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*Published in:*  
IET Generation, Transmission and Distribution

*DOI:*  
[10.1049/iet-gtd.2018.6612](https://doi.org/10.1049/iet-gtd.2018.6612)

Published: 01/01/2019

*Document Version*  
Peer reviewed version

*Please cite the original version:*  
Melgar-Dominguez, O. D., Pourakbari-Kasmaei, M., Lehtonen, M., & Mantovani, J. R. S. (2019). Voltage-Dependent Load Model-Based Short-Term Distribution Network Planning Considering Carbon Tax Surplus. *IET Generation, Transmission and Distribution*, 13(17), 3760–3770. <https://doi.org/10.1049/iet-gtd.2018.6612>

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# Voltage-Dependent Load Model-Based Short-Term Distribution Network Planning Considering Carbon Tax Surplus

ISSN 1751-8644  
doi: 0000000000  
www.ietdl.org

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**Abstract:** Nowadays, strategies to cope with environmental issues play a crucial role in the development of planning methodologies for electric distribution networks (EDNs). The primary goal of these methodologies is to find the equilibrium point, for which the EDN provides a high-quality service with the most environmentally-committed operation. Based on this fact, an alternative strategic approach for short-term EDN planning is presented in this manuscript. This decision-making scheme is based on classical investment actions to enhance the EDN performance in which the investment and operating costs, as well as the carbon tax surpluses, are minimized simultaneously. In this regard, unlike the traditional short-term planning methods and to explore a more accurate approach, the electricity demand is represented by the voltage-dependent exponential load model. By using this representation, substantial benefits related to energy saving can be achieved. To validate the proposed planning scheme, several test cases considering constant power demand and the voltage-dependent representation are widely studied on a 135-node distribution network. Additionally, the scalability of the proposed planning scheme is studied by using two medium distribution networks of 42- and 417-node. Numerical results confirm the robustness and applicability of the presented approach as an appropriate way of promoting to an efficient and low carbon tax surplus network.

## Nomenclature

### Sets and Indexes:

$\Omega^{cb}$	Set of capacitor bank (CB) capacities
$\Omega^c$	Set of conductors types
$L$	Set of circuits
$N$	Set of nodes
$T$	Set of time intervals
$Y$	Set of planning horizon
$b$	Index of reactive power capacity of CB
$c$	Index of conductor type
$m, n$	Indexes of nodes
$mn$	Indexes of circuits
$t$	Indexes of time intervals
$y$	Indexes of years

### Constants:

$\zeta_{t,y}^{ec}$	Energy cost at time interval $t$ and year $y$
$\zeta_{t,y}^{ct}$	Carbon tax cost at time interval $t$ and year $y$
$\zeta_b^{f/sw}$	Investment costs of fixed and switchable CBs with reactive power capacity $b$
$\zeta_{\hat{c},c}^{cr}$	Upgrading cost to replace the conductor $\hat{c}$ by new conductor $c$
$\zeta^{vr}$	Investment costs of voltage regulator (VR) device
$\alpha, \beta$	Parameters of exponential demand model
$\delta$	Weight to transform a year to time intervals (8760 h/T)
$\Phi, \bar{\Phi}$	Lower and upper bounds for substation power factor
$e^{co2}$	Emission coefficient
$\bar{I}^{(c)}$	Maximum current magnitude allowed at cable $c$
$l_{mn}$	Length of circuit $mn$
$\bar{N}^{cb}$	Maximum number of operations allowable over the time period of the installed CB

### $\bar{N}^{vr}$

$\bar{N}^{vr}$	Maximum number of operations allowable over the time period of the installed VR device
$P_{m,t,y}^D$	Nominal active power demand at node $m$ , time $t$ , and year $y$
$Q_b^D$	Specified reactive power of the CB with capacity $b$
$Q_{m,t,y}^D$	Nominal reactive power demand at node $m$ , time $t$ , and year $y$
$R_{mn}^{(c)}$	Resistance of conductor $c$ at circuit $mn$
$R\%$	Regulation range (%) of the VR device
$\bar{T}^P$	Maximum number of steps of VR device
$\bar{V}, \underline{V}$	Upper and lower voltage magnitude bounds
$V^o$	Nominal voltage magnitude
$X_{mn}^{(c)}$	Reactance of conductor $c$ at circuit $mn$
$Z_{mn}^{(c)}$	Impedance of conductor $c$ at circuit $mn$

### Continuous variables:

$\hat{I}_{mn,t,y}^{(c)}$	Square of current flow magnitude of selected conductor $c$ at circuit $mn$ , time $t$ , and year $y$
$P_{mn,t,y}^{(c)}$	Active power flow of selected conductor $c$ at circuit $mn$ , time $t$ , and year $y$
$P_{t,y}^{SS}$	Active power supplied by substation at time $t$ , and year $y$
$Q_{mn,t,y}^{(c)}$	Reactive power flow of selected conductor $c$ at circuit $mn$ , time $t$ , and year $y$
$Q_{m,t,y}^{cb}$	Reactive power delivered by the installed CB at node $m$ , time $t$ , and year $y$
$Q_{t,y}^{SS}$	Reactive power supplied by substation at time $t$ , and year $y$
$\hat{V}_{m,t,y}$	Square of voltage magnitude at node $m$ , time $t$ , and year $y$
$\check{V}_{n,t,y}$	Auxiliary variable to control $\hat{V}$ by the installed VR device at node $n$ , time $t$ , and year $y$

$U_{n,t,y}^{(k)}$  Auxiliary variable to linearize the operation of the VR device at tap position  $k$ , node  $n$ , time  $t$ , and year  $y$

*Binary and integer variables:*

$b_{n,t,y}^{tp(k)}$  Binary variable to determine the tap step  $k$  of the installed VR at node  $n$ , time  $t$ , and year  $y$

$M_{m,t,y}^{cb}$  Integer variable for number of modules in operation of the installed CB at node  $m$ , time  $t$ , and year  $y$

$\bar{M}_m^{cb}$  Integer variable for the maximum number of CB modules to be installed at node  $m$

$t_{n,t,y}^p$  Integer variable for the number of tap steps of the installed VR device at node  $n$ , time  $t$ , and year  $y$

$x_{m,b}^{cb}$  Binary variable that define the capacity  $b$  of the fixed CB to be installed at node  $m$

$z_{m,b}^{cb}$  Binary variable that defines the capacity  $b$  of the switchable CB to be installed at node  $m$

$u_{mn}^{(c)}$  Binary variable that defines the conductor type  $c$  to be replaced at circuit  $mn$

$w_n$  Binary variable that defines the VR device to be installed at node  $n$

## 1 Introduction

In recent years, strategies for mitigating environmental problems such as climate change have received significant attention in the planning studies of electric distribution networks (EDNs). In this regard, the transition to more efficient networks via multiple planning alternatives and environmental policies plays an essential role to fulfill the electricity demand increase requirements while at the same time promoting for a more sustainable network [1]-[3]. Traditionally, to guarantee the quality and reliability of the supplied energy by the EDN, some classical upgrading planning actions to provide adequate control over the voltage magnitude and reactive power flows, reductions of operational losses, among others are used in a short-term horizon [4]-[7]. On the other hand, to mitigate environmental problems, incentives have been adopted to promote the renewable energy-based sources integration [8].

