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# Article Random Field-Based Time-Dependent Reliability Analyses of a PSC Box-Girder Bridge

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**Abstract:** The parameters affecting structural reliability are usually regarded as independent random variables in the current reliability analyses of bridge structures while the randomness of these structural parameters in spatial distribution is neglected. To overcome this disadvantage, the random field model should be used to describe their probability distribution. In this paper, a structural reliability analysis method considering the effect of a random field is proposed, and its validity is verified by three numerical case studies. A prestressed concrete (PSC) box-girder bridge with a main span of 150 m is selected for demonstration, and the importance sampling (IS) method is applied to estimate its failure probability, in which the influences of shrinkage and creep, stress relaxation, and shear lag on the time-dependent performance of the structure are taken into account. In addition, the random fields of the structure are discretized by using the local average method (LAM). Finally, the effects of random field parameters, such as the number of discrete elements, correlation model, and correlation length, on the reliability of the box-girder bridge are discussed.

**Keywords:** long-span box-girder bridge; random field; local average method; important sampling; creep and shrinkage; time-dependent reliability

# 1. Introduction

The serviceability and durability of bridges are gradually reduced due to the influence of various deterioration factors during service, which may even lead to accidents involving safety [1,2]. Therefore, to provide the scientific basis for diagnosis of structural health and maintenance decisions to ensure the safe operation of bridges, it is necessary to evaluate the service status of bridges throughout their whole service lives.

Factors, such as material properties, load conditions, geometric characteristics, and steel corrosion, etc., show considerable randomness and have significant influence on the mechanical properties of prestressed concrete (PSC) bridges. Therefore, probabilistic methods have been used to evaluate the structural state in order to obtain rational results [3–5]. Based on the first-order reliability method (FORM), AI-Harthy [6] studied the reliability of several typical PSC beams, such as rectangular beams, T-beams, and  $\pi$ -beams, in American Concrete Institute (ACI) code and discussed the effects of secondary moment, eccentricity, and area of prestressed tendons on structural reliability. Du [7] analyzed the influence of steel corrosion on the reliability of the flexural bearing capacity of PSC bridges (T-type)

during the whole service process. Akgul [8] proposed a life-cycle performance evaluation method based on the structural reliability of PSC bridges. Steinberg [9] applied the Monte Carlo method to evaluate the reliability of high-performance PSC elements, including box girders, I-girders, and double-T girders. Nowak [10] compared the reliability levels of precast PSC bridges designed by three different codes: The Spanish Norma IAP-98 code, the Eurocode, and the AASHTO LRFD code. Du [11] also carried out some related studies. In addition, the time-dependent effects caused by concrete shrinkage and creep present a high degree of uncertainty due to the influences of various random factors. Guo [12] presented the time-dependent reliability of a PSC box-girder bridge by using the stochastic finite element method, in which the combined effects of creep, shrinkage, and steel corrosion were taken into account. Guo [13] also proposed a hybrid method to analyze the time-variant deflection reliability of a high-speed railway PSC bridge, and the research results could provide theoretical guidance for maintenance and rehabilitation of similar bridges.

In the aforementioned reliability analysis of PSC bridges, structural parameters are usually treated as uncorrelated random variables. In fact, the stiffness, distributed load, and material characteristics of bridge structures may change randomly with their locations, resulting in their probability distribution being described by the random field models. VanMarcke [14] proposed a simple and practical procedure of stochastic finite element analysis to obtain the statistical characteristics of the deformation of structural members whose material properties vary randomly along their axes. For concrete structures, Most [15] presented a probabilistic analysis method that took into account the material uncertainties modeled by means of random fields and failure due to cracking. Rahman [16] developed a stochastic meshless method for reliability analysis of linear-elastic structures with spatial variability of material properties, and the effectiveness of the developed method was verified by numerical examples. Stewart [17,18] studied the influence of the spatial variability in the local corrosion of steel on the reliability and vulnerability of reinforced concrete (RC) structures. The results show that spatial variability has a great effect on the failure probability of RC structures. However, few studies have considered the influence of random fields on the time-dependent reliability of PSC bridges. Cheng [19] presented a random field-based reliability methodology for a PSC box-girder bridge in which the inertia moment and live load were modeled as random fields and discretized by the midpoint method. Due to the use of beam elements in bridge modeling, the unique shear lag effect of the box-girder bridge could not be considered, and the uncertainty in shrinkage and creep was neglected in this study. However, previous studies show that the uncertainty in shrinkage and creep models has a significant influence on the evaluation of long-term responses of PSC bridges [20], which should be taken into account in the reliability analysis of PSC bridges.

