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Analytical Treatment of Distortion Effects on Fatigue Behaviors of Lightweight Shipboard Structures

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Highlights

- A novel notional-load approach is developed to model distortion effects on fatigue
- Nonlinear geometry effects are incorporated analytically in SCF formulation
- Proposed a distortion decomposition method to analyze distortions in structures
- Both lab- and full-scale test data show good correlation with the proposed method

Abstract

In this paper, a notional load method is presented for providing analytical treatment of complex distortion effects on fatigue behaviors of lightweight shipboard structures through a distortion decomposition technique. Its applications for analyzing secondary bending stresses caused by nonlinear interactions between four common distortion types induced by welding and remotely applied load are discussed in detail. In addition, two sets of lab-scale specimens and nine full-scale stiffened panel fatigue tests involving complex distortion shapes are also analyzed using the closed form analytical solutions developed. The analytically calculated stress concentration factor results are validated by direct finite element computations in all cases. Furthermore, fatigue test data obtained from both butt-welded thin plate lab specimens and full-scale stiffened panels are shown not only in a good agreement with one another, but also fully contained by the master S-N curve scatter band adopted by ASME Div. 2 since 2007.

Keywords

Secondary bending; Lightweight structures; Welding distortions; Stress concentration factors; Master S-N curve

1 Introduction

In recent years, the demand for structural lightweighting for surface ships has become more intensified as the industry strives to carry more payload at a higher speed with further improved fuel economy and reduced environmental emissions [1]. As reported by Huang et al. [2,3], plates with thicknesses equal to or less than 5mm have become increasingly dominant in lightweight shipboard structures in surface combatants, which have caused significant challenges in accuracy control in construction processes. Welding-induced distortions have become a major issue in the construction of lightweight ship structures [4,5,6]. Furthermore, the existing distortion tolerances in current code and standards stipulated by Class societies and regulators are mostly carried over from legacy requirements which were based on data and experiences associated thick-section structures (e.g. [7]) or decades old (e.g., MIL-STD-1698 [8]).

There have been plenty of research efforts carried out over the recent years on effects of the welding-induced residual stresses on structural integrity in the literature, e.g., by Dong and Brust [9], Dong et al. [10], and their co-workers [11-13], and others [14-16] in the context of pressure vessels and piping components. In contrast, discussions on the effects of welding-induced distortions on the structural integrity of lightweight shipboard structures have been rather limited. For instance, Antoniou [17] and Carlsen and Czujko [18] studied experimental observations on some specific types of distortions observed in the ship construction environment. These studies mainly focused on structural buckling strength under compressive loading applied on plates with thicknesses greater than 10mm and did not address how distortions influence fatigue lives of welded structures. In current fatigue assessment and fitness-for-service (FFS) procedures, there is essentially no procedure for assessing complex distortion effects on structural integrity except for some limited provisions given in BS 7910 [19] for treating both simple axial and angular misalignments in butt-welded joints. Distortion curvature effects on fatigue, which may generate significant secondary bending stresses in thin-section structures, have not been considered in any recognized Codes and Standards or recommended practices.

To address such a need, recent research efforts include the work by Lillemäe et al. [20,21] on the complex distortion effects on fatigue strength of butt-welded thin plate joints, Xing et al [22, 23, 24] on the effects of joint misalignments on fatigue failure mode transition behaviors in thin plate cruciform joints by performing both experimental and finite element studies. The common findings in these studies showed that fatigue behaviors in thin plate structures tend to show a great deal of scattering, much more so than thick plate joints, regardless of stress definitions used for data interpretation, e.g., hot spot stress, local notch stress, and structural stress methods, etc. All these findings highlight the fact that distortion modes involved must be properly considered as a part of the stress concentration computation.

The stress concentration caused by initial distortions or geometric imperfections observed in thin plate shipboard structures under fatigue loading was recently studied by several researchers.

Eggert et al. [25] and Lillemäe et al. [26] performed finite element (FE) analyses using FE models incorporating detailed distortion measurements of test specimens and found that both the shape and magnitude of distortions can have significant effects on stress concentration factors calculated. However, a generalized stress concentration analysis method for incorporating various distortion modes or types remains to be found. It is worth noting that Chan et al. [27, 28] and Gu et al. [29] presented a series of analytical solutions based on beam theory with presumed simple geometric imperfections described in the form of a half sine wave or a parabolic function. Even though these solutions cannot be directly applied for distortion problems of interest here, the analytical approach should be of interest for the present applications. Further along this line of analytical approach, Liew [30] introduced a notional load approach in which equivalent loads were applied against a nominally perfect geometry of a beam or a frame member to re-produce initial imperfections of interest for studying nonlinear deformation problems of beams. In a rather similar manner, Dong et al. [31, 32], recently presented an analytical treatment of distortion effects on secondary stress concentration development in stiffened panel structures in which notional loads were used to model nonlinear interactions between a lateral load and out-of-plane distortions. They used their analytically calculated stress concentration factors (SCF) and achieved a very good correlation of some available fatigue test data on thin plate butt-welded specimens exhibiting severe distortions [20,21].