The planning studies have been represented via multiple decision-making processes to offer a more flexible and efficient plan. In [9], an integrated planning model considering renewable energy sources and demand response programs was proposed. Different alternatives were explored to enhance the performance of the system where the carbon emission was mitigated via a monetary form. A strategic approach for the optimal allocation of renewable energy sources and reactive power support devices was investigated in [10]. This work defined an approach to minimize the distribution energy losses and the carbon emission, and consequently, enhance the voltage levels. In a similar way, a long-term planning allocation problem of renewable energy sources was proposed in [11]. From an economic point of view and to achieve a clean environment, this approach intended to minimize the carbon emission costs, while maximizing the incentives in renewable sources investments. Analogously, in [12], distributed generation (DG) units based on multiples renewable energy technologies were simultaneously optimized to improve the performance indices related to technical, economic, and environmental aspects. A strategic planning scheme, to enhance the performance of the EDN considering multiple short-term planning actions at the presence of renewable energy sources was presented in [13] where the environmental targets were addressed via the carbon cap policy. On the other hand, in [14], actions such as substation and feeder reinforcement and DG allocation was formulated to solve a long-term distribution planning problem. This work aimed at minimizing the investment costs, supplied energy costs, and costs of carbon emission taxes, simultaneously. Besides the economic and technical goals, these works investigated that by using several planning alternatives and environmental policies, reductions on pollutant emissions are achieved.

Indeed, the mitigation of carbon emissions in an EDN can be achieved with the integration of low-carbon technologies. However, from another perspective, the EDN demand could be managed to obtain energy savings, and consequently, reduction in carbon emissions results. This procedure consists of increasing the energy efficiency via voltage magnitude control for which the demand consumption and energy losses can be reduced [15]. In the operation stage, this procedure has been presented with benefits relevant to obtaining energy savings where the most economic EDN operation is guaranteed [16]-[18]. Alternatively, this procedure has been combined to optimize the location and sizing of renewable energy-based sources via a two-stage stochastic planning framework [19]. Similarly, a method that investigates the optimal energy storage size and location considering energy saving via the voltage level reduction approach was developed in [20]. To achieve such benefits and by considering that in medium voltage networks, the demand is predominantly voltage-dependent, a suitable demand representation that involves such voltage-dependent behavior is indispensable. In medium voltage networks studies, two static models such as the ZIP and exponential load models are highlighted [21]. It can be seen that for the sake of simplicity, the voltage-dependent behavior in the EDN demand has been rarely investigated in most planning studies. As expected, the solution to this kind of planning problems is highly sophisticated due to its combinatorial nature; thereby, several strategies have been developed to address such complexities [22]. Capacitor bank (CB) allocation, DG placement, and simultaneous placement of DG and CB in distribution network considering the load type of consumers, reliability, and economic-driven were studied in [23], [24] and [25], respectively. These studies show that by considering power-constant load value, for the consumers, or constant load value, for the feeder's failure, the obtained planning decisions result in weak network performance. In [26], a strategy for increasing the renewable energy hosting capacity and energy efficiency in EDNs was studied. In this work, in order to obtain a suitable approach, the EDN demand was characterized by using an exponential load model, for which, energy saving targets were achieved. An approach to simultaneously solve the feeders reconfiguration and CB allocation problems in an EDN was proposed in [27]. Results show that the investment plan could be significantly affected by using different load models, namely the voltage-dependent load and the power-constant load models. A two-layer co-planning strategy for DG and energy storage systems integration to enhance the voltage regulation in EDN was studied by [28]. In this study, besides meeting voltage regulation targets, energy saving purposes were also achieved by implementing conservative voltage regulation effects via a time-varying voltage-dependent load model. Reviewing the existing works reveal that there is no study on assessing the carbon tax surplus reduction in EDN by taking advantage of voltage-dependent behavior of loads, whereby substantial benefits in the planning stage can be explored as an alternative way to reduce the carbon emissions at the distribution level.

Therefore, in this manuscript, a traditional planning approach for EDNs is investigated under other economic and environmental perspectives. To formulate this planning strategy, traditional actions such as reinforcement of the existing circuits, sizing and placement of switchable and fixed CBs, and placement of voltage regulator (VR) devices are used to improve the EDN performance. Besides, to obtain a more accurate approach, the EDN demand is modeled via an exponential load representation. The proposed planning strategy minimizes the costs of the energy supplied by the substation and surpluses related to the carbon taxes subject to technical and operational constraints. Inherently, this strategy is formulated as a non-convex mixed integer nonlinear programming (NMILP) model that via appropriate linearization techniques is recast into a convex mixed integer linear programming (CMILP) model. Consequently, to validate the accuracy, flexibility, and robustness of the presented approach, a medium voltage distribution network of 135-node is tested under different study cases. Additionally, the scalability of the presented short-term planning scheme is validated by using two medium-voltage distribution networks of 42- and 417-node. Numerical results confirm the practicality and efficiency

of the proposed approach for improving the EDN performance and to obtain the best economic and environmentally-committed plan. This approach is implemented in AMPL while the commercial solver CPLEX is used to carry out its solution. Therefore, the technical contributions of this work can be categorized as follows.

- Proposing a practical tool to determine an effective short-term upgrading plan for distribution networks where several traditional investment actions are simultaneously considered to improve the EDN efficiency as well as to reduce carbon tax surplus and energy supplied costs.
- Proposing a mathematical programming model, to represent a complex decision-making process that besides considering the technical and operational constraints, takes into account the exponential load representation to model the electricity demand more practically. This problem is modeled as a nonconvex mixed integer nonlinear programming model (NMINLP) that more often than not makes severe obstacles in finding a high-quality solution.
- Recasting the original NMINLP model to a more flexible and practical CMILP optimization model. Since the obtained model is a linear-based convex model, therefore, finding the global optimal solution with a high computational efficiency is guaranteed.

The rest of this paper is organized as follows. In Sect. 2, first, the original NMINLP model to represent the short-term planning problem, and then, the recast process to obtain a CMILP model are duly presented. Sect. 3 presents the technical and financial information, assumptions, and case studies to validate the proposed approach. Numerical results are reported and analyzed in Sect. 4. Finally, the concluding remarks are presented in Sect. 5.

## 2 Problem Formulation And Solution Scheme

The presented planning scheme aims to determine an efficient and low-cost investment plan, which includes upgrading of the existing system by reinforcement of circuits as well as the placement of new assets. This section describes the mathematical formulation of the aforementioned problem adequately. This formulation represents a flexible and multi-choice strategy that simultaneously considers sizing, placement, type of CBs to be installed, placement of VR devices, and circuits to be reinforced as well as defines the optimal operation of the Volt-VAR control devices. Therefore, this strategy aims at enhancing the performance of an EDN in a short-term period where costs of the energy purchased by the substation and expenses related to the carbon tax are minimized simultaneously with the investment costs. As expected, the resulted model is inherently an NMINLP programming problem. To guarantee finding the globally optimal solution, appropriate linearization methods are used to represent this problem via a CMILP optimization model.

### 2.1 Non-convex Programming Model

The proposed planning problem is modeled in (1)–(20) where this formulation a) defines the placement, type (fixed or/and switchable), size and optimal operation of the reactive support devices to be installed; b) defines the optimal location of the VR devices to be installed and by specifying the tap position to control the voltage magnitude; and c) determines the circuits to be reinforced, by specifying the new conductor type. The objective function (O.F.) in this optimization model minimizes: 1) the overall cost of the energy purchased by the substation (1<sup>st</sup> term); 2) costs of the carbon emission tax (2<sup>nd</sup> term); 3) Investment cost due to fixed and switchable reactive power support devices, 3<sup>rd</sup> and 4<sup>th</sup> terms, respectively; 4) Investment cost due to upgrading EDN circuits (5<sup>th</sup> term); and finally, 5) Investment cost due to placement of VR devices

(6<sup>th</sup> term).

$$\begin{aligned} \min \sum_{t \in T} \sum_{y \in Y} \zeta_{t,y}^{ec} \delta P_{t,y}^{SS} + \sum_{t \in T} \sum_{y \in Y} \zeta_{t,y}^{ct} e^{co2} \delta P_{t,y}^{SS} + \\ \sum_{b \in \Omega^{cb}} \sum_{m \in N} \zeta_b^{fx} x_{m,b}^{cb} + \sum_{b \in \Omega^{cb}} \sum_{m \in N} \zeta_b^{sw} z_{m,b}^{cb} + \\ \sum_{c \in \Omega^c} \sum_{mn \in L} \zeta_{c,c}^{cr} I_{mn}^{(c)} u_{mn}^{(c)} + \sum_{n \in N} \zeta^{vr} w_n \end{aligned} \quad (1)$$

