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Published in:
Developments in the Collision and Grounding of Ships and Offshore Structures - Proceedings of the 8th International Conference on Collision and Grounding of Ships and Offshore Structures, ICCGS 2019

Published: 01/01/2020

Document Version
Peer-reviewed accepted author manuscript, also known as Final accepted manuscript or Post-print

Please cite the original version:
Gil, M., Montewka, J., Krata, P., Hinz, T., & Hirdaris, S. (2020). Semi-dynamic ship domain in the encounter situation of two vessels. In C. G. Soares (Ed.), *Developments in the Collision and Grounding of Ships and Offshore Structures - Proceedings of the 8th International Conference on Collision and Grounding of Ships and Offshore Structures, ICCGS 2019* (1 ed., Vol. 4, pp. 301-307). (Proceedings in Marine Technology and Ocean Engineering; Vol. 4). CRC Press.

Semi-dynamic ship domain in the vessels' encounter situation

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ABSTRACT: There exist numerous examples of ship domains that are rooted in the concept of Fujii and Goodwin. Initially, the idea was developed to evaluate the capacity of waterways by calculating the probability of accidents in the fairways. Over the years, the applicability of the conception has been significantly expanded, beyond its initial area of design. Nowadays, the ship domain is mainly used to assess the safety of navigation, where a domain violation in the encounter of two vessels is recognized as an unsafe operation. Thus, any situation that does not fracture the domain is considered as a safe one. Nevertheless, in the literature, there is a lack of proper justification for transferring the ship domain concept from the original application filed to the other, especially into the safety-critical areas. Therefore, in this paper we undertake an attempt to specify the basic requirements for a concept that can be used to evaluate the safety of ship-ship encounter. The idea reflects the dynamics of two vessels involved in the close approach situation, their maneuvering and operational characteristics. For that reason, we discuss the applicability of the ship domains for the purpose of the navigational safety assessment.

1 INTRODUCTION

Ensuring safe and efficient passage of a ship is one of the main tasks of the *Officer of the Watch* (OOW). However, the number of his duties including those non-navigational grows as soon as the ship draws into the coastal waters. In addition, the traffic is getting denser year by year, especially in the vicinity of port approaches, where the routes are merging and crossing, [1]. This means the available time for making decisions onboard a ship is shrinking and the chance for an erroneous behavior of the navigator rise then, [2]–[4]. Therefore, to reduce this phenomenon and improve the safety of navigation, numerous decision support tools are developed to assist the OOW in traffic monitoring and making decisions in the area of collision avoidance. The most widely used collision avoidance system (CAS) is the *Automatic Radar Plotting Aid* (ARPA). This technology should be able automatically to track, process, simultaneously display and continuously update information on at least 20 radar-detected targets, [5]. The proximity indicators used therein are called CPA *(closest point of approach*) and TCPA (time to CPA). However, the passing distance does not directly translate into the required area for the safe and efficient evasive maneuver, thus the professional knowledge and sea experience is needful. Another type of proximity indicator stems from the concept of a *ship domain*. Where the *ship domain can be thought of as the sea area around the ship which a navigator would like to keep free, with respect to other ships and fixed objects*, see [6]. The concept was not intended for collision avoidance purposes initially, thus its size may be insufficient for collision evasive action when the domain is infringed by another ship. Rather its usage was anticipated for waterway capacity evaluation and strategic risk assessment, see [6]–[9]. However, it migrated to the field of operational risk assessment and collision avoidance, as used by [10]–[15]. Simultaneously a concept of *arenas* has been introduced in [16], defined therein as *the area around the own ship which when infringed causes the mariner to consider whether to make a collision-evasive manoeuvre*. From the collision-avoidance perspective, a combination of the properly defined arena and domain improves the officer's situational awareness, whereas separately those concepts are of limited value, and may be even misleading.

Another important point to raise is the subjectivity of those proximity indicators. They refer to the comfort area defined by a navigator rather than a safety-critical area for a ship to perform evasive action. The difference between comfort and critical areas is substantial. Therefore, a navigator handling a ship should be aware of the safety area's dimension or reversely on its critical size. This information arises as a necessity when planning an evasive maneuver in an encounter, where the other, give-a-way vessel is not acting as supposed. This critical area depends on numerous factors, where the ship's dynamics is one of them, and only a few studies address this issue, see for example [17]–[22]. However, those models face serious limitations, by considering one type of maneuver for fixed rudder settings (turning circle at 20° angle), one type of ship, and presumably favorable stability and weather conditions.

