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### Castillo, Daniel; Al-Qadi, Imad

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### Mechanical Modelling of Asphalt Concrete Using Grid Division

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Keywords:	multiphase material, modelling, grid division, finite elements, microstructure

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# 1 MECHANICAL MODELLING OF ASPHALT CONCRETE USING GRID DIVISION

Abstract: In this paper, a simple method is introduced for the computational modelling of multiphase materials, and for the approximation of their mechanical response. The two-dimensional microstructures of six asphalt concrete specimens are selected; three of the specimens have 'low'-, and three have 'high' aggregate fraction. A grid is used to divide the surface of each microstructure into square cells. The procedure of grid division is applied from 1 up to 100 divisions per side (i.e., up to 10,000 cells in the grid-divided specimen). To obtain an approximation of the mechanical response of the microstructure, the properties of the cells are estimated using three simple interpolation rules between the properties of the two phases, i.e. asphalt matrix and rock aggregates. It is found that the interpolation rules can yield reasonably representative results depending on the aggregate fraction of the microstructures and the number of divisions/cells in the grid. The grid-divided specimens allow approximating the overall mechanical response of the microstructures, and characteristics such as strain concentrations, overall deformations, and resulting force.

*Keywords:* multiphase material, modelling, grid division, finite elements.

### **1. INTRODUCTION**

The process of modelling multiphase materials in two and three dimensions can become quite complex, depending on the accuracy desired in the geometric and mechanical description of the individual phases. Commonly, a modeling process requires laboratory resource-and computational cost-intensive efforts. This process may entail acquiring the geometry of multiple phases composing the material, i.e. through imaging or X-ray analysis of cored specimens (Masad et al. 1999, Gruber et al. 2012, Obaidat et al. 2017), as well as laboratory testing of individual phases to quantify their associated mechanical or physical characteristics of interest, such as moisture/thermal diffusivity, stiffness, surface free energy or linear viscoelastic parameters (Kim et al. 2005, Kim et al. 2008, Yokohama and Nakashima, 2005, Witthüser et al. 2006). The process may also involve the formulation or usage of heavy, time-consuming computational models, that need calibration to yield representative results. Understandably, such a process necessitates a trade-off between result accuracy and computational costs. Hence, alternatives to de-escalate the complexity of the modelling process should be considered, geared towards time- and cost-efficient models that consume fewer resources.

In this context, there is interest in the area of pavements to adequately characterize the geometry and mechanical behavior of the multi-phase materials that compose the pavement structures, as well as numerous variables that simultaneously affect their properties, such as temperature, moisture, stress regime and oxidation. To this end, complex models have been developed by several researchers to represent pavement materials and structures. Many of these models use the finite element (FE) method and focus particularly on the behavior of asphalt concrete (AC) mixtures. Such models have been developed to predict damage and visco-plastic behavior of material under loading (Darabi et al. 2011), aging (Rahmani et al. 2017), and fracture using the cohesive-zone theory (Aragão et al. 2010). Tire-pavement interaction has

44 also been considered, using measured three-dimensional contact stresses (Wang and Al-Qadi, 45 2009), including various tire configurations such as wide-base tires, and considering non-linear 46 anisotropy in the lower layers of the pavement structure (Hernandez et al., 2016). In addition, 47 the impact of uncertainty and variability on AC behavior has been studied, through random 48 correlated spatial variability and the random generation of micromechanical configurations, as 49 well as the usage of domain analysis methods to quantify 'bulk' damage potential (Castillo and 50 Al-Qadi, 2018; Caro et al., 2018; Gamez et al. 2018).

These models take advantage of the versatility of the FE method to provide a representative spatial discretization of the domain of interest, and to accurately consider appropriate constitutive behaviors of the composing phases of the materials or the layers of the structure. The computational models are based on previous validations of FE predictions and provide good matches with experimental results; for instance, using viscoelastic approximations for the behavior of the AC material has proved useful for modeling distresses, such as rutting (Elseifi et al. 2006). This constitutive model was also used to predict strains in full-depth flexible pavement sections under moving loads (Wang and Al-Oadi, 2008).