**Subject to:**

$$\begin{aligned} \sum_{nm \in L} \sum_{c \in \Omega^c} P_{nm,t,y}^{(c)} - \sum_{mn \in L} \sum_{c \in \Omega^c} (P_{mn,t,y}^{(c)} + R_{mn}^{(c)} \hat{I}_{mn,t,y}^{(c)}) + \\ P_{t,y}^{SS} = P_{m,t,y}^D \left( \frac{\sqrt{\hat{V}_{m,t,y}}}{V_o} \right)^{\alpha_m} \end{aligned} \quad (2)$$

$$\begin{aligned} \sum_{nm \in L} \sum_{c \in \Omega^c} Q_{nm,t,y}^{(c)} - \sum_{mn \in L} \sum_{c \in \Omega^c} (Q_{mn,t,y}^{(c)} + X_{mn}^{(c)} \hat{I}_{mn,t,y}^{(c)}) + \\ Q_{t,y}^{SS} + Q_{m,t,y}^{cb} = Q_{m,t,y}^D \left( \frac{\sqrt{\hat{V}_{m,t,y}}}{V_o} \right)^{\beta_m} \end{aligned} \quad (3)$$

$$\begin{aligned} \hat{V}_{m,t,y} - \check{V}_{n,t,y} = \\ \sum_{c \in \Omega^c} \left[ 2(R_{mn}^{(c)} P_{mn,t,y}^{(c)} + X_{mn}^{(c)} Q_{mn,t,y}^{(c)}) - (Z_{mn}^{(c)})^2 \hat{I}_{mn,t,y}^{(c)} \right] \end{aligned} \quad (4)$$

$$\check{V}_{n,t,y} \hat{I}_{mn,t,y}^{(c)} = (P_{mn,t,y}^{(c)})^2 + (Q_{mn,t,y}^{(c)})^2 \quad (5)$$

$$\underline{V}^2 \leq \hat{V}_{m,t,y} \leq \bar{V}^2 \quad (6)$$

$$0 \leq \hat{I}_{mn,t,y}^{(c)} \leq u_{mn}^{(c)} (\bar{I}^{(c)})^2 \quad (7)$$

$$-P_{t,y}^{SS} \tan(\cos^{-1} \Phi) \leq Q_{t,y}^{SS} \leq P_{t,y}^{SS} \tan(\cos^{-1} \Phi) \quad (8)$$

$$Q_{m,t,y}^{cb} = M_{m,t,y}^{cb} Q_b \quad (9)$$

$$0 \leq \bar{M}_m^{cb} \leq \sum_{b \in \Omega^{cb}} b x_{m,b}^{cb} + \sum_{b \in \Omega^{cb}} b z_{m,b}^{cb} \quad (10)$$

$$\bar{M}_m^{cb} \leq M_{m,t,y}^{cb} + \sum_{b \in \Omega^{cb}} b z_{m,b}^{cb} \quad (11)$$

$$0 \leq M_{m,t,y}^{cb} \leq \bar{M}_m^{cb} \quad (12)$$

$$\hat{V}_{n,t,d} = \left( 1 + R\% \frac{t_{n,t,y}^p}{\bar{I}^p} \right)^2 \check{V}_{n,t,y} \quad (13)$$

$$-w_n \bar{I}^p \leq t_{n,t,y}^p \leq w_n \bar{I}^p \quad (14)$$

$$\sum_{b \in \Omega^{cb}} x_{m,b}^{cb} \leq 1 \quad (15)$$

$$\sum_{b \in \Omega^{cb}} z_{m,b}^{cb} \leq 1 \quad (16)$$

$$\sum_{c \in \Omega^c} u_{mm}^{(c)} = 1 \quad (17)$$

$$\sum_{t \in T} \left| M_{m,t,y}^{cb} - M_{m,t-1,y}^{cb} \right| \leq \bar{N}^{cb} : \quad \forall t > 1 \quad (18)$$

$$\sum_{t \in T} \left| t_{n,t,y}^p - t_{n,t-1,y}^p \right| \leq \bar{N}^{vr} : \quad \forall t > 1 \quad (19)$$

$$F(x^{cb}, z^{cb}, u, w) \leq \Pi \quad (20)$$

$$\forall b \in \Omega^{cb}; c \in \Omega^c; m, n \in N; mn \in L; t \in T; y \in Y$$

The steady-state operation of an EDN is represented by the set of equations (2)–(5). The active and reactive power balance is shown in (2) and (3), respectively. The voltage magnitude drop at the circuit

$mn$  is determined by using (4), while (5) defines the square of the current flow magnitude at circuit  $mn$ , considering the square of the active and reactive power flows and the voltage magnitude. However, to obtain a high-quality service, operational conditions must be fulfilled in an EDN. These quality requirements are represented in the equations set (6)–(8). From these constraints, (6) and (7) represent the voltage magnitude bounds and the current magnitude capacity, respectively, while the substation power factor is controlled by (8). It is worth noting that the current flow capacity at the circuit  $mn$  is defined over the selected conductor type  $c$  and by using the binary variable  $u^{(c)}$ .

On the other hand, the reactive power injected by the CBs is defined by (9) where the maximum number of modules to be installed is determined by (10). In order to determine the capacity of the CB type (fixed or switchable), the product between the capacity  $b$  and binary variables  $x^{cb}, z^{cb}$  is used. In this regard, to optimally choose of the type of CB constraints (10) and (11) are used; fixed if  $M^{cb} = \overline{M}^{cb}$ , and switchable if  $M^{cb} < \overline{M}^{cb}$ . The operation of the CB modules is defined by the maximum number of modules to be installed in (12). The voltage magnitude at node  $n$  is controlled by (13), if a VR device is installed, the variable  $\hat{V}$  is controlled via the tap position  $tp$ , otherwise,  $\hat{V} = \check{V}$ . To adjust the tap position properly, the number of tap steps are bounded by (14). Logical constraints are defined for the capacity of each CB type and the selected conductor type in (15)–(17). (15) and (16) guarantee the capacity of fixed or switchable CB to be installed at node  $m$ , respectively, and (17) defines that only one conductor type is selected. The lifetime of reactive support and VR devices is significantly impacted by the number of operations over the entire period. Therefore, to preserve its lifetime, constraints (18) and (19) are used. Finally, to obtain an economic plan that meets the financial budgets of the distribution company, investment limits can be defined for each planning alternative by using (20).

## 2.2 Mixed-Integer Linear Programming Model

Although the aforementioned proposed model can be solved by several techniques such as nonlinear solvers, heuristic and metaheuristic methods, and global optimization algorithms, finding the global solution is not guaranteed, and it is highly dependent on the finite computer arithmetic and mild conditions to represent the problem [29]. Therefore, by using appropriate linearization techniques, this model is recast into a solver-friendly approximated mixed-integer linear model.