In our earlier works [24], [25], we present a model determining the critical area for a Ro-pax ship, accounting for her dynamics, preselected stability conditions and simplified encounter type. The models stem from the concept of the *Minimum Distance To Collision* (MDTC), as introduced in [26], [27]. Therefore in this paper, we present a significantly improved model determining *Collision Avoidance Dynamic Critical Area* (CADCA) around own ship that needs to be kept free from other objects steaming towards a potential collision, to ensure the safe passage of the ship in an encounter. As a case study, we demonstrate the safe area for a container ship.

The structure of the paper is as follows: Section 2 introduces the concept of the safe area around own ship, Section 3 presents the methods adopted in the study and the developed model. In Sections 4 and 5, the results are elaborated and discussed, while Section 6 concludes.

2 CADCA CONCEPT

To verify ship domain concept for the purpose of collision evasion, it has been necessary to validate it in different operational conditions such as ships' encounter scenarios, rudder settings or initial longitudinal speed of the vessel. Then, execution of a large number time-consuming computations has been required, to scrutinize whether and how the ship's domain changes its shape and limits under different conditions. Therefore, the simulation method of the study was carefully selected to provide a representative dataset of computations in the most efficient and available way.

The main requirement posed to the computer simulation software was providing the highly efficient calculations in accordance with anti-collision indicators methodology adopted by authors. The measure selected to compute the minimal ship's safety domain in semi-dynamic conditions was the *Minimum Distance To Collision* (MDTC) introduced in [25], [28]–[30]. The *semi-dynamic* denotes the lack of weather impact such as waves and wind on the hydrodynamics of the vessel. Calculation of this criterion allows for determining the minimum distance thus, also the last moment, when the vessel is obligated to execute the maneuver. The avoidance of collision in the MDTC is still possible with regard to the hydrodynamics of the vessels and specific of their encounter.

The hydrodynamic effects can realistically beget due to the utilization of the external software called *LaiDyn*. This application generates vessels' trajectories which take into account the 6DoF motion model, see for example [31], [32]. Turning circles are obtained for various initial settings of the ship's model like its speed or rudder angle. It is also doable to prepare different loading and stability conditions of the vessel and simulate various hydrometeorological conditions. Their impact on the ship's hull and its motions could be also included, but these were not considered in this research.

Simulations of large number semi-dynamic scenarios using *LaiDyn* models' trajectories as input files, allow determining a broad set of MDTC values. Obtained distances have been used to build-up a shape of the area which surrounds the own ship and indicates the last moment to avoid a collision. For a particular simulation case, limits of the envelope differ in various conditions. This determined semi-dynamic ship domain is called *Collision Avoidance Dynamic Critical Area* (CADCA).

Thereby, based on MDTC criterion minimum distances between the target and the own ship in encounter situation have been determined. Then, using a cloud of the points critical areas were delimited. The initial concept was developed and refined by including various operational aspects of the vessel like her forward speed or rudder angle. This results in the area that partly considered alternation of ship's parameters. In a further approach, the CADCA would be raised from semi to full dynamic variant by including stability issues and external disturbances that derived from the weather. In the version presented in this paper, the CADCA is taking into account type of ships' encounter (angular arrangement of the vessels), type of evasive maneuver (directions of the turns), initial speed of the vessels, and various rudder angles.

3 CADCA SIMULATOR

Due to the need to prepare extensive dataset of ships' encounter simulations, a computer application has been developed. The algorithm of CADCA simulator presented in Figure 1 determining in an efficient way navigational parameters of ships at different stages of vessels' encounter (such as bearing, distance, headings). The application computes the MDTC values and prepares various types of charts for filtered simulations' cases as output files.

The main principle of simulator's operation is to determine four limiting collision arrangements using mathematical projections of simplified ships' hulls. Layouts correspond to extreme mutual positions of the vessels (tangents to the bow and stern of each ship respectively). Afterward, vessels are moved backward apart with regard to their speeds. For each time step that suits to the data interval of trajectories files, the software attempts to lay the vessel's 6DoF track generated from the *LaiDyn*. Ships proceed on new trajectories, while software verifies if vessels would collide again. If the collision occurs, the algorithm executes in the loop another time step and moves ships backward again. For each time step application validates the opportunity of collision avoidance for given simulation settings.