In the context of the FE model, the domain of interest (i.e. the multi-phase geometry) commonly needs a finer discretization around irregularities in the material, near which critical concentrations of stresses and strains are expected to develop. Moreover, the need to consider thousands of irregularly-shape aggregates and air voids is a shortcoming. The presence of these numerous irregularities and inclusions is responsible for a significant part of the computational cost of the modeling process; characteristics such as the low thickness of the asphalt matrix between highly angular aggregates, or a high density of air voids over a small area, are directly responsible for inducing finer-than-average meshes. Considering these characteristics results not only in a greater number of finite elements, but it also hinders the meshing process and may result in non-suitable element shapes that compromise the results (i.e. leading to ill-conditioned matrices in the FE model). Moreover, the modelling of pre-existing cracks within the material,
as well as additional parameters inherent to the model such as interaction laws between the
phases, add to the complexity and associated computational cost of the models.

An alternative that has been used to address these challenges is multiscale modeling. A multiscale model alternates between a local, heterogeneous representation of the microstructure of the material, and a global, statistically homogeneous model of a structure. The scales are coupled two-way and linked through homogenization; multiscale models can include nonlinear behavior and fracture, and significantly reduce computational times using parallelization techniques (Kim et al. 2013). The local representation should be large enough to be representative of the global behavior of the mixture, but still small enough to be computationally efficient. This smallest practical unit is called a representative volume element (RVE); for instance, an RVE of 60 mm was found for a Superpave mixture with 12.5 mm nominal maximum aggregate size (NMAS) using digital image correlation (Kim et al. 2010), and in general, statistical homogeneity in the fraction of inclusions is observed from sizes greater than 2-3 times the NMAS, at least in two dimensions (Ozer et al. 2016). Multiscale experimental data shows different levels of sensitivity depending on the scale (Underwood and Kim, 2013), through temperature and frequency sweep tests at the scales of binder/mastic, fine aggregate matrix, and asphalt mixtures.

While the approach presented herein is not multiscale, it certainly shares many of the basic characteristics of multiscale modeling. These include discretizing phases at different scales, the search for a scale/size that is representative of the actual overall response, homogenizing mechanical/physical properties of the material, and in general, offering a computationally efficient alternative to model phenomena over a highly heterogeneous domain. The main difference between the two approaches is the presence of a *range* of scales (multiscale modeling uses 'leaps' between scales, whereas in the presented approach the range is

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94 continuous), and therefore the lack of feedback between scales.

95 This paper offers an intermediate approach, half-way between micromechanics and continuum-damage based modeling, to approximate the response of two-dimensional 96 specimens of AC. By discretizing their phases (aggregates and matrix) and estimating 97 mechanical properties over a grid, the complexity of the model is reduced without neglecting 98 99 the presence and effects of irregularities and inclusions in the domain.

101 2. METHODOLOGY

102 The present study follows four steps to obtain a range of estimates of the response of asphalt 103 concrete (AC) microstructure:

- 104 - Step 1: Superimpose a grid on the surface of the microstructure, to divide it into cells.
- 105 - Step 2: Determine the aggregate fraction per cell, i.e. the percentage area of the cell that is 106 covered by the aggregate phase.
- 107 - *Step 3*: Estimate the mechanical properties cell-wise, interpolating between the properties 108 of cells with 0-100% aggregate fraction.
  - *Step 4*: Create an FE model of the grid-divided specimen in axial compression. 109

110 Three alternatives are tested to estimate the mechanical properties of the cells in Step 3 (see 111 Section 2.3), which involve simple interpolation rules between the properties of the two 'pure' 112 phases; rock aggregates and matrix. Herein, the aggregate fraction was selected as the 113 determining parameter of mechanical behavior, since the (instantaneous) stiffness of the coarse 114 aggregates can be two or three orders of magnitude greater than that of the binder. The steps 115 are described next.