In this work, the EDN demand is mathematically represented via the exponential load model, as shown by (2) and (3), active and reactive power demand, respectively. These terms represent nonlinear expressions such as  $\sqrt{\hat{V}}^\alpha$ , and  $\sqrt{\hat{V}}^\beta$  for each constraint. Therefore, to obtain a tractable problem, these terms are linearized around an estimated operation point ( $V^{*2}$ ) as can be seen in (21).

$$\hat{V}_{m,t,y}^{\frac{\alpha_m}{2}} \approx \frac{\partial \hat{V}_{m,t,y}^{\frac{\alpha_m}{2}}}{\partial \hat{V}} \bigg|_{V_{m,t,d}^*} (\hat{V}_{m,t,y} - V_{m,t,y}^{*2}) \approx \frac{\alpha_m}{2} V_{m,t,y}^{*(\alpha_m-2)} (\hat{V}_{m,t,y} - V_{m,t,y}^{*2}) \quad (21)$$

Accordingly, by using (21), the active and reactive power demand in (2) and (3) are replaced by (22) and (23).

$$P_{m,t,y}^D \left( \frac{\sqrt{\hat{V}_{m,t,y}}}{V^o} \right)^{\alpha_m} \approx \frac{P_{m,t,y}^D}{(V^o)^{\alpha_m}} \frac{\alpha_m V_{m,t,y}^{*(\alpha_m-2)}}{2} (\hat{V}_{m,t,y} - V_{m,t,y}^{*2}) \quad (22)$$

$$Q_{m,t,y}^D \left( \frac{\sqrt{\hat{V}_{m,t,y}}}{V^o} \right)^{\beta_m} \approx \frac{Q_{m,t,y}^D}{(V^o)^{\beta_m}} \frac{\beta_m V_{m,t,y}^{*(\beta_m-2)}}{2} (\hat{V}_{m,t,y} - V_{m,t,y}^{*2}) \quad (23)$$

Constraint (5) stands for the steady state operation of the EDN, its non-convexity is a result of the product of two continuous variables (on the left side) equal to the quadratic terms (on the right side). In this regard, methods based on conic approach [30] and linearization techniques [31] have been explored to cope with this complicated expression. Therefore, in this work to recast the product of these variables, a voltage magnitude estimated value ( $V^*$ )<sup>2</sup> can be used to approximate  $\hat{V}$  where this estimated value is approximated by the nominal voltage magnitude ( $V^* = V^o$ ) corresponding to the initial operating point. On the other hand, the quadratic terms are linearized by the well-known piecewise linear method, for which these terms are divided into straight segments [32]. Thereby, by using this linearization technique constraint (5) is recast into (24).

$$V^* \hat{I}_{mn,t,y}^{(c)} = g(P, \overline{V}\overline{I}, \overline{\lambda}) + g(Q, \overline{V}\overline{I}, \overline{\lambda}) \quad (24)$$

From this expression, each quadratic term  $P^2$  and  $Q^2$  is defined by the generic function  $g$  where each term is represented by  $\rho$  and linearized by (25)–(29). In this equations, the length of each segment of the power flow is determined by  $\overline{V}\overline{I}$  and  $\overline{\lambda}$ , for which  $\overline{\lambda}$  represents the maximum number of segments to be used. For each power flow variable  $P$  and  $Q$ , two nonnegative auxiliary variables are used, as (26). The absolute value of power flow variables is represented as the sum of these auxiliary variables that is equal to the summation of discretization variables in (27). These discretization variables are limited by (28), while the slope of each segment is calculated via (29).

$$\rho^2 \approx g(\rho, \overline{V}\overline{I}, \overline{\lambda}) = \sum_{r=1}^{\overline{\lambda}} m_r^\rho \Delta_r^\rho \quad (25)$$

$$\rho^+ - \rho^- = \rho \quad (26)$$

$$\rho^+ + \rho^- = \sum_{r=1}^{\overline{\lambda}} \Delta_r^\rho \quad (27)$$

$$0 \leq \Delta_r^\rho \leq \frac{\overline{V}\overline{I}}{\lambda} \quad \forall r = 1 \dots \overline{\lambda} \quad (28)$$

$$m_r^\rho = (2r - 1) \frac{\overline{V}\overline{I}}{\lambda} \quad \forall r = 1 \dots \overline{\lambda} \quad (29)$$

Furthermore, expression (13) contains the product between the square value of the integer variable  $t^{p^2}$  and the continuous value of  $\check{V}$ . This product results in a complex nonlinear equation. Therefore, to linearize this complicated expression,  $t^{p^2}$  is represented by binary variables  $b^{tp}$  in the set  $k = 1..2\overline{T}^p$ , [17]. Additionally, the variable  $U$  is used to represent the product between  $t^{p^2}$  and  $\check{V}$ . Finally, (13) is substituted by the equations set (30)–(34).

$$\hat{V}_{n,t,y} = \check{V}_{n,t,y} (1 - R\%)^2 + \sum_{k=1}^{2\overline{T}^p} \left[ \frac{R\%}{\overline{T}^p} \left( \frac{(2k-1)R\%}{\overline{T}^p} + 2(1-R\%) \right) U_{n,t,y}^{(k)} \right] \quad (30)$$

$$\underline{V}^2 (1 - b_{n,t,y}^{tp(k)}) \leq \check{V}_{n,t,y} - U_{n,t,y}^{(k)} \leq \overline{V}^2 (1 - b_{n,t,y}^{tp(k)}) \quad (31)$$

$$\underline{V}^2 b_{n,t,y}^{tp(k)} \leq U_{n,t,y}^{(k)} \leq \overline{V}^2 b_{n,t,y}^{tp(k)} \quad (32)$$

$$b_{n,t,y}^{tp(k)} \leq b_{n,t,y}^{tp(k-1)} \quad \forall k > 1 \quad (33)$$

$$\sum_{k=1}^{2\overline{T}^p} b_{n,t,y}^{tp(k)} - \overline{T}^p = t_{n,t,y}^p \quad (34)$$

In summary, classical linearization techniques have been applied to recast the original short-term asset upgrading planning problem into an CMILP model. The exponential load model, utilized to represent the EDN demand, has been linearized around an estimated operation point, and consequently, linear expressions have been

**Table 1** Information of electricity demand, energy cost, and cost of carbon emission tax for each time interval

$t$	Demand (pu)	Energy (\$/MWh)	CO2 (\$/Ton)	$t$	Demand (pu)	Energy (\$/MWh)	CO2 (\$/Ton)
1	0.39	38.00	15.00	13	1.00	85.00	55.00
2	0.44	38.00	15.50	14	1.00	87.00	55.00
3	0.46	47.00	16.00	15	0.98	85.00	53.00
4	0.46	50.00	18.00	16	0.97	83.00	49.00
5	0.57	53.00	22.00	17	0.98	71.00	53.00
6	0.64	57.00	35.00	18	0.94	67.00	43.00
7	0.72	58.00	38.00	19	0.86	63.00	39.00
8	0.83	62.00	41.00	20	0.72	60.00	33.00
9	0.86	63.00	43.00	21	0.60	57.00	28.00
10	0.94	64.00	45.00	22	0.51	47.00	26.00
11	0.99	66.00	52.00	23	0.47	43.00	21.00
12	0.98	70.00	50.00	24	0.38	40.00	16.00

determined in (22) and (23). Additionally, the product of two continuous variables and the quadratic terms in (5) have been linearized, and as a result, the square of the current flow magnitude has been calculated by (24). Finally, expression (13) that defines the operation of a VR, has been recast by using the linear expressions given in (30)-(34). Therefore, by applying these linearization techniques for the original problem presented by (1)-(20), a tractable problem has been obtained that facilitates the utilization of classical optimization methods in achieving the optimal global solution with high computational efficiency.

### 3 Assumptions and Case Studies

In order to validate the applicability of the proposed planning framework, this section presents the technical and financial information, assumptions, and test cases applied to a 135-node medium distribution network. The proposed planning strategy was implemented in the modeling language AMPL, and the well-known solver CPLEX was used to carry out the solution on an Intel i7 7700 computer with 3.6GHz CPU and 16.0 GB RAM.