When the algorithm reaches the end of a loaded ship's trajectory, it means that vessels can safely pass each other. Thus, they avoid contact with their hulls. The simulator stops the execution of further time steps when the first opportunity of passing occurs. Even, if the space between the vessels will be too small to provide safe collision-avoidance in conditions of routine operation. It results among others from a lack of consideration of the hydrometeorological conditions. In the presented version of CADCA concept not all dynamic motions of the vessel and all environmental factors are taken into account.

Therefore, it was necessary to additionally enlarge the hulls' projections by safety margin to provide a safer passage and imitate real liminal distance for passing or overtaking two vessels. To add an additional buffer, empirical factors for ship-ship interactions used by PIANC (*The World Association for Waterborne Transport Infrastructure*) were considered, [33] and implemented into the software. Thus, the violation of the safety margin, not the particular hull projection was considered by computer application as a collision.

In Figure 2, the concept of simulator operation (collision of two vessels, execution of time steps, and laying the loaded trajectory) for a particular scenario is presented. Therein the sample case where own ship (blue) executes turn to the starboard side (rudder set to 25.0° stb.) accordingly to COLREGs (*The International Regulations for Preventing Collisions at Sea 1972*) - [34] - in a crossing situation. At the same time, the target vessel (red) does not follow the rules and keeps her course.

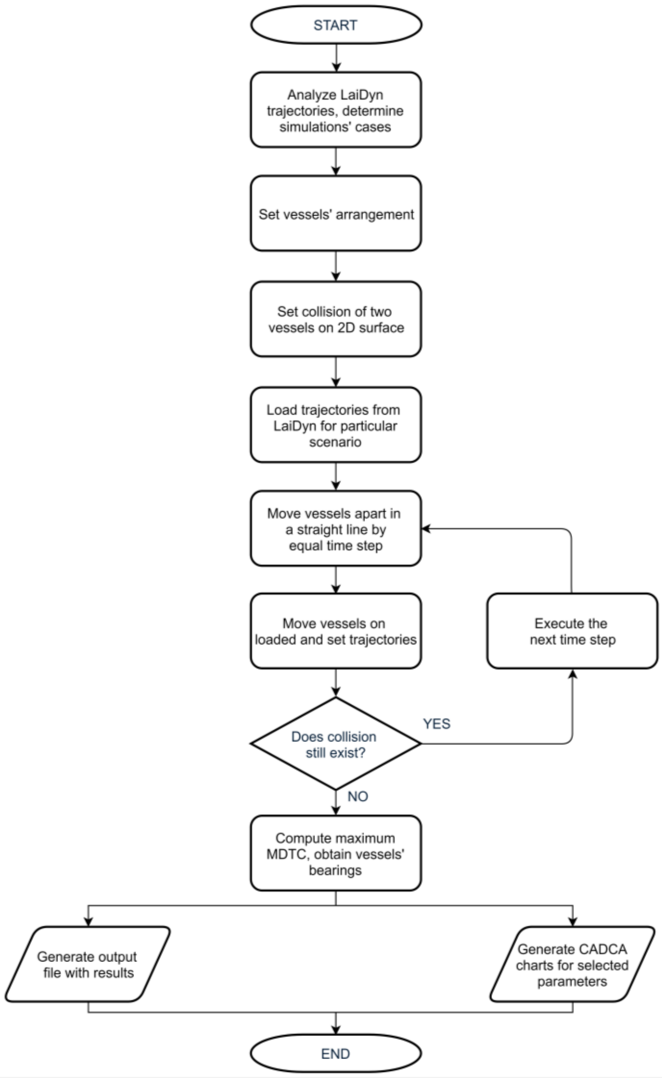

Figure 1 - Flowchart of CADCA simulator.