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#### 2.1. Division of the microstructure into cells, using a grid (Step 1) 117

#### 118 2.1.1. Selection and mechanical properties of the AC specimens

The objective of the first step is to partition the surface of the specimen into cells. A 'specimen' is understood as a microstructure of AC, i.e. a two-dimensional configuration of aggregate particles embedded in a homogeneous matrix of fine aggregates, asphalt binder and air. For simplicity, in this study the specimens are composed of two phases only, namely coarse aggregates and asphalt matrix. For simplicity, in this application the phases are not susceptible to damage, i.e. there is no reduction in the ability to sustain stresses.

Two values of aggregate fraction are considered in this study: a 'low' aggregate fraction of 52.7%, and a 'high' aggregate fraction of 66.1%. The values of aggregate fraction tend to vary over a relatively wide range. The selected values are within the range of two sources (You et al., 2009, Zelelew and Papagiannakis, 2011). For each aggregate fraction, three AC microstructures were randomly generated, for a total of six microstructures of AC. Two values of aggregate fraction are used because the aggregate phase is expected to have a major influence in the mechanical response of the microstructures, due to the high stiffness of the rocks. The microstructures are squares with 100-mm sides; the shape and size were arbitrarily selected, to be within the scale of laboratory specimens. The microstructures are presented in Figure 1.



and random elongation. The MG is capable of achieving various levels of aggregate angularityand microstructure forms (Castillo *et al.* 2015).

The objective of having three microstructures for each aggregate fraction (i.e., replicates) is to assess the consistency of the results; in other words, to observe if the behavior presented by any of the specimens is representative of their corresponding aggregate fraction. A key for this assessment is consistency among microstructures, i.e. gradation of the particles is the same for all microstructures, and the two levels of aggregate fraction are kept almost constant among specimens. Herein, consistency was achieved by the procedure of random generation of the microstructures provided by the MG. The aggregate size of the microstructures presented in Figure 1 ranges from 1.18 mm (sieve #16) to 19 mm (sieve 3/4''). 

158 2.1.2. Grid pattern

Figure 2 presents three possible 'tiling patterns,' or ways to divide a two-dimensional domain using a grid of cells with the same size and/or shape; squares, equilateral triangles, and regular hexagons. As observed, there are many ways of dividing a domain into cells; consider that the cells could also be combined to create arrays of cells of the same shape and different size.



Figure 2. Three possible ways to divide a plane using a regular grid. (a) Square cells, (b)
Triangular cells, (c) Hexagonal cells.

Although any tiling pattern could be used to divide the specimens, in this study the microstructures are divided into square cells of equal size, which is perhaps the simplest way to divide a plane and the most computationally favorable. Using this pattern, the grid resolution can easily be doubled, which simplifies increasing the divisions (i.e. the 'next' grid with more divisions is relatively easy to produce). Also, all even- and odd- divided specimens will always have some nodes in the same position. 

### 2.2. Cell-wise determination of the aggregate fraction (Step 2)

The six microstructures were divided using grids, in the (full) range from 1 to 100 divisions, i.e. partitioning them into 1 to 10,000 cells. Selected divisions corresponding to microstructures L1 and H1 from Figure 1 are presented in Figure 3. 



Notice that quite early in the division process, some cells are already covered completely by aggregates, and for higher numbers of divisions (or microstructures with low aggregate fraction) there are also cells that comprise only asphalt matrix. These cells may prove critical to the overall response of the specimen, since they will perform purely as aggregates or matrix, without any kind of interpolation.

The grid-divided specimens from Figure 3 can be interpreted as representations of the microstructures at different 'resolutions' (scales), ranging from very coarse (low resolution, less than 10-20 divisions of a 100-mm side) to detailed (about 30-40 divisions and upwards, for the scale used in this study). The maximum number of divisions in the grid (100) was selected arbitrarily; as the number of divisions increases, the original aggregate microstructure becomes more discernible.