In this study, a random field-based time-dependent reliability method was developed, which is the first combination of random field method and time-dependent analysis for PSC box-girder bridges. Besides, shell elements rather than a beam element were used for accurate modeling of the box-girder bridge, and accordingly the random field analysis of PSC box-girder bridges was firstly extended to two-dimensional. Considering the high computational cost for such a stochastic finite element analysis, the importance sampling (IS) method was adopted to obtain rational efficiency. A case study on the Jinghang Canal Bridge, a three-span continuous PSC box-girder bridge with a main span of 150 m in China, was made as a demonstration.

# 2. Basic Theory of Random Field

#### 2.1. Discretization of Random Field.

The random field model H(w), denoted by  $\{H(w), w \in \Omega\}$ , represents the variability in the spatial distribution of structural parameters, and  $\Omega$  indicates the definition domain of the random field [21]. Considering the effects of a random field in structural reliability analysis, the random field should be discretized into a set of random variables. At present, several methods have been developed for the discretization of random fields, including the midpoint method [22], interpolation method [23], local

average method (LAM) [24,25], weighted integral method [26], orthogonal expansion method [27], and optimal linear evaluation method [28]. However, each method does not have absolute advantages over other methods. The LAM means that the average characteristic of a random field in any discrete element represents its probability distribution in this element. This method can achieve the balance between calculation accuracy and efficiency and can be easily programmed in finite element software; therefore, it is widely used in practice. In this paper, the method is used to discretize the random fields of a structure. The applications of the LAM in the discretization of one-dimensional (1D) and two-dimensional (2D) random fields are described as follows.

# 2.1.1. 1D Random Field

H(x) represents a 1D uniform random field, which is discretized by linear elements, as shown in Figure 1, and then the random field is discretized into *n* random variables.  $L_i$  denotes a linear element with a midpoint coordinate of  $x_i$  and a length of  $l_i$ . The local average of H(x) defined on the element is expressed as follows:

$$H_i = H_i(x_i) = \frac{1}{l_i} \int_{x_i - l_i/2}^{x_i + l_i/2} H(x) dx.$$
 (1)

 $H_j$  represents the local average of H(x) defined on the element  $L_j$ , and the degree of correlation between  $H_j$  and  $H_i$  is expressed in terms of covariance:

$$Cov(H_i, H_j) = \frac{\sigma^2}{2l_i l_j} \sum_{k=0}^3 (-1)^k l_k^2 \Gamma^2(l_k),$$
(2)

where  $l_k$  (k = 0, 1, 2, 3) indicates the relative position between the elements,  $L_i$  and  $L_j$  (see Figure 1). Parameter  $\Gamma^2(l_k)$  is the reduction coefficient of variance, which represents the reduction degree of variance of the local average with respect to the point variance of the random field and is related to the correlation model,  $\rho(\tau)$ . There are four main types of  $\rho(\tau)$ , including the triangular (T) model, exponential (Exp) model, second order auto-regressive (AR) model, and Gaussian (G) model, whose specific expressions and corresponding reduction coefficients of variance can be found in [29]. Figure 2 illustrates the curve of  $\Gamma^2(l_k)$  changing with  $l_k/L$ , in which L is the correlation length of the random field. Figure 2 shows that the variation in  $\Gamma^2(l_k)$  with  $l_k/L$  is basically identical under different correlation models and that the differences between the curves are small. This indicates that the LAM of random field discretization is insensitive to the type of correlation models, which is another reason for the wide application of this method in practical engineering.