#### 3.1 Assumptions

The proposed planning method considers annual operating conditions via the time intervals representation. For instance, a year is constituted by 8760h where for each operating hour, the corresponding conditions for EDN demand, energy costs, and carbon emission taxes can be described. In practical terms, this representation could be an intractable problem to solve with high computational efficiency. In this regard, this work characterizes these operating conditions via 24-time intervals ( $t_s$ ). Therefore, for exploratory purposes, a short-term planning horizon of 1 year is considered. The demand levels, energy costs, and costs related to carbon emission tax are associated for each  $t$ . In order to properly analyze the model, the main assumptions of the proposed planning framework are as follows.

- For the planning horizon, the initial demand is increased by 5%.
- Reinforcement cost of overloaded circuit is limited to \$30k.
- Asset costs related to the CB and VR allocation are limited to \$20k and \$32k, respectively.
- An emission coefficient  $e^{CO_2}$  of 0.95 kg CO<sub>2</sub>/kWh is considered [33].
- Variability of the energy cost and carbon emission tax are considered for each  $t$ . These values are shown in Table 1 [33], [34].

#### 3.2 Financial and Technical Information

A 135-node medium voltage distribution network, presented in Fig. 1, is analyzed in detail to validate the proposed approach. This test system contains one substation, 134 nodes with 100 load nodes, and 133 circuits with different technical specifications for each conductor. The total conventional demand for the system is

**Table 2** Values of Parameters of the Exponential Load Model

NODE	$\alpha$	$\beta$
12, 16, 24, 36, 39, 46, 53, 58, 60, 61, 62, 66, 69, 72, 73, 77, 86, 94, 97, 99, 124, 125	0.72	2.96
3, 5, 6, 7, 8, 9, 13, 14, 19, 29, 41, 42, 43, 45, 50, 64, 65, 71, 79, 83, 84, 87, 88, 89, 96, 100, 104, 105, 112, 115, 116, 117, 118, 121	1.25	3.50
15, 17, 20, 21, 22, 25, 27, 30, 31, 32, 33, 34, 37, 44, 47, 49, 51, 54, 55, 56, 59, 68, 70, 74, 75, 76, 80, 81, 85, 91, 93, 101, 102, 106, 109, 110, 111, 122, 126, 127, 129, 131, 133, 134	0.18	6.00

**Table 3** Installation Costs of Types of CBs

CB Installation Cost		
$Q_b$ (kVar)	Fixed (\$)	Switchable (\$)
300	4,950.00	7,450.00
600	5,150.00	7,650.00
900	6,550.00	9,550.00
1200	7,500.00	10,150.00
1500	8,075.00	10,950.00

**Table 4** Technical Specifications and Upgrading Costs of Conductors

Conductors data			Upgrading costs ( $10^3$ \$/km)				
Type	$R + jX$ ( $\Omega$ /km)	$\bar{I}$	C2	C3	C4	C5	C6
C1	1.1440 + j0.8762	130	7.50	13.50	21.50	29.50	37.50
C2	0.9963 + j0.7133	175	-	11.00	18.50	25.50	34.00
C3	0.7618 + j0.7077	235	-	-	14.00	22.00	29.00
C4	0.5995 + j0.6610	365	-	-	-	17.50	25.00
C5	0.3692 + j0.4150	495	-	-	-	-	20.50
C6	0.3209 + j0.3554	615	-	-	-	-	-

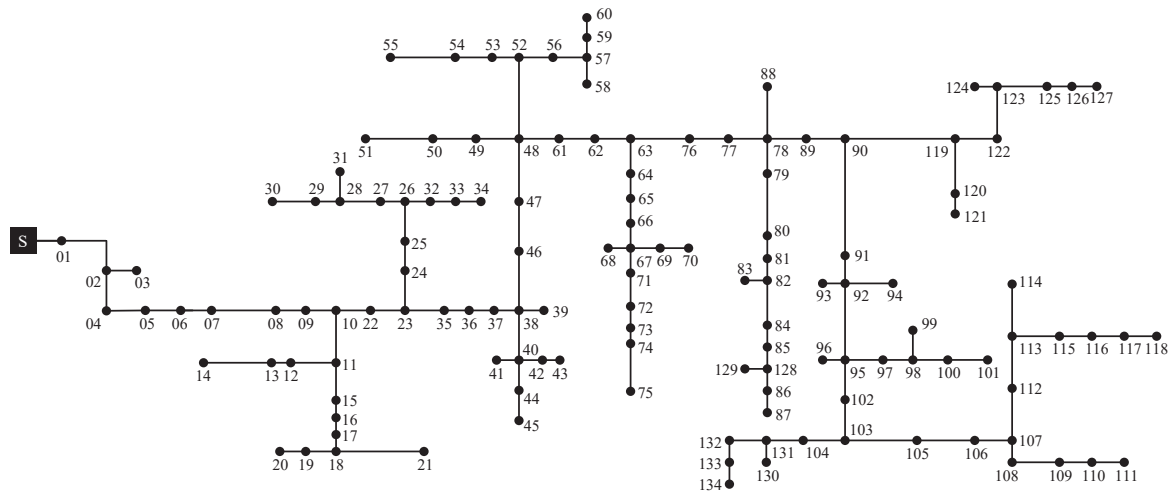
6.499 MW and 2.769 MVar. The system operates with the nominal voltage level of 13.8 kV where for energy quality requirements, the upper and lower bounds are set into 1.05 and 0.95 p.u., respectively. Additionally, the substation power factor can vary between 0.95 and 1.0 where the delivered reactive power depends on the active power purchased from the substation [4].

In this work, for didactic purposes, some modification has been done on the original EDN. The EDN demand, by using several values for the parameters of the exponential load model, was characterized in residential, commercial, and industrial consumers. This information is presented in the Table 2 [35].

On the other hand, in order to provide higher flexibility in the EDN operation, fixed and switchable CBs are considered as a viable option to provide reactive support where the optimal choice depends on the network requirements. The technical and financial information used in this work is summarized in Table 3. In this table, the reactive power capacity of each type of CB is associated with its respective installation cost. The installation cost of VR devices is set to \$15,800 with a regulation range of 10%. A reinforcement plan is proposed via conductors upgrading of the EDN circuits. For this action, the technical specification and financial information are presented in Table 4. From this table, six different types of conductors are considered where an upgrading cost is associated with the replacement of a specific conductor type by the new one. This technical and financial information was derived from [4].

#### 3.3 Case Studies

In order to validate and highlight the applicability of the proposed planning strategy, a base case before planning via a conventional optimal power flow is analyzed to establish the EDN conditions under the new demand requirements. To improve the EDN performance of this base case, three cases under two different conditions are carried out. For the first condition, nominal values for active and reactive power are considered. In other words, the network demand is modeled as constant power ( $\alpha = 0, \beta = 0$ ). In contrast, under the second condition, the exponential demand model



**Fig. 1:** Diagram of the 135-node distribution network

is considered by using the values shown in Table 2. These cases are explained in detail as follows.

- Case I: Optimal location, sizing, and operation of CBs and VRs. In this case, the conductor reinforcement option is not available.
- Case II: Sizing, placement, type, and operation of CBs and conductor replacement are considered. In this case, the VR allocation is disregarded.
- Case III: To solve the proposed asset upgrading planning model, this case takes into account all the planning actions such as conductor replacement, and optimal placement, sizing, and operation of CBs and VRs.

To differentiate the first and the second condition, the case studies are represented by extensions .p (Case I.p, Case II.p, and Case III.p) and .e (Case I.e, Case II.e, and Case III.e) for constant power and exponential load model cases, respectively.

## 4 Numerical Results And Discussion

This section summarizes the numerical results found by the proposed methodology for which several planning schemes are determined to improve the efficiency of the 135-node distribution network. Results are classified for each proposed condition and are duly discussed to highlight the novelty of the proposed approach.