Because of the complex geometry of the aggregates, calculating the aggregate fraction per cell can be challenging. The uniform grid frequently isolates highly irregular partitions of aggregates and matrix, and these areas become difficult to account for. Additionally, the increased number of cells is a challenge, even for a specimen with relatively low divisions. In this study, an automated meshing procedure was devised using (1) custom Python scripts and (2) the built-in meshing algorithm from the general-purpose FE software Abaqus, as follows. The matrix phase of the specimens was first isolated and partitioned using the grids, and the resulting (partitioned) matrix phase was meshed with finite elements. The presence of the partitions guaranteed that (1) no finite element belonged to more than one cell, and (2) the full domain of the matrix phase was covered with elements. The area of the elements within each cell was then accumulated (matrix area per cell), and the aggregate area per cell was obtained by subtraction.

### **2.3.** Interpolation rules to estimate the mechanical properties of the cells (Step 3)

The data resulting from the grid division (Step 2) was used as part of a simplified methodology to approximate the behavior of the microstructures. Considering that the aggregate fraction per cell is known for all (600) grid-divided specimens, an estimate of the mechanical properties of the cells was calculated based on the elastic and linear viscoelastic properties of the 'pure' aggregate and matrix phases, respectively, following three interpolation rules.

The aggregate phase was considered to be fully elastic, with a Young's modulus of 25 GPa and a Poisson's ratio of 0.16. These values are within the ranges reported for limestone (Shakiba et al. 2015, Yasar et al. 2004, Hart and Wang, 1995). On the other hand, the matrix phase was considered linear viscoelastic, with instantaneous relaxation modulus ( $E_o$ ) equal to 112 MPa at 25 °C, Poisson's ratio 0.40, and Prony parameters reported by Kim et al. (2005). The parameters are presented in Table 1. The same set of relaxation parameters is applied to all viscoelastic cells of material, in normalized form.

Table 1. Viscoelastic mechanical parameters of the asphalt matrix, from Kim et al. (2005).

<i>E</i> 1 [kPa]	73,173	$\eta_1$ [kPa·s]	115
$E_2$	25,506	$\eta_2$	418
$E_3$	10,847	$\eta_3$	1,079
$E_4$	2,078	$\eta_4$	1,529
$E_5$	332	$\eta_5$	2,249
$E_6$	53.4	$\eta_6$	3,513
$E_7$	8.44	$\eta_7$	5,430
$E_8$	1.33	$\eta_8$	8,664
$E_{9}$	0.252	$\eta_9$	20,406
$E_{\infty}$	0.122		
V	0.40		

In any of the three interpolation rules, a cell with 100% aggregate fraction will behave as a

 homogeneous block of rock (elastic). Conversely, a cell with 0% aggregate fraction is modelled
as a block of matrix (linear viscoelastic). For all other cells with aggregate fractions in the 0100% range, three simple interpolation rules between the two phases are tested. An example
showcasing the three alternatives is presented in Figure 4 for a grid-divided specimen with 40
divisions, i.e. 1,600 cells (microstructure 'L2' in Figure 1).



**Figure 4.** Three simple interpolation rules to estimate the mechanical properties of the cells.

234 2.3.1. Rule 1. '50/50'

In the first interpolation rule, the mechanical response of a cell is decided using a simple true/false criterion (only two possible outcomes); if the aggregate fraction of the cell is less than 50%, it will behave as pure matrix; on the contrary, if it is over 50%, it will be modelled as pure aggregate. In this way, a cell with 25% aggregate fraction would behave as pure matrix (viscoelastic, instantaneous modulus  $E_o = 112$  MPa), and a cell with 75% aggregate fraction would behave as pure aggregate (elastic, modulus E = 25,000 MPa). This interpolation rule is considered to be the simplest approximation to the actual response of the microstructure.

### 243 2.3.2. Rule 2. Linear interpolation

A linear interpolation was proposed as the logical step following the aforementioned alternative. In this rule, if the aggregate fraction is not 100%, the cell is modelled linear viscoelastic and its value of instantaneous modulus  $E_o$  is found by linear interpolation between the  $E_o$  of the matrix and the Young's modulus of the aggregates, i.e., an intermediate modulus between matrix and aggregates, proportional to the aggregate content of the cell. Only if the aggregate fraction is equal to 100% would the cell be modelled as pure rock. For instance, a couple of cells with aggregate fraction 25% and 75% would be modelled linear viscoelastic, with instantaneous modulus  $E_{o}$  6,300 MPa and 18,800 MPa, respectively.