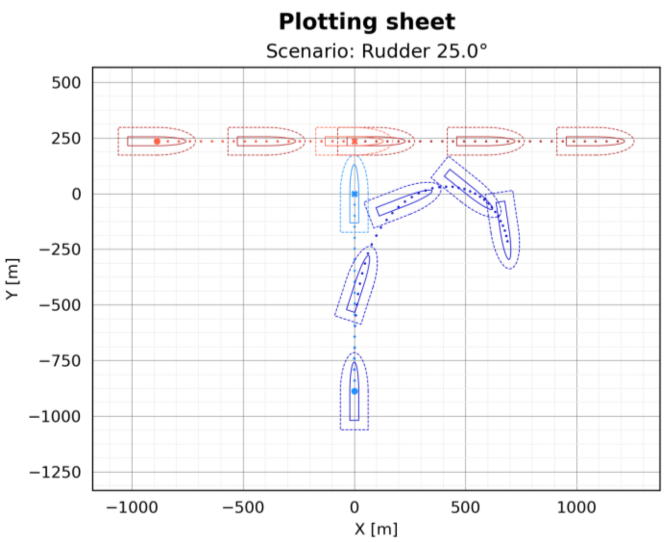

Figure 2 - Example plotting sheet with ships' hulls and safety margin projection.

The algorithm computed the last possibility (so the first option to plot complete trajectory) to execute the maneuver when ships are proceeding with an initial speed of 16.0 knots. The lighter ships' colors on the chart indicate the principle of the algorithm operation from the beginning of the simulation, so moving ships backward apart from initial collision arrangement to the position of MDTC.

Finally, for determined minimal distances simulator creates output files that contain numerical results of calculations, as well as the plots with an obtained semi-dynamic version of CADCA for different parameters of the simulation cases.

4 RESULTS

The simulations have been executed for one ship's model (used as own ship and the target respectively) and:

- 13 settings of the rudder (midship and from 5° to 30° for each side in 5° step),
- 6 different initial speeds (from 10 to 20 kts in 2 kts step which represents every 10%),
- and 36 target's headings (from 0 to 350° with 10° step).

Loading condition of the vessel has not been changed in the performed simulations. There was a lack of environmental disturbances like the impact of wind or waves. Characteristic of 6DoF vessel's model selected to the research is presented in Table 1.

Table 1 - Basic characteristic of the selected model.

Type	LOA [m]	Beam [m]	Draft [m]	Mass [t]
Container	262.0	40.0	12.3	76 028

Totally, more than 41 thousands of simulations for various ship's operational parameters have been conducted. The final post-processed output of those, are areas surrounding own vessel. Their limits are borders of CADCAs which splitting up the entire analyzed area into two subregions. The first is the CADCA, which means the area inside the envelope where the evasive maneuver is no longer feasible (vessels approach too close each other to successfully undertake an action). The second is area remaining outside the CADCA envelope that ensures the possibility to execute the effective evasive maneuver. The graphical presentation of the CADCA utilizes the radar plot that depicts the obtained area versus the relative bearing to the target ship.

To verify if CADCA is varying according to various parameter settings, several data breakdowns were prepared. The detailed information about a particular simulation case is presented in Table 2 with provided references to related figures.

Table 2 – Summary of presented simulations

Figure no.	3	4	5	6	7
OS initial heading [°]	000 (North)				
TS initial heading [°]	[000, 350] for 10° step				
Ships' speeds [kts]	16	16	[10, 20] 2kn step	16	10 & 20
OS rudder angle [°]	[5, 30] 5° step	5 & 30	15	15	25
TS rudder angle [°]	0 (midship)				
OS direction of turn	starboard side				port & stb. side
TS direction of turn	not applicable (ship keeps her course and speed)				

4.1 *Rudder settings*

One of the purposes of prepared simulations was determining how different rudder settings affect CADCA. To present this, the results were filtered out just for selected cases where the speed of the vessel equals 16.0 kts. Target vessel keeps her course and speed on different 36 initial headings. The own ship initially proceeds on the northern course (000°) and she executes a turn to starboard side for all considered rudder angles.

CADCA for 6 various settings of own ship's rudder (color-coded) and all bearings to the target (outer scale) is depicted in Figure 3. As the vessel's rudder angle increases the area of surface limited by CADCA decreases respectively.

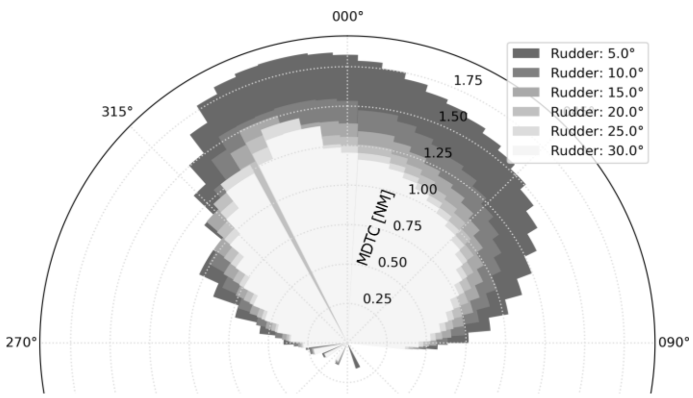

Figure 3 – CADCA for 6 various rudder angles presented for the bearings to the target.