### 4.1 Base Case

This case aims at evaluating the impact of the demand growth in the 135-node distribution network. To determine this impact, an optimal power flow tool is used. Results show a violated system in technical aspects such as voltage magnitude, current flow, and power factor limits. The minimum voltage profile is depicted in Fig. 2. As can be seen, for this instance the lower voltage bound has not been satisfied. Additionally, the minimum value for the system power factor is 0.918, and the cost of the supplied energy by the substation is \$2987.59k. Under this operating circumstance, the environmental condition of the EDN reveals an annual pollutant emission of 43.78 kTon with a total carbon tax of \$1763.84k. In order to improve this violated performance, three cases under different conditions are analyzed in the following subsections.

### 4.2 Constant power model

Traditionally, for practicality in planning studies, the demand in the EDN is modeled as constant power where the nominal values of active and reactive power demand are used. Therefore in this subsection, the proposed test cases are analyzed considering this

nominal values in which the numerical results are duly discussed and compared with the Base Case.

**4.2.1 Case I.p:** In this case, classical alternatives for voltage and reactive power control is considered to improve the performance of an initially violated 135-node network. The obtained solution under this instance reveals a total cost of \$4,748.16k. This solution suggests investment actions such as a) allocation of three fixed CBs with a cost of \$16.45k, b) allocation of a VR with a cost of \$15.8k. The cost of the energy supplied by the substation is \$2965.58k, while the annual pollutant emission is 43.47 kTon with a carbon tax of \$1750.33k.

Comparing with the Base Case, the solution obtained for case I.p demonstrates that the technical operating conditions have been fulfilled. Besides improving the technical requirements, reductions of 0.74% and 0.77% in energy cost and carbon tax are obtained, respectively.

**4.2.2 Case II.p:** Similar to Case I.p, two planning actions are considered to solve the asset upgrading planning problem. For this case, the conductor reinforcement option has been simultaneously optimized taking into account the placement and sizing of reactive power support devices. The total cost obtained for this case is \$4718.12k that contains the cost of the supplied energy by the substation, \$2936.96k, the costs related to the carbon taxes, \$1732.94k, and the investment costs, \$48.22k. The investment costs correspond to the actions such as a) allocation of a fixed and switchable CBs, \$8.075k and \$10.15k, respectively; and b) upgrading 6 circuits with new conductor types, \$30.00k.

Analogously to Case I.p, technical and operating conditions have been fulfilled with the proposed upgrading plan. Comparing the solution obtained from this case with the Base Case, reductions about 1.69% and 1.75% for energy costs and carbon tax have been obtained, respectively. Comparing these reductions with the solution of Case I.p, it is concluded that significant financial decreases can be obtained with the replacement of the overloaded circuits' conductor.

**4.2.3 Case III.p:** To validate the proposed planning framework as a multi-choice strategy, this case considers all the planning actions simultaneously. The solution of this case presents a total cost of \$4735.41k where \$2959.90k is for the energy cost, \$1746.89k stand for the costs related to the carbon tax, and the investment cost is \$28.62k. The obtained solution of this case proposes the following actions: a) allocation of a fixed CB with \$8.075k; b) allocation of a VRs with \$15.80; and c) 2 circuits have been selected for conductors upgrade with a cost of \$4.75k.

It worth noting that in the solution found in this case, the investment cost compared to Case I.p and Case II.p has been increased by about 11.26% and 40%, respectively. Besides improving the performance of the initially violated 135-node



distribution network, comparing this solution with the base case demonstrates financial reductions of 0.93% and 0.96% related to the energy cost and carbon tax, respectively.

### 4.3 Exponential demand model

In order to obtain a more accurate analysis and by considering that the loads in the distribution networks are dependent on the voltage, the proposed approach is validated considering the developed exponential demand model. Thereby, under this condition the test cases I–III are studied in detail to highlight the applicability of the proposed model.

**4.3.1 Case I.e:** Similar to Case I.p, the same planning actions are considered in this case. The obtained solution of this case reveals a total cost of \$4654.19k. This cost contains \$2906.88k related to the cost of the energy supplied by the substation, \$1715.61k pertaining with the carbon tax corresponding to 42.61 kTon of emission of the EDN, and \$31.70k is the investment cost. Therefore, to maximize the efficiency of the network, the following actions have been proposed: a) allocation of a fixed and a switchable CB with \$4.95k and \$10.95k, respectively; and b) allocation of a VR with an investment cost of \$15.80k.

Comparing this case with the base case, it can be highlighted that the technical and operating conditions have been fulfilled with low operating cost. In this regard, reductions of 2.71% and 2.73% in energy cost and carbon tax have been obtained, respectively. On the other hand, compared to Case I.p, with about 1.98% decrease (about \$95k), this case presents a more economical plan. This significant difference, presented in the Table 6, can be seen in costs related to the energy (around of \$60k lower), carbon tax (around \$35k lower), and investment cost (around of \$0.60k lower).

**4.3.2 Case II.e:** In the same way, the solution of this case shows a total cost of \$4632.35k, where \$2887.14k is the cost of the energy supplied by the substation, the cost of the carbon tax is \$1703.06k correspond to 42.36 kTon of emission, and \$42.36k of the investment costs. This asset upgrading plan presents actions such as a) allocation of two switchable CBs with a cost of \$18.60k, and b) 7 overloaded circuits have been identified to replace their initial conductors with new ones, this upgrading action shows a cost of \$23.55k.

This case, compared with the base case, shows significant reductions in both the energy cost and cost related to the carbon tax, respectively about 3.36% and 3.44%. Another interesting aspect to be highlighted is that the total cost obtained for this case is 1.82% lower than the total costs of Case II.p (see Table 6). This difference is related to the energy cost (around \$50k lower), carbon tax (around of \$30k lower), and investment cost (around of \$6.07k lower).

**4.3.3 Case III.e:** Finally, in this case, all the planning actions are considered simultaneously with the exponential demand model. This case aims to investigate the proposed planning scheme via a multi-choice strategy and its impact on the system plan by studying the voltage dependence demands in the EDN. The obtained solution for this case demonstrates that the cost of energy supplied by the substation is \$2897.58k. This cost is related to the carbon tax, \$1709.71k, and the investment \$31.75k. The investment cost stands for the actions such as a) allocating a fixed CB with \$8.075k cost.; b) allocating a VR with \$15.80k cost, and c) replacement of conductor of 3 circuits with \$7.88k cost.

From this result, it can be highlighted that compared to the base case, reductions of 3.01% and 3.07% have been obtained for energy cost and carbon tax, respectively. Additionally, the objective function, energy cost and carbon tax are 2.04%, 2.11% and 2.13% lower than values obtained from Case III.p (see Table 6). Meanwhile, to obtain these reductions, the solution of this case determines an investment cost of 9.82% higher than the value obtained from case III.p. This value is defined by investment cost related with new conductor types to reinforcement the EDN circuits, this information is given in Tables 5 and 7.

In summary, the developed methodology has found several feasible plans under different conditions to maximize the efficiency

**Table 5** Summary Cost ( $10^3$ \$) Of The Test Cases

Case	O.F.	Energy Cost	Carbon tax	Investment Cost		
				CB	VR	Conductors
Base	4751.43	2987.59	1763.84	–	–	–
I.p	4748.16	2965.58	1750.33	16.45	15.80	–
II.p	4718.12	2936.96	1732.94	18.22	–	30.00
III.p	4735.41	2959.90	1746.89	8.075	15.80	4.75
I.e	4654.19	2906.88	1715.61	15.90	15.80	–
II.e	4632.35	2887.14	1703.06	18.60	–	23.55
III.e	4639.04	2897.58	1709.71	8.075	15.80	7.88