Figure 1. Discretization of a 1D random field.

# 2.1.2. 2D Random Field

H(x, y) represents a 2D uniform random field, and the rectangular elements are used to divide the distribution area of the random field, as shown in Figure 3.  $A_i$  denotes a rectangular element with centroid coordinates of  $(x_i, y_i)$  as well as the lengths of  $l_{xi}$  and  $l_{yi}$  in two directions (x and y), respectively. The local average of H(x,y) defined on this element is expressed as follows:

$$H_{i} = H_{i}(x_{i}, y_{i}) = \frac{1}{l_{xi}l_{yi}} \int_{x_{i}-(l_{xi}/2)}^{x_{i}+(l_{xi}/2)} \int_{y_{i}-(l_{yi}/2)}^{x_{i}+(l_{yi}/2)} H(x, y) dx dy,$$
(3)

where  $H_j$  represents the local average of H(x, y) defined on the rectangular element,  $A_j$ , with the lengths of  $l_{xj}$  and  $l_{yj}$  in the direction of x and y, respectively. The covariance between  $H_j$  and  $H_i$  is expressed as follows:

$$Cov(H_i, H_j) = \frac{\sigma^2}{4l_{xi}l_{yi}l_{xj}l_{yj}} \sum_{k=0}^3 \sum_{l=0}^3 (-1)^{k+l} (l_{xk}l_{yl})^2 \Gamma^2(l_{xk}, l_{yl}),$$
(4)

where  $l_{xk}$ ,  $l_{yl}$  (k, l = 0, 1, 2, 3) describe the relative position between  $A_i$  and  $A_j$ , as shown in Figure 3.



Figure 2. Reduction coefficient of variance vs. the correlation model.



Figure 3. Discretization of a 2D random field.

The discretization of a random field can be different from that of a structural finite element model. Generally, a random field element can contain several finite elements [30]. When the mesh is determined, the distribution characteristics of the random field on each element are represented by their local average on the element. Thus, a set of random variables is obtained, and their statistical properties are reflected by mean  $E(H_i)$ ,  $Var(H_i)$ , and  $Cov(H_i, H_i)$ .

# 2.2. Orthogonal Transform.

As mentioned above, the random field is discretized into a set of correlated random variables based on the LAM (e.g.,  $[H] = [H_1, H_2, ..., H_n]^T$ ). However, these random variables cannot be directly used in the stochastic finite element analyses. The correlated random variables should be transformed into the independent random variables by means of orthogonal transform. The covariance values

calculated by Equation (2) or Equation (4) for the random variables in the vector [*H*] constitute the matrix  $[C_H]$ , as shown below:

$$[C_{H}] = \begin{bmatrix} cov(H_{1}, H_{1}) & cov(H_{1}, H_{2}) & \cdots & cov(H_{1}, H_{n}) \\ cov(H_{2}, H_{1}) & cov(H_{2}, H_{2}) & \cdots & cov(H_{2}, H_{n}) \\ \vdots & \vdots & \ddots & \vdots \\ cov(H_{n}, H_{1}) & cov(H_{n}, H_{2}) & \cdots & cov(H_{n}, H_{n}) \end{bmatrix}.$$
(5)