In this paper, we present a more general analytical method for computing SCF caused by various common forms of welding-induced distortions and their interactions with a lateral load (perpendicular to weld direction). Starting with some of the typical distortion shape presented in [31,32], we introduce a classic Euler-Bernoulli beam model with notional loads that are used to re-produce various distortion modes. Then, an imperfect beam based on Timoshenko beam-column theory is introduced and solved by taking advantage of the notional loads for modeling nonlinear interactions between a distortion mode and lateral load applied. A series of analytical SCF solutions are then presented for studying fatigue behaviors observed in thin plate fatigue test specimens both at lab-scale and full-scale levels. Finally, for the treatment of complex distortions such as those observed in full-scale shipboard components, a general distortion data interpretation procedure is also presented for taking advantage of the analytical SCF solutions developed. It is found that the analytical SCF solutions developed in this study are effective for interpreting fatigue test data available for both thin plate lab-scale and full-scale components.

2 Analytical Treatment

2.1 Assumptions

The analytical developments presented in this section are based on the following assumptions which will be further validated by FE analysis at the end of this section:

a) A transverse section of a butt-seam welded or stiffened shipboard panel follows a beam theory (consistent with strip beam theory often used for analyzing ship structures)

b) Beam material is assumed to follow elastic material behaviors within the loading range of concern, and the beam distortion and lateral load interactions consider geometric nonlinearity.

c) The magnitude of pre-existing beam distortions or imperfections is small compared to beam length.

d) The initial beam distortions can be well fitted by cubic Hermite splines.

e) Transverse shear deformation is negligible.

2.2 Method of Notional Loads

Consider the interactions between a pre-existing distortion mode $v_0(x)$ of a beam and its axial load P, as depicted in Fig. 1a, where P is considered positive when it generates tensile stress in the beam, and $v_1(x)$ represents the unknown deflection of the beam caused by P. Note that the square symbols (see Fig. 1) in this paper represents the boundary condition in which rotations are fixed but the translations in all directions are free. The classic governing differential equation of such a beam with imperfections described by $v_0(x)$, incorporating geometric nonlinearity, is given as [33],

$$EI\frac{d^4v_1}{dx^4} - P\frac{d^2\left(v_0+v_1\right)}{dx^2} = 0 \tag{1}$$

in which E is material Young's Modulus and I represents the moment of inertia. And the secondary bending moment M_1 can be expressed as:

$$M_1 = EI\frac{d^2v_1}{dx^2} \tag{2}$$

Eqn. (1) can be solved directly with a prescribed simple initial distortion shape function $v_0(x)$, e.g., a simple one-half sine wave [28, 29] or a parabolic shape [27]. For more complex distortion shapes, however, Eqn. (1) often becomes difficult to solve in closed form. This difficulty can be overcome if the concept of notional loads [30] is introduced. As such, based on assumption d) in Section 2.1, the initial distortion $v_0(x)$ can be considered as being caused by a set of notional loads acting on a linear Euler-Bernoulli beam, (e.g., Fig. 1b). The resulting $v_0(x)$ shall satisfy:

$$EI\frac{d^4v_0}{dx^4} = 0 \tag{3}$$

Note that displacement boundary conditions on the linear beam can be imposed in a manner that best represents a pre-existing distortion shape of interest and are independent of the ones prescribed for the imperfect beam (Fig. 1a). To mathematically enable superposition between the imperfect beam problem and the linear beam problem, we can add or release boundary restraints of the linear beam by replacing them with statically equivalent notional loads. As such, the constrained displacements or rotations are consistent between the linear beam and the imperfect beam, as illustrated in Fig. 1c.

Then, through a superposition of the two problems described in Fig. 1a governed by Eqn. (1) and Fig. 1c governed Eqn. (3), the resulting governing equation becomes:

$$EI\frac{d^4\left(v_0+v_1\right)}{dx^4}-P\frac{d^2\left(v_0+v_1\right)}{dx^2}=0 \tag{4}$$

By denoting $v = v_0 + v_1$, Eqn. (4) becomes

$$EI\frac{d^4 v}{dx^4}-P\frac{d^2 v}{dx^2}=0 \tag{5}$$

which is the governing equation of a geometrically nonlinear beam with perfect nominal geometry (Fig. 1d) subjected to the notional loads on the nominally perfect beam (Fig. 1c) and the axial force P applied to the imperfect beam (Fig. 1a).

Hence, the imperfect beam problem in Fig. 1a can be solved by first determining the loading pattern of notional loads on a linear beam (Fig. 1b and 1c) and their values, then solving the nonlinear perfect beam problem in Fig. 1d, which allows the determination of $v_1 = v - v_0$ sought. With such a procedure, we can either solve the problem by obtaining the solution to the homogeneous equation Eqn. (5) or use existing solutions to corresponding nonlinear perfect beam problems, avoiding solving the nonhomogeneous equation Eqn. (1) for every possible distortion shape of concern.