Execution of the turn to the starboard side with the least available rudder angle 5° results in the CADCA's diameter almost 0.5 Nm larger (~865 m for the worst target's headings: 140-150°) than for the rudder setting 30°. Thus, the evasive maneuver should be carried out earlier respectively to the vessel's speed. The mentioned cases are depicted in Figure 4.

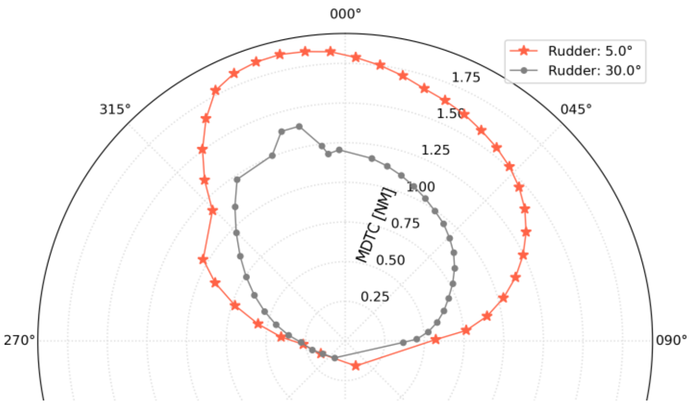

Figure 4 – CADCA envelope for two extreme rudder angles presented for the bearings to the target.

4.2 *Ships' speed*

The initial speed of the vessels was also taken into consideration as a parameter that could have a significant impact on CADCA's size and limits. To verify how ship's speed affects, the simulation scenario with the average rudder angle setting was selected (15°). The turn of the own ship has been executed to the starboard side that is a typical evasive maneuver according to the COLREGs. The target vessel was keeping her course and speed. The set of 36 different initial headings starting from the North with 10° step was simulated. Both ships had the equal initial speed that varies for each simulation from 10 kts (half ahead) to 20 kts (full ahead).

The obtained results indicate that initial vessel's speed has an insignificant impact on CADCA's limits. As depicted in Figure 5, for the vast majority of obtained values, MDTC is similar regardless of the ship's speed. Only for several bearings, the considerable difference in the individual MDTC values can be noticed, ranging around 0.4 Nm between extreme values in the worst scenario.

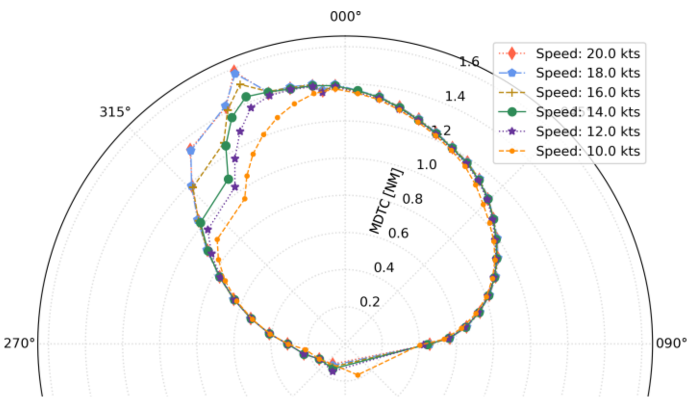

Figure 5 – CADCA presented as MDTC vs bearing to the target for various initial speeds of both vessels.

For the cases where some visible difference in the values for particular bearings exist, it could be noticed that when a vessel's speed increases, the maneuver has to be executed earlier not only in terms of time, which is quite obvious, but also commenced in a larger distance. The determined liminal distances of vessels approach presented vs. bearings to the target (Figure 5) confirm the general rule about vessels' collision avoidance. MDTC increases significantly ahead from the own ship's beam. Thus, the most unsafe cases concern the situations when the target is located in the forward sectors. CADCA for one selected vessels' speed (16 kts) and a variety of bearings to the target are depicted in Figure 6.