From Equation (5), it can be seen that the  $[C_H]$  is a real symmetric matrix. According to the theory of linear algebra, the matrix [A] can be expressed as:

$$[A]^{T}[C_{H}][A] = [C_{\lambda}], \tag{6}$$

$$\begin{bmatrix} C_{\lambda} \end{bmatrix} = \begin{bmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ & & \ddots & \\ 0 & & & \lambda_n \end{bmatrix},$$
(7)

where  $[C_{\lambda}]$  denotes the *n* order diagonal matrix composed of the eigenvalues ( $\lambda_i$ , *i*=1, 2, ..., *n*) of the covariance matrix  $[C_H]$ , as shown in Equation (7); [A] denotes the *n* order orthogonal matrix constructed by the eigenvectors of the matrix  $[C_H]$ ; and [Y] is a vector consisting of *n* random variables  $Y_i$  (*i*=1, 2, ..., *n*), which obey a normal distribution:

$$[Y] = [A]^T [H]. \tag{8}$$

From Equation (8), the mean values of the random variables in [Y] are described as Equation (9):

$$\mu_{Y} = E[Y] = E[A^{T}H] = A^{T}E[H].$$
(9)

The matrix  $[C_Y]$  indicates the autocorrelation among the random variables in [Y]:

$$[C_{Y}] = E([Y][Y]^{T}) = E([A^{T}H][A^{T}H]^{T}) = A^{T}E([H][H]^{T})A = A^{T}C_{H}A = [C_{\lambda}].$$
 (10)

From Equation (10), it can be seen that the random variables in [Y] are unrelated to each other, and the variance of random variable,  $Y_i$ , corresponds to the main diagonal element in the matrix  $[C_{\lambda}]$ , namely,  $\sigma^2[Y_i] = \lambda_i$ . At this point, the [*H*] can be expressed by independent random variables in [Y]:

$$[H] = [A][Y]. (11)$$

Based on the probability distribution of the random variable in [Y], samples of random variables are generated. Then, they are substituted into Equation (11) to obtain the sets of samples of the discrete elements in the random field corresponding to random variables [H], whose attributes are assigned to the relevant elements of the structural finite element (FE) model for reliability evaluation.

# 3. Structural Reliability Considering Random Field

A critical issue of structural reliability analysis is to accommodate the calculation accuracy and efficiency. For large and complex structures with very small failure probabilities, the direct Monte-Carlo sampling (MCS) method requires a large number of random samples to ensure estimation accuracy, which leads to low calculation efficiency and is difficult to apply in practice. To minimize the required number of samples, an IS method [31] is used to analyze the structural reliability. The basic idea of this method is to change the sampling center, which would be established at the most likely failure

point of the structure (i.e., "checking point") to improve the chance of the sample points falling into the failure region.

The other independent random variables of the structure are assumed as  $R_j$  (j = 1, 2, ..., m) represented in the vector [R] = [ $R_1, R_2, ..., R_m$ ]. The total random variables considered in the reliability analysis constitute the vector [X] = [H] + [R], including the random variables discretized from random fields. The failure probability based on the IS method can be written in the following form:

$$P_{f} = \int_{-\infty}^{+\infty} \frac{I[G(X)]f(X)}{\rho(X)} \rho(X) dX = E \bigg\{ \frac{I[G(X)]f(X)}{\rho(X)} \bigg\},$$
(12)

where G(X) = performance function of the structure; f(X) = joint probability density function; and  $\rho(X)$  = IS density function. The specific expressions of these three functions are shown below. I[G(X)] denotes the indicator function, where I[G(X)] = 1 if  $[G(X)] \le 0$  and I[G(X)] = 0 if [G(X)] > 0. Applying the probability distribution of  $\rho(X)$ , the N sets of samples for the random variables  $X_k$  (k = 1, 2, ..., N) are generated, and then the failure probability of the structure can be estimated as follows:

$$\hat{P}_f = \frac{1}{N} \sum_{k=1}^{N} \left\{ \frac{I[G(X_k)]f(X_k)}{\rho(X_k)} \right\}.$$
(13)

For the IS method,  $\rho(X)$  plays a crucial role in the calculation accuracy and efficiency of structural reliability, and its sampling center is established at the checking point. When the performance function of a structure is described as an explicit function of random variables, the checking point is obtained by solving a constrained optimization problem. However, the performance function is usually implicit in the probabilistic FE method. Moreover, when the reliability analysis takes into account the influence of the random field, the number of random variables will be generally very large after the discretization of the random fields. As a result, the structural performance function will hardly be expressed as the explicit function of random variables. The approximate checking point can be found via a search process based on a certain number of Monte-Carlo samplings (MCSs) and cyclic finite element analysis, as presented in Figure 4.