It can probably be foreseen that the grids with few divisions will result in grossly nonrepresentative responses; however, calculating them is of interest since their results will allow assessing whether the alternatives produce more refined results with increasing divisions, if applicable.

257 2.3.3. Rule 3. Exponential interpolation

Finally, an exponential interpolation was proposed as a slightly more refined, yet still extremely simplified approximation to estimating the properties of the cells. The idea of testing an exponential interpolation comes, once again, from the difference in magnitude between aggregates and matrix. From the linear interpolation, it may be noticed that there could be an overestimation of the contribution of the aggregates, since it is more likely that the controlling behavior is viscoelastic (due to the matrix), even at relatively large aggregate fractions. To counteract this effect, the exponential interpolation assigns a lighter 'weight' to the influence

265 of the aggregates when compared to the linear alternative.

A curve of the following shape was used (moduli in MPa):

$$E_{a}^{cell} = E_{a}^{matrix} e^{k*AGF} \tag{1}$$

where the instantaneous modulus of a cell  $(E_{o cell})$  is a function of the modulus of the matrix phase ( $E_{o matrix}$ ), the aggregate fraction of the cell (AGF, ranging from 0 to 1) and a constant k that ensures the fit, i.e. if the aggregate fraction is 100% then  $E_{o cell}$  equals the modulus of the rock. For the moduli used in this study, k was equal to 5.4081. Using the exponential interpolation, for instance, a cell with 25% aggregate fraction would behave viscoelastic with an instantaneous modulus  $E_o$  equal to 433 MPa (instead of more than 6,000 MPa in the linear interpolation), and a cell with 75% aggregate area will have  $E_o$  equal to 6,468 MPa (instead of 18,800 MPa, linear). If the cell is 100% aggregate, it will be modelled as pure rock. 

### **2.4.** Conditions for the finite element simulations

As presented in the above sections, six AC microstructures (Figure 1) will be divided using a grid from 1 to 100 divisions, to create 100 grid-divided specimens per microstructure. Also, there are three interpolation rules per grid-divided specimen, which gives a total of 1,800 simulations. Six additional simulations are carried out for the microstructures with no grid-like divisions (i.e. traditional meshing of the microstructures in Figure 1), to provide a comparison and contrast with the results of the grid-divided specimens.

The specimens were modelled in FE under a fixed loading scheme. A total displacement of 0.1 mm in monotonic compression was applied during 0.5 sec on a rigid plate tied to the top of 12.2 the specimens. In the simulations, the vertical movement at the base of the specimens was 13.2 restricted, and the plate that applied the displacement moved downwards along the vertical axis 14.5 only, with no rotation.

A uniform target mesh size of 0.5 mm was selected for both the grid-divided specimens and

the six 'original', undivided microstructures. The grid-divided specimens were meshed with rectangular elements (about 40,000), and the microstructures were meshed with triangular elements (around 90,000). Nevertheless, consider that the meshing of the grid-divided specimens is completely neat, since there are no inclusions or irregularities in the geometry (i.e., square elements meshing square subdivisions on a square specimen). Therefore, a model as 'simple' as those presented in Figure 3 could easily be meshed with only a handful FE, greatly reducing its computational cost.

**3. RES** 

## 3. RESULTS AND ANALYSIS

The analysis of results is presented in three stages. First, the strain in the deformed specimens provides an overview of the differences between the three alternatives. Then, a more detailed analysis of the force-displacement curves gives insight into the accuracy associated with each of the alternatives. Finally, a summary of the force at the end of all the simulations is presented.

### **3.1.** Strain in the deformed specimens

Figure 5 presents a selection of grid-divided specimens for one microstructure of AC ('L1' from Figure 1) following the three interpolation rules between the mechanical properties of the matrix and the aggregates. The deformations are plotted with a factor of 30. The grid divisions in Figure 5 correspond to those presented in Figure 3, from 10 divisions upwards.