3 Analytical Solutions to Common Distortion Types

3.1 Distortions in Stiffened Panels

According to the detailed distortion investigations by Dong [12,13] and Yang and Dong [6], two typical distortion modes are dominant in lightweight shipboard structures, as illustrated in Fig. 2. One is referred to as buckling type, resulted from structural instability behaviors triggered by compressive residual stresses. Fig. 2a shows a LIDAR (a short form of LIght Detection and Ranging) image of a 16'×20' (4.877m×6.096m) stiffened panel, which clearly exhibits well-defined checker-board pattern shown at the lower half of the image. The other type is referred to as angular distortion, which is depicted in a sketch for clarity as shown in Fig. 2b. If a transverse cross-section, say along Section A-A, is considered, the two types of distortions can be depicted in Figs. 2c and 2d, respectively. Within one stiffener spacing *l*, it can be seen that buckling distortion is defined as one-half sinusoidal waveform, while the angular distortion has one cosine waveform with no rotation at stiffener locations. The amplitude or peak distortion values for both cases are given as δ_0. In this study, strip beam theory is assumed to be applicable for simplicity, and the beams mentioned in this paper are all in unit width.

3.1.1 Angular Distortions

According to Fig. 1d, an imperfect beam model representing a typical angular distortion within one stiffening spacing of l is depicted in Fig. 3a, in which two beam ends are restrained under embedded conditions. Such an initial distortion shape, as discussed in Section 2.2, can be represented by a linear beam subjected to a concentrated notional force F_0 at beam mid-span, as shown in Fig.

3b. The magnitude of F_0 can be obtained through classic beam theory by setting beam mid-span deflection δ_0. The distortion field $v_0(x)$ can then be obtained from the linear beam theory.

Using the procedure presented in Section 2.2, the secondary bending stress induced by the angular distortion as a result of lateral load P can be expressed as the bending stress concentration factor k_b (see Appendix A for the detailed solution process) at the stiffener location (i.e., x=0, y=t/2):

$$k_\mathrm{b} = \begin{cases} -\dfrac{144}{(\lambda l)^2}\dfrac{\delta_0}{t}\left(\dfrac{4}{\lambda l}\dfrac{\cosh\dfrac{\lambda l}{2}-1}{\sinh\dfrac{\lambda l}{2}}-1\right) & P>0, \lambda=\sqrt{\dfrac{P}{EI}} \\ -\dfrac{144}{(\lambda l)^2}\dfrac{\delta_0}{t}\left(\dfrac{4}{\lambda l}\dfrac{\cos\dfrac{\lambda l}{2}-1}{\sin\dfrac{\lambda l}{2}}+1\right) & P<0, \lambda=\sqrt{\dfrac{-P}{EI}} \end{cases} \tag{6}$$

Note that, unless otherwise stated, all k_b solutions refer to the top surface (i.e., y=t/2) in the rest of the paper and that the second equations given in all k_b expressions are valid before the compressive axial loading magnitude reaches the model's Euler's critical load beyond which buckling occurs.

3.1.2 Buckling Distortions

Similarly, the buckling distortion shape illustrated in Fig. 2c can be represented as the deflection of a beam with two pinned ends and a concentrated notional force in the middle, as illustrated in Fig. 4b. The corresponding imperfect beam problem is illustrated in Fig. 4a, and the corresponding nonlinear perfect beam model is given in Fig. 4c. Then, k_{b} due to secondary bending resulted from the buckling distortion mode with respect to the stiffener (x=0) can be then obtained as

$$k_{\mathrm{b}} = \begin{cases} 18\left[\dfrac{\delta_0}{t}\dfrac{\cosh\dfrac{\lambda l}{2}}{\lambda l\sinh\dfrac{\lambda l}{2}} - \dfrac{8}{(\lambda l)^2}\dfrac{\delta_0}{t}\left(\dfrac{\cosh\dfrac{\lambda l}{2}-1}{\lambda l\sinh\dfrac{\lambda l}{2}}\right)\right] & P>0,\ \lambda=\sqrt{\dfrac{P}{EI}} \\ -18\left[\dfrac{\delta_0}{t}\dfrac{\cos\dfrac{\lambda l}{2}}{\lambda l\sin\dfrac{\lambda l}{2}} + \dfrac{8}{(\lambda l)^2}\dfrac{\delta_0}{t}\left(\dfrac{\cos\dfrac{\lambda l}{2}-1}{\lambda l\sin\dfrac{\lambda l}{2}}\right)\right] & P<0,\ \lambda=\sqrt{\dfrac{-P}{EI}} \end{cases} \tag{7}$$

3.2 Distortions in Butt-Welded Plates

Lillemäe et al. [20] reported some interesting fatigue tests on lab-scale butt-welded specimens with distortions characterized as shown in Fig. 5. Detailed axial misalignments e, angular distortions measurements in terms of $\alpha_{\mathrm{L},1}$ and α_{G}, as defined in Fig. 5, are also given in [20]. As a part of this study, $\alpha_{\mathrm{L},2}$ is also measured and used for test data analysis.