As shown in Figure 4, the proposed reliability analysis method considering the influence of the random field mainly includes three steps: Discretization of the random field, determination of the checking point, and calculation of the failure probability. First, the random field of a bridge structure should be discretized into a set of correlated random variables by the LAM (represented as  $[H] = [H_1, H_2, ..., H_n]^T$ ), and then the covariance matrix  $[C_H]$  of [H] can be obtained by Equation (2) or Equation (4). The eigenvalue analysis of  $[C_H]$  can obtain the eigenvalue matrix  $[C_{\lambda}]$  and the orthogonal matrix [A] composed of eigenvectors. Afterwards, the random variables in [H] are represented by the uncorrelated random variables in [Y] with orthogonal transformation, as shown by Equation (8). The mean and variance of  $Y_i$  are shown in Equation (9) and Equation (10), respectively.

*M* sets of samples for  $\underline{Y}_k$  (k = 1, 2, ..., M) are generated according to the probability distribution of each variable  $Y_i$  in [*Y*], and the samples of the random field [*H*], expressed as  $\underline{H}_k$ , are obtained by substituting  $\underline{Y}_k$  into Equation (11). Similarly, *M* sets of samples for the random variables in [*R*] are generated based on their distributions, represented as  $\underline{R}_k$  (k = 1, 2, ..., M). Each set of samples  $\underline{H}_k$  and  $\underline{R}_k$  are substituted into the FE model of the bridge to calculate the values of G ( $\underline{H}_k, \underline{R}_k$ ). Among the *M* sets of samples  $\underline{H}_k$  and  $\underline{R}_k$  with G ( $\underline{H}_k, \underline{R}_k$ )  $\leq 0$ , the set of samples  $\underline{H}^*$  and  $\underline{R}^*$  corresponding to the maximum f ( $\underline{H}_k, \underline{R}_k$ ) would be treated as a candidate checking point; otherwise, the set of samples  $\underline{H}^*$ and  $\underline{R}^*$  corresponding to the minimum G ( $\underline{H}_k, \underline{R}_k$ ) would be selected as the candidate checking point. Then, the  $\underline{Y}^*$  corresponding to  $\underline{H}^*$  can be obtained based on Equation (8). Subsequently, the probability distributions for each random variable  $Y_i$  in [*Y*] and  $R_j$  in [*R*] are modified by replacing the mean value  $\mu_i$  (i = 1, 2, ..., n) with  $Y_i^*$  and  $\mu_j$  (j = 1, 2, ..., m) with  $R_j^*$ . *M* sets of samples for the random variables are generated again to search for the checking point, and FE analysis is carried out to obtain the new checking point,  $\underline{H}^*$  and  $\underline{R}^*$ . The process of searching for the checking point is repeated until the following conditions are satisfied:  $G(\underline{H}^*, \underline{R}^*) \approx 0$ , which guarantees that the checking point is located on the failure boundary of the structure; and the increment in  $H_i^*$  and  $R_i^*$  between two iterations is less than the predefined errors,  $\varepsilon_i$  and  $\tau_j$ , respectively. The final  $\underline{H}^*$  (or  $\underline{Y}^*$ ) and  $\underline{R}^*$  can be used as the approximate checking point.



Figure 4. The procedure of structural reliability analysis considering a random field.