A clear contrast is observed between the alternatives in the specimens with fewer grid divisions (~20 or less). In rule 1 ('50/50') there is a marked difference between the stiff, elastic blocks of rock, and the blocks of matrix, which are very soft and viscous in comparison. The specimens with linear and exponential interpolation exhibit the presence of the aggregates as well, but their overall behavior is more uniform, and the deformations of the vertical (free) edges of the specimens are smaller than in the first alternative.



Figure 5. Maximum principal strain for selected grid-divided specimens, three interpolation rules. The corresponding microstructure is 'L1' from Figure 1. Notice the difference in the scale among alternatives; results of the full microstructure are identical and differ only in the color scale. 

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When the number of divisions starts to increase, the behavior of the grid-divided specimens becomes more consistent, and similar traits appear among alternatives. The shapes of the aggregates begin to emerge, and strain concentrations gradually converge to similar values; for example, in the three alternatives the strain on the 'paths' between aggregates already has values that are comparable to the microstructural results (lower part of Figure 5 – there is only one microstructure, presented with three color scales for comparison within the alternatives) as early as 30 divisions. In general, by 50-100 subdivisions the actual shapes of the aggregates, the 'paths' of high strain in the matrix and the strain concentrations from the original microstructure are observable in the grid-divided specimens. The improvement in the representation of certain characteristics of the actual behavior is a consequence of the increase in 'resolution,' i.e. the accuracy of the geometric representation of the aggregates.

When comparing the results from the grid-divided specimens with the response of the associated microstructure, the three alternatives seem to be converging towards the microstructural response with an increasing number of grid divisions. The convergence is happening at different rates, however; the exponential interpolation appears to offer a closest match with the microstructure 'earlier' (at a lower number of divisions) than the other two alternatives.

The methodology used herein produces simplified models in terms of computational cost. Given the conditions in this study, grid-divided specimens were two-dimensional and had less than 50,000 elements. It was possible to (a) create, (b) run and (c) analyze (extract data) from a considerable number of simulations in bulk (1,800), in an estimated cumulative time of 3-4 days, using a common office computer. Hence, the development of one simulation would take approximately 3 minutes to run, from start to finish. In contrast, three-dimensional full microstructural models are considerably heavier, usually with more than several hundred thousand finite elements. Each may take more than one day to create and a similar time to run

3.2.

'H1'.

(Chen et al. 2018, You et al. 2012).

**Force-displacement curves** 

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To find out whether the alternatives offer a reasonable representation of the overall response

of the microstructure or not, the analysis continues towards the force-displacement curves of

the grid-divided specimens. These curves show the force sustained by the specimen under

displacement (in compression) during the total time of the simulation. Figure 6 presents the

force-displacement curves of 100 grid-divided specimens of two microstructures, 'L1' and

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Figure 6. Force-displacement curves of 100 grid-divided specimens of two microstructures,
L1 and H1, for the three interpolation rules. Curves from 50 to 100 divisions are colored. Notice
the difference in vertical scale between specimens.

359 Clearly, the results of the three interpolations tend to be far from being an accurate 360 representation of the microstructural models. However, an overall consistent behavior is 361 observed, where the three alternatives are converging towards the actual response, and at a 362 medium to high number of divisions, in rules 1 and 3 the force is even within the same order 363 of magnitude as in the original microstructures.

It was expected that the 50/50 interpolation rule would grossly overestimate the stiffness of the grid-divided specimens with few divisions/cells, because every cell with medium to high aggregate fraction would behave as pure aggregate. This was particularly true for the microstructures with high aggregate fraction, as it can be observed that the curves converge towards the real response much slowly for the 'H1' grid-divided specimens than for the 'L1'. However, and perhaps surprisingly, at high numbers of divisions/cells the results obtained from the 50/50 interpolation rule are actually better than the linear approach. Initial results with less divisions/cells are rougher, but shortly after 10 divisions (low aggregate fraction) and around 60 divisions (high aggregate fraction), the force-displacement curves from the 50/50 rule are effectively better approximations to the real response than the linear. At 100 divisions, 50/50 results for the low aggregate fraction are even closer to the actual response than the exponential, which offered the best approximation overall. In general terms, while a perfect fit is not obtained, it is interesting to note that such a coarse simplification is still capable, to a point, of representing the results of the original microstructure.