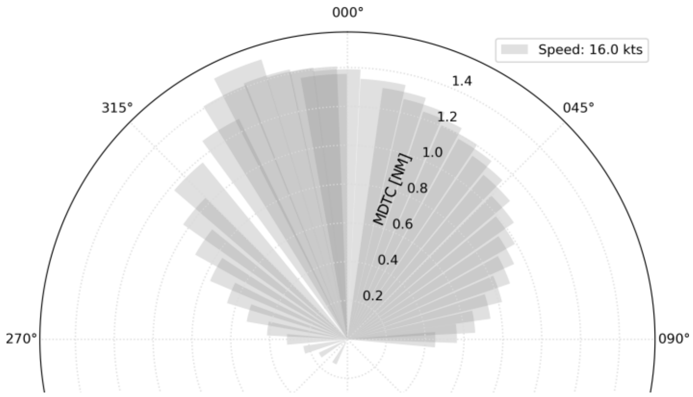

Figure 6 - MDTC vs bearing to the target for initial speeds 16 kts of both vessels.

4.3 *Ship's turning direction*

CADCA was computed also for different directions of own ship's turn, for port and starboard sides respectively. The simulation scenario concerns the situation where the target vessel keeps her course and speed, but own ship makes a turn to each side. Both vessels have the same initial forward speeds i.e. 10 and 20 kts, which correspond to the half ahead and full-sea speeds in engine order telegraph. In this scenario own ship sets the rudder angle to 25°.

As depicted in Figure 7, the limits of the determined area vary depending on the direction of the ship's turn for half ahead, as well as for full ahead speed. It results from the maneuvering characteristic of the container vessel's model used in the study. According to the turning circles presented in the documentation normally posted on the navigational bridge (e.g. wheelhouse poster), one side of the turning circle is, in general, wider than the second one. The difference in tactical diameters in ship's circulation results from the direction of propeller's rotation. The vessel used in the research is equipped in a propeller that rotates clockwise. Therefore, the area obtained for the turn to the starboard side is also larger.

The CADCA determined for different sides of the ship's turn slightly differs. Nonetheless, noteworthy is that the shape and the limits are almost a mirror reflection of the other side.

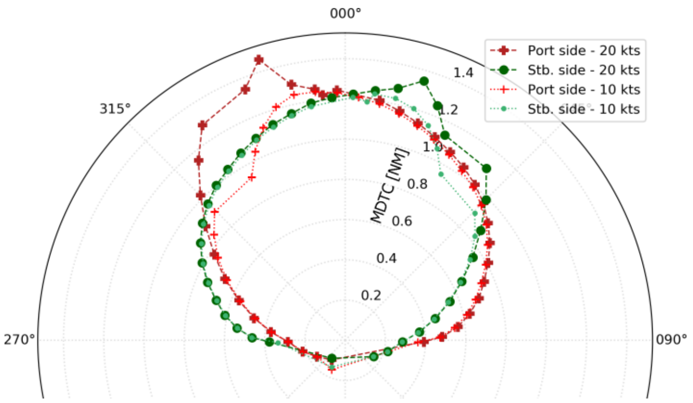

Figure 7 – CADCA as MDTC vs bearing to target for both turning sides. Rudder angle 25°, speed 10 and 20 kts.

5 DISCUSSION

The presented application for CADA calculation faces some limitations that opens up the areas for future research. Presently, it is able to simulate the same values of the vessels' speeds or rudder angles for both ships. The parameters should be separated for the own ship and the target to obtain an aggregate database of combined results. Firstly, it could result in providing a real-time reaction and allows for remodeling CADCA in case of change of vessels' parameters. Secondly, this would allow for simulation of cases, where own ship and target vessel proceed with different speeds. The analysis of the difference between relative speeds of the vessels could be valuable for the further development of CADCA concept.

Noteworthy, in real conditions, especially in rough seas it could be unacceptable to execute hard to port or starboard maneuver in the close-quarters situation. Such maneuvers could lead to generating of the excessive heel during the ship's turning. Thus, the rudder setting should correspond to the vessel's speed, so this fact ought to be considered during the evasive maneuver planning.

Therefore, another parameter of the vessel's motion model affecting the dimension of CADCA should be considered. These refer to stability and loading conditions, which determine those simulations from the dataset that should not be considered because of generating excessive heel during turning. Another issue is the utilization of *LaiDyn* trajectories developed for varying weather conditions, determining the behavior of the ship.

