &= \text{rand}_d \times (T^t - TF \times M_s^t) \\
X_{\text{new}}^{t+1} &= X_{\text{old}}^t + X_{\text{diff}}^t
\end{align*}
\]  

(A15) 

(A16)

where \(T^t\) is the teacher at \(t^{th}\) iteration; \(M_s^t\) is the average of students’ knowledge at \(t^{th}\) iteration; \(TF\) is the teaching factor; and \(X_{\text{diff}}^t\) is the difference between the level of teacher’s knowledge and the average of the level of the classroom at iteration \(t\).

**Learning Phase:** Students can enhance their level of knowledge using two different ways. These methods are including learning by receiving from the teacher as well as learning by interacting with other students. To this end, the interaction and dynamic behavior of one student with the other ones can be formulated as Eq. (A17).

\[
X_{\text{new}} = X_{\text{old}} + \text{rand}_d(X_m - X_n)
\]

(A17)

where \(X_m\) and \(X_n\) are \(m^{th}\) and \(n^{th}\) student, respectively; and \(X_{\text{old}}\) and \(X_{\text{new}}\) are the old and new student in the population, respectively.

In the TLBO algorithm, after computing the new students, a comparison process should be carried out between the fitness of the student in the current and previous iterations. Therefore, if the new value is less than the old one, the old student can be substituted by the new one, and this procedure will continue for a specific number of iterations, \(K_{\text{max}}\).

b) MTLBO algorithm

The proposed structure of MTLBO algorithm is designed according to the SAPMO strategy; therefore, the mutant vector for this algorithm can be written as follows.

\[
X_{\text{TLBO}}^M = X_{\text{new}}^t + \text{rand}_d \times (X_2^t - X_3^t) + \text{rand}_d \times (X_1^t - X_4^t)
\]

(A18)

where \(X_1^t, X_2^t, X_3^t\) and \(X_4^t\) are mutant vectors, which are randomly chosen from the initial population; and \(X_{\text{TLBO}}^M\) is the mutant vector of TLBO algorithm.
6.2.5. ICA algorithm

The ICA works based on social evolutions. In the ICA optimization algorithm, each particle in the population is known as a country in which these particles are split into two major categories, including colony and imperialist. Some of the best particles in the population are considered as imperialists, and the rest of them are taken into consideration as colonies. These colonies belong to imperialists in which each empire can control a set of them. In this regard, if the number of imperialists is equal to $n_{\text{imperialist}}$ and the number of colonies is equal to $n_{\text{colony}}$, then the size of the population, i.e., $PS$ is equal to $n_{\text{country}} = n_{\text{imperialist}} + n_{\text{colony}}$. Indeed, for a $d$-dimensional optimization problem, the ICA includes four sub-routines. These mentioned sub-routines are generating an initial population randomly, modeling the absorption policy, calculating the power of a sample empire, and modeling the process of imperialistic competition [71], which are formulated as (A19)–(A27).

$$PS = [\text{country}_1, \text{country}_2, ..., \text{country}_p, ..., \text{country}_d]$$ \hspace{1cm} (A19)

$$COC_p = f(p)$$ \hspace{1cm} (A20)

$$NCOI_p = \max(c_p) - COI_p$$ \hspace{1cm} (A21)

$$NPOI_p = \frac{COI_p}{\sum_{p=1}^{n_{\text{imperialist}}} COI_p}$$ \hspace{1cm} (A22)

$$INC = \text{round}(NPOI_p \times n_{\text{colony}})$$ \hspace{1cm} (A23)

$$x \sim U(0, \beta \times D)$$ \hspace{1cm} (A24)

$$\theta \sim U(-\gamma, +\gamma)$$ \hspace{1cm} (A25)

$$TCOE_p = COI_p + \xi \times \text{mean(COCOE}_p\text{)}$$ \hspace{1cm} (A26)

$$NTCOE = \max(TCOE_p) - TCOE$$ \hspace{1cm} (A27)

where $\text{country}_p$ is the $p^{th}$ particle or variable, which must be minimized and $d$ is the dimension of optimization problem; $COC_p$ is the cost of $p^{th}$ country; $COI_p$ is the cost of $p^{th}$ imperialist; $NCOI_p$ is the normalized cost of $p^{th}$ imperialist; $NPOI_p$ is the normalized power of $p^{th}$ imperialist; $INC$ is the initial number of colonies; $x$ is the fortuitous number with steady distribution; $D$ is the distance between imperialist and colony; $\beta$ is a positive number in the range $[1, 2]$; $U$ is the uniform distribution; $\theta$ is an algorithm parameter with uniform distribution; $\gamma$ is a parameter related to the searching area; $TCOE_p$ is the total cost of $p^{th}$ empire; $\xi$ is a positive number in the range $[1, 2]$; $COCOE_p$ is the cost of colonies related to $p^{th}$ empire; and $NTCOE_p$ is the normalized total cost of $p^{th}$ empire.