While the results from the linear interpolation are more consistent than the '50/50', they fail to approach the actual response at an acceptable rate. At 100 divisions, this alternative is the farthest of the three from the microstructural results. Interpolating linearly effectively overestimates the contribution of the aggregates to the mechanical response of the cells, misrepresenting the overall behavior of the specimen. While this is corrected as the divisions increase, the contribution of the cells does not increase linearly with aggregate fraction, even when it does eventually approach the elastic modulus of the rock. On the other hand, the exponential interpolation rule is clearly the better alternative of the three. It converges quickly and is consistently closer to the real response than the other alternatives, at low and high aggregate fractions, even at very low number of divisions/cells. The exponential interpolation is more effectively balancing the mechanical contribution of the cells, and it properly represents the viscoelastic behavior that dominates at low aggregate fractions.

Figure 7 presents a summary of the 100-division specimens of the six microstructures, for the three interpolation rules. It can be observed that while the specimens with the same aggregate fraction yield unique results, their behavior is quite consistent (among specimens with the same aggregate fraction); for instance, the curves from microstructures L1, L2 and L3 differ in value, but are close together (they have the same aggregate fraction) and differentiated Reiczonz for each of the interpolation rules.



Figure 7. Force-displacement curves from the grid-divided specimens (100 divisions only) of
the six microstructures. Specimens with (a) low aggregate fraction, and (b) high aggregate
fraction.

**3.3.** Force at the end of the simulation

An analysis of the force at the end of the simulation offers an interesting insight of the capabilities of the interpolation alternatives. A summary of the force at the end of the simulation for all six microstructures (L1, L2, L3, H1, H2, H3) and grid divisions (1 to 100) is presented in Figure 8.



Figure 8. Force at the end of the simulation (t = 0.5 sec, displacement 0.1 mm) for all grid-divided specimens; six microstructures (L1-2-3, H1-2-3), three interpolation rules (50/50, Linear, Exp.). (a) Low aggregate area, (b) High aggregate area. Notice the difference in vertical scale.

Figure 8 provides a clear view of the behavior of the interpolation rules, and the consistency among the simulations. The results from the 50/50 interpolation have a far greater variability, but together with the linear and exponential approximations, their values are consistent among 

specimens with the same aggregate fraction (i.e. L1, L2 and L3; H1, H2 and H3). The results are clearly differentiated between interpolation rules, and the nature of the trends is distinct at low and high aggregate fractions, confirming the dominant influence of this parameter in the mechanical response of the specimens. Overall, among the range of divisions selected in this study (i.e. 1 to 100 divisions of a 100-mm side) the alternatives offered a better approximation to the real response for the microstructures with low aggregate fraction; nevertheless, they converge towards the real response for both low and high aggregate fractions, given enough divisions on the grid.

The increased variability among the specimen results for the 50/50 approach may be related to the fact that a *step* function was used to assign their properties (see Figure 4); i.e. the change between elastic and viscoelastic constitutive behaviors is sudden. Therefore, the response of a cell with 49% aggregate fraction may differ widely from that of a cell with 51% aggregate fraction; on the other hand, the transition between these behaviors is smooth for the linear and exponential interpolation rules.

An interesting trend appears in the linear and exponential interpolation rules. In the linear alternative, the force of the grid-divided specimens with low aggregate area (Figure 8a), has an apparent bi-linear trend with the number of divisions in the grid; it starts at 96 kN for 1 division, decreases to around 90 kN for 20-25 divisions, and then it reaches a local maximum; it decreases more rapidly after this point. The trend for the exponential rule increases from 1 to ~25 divisions, presents a maximum (~16 kN), and then it decreases, approaching the correct value of the microstructures. A similar trend was clearly observed at high aggregate area (Figure 8b) for both linear and exponential rules; there is a peak in the force sustained by the grid-divided specimens at ~40 divisions.