The simulator used for the research should be optimized to provide high-efficiency computation time of further parameters combination. Different ranges of vessels' speeds or rudder angles will cause the generation of hundreds of thousands of additional combination. Correlation of ship motion parameters with weather parameters (several directions of wind and wave separately, wind speeds, wave heights) will even enlarge this dataset. Thus, the software should be optimized as much as possible, to provide an efficient way of big data simulations.

Such a complex approach to the determination of critical area in collision avoidance could result in introducing the full-dynamic concept that will change CADCA limits during the encounter. Preparation of a large number of simulations using computer application for various combination of ship's, encounter, and weather parameters could be used to prepare ship's meta-model or to implement it into the decision support system.

6 CONCLUSIONS

The aim of this paper is to present a model determining *Collision Avoidance Dynamic Critical Area* (CADCA) around own ship that must be kept free from other objects on the collision course. CADCA changes the limits and shapes according to the situation, primarily due to the ship's motion or encounter parameters such as rudder settings, forward speed, target's headings or turning sides. Therefore the concept seems to be reasonable and justified in the light of the navigation routine. In such a dynamically changing environment like sea or ocean where maritime transportation takes place, safety issues are crucial and any attempt aiming at the improvement of the safety of navigation could be valuable for seafarers. As presented in the results, the modification of one of the components values mostly causes also the change of the moment of evasive maneuver execution.

The future works that will be focused on introducing further parameters of the ship's dynamic (stability issues), as well as the environment (weather disturbances), allowing the development of meta-model. The latter could be implemented into the navigational decision support system.

7 ACKNOWLEDGMENTS

The work presented here was carried out under project FLooding Accident REsponse (FLARE), number 814753, under H2020 program, funded by the EU.

8 REFERENCES

[1] UNCTAD, "Review of maritime transport 2018," United Nations Conference on Trade and Development, ISBN 978-92-1-112878-9, 2018.

[2] Corporate Risk Associates, "A user manual for the nuclear action reliability assessment (NARA) human error quantification technique. Report No 2," Surrey, UK, 2011.

[3] Z. L. Yang, S. Bonsall, A. Wall, J. Wang, and

M. Usman, "A modified CREAM to human reliability quantification in marine engineering," *Ocean Eng.*, vol. 58, no. null, pp. 293–303, Jan. 2013.

[4] M. C. Leva, P. Friis-Hansen, E. S. Ravn, and A. Lepsoe, "SAFEDOR: A practical approach to model the action of an officer of the watch in collision scenarios," in *European safety and reliability conference; Safety and reliability for managing risk; ESREL 2006*, 2006, pp. 2795–2804.

[5] IMO, *SOLAS Consolidated Edition 2014*, 6th Ed. London: International Maritime Organization, 2014.

[6] E. M. Goodwin, "A Statistical Study of Ship Domains," *J. Navig.*, vol. 28, no. 03, pp. 328–344, 1975.

[7] Y. Fujii and K. Tanaka, "Traffic Capacity," *J. Navig.*, vol. 24, no. 04, pp. 543–552, Jan. 1971.

[8] E. M. Goodwin, "A Statistical Study of Ship Domains," *J. Navig.*, vol. 26, no. 01, p. 130, 1973.

[9] T. G. Coldwell, "Marine Traffic Behaviour in Restricted Waters," *J. Navig.*, vol. 36, no. 03, pp. 430–444, Nov. 1983.

[10] S.-L. Kao, K.-T. Lee, K.-Y. Chang, and M.-D. Ko, "A Fuzzy Logic Method for Collision Avoidance in Vessel Traffic Service," *J. Navig.*, vol. 60, no. 1, pp. 17–31, 2007.

[11] Z. Pietrzykowski and J. Uriasz, "The Ship Domain – A Criterion of Navigational Safety Assessment in an Open Sea Area," *J. Navig.*, vol. 62, no. 01, pp. 93–108, Dec. 2009.

[12] N. Wang, "An Intelligent Spatial Collision Risk Based on the Quaternion Ship Domain," *J. Navig.*, vol. 63, no. 04, pp. 733–749, Sep. 2010.

[13] A. Lopez-Santander and J. Lawry, "An Ordinal Model of Risk Based on Mariner's Judgement," *J. Navig.*, pp. 1–16, Sep. 2016.