To demonstrate how the analytical procedure described in Section 2.2 can be used for characterizing the distortion types shown in Fig. 5, axial misalignment e is not discussed in this study since the solutions under various conditions can be found from the recent work by Xing and Dong [24]. As far as the angular distortions shown in Fig. 5 are concerned, they can be assumed to be symmetric about the weld centerline and thus only one half of the specimen needs to be considered, as depicted in Fig. 6a. Furthermore, the distortions involved in Fig. 5 can be decomposed into two simple distortion modes: global angular distortion (Fig. 6b), which is typically referred to as angular misalignment, e.g., in BS 7910 [19], and local angular distortion (Fig. 6c).

3.2.1 Global Angular distortion

The global angular distortion shown in Fig. 6b does not involve any curvature as pre-existing distortion. Therefore, no notional load needs to be considered when examining its interaction with a beam axial load P, according to the method described in Section 2.2. The equivalent nonlinear beam model corresponding to the clamped-end condition is given in Fig. 7 with the global angular distortion defined as θ_{G}. Note that the sign conventions for the rotations throughout the rest of this paper follows the right-hand rule, which is also given in Fig. 7. It then can be shown that stress concentration factor k_{b} with respect to the weld location (x=0) can be solved as:

$$k_{\mathrm{b}} = \begin{cases} 6\theta_{\mathrm{G}} \dfrac{l}{t}\left(\dfrac{\cosh \lambda l - 1}{\lambda l \sinh \lambda l}\right) & P > 0,\ \lambda = \sqrt{\dfrac{P}{EI}} \\ -6\theta_{\mathrm{G}} \dfrac{l}{t}\left(\dfrac{\cos \lambda l - 1}{\lambda l \sin \lambda l}\right) & P < 0,\ \lambda = \sqrt{\dfrac{-P}{EI}} \end{cases} \quad (2)$$

It is worth pointing out that Eqn. (2) is exactly the same as the one given in BS 7910 [19] for computing secondary stress caused by angular misalignment with clamped-end conditions. However, the source of this solution is not given in BS 7910 [19]. This confirms the validity of our approach as described in Section 2.2.

3.2.2 Local Angular Distortion

The treatment of local angular distortion depicted in Fig. 6b is shown in Fig. 8, assuming that the distortion curvature is simple and can be fully described by the rotations at both ends θ'_1 and θ'_2. Then, this type of local angular distortion can be modeled by a tilted cantilever beam loaded with a notional force and a notional moment at the free end, as shown in Fig. 8b. The relationships between the initial rotations θ'_1, θ'_2 and the notional loads can be determined by classic beam theory as

$$\begin{aligned} F_0 &= -\frac{6EI}{l^2}\left(\theta_1' + \theta_2'\right) \\ m_0 &= \frac{2EI}{l}\left(\theta_1' + 2\theta_2'\right) \end{aligned} \tag{9}$$

Following the procedure described in Section 2.2, one can show that, for the nonlinear perfect beam shown in Fig. 8c, k_b at weld location (x=0) can be expressed as

$$k_\mathrm{b} = \begin{cases} 6\dfrac{l}{t}\left\{ \begin{aligned} &\theta'_1\left[\frac{6}{(\lambda l)^2}\frac{\cosh\lambda l - 1}{\lambda l \sinh\lambda l} + \frac{\cosh\lambda l}{\lambda l\sinh\lambda l} - \frac{4}{(\lambda l)^2}\right] \\ &+\theta'_2\left[\frac{6}{(\lambda l)^2}\frac{\cosh\lambda l - 1}{\lambda l\sinh\lambda l} - \frac{1}{\lambda l\sinh\lambda l} - \frac{2}{(\lambda l)^2}\right] \end{aligned} \right\} & P > 0, \lambda = \sqrt{\dfrac{P}{EI}} \\ 6\dfrac{l}{t}\left\{ \begin{aligned} &\theta'_1\left[\frac{6}{(\lambda l)^2}\frac{\cos\lambda l - 1}{\lambda l\sin\lambda l} - \frac{\cos\lambda l}{\lambda l\sin\lambda l} + \frac{4}{(\lambda l)^2}\right] \\ &+\theta'_2\left[\frac{6}{(\lambda l)^2}\frac{\cos\lambda l - 1}{\lambda l\sin\lambda l} + \frac{1}{\lambda l\sin\lambda l} + \frac{2}{(\lambda l)^2}\right] \end{aligned} \right\} & P < 0, \lambda = \sqrt{\dfrac{-P}{EI}} \end{cases} \tag{10}$$

3.3 Validation Using Finite Element Solutions

To valid the solutions developed in Sections. 3.1 and 3.2, including the assumptions introduced, four finite element imperfect beam models incorporating the distortion shapes considered in the previous sections are shown in Fig. 9. ABAQUS “B21” beam element were used and nonlinear geometry effects are considered in all these models. All these beam models have rectangular cross-sections with unit width. The material used in the finite element analysis corresponds to a typical structure steel whose properties are given in [20] (with Young's Modulus being 210000MPa and Poisson ratio being 0.3). The axial load for the models in Fig. 9a and Fig. 9b varies from P = -317.5N $(\sigma_n = -50\text{MPa})$ to P = 1587.5N $(\sigma_n = 250\text{MPa})$, while the minimum axial load for the models in Fig. 9c and Fig. 9d is set as P = -114.3N $(\sigma_n = -18\text{MPa}$ $)$. The FE-based k_b at weld location (x=0) are calculated and compared with analytical solutions in Fig. 10, demonstrating an excellent agreement between the analytical and FE methods for the entire axial load range.