For better understanding, the geometrical representation of colonies mobility toward an imperialist is illustrated in Fig. A4.
Fig. A4. Displacement of colonies toward imperialist in the ICA

6.2.6. **HMPSO-DE algorithm**

Since the PSO and MPSO algorithms were explained in detail in the sub-sections 6.2.2, in this subsection the DE algorithms are explained in detail.

**a) DE algorithm**

The DE optimization algorithm is a simple and effective algorithm, which has been shown an eligible and appropriate performance in most of the optimization problems and under most circumstances. In this algorithm, first, an initial population is generated randomly based on uniform distribution. Then, to diversify the current population and generate a new population (generation), three different operators are applied. Integrally, for a \( d \)-dimensional problem, DE algorithm is including four sub-sections, initialization, mutation, crossover, and selection. This optimization algorithm can be mathematically modeled as (A28)–(A31), where (A28) and (A29) are corresponding to the initialization and mutation processes and (A30) and (A31) are corresponding to crossover and selection operators, respectively [72].

\[
X_{p}^{j} = X_{p}^{\text{min}} + UD \times (X_{j}^{\text{max}} - X_{j}^{\text{min}}); \forall p, j \\
X_{p,m}^{G} = X_{a}^{G} + CP(X_{b}^{G} - X_{c}^{G}); \forall p, a \neq b \neq c \neq i \\
X_{p,t}^{G} = \begin{cases} 
X_{p,m}^{G} & UD_{t} \leq C_{CO} \\
X_{p}^{G} & \text{Otherwise} 
\end{cases}; \forall p, j \\
X_{p}^{G+1} = \begin{cases} 
X_{p,t}^{G+1} & f(X_{p,t}^{G+1}) \leq f(X_{p}^{G}) \\
X_{p}^{G} & \text{Otherwise} 
\end{cases}; \forall p
\]

where \( X_{j}^{\text{min}} \) and \( X_{j}^{\text{max}} \) are the minimum and maximum values of \( j \)-th control variable, respectively; \( UD \) is a uniformly distributed number in the range \([0, 1]\); \( X_{p,m}^{G} \) is the mutant vector in generation \( G \); \( X_{a}^{G} \), \( X_{b}^{G} \) and \( X_{c}^{G} \) are randomly selected vectors in generation \( G \); \( a \), \( b \), and \( c \) are randomly selected indices; and \( CP \) is the control parameter of the DE algorithm in range of \([0, 2]\).

**b) HMPSO-DE algorithm**

In order to improve the searching ability of the MPSO algorithm as well as enhancing the escaping capability from an undesirable local optimal solution, this paper introduces a novel
hybrid configuration of MPSO and DE algorithms. The DE algorithm is hybridized with the MPSO algorithm to diversify the population drastically and improve the obtained results. Fig. A5 displays the flowchart of the aforementioned hybridization process in this paper.

Fig. A5. Hybridization process of MPSO and DE algorithms in this paper

7. Appendix B

Essential information regarding different generation plans G1 to G4 on IEEE 24-bus test system are tabulated in Table B1.

<table>
<thead>
<tr>
<th>Bus number with</th>
<th>Generation Plans</th>
</tr>
</thead>
<tbody>
<tr>
<td>generation unit</td>
<td>G1</td>
</tr>
<tr>
<td>1</td>
<td>576</td>
</tr>
<tr>
<td>2</td>
<td>576</td>
</tr>
<tr>
<td>7</td>
<td>900</td>
</tr>
<tr>
<td>13</td>
<td>1773</td>
</tr>
<tr>
<td>15</td>
<td>645</td>
</tr>
<tr>
<td>16</td>
<td>465</td>
</tr>
<tr>
<td>18</td>
<td>1200</td>
</tr>
<tr>
<td>21</td>
<td>1200</td>
</tr>
<tr>
<td>22</td>
<td>900</td>
</tr>
<tr>
<td>23</td>
<td>315</td>
</tr>
</tbody>
</table>

References