439 The results suggest an interesting property of the microstructure, which is able to reach a
 440 'maximum potential stiffness' at a certain number (or range) of grid divisions. This value seems

to be dependent on the aggregate fraction of the specimen (higher for increased aggregate fraction), and it is probably related to the ratio of the number of pure aggregate/pure matrix cells in the grid-divided specimen, as well as to the maximum nominal size of the gradation of the microstructures. Additionally, in the exponential alternative at both low and high aggregate fractions, this maximum stiffness was roughly twice the actual response of the microstructure. Results from the 50/50 interpolation rule did not follow this pattern.

 

### 448 4. SUMMARY AND CLOSING REMARKS

A simple method was proposed to model the mechanical response of six microstructures of asphalt concrete, three with low and three with high aggregate fraction. This procedure allows for a rough estimate of the actual response of a microstructure, based only on the mechanical properties of its composing phases.

The surface of the microstructures was divided into cells using grids with a range of 1 to 100 divisions (i.e. 1 to 10,000 cells). The aggregate fraction of the cells was used to estimate their mechanical properties, using three simple interpolation rules between the properties of the asphalt matrix and the rock aggregate: (1) 50/50, either matrix or aggregate, (2) linear interpolation, and (3) exponential interpolation.

The results were consistent among different microstructures with the same aggregate fraction, validating the use of this parameter as part of the three interpolation rules. While the approximations are less accurate for microstructures with 'high' aggregate fraction, the three alternatives converged towards the original microstructural response with an increasing number of divisions in the grid. Overall, the exponential interpolation rule was clearly the best and closest to the actual response of the microstructure. The '50/50' alternative yielded more variable results, but in some conditions it behaved better than the more 'refined' linear rule (low aggregate fraction, medium to high number of divisions). In fact, results suggest that it is

possible to identify a cell-wise threshold for aggregate fraction (i.e., instead of 50% in the 50/50
interpolation rule) to generate simple interpolation rules that adjust better to the microstructural
response. Additionally, the linear and exponential alternatives exhibited a local maximum in
the force resisted by the grid-divided specimens, at the same number of divisions (25 divisions
for low, 40 for high aggregate fraction).

Although the results of this study are an approximation to the real microstructural response, it is remarkable that by using simple interpolation rules between only a couple of points ( $E_{a}$  of the matrix, and Young's modulus of aggregates) it was possible to approximate a behavior that is inherently highly complex, and that is affected by the heterogeneity of the microstructure. Even though the grid-divided specimens lack a microstructure, to some extent their behavior can approximate that of the actual specimens of asphalt concrete, including characteristics such as strain concentrations and 'paths', overall deformations, and resulting force. Note that at no point during the analysis were bulk AC properties inputted into the models.

Potential applications for the methodology presented herein include modelling high volumes of mineral-based materials where micromechanical approaches are not practical due to high computational cost, i.e. full pavement cross-sections. Including microstructural characteristics in such conditions (i.e. aggregates, air voids, and/or cracks and other irregularities in full layers), is currently computationally expensive and may require several millions of finite elements for adequate meshing. The proposed methodology may offer a simplified way to approach these models.

The method of grid division is applicable to any multi-phase microstructure, whether acquired by image analysis (real microstructures) or by a packing or otherwise computational algorithm, random or not (virtual microstructures), and the same approach of calculating aggregate fraction in the cells holds in three dimensions. Instead of modulus, any other property may be estimated (e.g. coefficient of thermal or moisture diffusion, with different interpolation 491 rules) to model a variety of physical processes. Although the methodology is illustrated herein 492 for intact materials, cohesive damage in AC may be considered by replacing the viscoelastic 493 with a damage-encompassing constitutive behavior. Also, as it was mentioned before, the grid 494 does not need to be composed of squares; uneven shapes, or tessellations like Voronoi could 495 be used as well.

Accurate computational models of multi-phase materials can easily become 'heavy'; some may take hours or even days to run, using special-purpose servers and supercomputers. In this regard, techniques for 'de-scalation' of complexity such as the one presented in this study may prove useful, enhancing computational efficiency while maintaining reasonable accuracy in the results.

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