4 Applications in Fatigue Test Data Analysis

4.1 Lab-Scale Butt-Welded Specimens

Some fatigue test results on lab-scale butt welded specimens (3mm in thickness) were reported in [20], in which detailed geometric nonlinear finite element analysis of these specimens with measured distortions was also performed. In these cyclic tensile fatigue tests, rotations at grip positions were fixed during testing and special clamping system was used to avoid additional bending from clamping. They evaluated the feasibility of using either surface extrapolated hot spot stress or local notch stress method recommended by IIW (Hobbacher, [34]) and the results are shown in Fig. 11 for later comparison purpose. Although the laser weld test data seems to show a reasonable trend (with standard deviation of 0.115) while the arc weld test data show a wide spread in fatigue lives (about a factor of 10 with a standard deviation of 0.335) at a rather similar stress range level. This can be attributed to significantly larger distortions in arc-welded specimens than laser-welded specimens, as discussed in [20]. The combined standard deviations are 0.275 and 0.277 in terms of hot spot stress and notch stress, respectively.

By considering the test clamping conditions as well as the distortions involved, these lab-scale specimens can be modeled as the imperfect beam illustrated in Fig. 12. Through a comparison between Fig. 5 and Fig. 12, the butt weld is located at x=0, and the beam end angles are $\theta_1 = \alpha_{\mathrm{L},1}/2,$ and $\theta_{\mathrm{G}} = \alpha_{\mathrm{G}}/2$, where α_{G} and $\alpha_{\mathrm{L},1}$ are defined in Fig. 5 and given in [20]. Note that simplified distortion shapes were assumed in [20] and $\theta_2 = \alpha_{\mathrm{L},2}$ values were not given, thus a modified local angular distortion model is introduced to accommodate such assumption.

Based on the development in Section 3.2, such distortions are first decomposed into global angular distortion and local angular distortion as in Fig. 6; thus, the global angular distortion is θ_{G} and the initial rotations in the local angular distortion θ'_1 and θ'_2 is obtained by $\theta'_1 = \theta_1 - \theta_{\mathrm{G}}$ and $\theta'_2 = \theta_2 - \theta_{\mathrm{G}}$. Since the θ_2 values are not available, we adjust the model by setting $m_0 = 0$ in the linear beam model shown in Fig. 8a, leading to $\theta'_2 = -\theta'_1/2$. Then, the distortion induced stress concentration factors $k_{\mathrm{b,global}}$ and $k_{\mathrm{b,local}}$ can be obtained through Eqn. (8) and (9), respectively. However, through a close examination of the specimens' pictures, we found that the distortion shape of several arc-welded specimens cannot be well represented by the adjusted model and thus their $\alpha_{\mathrm{L},2}$ values are measured specifically in this study, as summarized in Appendix B. The measurement of $\alpha_{\mathrm{L},2}$ was done by fitting the distortion profiles from the 2D section measurements, which were generated in [20], using cubic splines and taking the slope at the clamped end. These specimens are then treated using the approach in Section 3.2 without the adjustment discussed above.

Based upon Eqn. (5), we can see that the method of superposition is applicable for geometric-nonlinear beams as long as the beams have the same length *l*, same bending rigidity *EI*, and are subjected to the same axial load *P*, resulting in $k_{\mathrm{b}} = k_{\mathrm{b,global}} + k_{\mathrm{b,local}}$ for each specimen. The bending stress concentration caused by axial misalignment (e) is calculated separately using $k_e = 3e/t$ according to [24] using detailed e measurements given in [20]. The equivalent structural stress range parameter adopted by ASME Div. 2 Code since 2007 (see Dong [36, 37, 38]) can then calculated as:

$$\Delta S_s = \frac{\Delta\sigma_s}{t^{\frac{2-m}{2m}} I(r)^{\frac{1}{m}}} \tag{11}$$

where σ_s is calculated by

$$\sigma_s = (1 + k_e + k_b) \times \sigma_n \tag{12}$$

in which σ_n is the nominal stress. In Eqn. (11), t is the actual thickness of the specimen at failure locations observed in fatigue tests, m is given as 3.6; and $I(r)$ is a dimensional polynomial function of bending ratio $r = (k_e + k_b)\sigma_n/\sigma_s$, as given in [38]. The structural stress range $\Delta\sigma_s$ in Eq. (11) becomes simply $\Delta\sigma_s = \sigma_{s,\max} - \sigma_{s,\min}$.

With the equivalent structural stress range in Eqn. (11), the same fatigue test results given in Fig. 11 are replotted in Fig. 13b, labeled as "lab-scale" specimens. It can be seen that the same test data not only show a significantly improved correlation with a standard deviation of 0.202, but also exhibit a clearly defined slope. For comparison purpose, the nominal stress range-based plot of the same test data is also given in Fig. 13a and the master S-N curve scatter band from ASME [39] as dashed lines in Fig. 13b. It is interesting to note that in Fig. 13b that the butt-welded lab-scale specimen data fall within 2007 ASME's master S-N curve scatter band which represents about 1000 large scale fatigue tests with plate thickness varying from 5mm up to over 100mm.