**Table 6** Comparing solutions for each test case by using constant power and exponential demand model

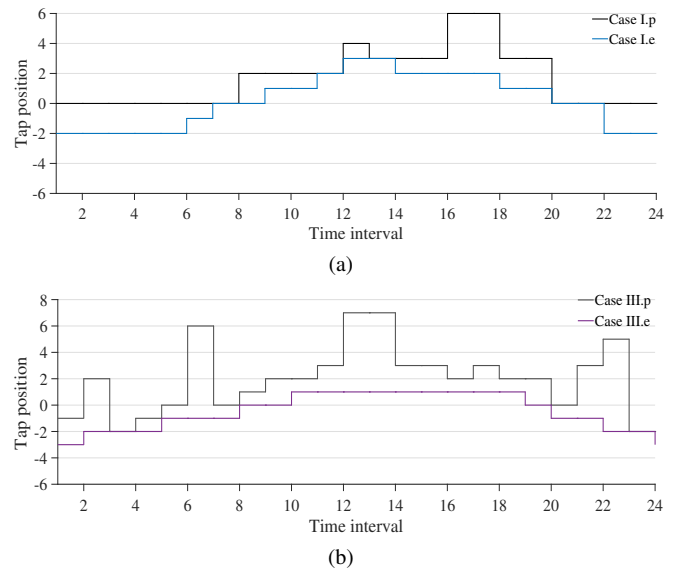
Case	O.F. ( $10^3$ \$)	Energy Cost ( $10^3$ \$)	Carbon Tax ( $10^3$ \$)	Investment Cost ( $10^3$ \$)	Solution Time (s)
I.p	4748.16	2965.58	1750.33	32.25	10.67
I.e	4654.19	2906.88	1715.61	31.70	61.98
% Diff.	1.98%	1.98%	1.98%	1.70%	82.78%
II.p	4718.12	2936.96	1732.94	48.22	178.94
II.e	4632.35	2887.14	1703.06	42.15	224.16
% Diff.	1.82%	1.69%	1.72%	12.59%	20.17%
III.p	4735.41	2959.90	1746.89	28.63	339.94
III.e	4639.04	2897.58	1709.71	31.75	929.56
% Diff.	2.04%	2.11%	2.13%	9.82%	63.43%

of the 135-node distribution network. In this regard, the financial results for each case are presented in Table 5. This table provides, for each case, the values of the objective function (O.F.), energy cost, cost of the carbon tax, and investment cost for each planning alternative. Note that, compared to the base case, substantial reductions in costs of energy and carbon taxes have been obtained. On the other hand, it is observed that the energy cost and carbon taxes are decreased for each case. However, comparing Case II and III, costs of energy and carbon taxes are increased by 0.77% and 0.79% for constant power condition, while under exponential load model condition the increments are about 0.36% and 0.38%. Notwithstanding, for each condition, the investment cost of Case III is around 40% and 24% lower than Case II. Such reductions are obtained due to VR allocation, for which the investment costs in CB allocation and conductor replacement are decreased (see Table 5). Additionally, Table 6 summarizes the percentage of difference (% Diff.) of O.F., energy cost, carbon tax, investment cost, and the solution time for each case. As can be seen, by considering the exponential demand representation, economic benefits of 1.98%, 1.82%, and 2.04% for the O.F. of Cases I, II, and III were achieved, respectively. Meanwhile, the solution times for Cases I, II, and III have been increased in 82.78%, 20.17%, and 63.43%, respectively.

The presented planning strategy found a necessary action set to improve technical, operational, and environmental conditions of the 135-node distribution network. These actions are a) fixed and switchable CB to be allocated at node  $m$  with nominal reactive power ( $Q$ ); b) VR device to be allocated at node  $n$  for controlling the voltage magnitude; and finally, c) upgrading the initial conductor of the overloaded circuit  $mn$  ( $C_x$ ) by the new selected conductor ( $C_X$ ). These investment actions corresponding to each case are summarized in Table 7. These reinforcement plans show a substantial improvement in the voltage magnitude profile of the 135-node distribution network as can be seen in Fig. 2. In the companion subfigures (Fig. 2.a) and (Fig. 2.b), the base case (red profile) is compared to the cases with the constant and exponential load model representation. It is worth noting that, by considering the traditionally demand representation (constant power), the technical performance of the EDN is guaranteed with an economical and environmentally committed operation. Nevertheless, a voltage-dependent load model, as an accurate representation of the EDN demand, demonstrates how overall energy cost and cost of the carbon tax can be significantly reduced compared to the traditionally demand representation. From this representation, the voltage magnitude profile in Fig. 2.b tends to the lower bound.

**Table 7** Proposed Investment Plan in CB and VR Allocation, and Conductor Reinforcement For Every Case

Case	Proposed Upgrading plan				
	Fixed CB $m(Q)$	Switc. CB $m(Q)$	VR ( $n$ )	Circuit/Conductor	
				$mn$ (Initial)	New
I.p	90, 95 (300kVAr) 107 (900kVAr)	–	23	–	–
II.p	90 (1500kVAr)	107 (900kVAr)	–	01-02 (C4) 04-05 (C4) 06-07 (C4) 07-08 (C4) 09-10 (C4) 22-23 (C4)	C6 C5 C6 C6 C6 C6
III.p	90 (1500kVAr)	–	23	01-02 (C4) 08-09 (C4)	C5 C6
I.e	107 (300kVAr)	90 (1500kVAr)	23	–	–
II.e	–	103 (1500kVAr)  107 (600kVAr)	–	02-04 (C4) 05-06 (C4) 06-07 (C4) 07-08 (C4) 08-09 (C4) 10-22 (C4) 35-36 (C4)	C5 C6 C5 C6 C5 C6 C5
III.e	107 (1500kVAr)	–	23	04-05 (C4) 06-07 (C4) 08-09 (C4)	C5 C5 C5



**Fig. 3:** Scheduling tap position for cases a) I.p and I.e, b) III.p and III.e

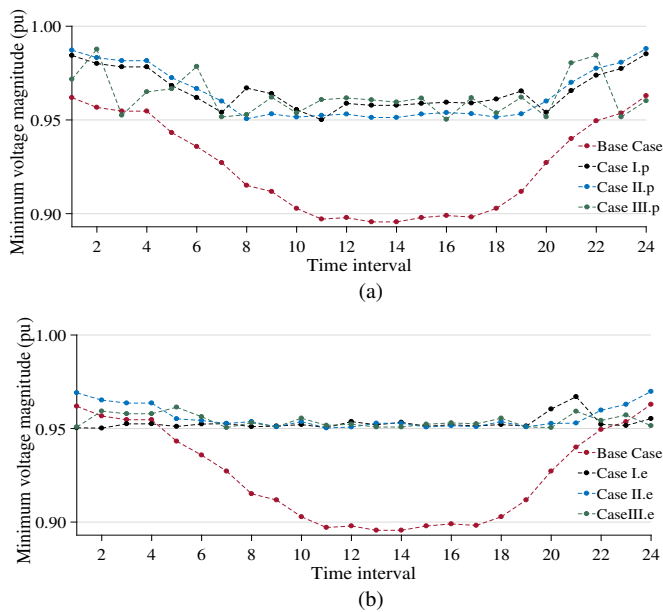
device presents a higher voltage level reduction to achieve a better economically and environmentally committed plan.

Analogously to the VR device operation, the proposed methodology defines an optimal reactive power dispatch for the installed CBs. In the solution obtained for Cases I.p, III.p, and III.e, fixed CBs have been installed to provide continuous reactive power support of 1500 kVAr. In Case II.p, two types of CBs have been installed, the optimal scheduling of these devices is shown in Fig. 4.a. It can be seen that a fixed CB installed at node 90 (CB-90 in the figure) has been simultaneously coordinated with the switchable CB installed at node 107 (CB-107 in the figure). In a similar way, the optimal operation for the installed CBs in Cases I.e and II.e has been determined by the proposed methodology. These scheduling is shown in Figures 4.b and 4.c, respectively. From these results, it can be concluded that by using a voltage-dependent demand model, the decision-making process related to planning and operating schemes are substantially altered to improve the technical, economic, and environmental aspects.