[14] N. Wang, "A Novel Analytical Framework for Dynamic Quaternion Ship Domains," *J. Navig.*, vol. 66, no. 02, pp. 265–281, 2013.

[15] R. Szłapczyński and J. Szłapczyńska, "An analysis of domain-based ship collision risk parameters," *Ocean Eng.*, vol. 126, pp. 47–56, Nov. 2016.

[16] P. V Davis, M. J. Dove, and C. T. Stockel, "A Computer Simulation of Marine Traffic Using Domains and Arenas," *J. Navig.*, vol. 33, no. 02, pp. 215–222, 1980.

[17] B. A. Colley, R. G. Curtis, and C. T. Stockel, "Manoeuvring Times, Domains and Arenas," *J. Navig.*, vol. 36, no. 02, pp. 324–328, 1983.

[18] R. Curtis, "A ship collision model for overtaking," *J. Oper. Res. Soc.*, vol. 37, no. 4, pp. 397–406, 1986.

[19] J. Montewka, F. Goerlandt, and P. Kujala, "Determination of collision criteria and causation factors appropriate to a model for estimating the probability of maritime accidents," *Ocean Eng.*, vol. 40, pp. 50–61, Feb. 2012.

[20] J. Zhang, X. Yan, X. Chen, L. Sang, and D. Zhang, "A novel approach for assistance with anti-collision decision making based on the International Regulations for Preventing Collisions at Sea," *Proc. Inst. Mech. Eng. Part M J. Eng. Marit. Environ.*, vol. 226, no. 3, pp. 250–259, Feb. 2012.

[21] A. Łukaszewicz, "Określanie odległości krytycznej podjęcia manewru ostatniej chwili zgodnie z przepisami konwencji COLREG," in *Międzynarodowa konferencja naukowa Transport XXI wieku*, 2007, pp. 389–396.

[22] R. Szlapczynski and P. Krata, "Determining and visualizing safe motion parameters of a ship navigating in severe weather conditions," *Ocean Eng.*, vol. 158, 2018.

[23] MAIB, "Report on the investigation into the listing, flooding and grounding of Hoegh Osaka Bramble Bank, The Solent, UK on 3 January 2015," Southampton, 2016.

[24] P. Krata and J. Montewka, "Assessment of a critical area for a give-way ship in a collision encounter," *Arch. Transp.*, vol. 34, no. 2, 2015.

[25] J. Montewka and P. Krata, "Towards the assessment of a critical distance between two encountering ships in open waters.," *Eur. J. Navig.*, vol. 12, pp. 7–13, 2014.

[26] J. Montewka, T. Hinz, P. Kujala, and J. Matusiak, "Probability modelling of vessel collisions," *Reliab. Eng. Syst. Saf.*, vol. 95, no. 5, pp. 573–589, May 2010.

[27] J. Montewka, F. Goerlandt, and P. Kujala, "Determination of collision criteria and causation factors appropriate to a model for estimating the probability of maritime accidents," *Ocean Eng.*, vol. 40, pp. 50–61, Feb. 2012.

[28] J. Montewka, F. Goerlandt, and P. Kujala, "Determination of collision criteria and causation factors appropriate to a model for estimating the probability of maritime accidents," *Ocean Eng.*, vol. 40, pp. 50–61, 2012.

[29] J. Montewka, F. Goerlandt, and P. Kujala, "A new definition of a collision zone for a geometrical model for ship-ship collision probability estimation," in *9th International Navigational Symposium on Marine Navigation and Safety of Sea Transportation*, 2011.

[30] P. Krata, J. Montewka, and T. Hinz, "Towards the assessment of critical area in a collision encounter accounting for stability conditions of a ship," *Pr. Nauk. Politech. Warsz. Transp.*, no. z. 114, pp. 169--178, 2016.

[31] J. Matusiak and C. Stigler, "Ship Roll Motion in Irregular Waves During a Turning Circle Maneuver," in *11th International Conference on Stability of Ships and Ocean Vehicle*, 2012, pp. 307–313.

[32] J. E. Matusiak, "Dynamics of Rigid Body," *Aalto Univ.*, 2017.

[33] PIANC, "MarCom WG 121: Harbour Approach Channels - Design Guidelines," 2014.
[34] *COLREG : Convention on the International Regulations for Preventing Collisions at Sea, 1972 : Consolidated Edition 2003*. London: IMO, 2010.