4.2 Full-Scale Stiffened Panels

Lillemäe et al. [21] also conducted detailed distortion measurements and fatigue tests of full-scale stiffened specimens (see Fig. 14). Prior to fatigue testing, the distortion profiles were measured and documented for a total of nine specimens (see Fig. 14b) along mid-width, as summarized in Fig. 15. The fatigue tests were conducted at a load ratio of R=0.1. In what follows, a procedure for taking advantage of the analytical approach given in Section 2.2 will be discussed.

4.2.1 Distortion profile characterization

As illustrated in Fig. 2, there exists a characteristic length scale in terms of stiffener spacing (l) for characterizing welding-induced distortions in stiffened shipboard panels. With this consideration, a characteristic distance of two-stiffener spacing $(2l)$ or one spacing (l) on one side of the butt weld is considered as shown in Fig. 15.

As a result, distortion profiles on one side of the butt weld are considered for further analysis.

Upon further inspections, the distortion profiles within one characteristic length l (see Fig. 15) can be represented by a characteristic profile illustrated in Fig. 16, which is used as the initial imperfections of a beam, as discussed in Section 2.2, with the left end (weld location) embedded and the rotation fixed at the right end. As such, θ_1, θ_2 and θ_{G} are parameters that can be adjusted to provide the best fit of the distortion profiles shown in Fig. 15. It is worth noting that the initial distortion profile described in Fig. 16 is the same as the one shown in Fig. 12. Thus, the local and global angular distortion modes discussed in Section 3.2 can also be used to model such distortion.

Without losing generality, consider the distortion profile corresponding to Specimen 334 (see Fig. 15); the corresponding measured distortion profile (see the solid line in Fig. 17) can be reasonably fitted into a third order polynomial model, i.e., $v_0(x) = a_0 + a_1x + a_2x^2 + a_3x^3$ (see the dashed lines in Fig. 17). Because the beam deflection curve corresponding to the model in Fig. 16 is also a cubic polynomial function based on the classical beam theory, the good agreement shown in Fig. 17 should not be surprising at all. In the same manner, a cubic polynomial representation for all other distortion profiles in Fig. 15 can be established for further analytical treatment in secondary bending stress calculations.

4.2.2 SCF Calculation and FE Validation

With the distortion function $v_0(x)$ given by the third order polynomial (see Fig. 17), beam end rotations θ_1, θ_2 can then be obtained by $\theta_1 = v'_0(0)$ and $\theta_2 = v'_0(l)$. The corresponding global angular distortion is given by $\theta_{\mathrm{G}} = [v_0(l) - v_0(0)]/l$. Similar to the procedure discussed in 4.1, the local angular distortions θ'_1, θ'_2 are obtained by $\theta'_1 = \theta_1 - \theta_{\mathrm{G}}$ and $\theta'_2 = \theta_2 - \theta_{\mathrm{G}}$. Then, stress concentration factors corresponding to the global and local angular distortions can be directly obtained using Eqns. (2) and (9) in Section 3.2, referred to as $k_{\mathrm{b}} = k_{\mathrm{b,global}} + k_{\mathrm{b,local}}$.

For validation purpose, two shell element models are used here. One is a full-scale specimen model shown in Fig. 18a and the other is a local model with only one stiffener spacing on each side of the transverse butt weld (or "$2l$ model in Fig. 18b). In both cases, the actually measured distortion fields provided in [21] were mapped onto these models as coordinate changes in z-axis before remote tension loading was applied. The k_{b} values were calculated using the mesh-insensitive method [35] by means of a matrix equation that transforms nodal forces/moments from an FE calculation to nodal line force/moments at a specified remote tension load level or nominal stress (σ_n) level.

Both FE and analytical results of k_{b} are compared in Fig. 18b. The two FE solutions are consistent with each other over the entire remote load (i.e.,σ_n) range evaluated, suggesting the use of a characteristic length of $2l$ is a reasonable assumption. When the applied nominal stress is greater than 50MPa, the analytical results are consistent with the FE results, being slightly higher (about 5%). In a rather low nominal stress region, say below about 30MPa, the strip beam model seems too flexible, resulting in an under-estimated k_b. It should be noted that such an under-estimation in low nominal stress regime tends to have a limited impact on the structural stress range calculated since an error in $\sigma_{s,\min}$ is also scaled by a small $\sigma_{n,\min}$ value. Therefore, the results in Fig. 18b further justifies the approach proposed here by considering a strip beam model representing a given longitudinal panel through-thickness section.