#### 4.5 Scalability of the presented approach

This subsection validates the scalability of the proposed planning strategy, by using two distribution test networks of 42- and 417-nodes. For detail information, technical data of these systems can be found in [36]. The 42-node distribution system contains one substation with the nominal voltage of 13.8 kV, 23 load nodes with conventional demand of 11.873 MW and 3.898 MVar, and 41 circuits with different types of conductors. On the other hand, the 417-node distribution network contains 3 substations, 327 demand nodes, and 414 circuits. The conventional demand for this system is 27.37 MW and 13.24 MVar, and the nominal voltage is 10 kV. For practical purposes, the same energy quality requirements defined for the 135-node EDN has been adopted for these systems. Additionally, the same values of energy cost, demand profile, carbon emission tax, and emission coefficient have also been considered. Finally, to validate and highlight the scalability of the presented planning framework, Case III in which all the decision variables are taken into account are considered under constant power and exponential demand model conditions.

**4.5.1 42-node distribution network:** Under constant power demand condition, the optimal short-term plan is achieved with O.F. of \$8554.70k that is divided into a) the cost of the energy supplied by the substation, \$5347.03k; b) the cost of carbon taxes corresponding to 78.44 kTon of emission, \$3154.62k; and c) the investment

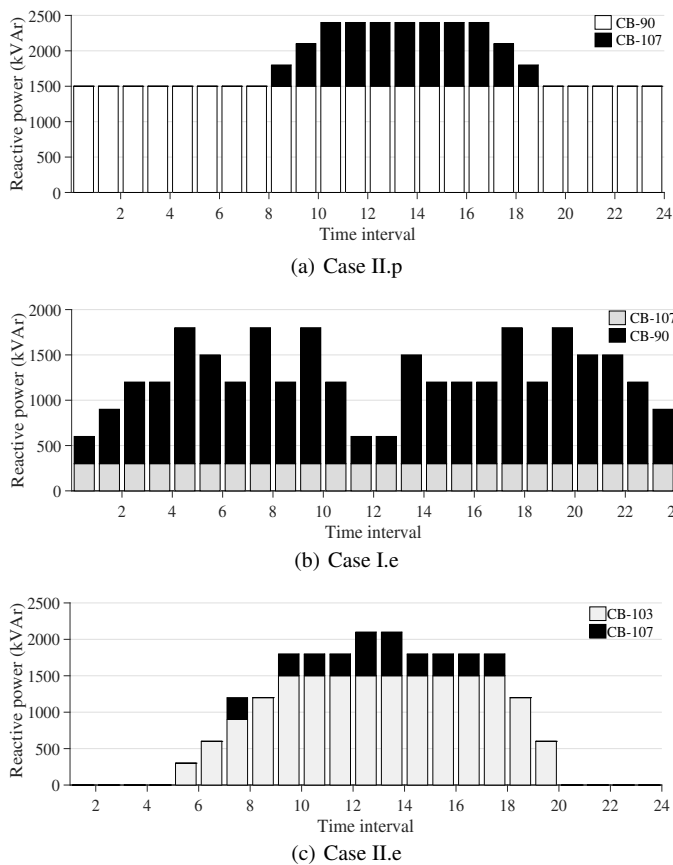


**Fig. 2:** Minimum voltage magnitude profile for demand representation using a) constant power, and b) exponential load model

This is mainly because of obtaining the most economical plan by reducing energy costs and carbon taxes.

#### 4.4 Operating control scheme

Besides finding the optimal location, size, and type of devices to be installed in the network, the presented model defines the optimal operating schedule for these devices. In other words, the tap position of the VR device and the number of modules in operation of the CB for each time interval are duly optimized, this fact can be seen in the Figures 3 and 4. Fig 3 and 4 represent the optimal tap position of the installed VR and the reactive power delivered by the installed CBs for each case, respectively. In both figures 3.a and 3.b, when the exponential load model is considered, the tap positions of the VR



**Fig. 4:** Scheduling reactive power delivered by the installed CBs for a) Case II.p, b) Case I.e and c) Case II.e

costs, \$53.05k, corresponds to the alternatives such as allocation of two fixed CBs (\$24.22k) and conductor replacement of 3 circuits (\$28.83k). The results under the second condition define a total cost of \$8319.07k which contains the cost of supplied energy, \$5207.71k, the carbon tax, \$3070.39k due to 76.47 kTon of emission, and the investment cost, \$40.97k. To attend the requirement of this condition, this investment cost is defined by alternatives such as allocation of a fixed and a switchable CB with a cost of \$15.30k and reconductoring of 2 EDN circuits with a cost of \$25.67k. It is worth noting that, economic benefits can be achieved comparing the solution of Case III.p with Case III.e. This fact is summarized in Table 8, where the percentage of difference in O.F., energy costs, carbon taxes, and investment costs are presented. In addition, from this table, it can be seen that the solution time for Case III.e is higher than Case III.p.

**4.5.2 417-node distribution network:** For this system, the constant power condition reveals an optimal short-term plan where the cost of the energy supplied by the substation is \$12181.49k, while the carbon tax is \$7184.16k correspond to 178.178.79 kTon of emission, and finally an investment cost of \$42.02k. To reinforce the 417-node EDN, the planning strategy defines alternatives such as allocation of 4 fixed CBs with a cost of \$29.37k and conductor replacement of 7 circuits with a cost \$12.65k. Additionally, the second condition proposes a more economical investment plan with an energy cost of \$12028.09k, a carbon tax of \$7009.70k corresponding to 176.65 kTon, and an investment cost of \$26.38k. Unlike the previous condition, this solution determines the allocation of 3 fixed CBs and conductor replacement of 3 circuits. In summary, economic aspects and solution time for Case III.p and .e are compared in the Table 8 where the energy cost, carbon tax surplus, and investment cost shows reductions about \$150.00k, \$9.00k and \$1.60k, respectively.

**Table 8** Comparing solutions of 42- and 417-node test systems for Case III.p and Case III.e

EDN	Case	O.F. (10 <sup>6</sup> \$)	Energy Cost (10 <sup>6</sup> \$)	Carbon Tax (10 <sup>6</sup> \$)	Investment Cost (10 <sup>6</sup> \$)	Solution Time (s)
42- node	III.p	8.55	5.35	3.15	0.053	73.00
	III.e	8.32	5.20	3.07	0.041	135.05
	% Diff.	2.79%	2.80%	2.54%	22.64%	45.95%
417- node	III.p	19.40	12.18	7.18	0.042	2041.97
	III.e	19.14	12.03	7.09	0.026	5625.33
	% Diff.	1.34%	1.26%	0.80%	38.09%	63.70%

## 5 Conclusions

In this manuscript, a practical model for the short-term asset upgrading planning of electric distribution networks has been proposed. A set of actions have been identified to ensure the quality of the supplied energy guaranteeing the optimal performance of the network. Besides improving the system technical aspects with the most economical plan, an environmentally committed system has been obtained by applying carbon tax policy.

Unlike the traditional asset upgrading approaches, in this work, the exponential load model representation has been duly explored. Numerical results prove that, compared to the traditionally constant demand model, this representation reduces substantially the planning costs, especially costs of energy supplied by the substation and costs related to the carbon taxes. Therefore, more economical and environmentally committed plans can be obtained by using an appropriate demand representation.

In order to enrich the applicability of the proposed planning approach, demand management schemes have been investigated as the prospects of further works. The demand management in addition to considering the carbon tax surplus enables the dynamic participation of the consumers, and consequently, results in carbon footprint mitigation. Additionally, the presented planning scheme could be adapted to represent long-term expansion planning problem by considering investment actions with a long-term vision.

## 6 Acknowledgments

The authors would like to thank the Brazilian institutions FAPESP (Grant No: 2018/12422 and 2015/21972-6), CAPES (code finance 001) and CNPq (Grant No 305318/2016-0) for the financial support.

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