4.2.3 Fatigue Data Correlation

With analytically calculated k_b values for all nine full-scale fatigue test specimens under loading ranges documented in [21], and k_e which is computed in the same manner as in Section 4.1 based on the axial misalignment measured from Fig. 15, the test data can be represented using the equivalent structural stress range given in Eqn. (11) corresponding to fatigue crack locations (see [21]) for data correlation purpose. The results are given in Fig. 13b, labeled as "full-scale" specimens. The nine full-scale test data surprisingly correlate well with one another, forming a narrow scatter band near the ASME master S-N curve mean line. The standard deviation (STD) of the nine data is calculated as 0.198. In contrast, the nominal stress range based plot in Fig. 13b for the same set of the data shows no clearly defined trend. Furthermore, both full-scale and lab-scale tests in Fig. 13b fall within the ASME master S-N curve's mean ± 2STD scatter band [38], suggesting the validity of both sets of test data and applicability of the ASME master S-N curve for fatigue evaluation of lightweight shipboard panel structures.

5 Conclusions

In this paper, a notional load method is presented for providing analytical treatment of complex distortion effects on fatigue behaviors of lightweight shipboard structures through a distortion decomposition technique. Its applications for analyzing secondary bending stresses caused by nonlinear interactions between four common distortion types induced by welding and remotely applied load are discussed in detail. In addition, two sets of lab-scale specimens and nine full-scale stiffened panel fatigue tests involving complex distortion shapes are also analyzed using the closed form analytical solutions developed. The analytically calculated stress concentration factor results are validated by direct finite element computations in all cases. Furthermore, an excellent agreement in fatigue test data is achieved not only between butt-welded thin plate lab specimens and full-scale stiffened panels but also with the traction structural stress based master S-N curve scatter band adopted by ASME Div. 2 since 2007. Some of the specific findings are worth noting, including:

(a) With the proposed method of notional loads, the imperfect beam problem is converted into a nonlinear perfect beam problem. As a result, existing nonlinear perfect beam solutions with a specified loading pattern can be used for deriving closed-form analytical k_b solutions for typical distortion modes of interest

(b) With such an analytical approach, only a few distortion measurements are needed for evaluating fatigue performance of weld joints in lightweight structures, significantly reducing the needs for full-field distortion measurements and their mapping onto a structural FE model

(c) Welding-induced distortions are shown to have significant effects on fatigue behaviors in welded thin-plate structures. Without appropriate treatment for secondary bending stresses, available test data cannot be correlated with existing data that support existing Codes and Standards (see Figs. 11 and 13b). The analytical approach presented in this paper proves effective for interpreting fatigue test data obtained in welded thin plate components

(d) The very fact that thin-plate test data (lab-scale and full-scale specimens) fall into the scatter band of the master S-N curve adopted by ASME Div. 2 suggests not only their relationship to existing thick plate fatigue test data, but also the applicability of the master S-N curve method for fatigue evaluation of lightweight structures

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Appendix

Appendix A

Detailed Solution Process for Angular Distortion Mode

Taking advantage of the symmetry condition at $x = l/2$, one can write the boundary conditions with respect to the governing equation given in Eqn.(3) corresponding to the linear beam without imperfections (see Fig. 3b), as:

$$v_0(0) = 0, \quad v_0'(0) = 0, \quad v_0'\left(\frac{l}{2}\right) = 0, \quad EI\left.\frac{d^3 v_0}{dx^3}\right|_{x=\frac{l}{2}} = \frac{F_0}{2}, \quad v_0(z) = v_0(l-z) \qquad \text{(A.1)}$$

Then, the solution that satisfies the above boundary conditions can be found in classical Mechanics of Materials textbooks [40]. By setting $\delta_0 = v_0(l/2)$, the corresponding notional load can be obtained as $F_0 = 192EI\delta_0/l^3$. Next, by a close examination of the nonlinear beam model with the initial imperfections prescribed by the now known $v_0(x)$ in Fig. 3a, its boundary conditions are given as:

$$v_1(0)=0, \quad v_1'(0)=0, \quad v_1'\left(\frac{l}{2}\right)=0, \quad EI\left.\frac{d^3v_1}{dx^3}\right|_{x=\frac{l}{2}}=0 \quad v_1(z)=v_1(l-z) \tag{A.2}$$

From Eqns. (A.1) and (A.2), it can be see that the boundary conditions in terms of both displacements and rotations are the same between the linear beam and the imperfect beam model. Then, Eqns. (A.1) and (A.2) are combined and forms the boundary conditions for $v(x)$ shown in Fig. 3c (i.e., an equivalent nonlinear beam without initial imperfections):

$$v(0)=0, \quad v'(0)=0, \quad v'\left(\frac{l}{2}\right)=0, \quad EI\left.\frac{d^3v}{dx^3}\right|_{x=\frac{l}{2}}=\frac{F_0}{2} \quad v(z)=v(l-z) \tag{A.3}$$

The final nonlinear beam deflection $v(x)$ can be obtained by solving Eqn. (5) with the above boundary conditions given in Eqn. (A.3). By substituting $F_0 = 192EI\delta_0/l^3$ into Eqn. (A.3), $v(x)$ corresponding to the beam span from x=0 to x=l/2 is given as:

$$v=\begin{cases}\dfrac{96\delta_0}{\lambda^3 l^3}\left[-\sinh\lambda x+\dfrac{\cosh\dfrac{\lambda l}{2}-1}{\sinh\dfrac{\lambda l}{2}}\left(\cosh\lambda x-1\right)+\lambda x\right] & P>0,\lambda=\sqrt{\dfrac{P}{EI}}\\ \dfrac{96\delta_0}{\lambda^3 l^3}\left[\sin\lambda x+\dfrac{\cos\dfrac{\lambda l}{2}-1}{\sin\dfrac{\lambda l}{2}}\left(\cos\lambda x-1\right)-\lambda x\right] & P<0,\lambda=\sqrt{\dfrac{-P}{EI}}\end{cases} \tag{A.4}$$

and the expression for the beam span from *x*=*l*/2 to *x*=*l* can be obtained by substituting x with $l-x$ in Eqn. (A.4), as a result of symmetry with respect to *x*=*l*/2.

The distortion-induced secondary moment at the weld location $M_1(0)$ is found as:

$$M_1(0)=EI\left.\frac{d^2v}{dx^2}\right|_{x=0}-EI\left.\frac{d^2v_0}{dx^2}\right|_{x=0}=\begin{cases}\dfrac{24P\delta_0}{\lambda^2l^2}\left(\dfrac{4}{\lambda l}\dfrac{\cosh\dfrac{\lambda l}{2}-1}{\sinh\dfrac{\lambda l}{2}}-1\right) & P>0\\ \dfrac{24P\delta_0}{\lambda^2l^2}\left(\dfrac{4}{\lambda l}\dfrac{\cos\dfrac{\lambda l}{2}-1}{\sin\dfrac{\lambda l}{2}}+1\right) & P<0\end{cases} \tag{A.5}$$

The resulting k_b at beam the top surface $(y=t/2)$ at the weld location $(x=0)$ becomes:

$$k_\mathrm{b}=\frac{\sigma_b}{\sigma_n}=\frac{-6M_1(0)/t^2}{P/t}=-\frac{6M_1(0)}{Pt} \tag{A.6}$$

which yields Eqn. (6).

Appendix B. Local angular misalignment measurements of $\alpha_{\mathrm{L},2}$

See Table B1

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Fig. 1. Procedure for solving imperfect beam problems using the method of notional loads: (a) imperfect beam; (b) linear beam subjected to notional loads; (c) linear beam with adjusted boundary conditions; (d) nonlinear perfect beam with notional loads.

Fig. 2. Two major distortion types in thin plate structures [**Error! Reference source not found.**].

Fig. 3. Beam models used for modeling interactions of angular distortion with load P: (a) imperfect beam; (b) linear beam; (c) nonlinear perfect beam.

Fig. 4. Beam models used for modeling interactions of buckling distortion with load P: (a) imperfect beam; (b) linear beam; (c) nonlinear perfect beam.

Fig. 5. Angular distortion definitions for butt-welded thin plate specimens [**Error! Reference source not found.**].

Fig. 6. Decomposition of complex angular distortions in butt-welded plate shown in Fig. 5: (a) a general distortion shape; (b) global angular distortion; (c) local angular distortion.

Fig. 7. Nonlinear beam model: global angular distortion.

Fig. 8. Beam models used for local distortion of butt-welds: (a) imperfect beam; (b) linear beam; (c) nonlinear perfect beam.

Fig. 9. FE beam models used for validating the analytical solutions: (a) angular distortion; (b) buckling distortion; (c) local and (d) global angular distortion of butt-welds.

Fig. 10. Comparison of stress concentration factors (k_b) results between FE and analytical solutions: (a) angular distortion and

buckling distortion; (b) local and global angular distortions of butt-welded plate specimens.

Fig. 11. Test data correlation using nonlinear geometry FEA calculated stress (taken from [**Error! Reference source not found.**]): (a) IIW's surface extrapolation based hot-spot stress method; (b) IIW's effective notch stress method.

Fig. 12. Imperfect beam model for modeling lab-scale specimens

Fig. 13. Data correlation using: (a) nominal stress range; (b) equivalent structural stress range given in 2007 ASME master S-N curve incorporating analytically calculated k_{b} due to global and local angular distortions and k_e caused by axial misalignments.

Fig. 14. Full-scale stiffened panel (4-mm thick base plate) and full-scale fatigue test specimen containing a hybrid laser butt-weld [21]:

(a) Full scale stiffened panel; (b) Illustration of full-scale fatigue test specimen extracted from (a) for distortion measurements and fatigue testing

Fig. 15. Out-of-plane distortion profiles measured along mid-width line of nine full-scale fatigue specimens prior to fatigue testing [21] (Note that transverse butt weld is located at $x = 0$)

Fig. 16. Characteristic distortion profile serving as initial beam imperfections for treatment of distortions in full-scale fatigue specimens

Fig. 17. Cubic polynomial fitting of measured distortions (Specimen 334, Right side)

Fig. 18. Validation of analytically calculated k_{b} using FE models incorporating actual measured distortions: (a) Full-scale and

characteristic length based FE models used; (b) Comparison of k_{b} results at weld toe at mid weld length

Table B1. Measured $\alpha_{\mathrm{L},2}$ values for lab-scale specimens

Specimen no.	$\alpha_{\mathrm{L},2}$ deg
Arc 7	-1.71
Arc 9	-0.69
Arc 10	-1.90
Arc 11	-1.89