The failure probability,  $P_f$ , can be estimated based on Equation (13). Specifically, the *N* sets of samples  $\underline{Y}_k$  and  $\underline{R}_k$  (k = 1, 2, ..., N) are generated from probability distributions centered on the final approximate checking point, and sets of values of  $\underline{H}_k$  can be obtained by substituting  $\underline{Y}_k$  into Equation

(11). Then, the structural FE analysis is carried out based on each set of samples  $\underline{H}_k$  and  $\underline{R}_k$ , and the corresponding value for  $G(\underline{H}_k, \underline{R}_k)$  is calculated. In Equation (13),  $f(X_k)$  is expressed as the product of the probability density functions for the uncorrelated random variables in [Y] and [R] as follows:

$$f(\underline{H}_k, \underline{R}_k) = f(\underline{Y}_k, \underline{R}_k) = \prod_{i=1}^n f_i(Y_{i,k}) \prod_{j=1}^m f_j(R_{j,k}) = \prod_{i=1}^n f_i(Y_{i,k}, \mu_i, \sigma_i) \prod_{j=1}^m f_j(R_{j,k}, \mu_j, \sigma_j)$$
(14)

$$\rho(\underline{H}_{k},\underline{R}_{k}) = \rho(\underline{Y}_{k},\underline{R}_{k}) = \prod_{i=1}^{n} \rho_{i}(Y_{i,k}) \prod_{j=1}^{m} \rho_{j}(R_{j,k}) = \prod_{i=1}^{n} f_{i}(Y_{i,k},Y_{i}^{*},\sigma_{i}) \prod_{j=1}^{m} f_{j}(R_{j,k},R_{j}^{*},\sigma_{j})$$
(15)

where  $f_i(\cdot)$  and  $f_j(\cdot)$  are the probability density functions for the random variables  $Y_i$  and  $R_j$ , respectively;  $Y_i^*$  and  $R_j^*$  denote the distribution centers for  $Y_i$  and  $R_j$  in the IS function  $\rho(X)$ , respectively. As shown in Figure 4, the parameters *M* and *N* are equal to 20 and 500, respectively, and there are 35 iterations to obtain the approximate checking point.

Several numerical examples are given to verify the validity of the proposed reliability analysis method that considers the effect of a random field, and the results are compared with those of other methods.

# 4. Verification of Numerical Examples

# 4.1. Example One

A fixed-end beam with a span of 20 m is considered in this example, as shown in Figure 5. The elastic modulus of the concrete (E) and the section moment of inertia (I) of the beam are modeled as a 1D Gaussian random field, whose statistical characteristics are presented in Table 1. The purpose of this case is to calculate the deflection reliability of a beam subjected to a distributed load q = 1 kN/m, and the performance function can be defined as:

$$G = 1.05\overline{\delta} - \delta_0,\tag{16}$$

where  $\overline{\delta}$  represents the deflection at the middle site of the beam when the mean values of the random variables are taken; and  $\delta_0$  denotes deflection at the initial point under loads.



Figure 5. Fixed-end beam subjected to a distributed load.

Table 1. Statistical properties of random fields.

Random Field	Mean	Coefficient of Variation (COV)	Distribution	Correlation Model	Correlation Length
E (MPa)	10	0.01	Normal	Exp	10 m
I (m <sup>4</sup> )	0.6667	0.01	Normal	Exp	10 m

Based on the method of local average discretization, the meshes of the random fields, *E* and *I*, are the same as those of the FE model of the beam. The deflection reliability of the beam is calculated by using the proposed method considering the effects of the random field, and the results are compared with those in [32,33], as shown in Figure 6.



Figure 6. Deflection reliability vs. the number of finite elements.

As seen in Figure 6, the reliability indexes calculated by this proposed method are in agreement with the results obtained from Deng [32] and Hisada [33], which reveals that the proposed method in this paper is effective and correct. Specifically, Deng adopts the perturbation stochastic finite element (PSFE) method to calculate structural reliability, and solving the gradient vector of the implicit function to random variables is the key step of the PSFE method. This process is more complicated because the existing FE program needs to be reformed. However, the calculation procedure of the proposed method is relatively simple.

Compared with the method adopted by Hisada [33], the method proposed in this paper has the advantage of a faster convergence speed and can achieve higher accuracy with fewer discrete elements. As shown in Figure 6, the number of discrete elements of a random field has a certain influence on the structural reliability, and the reliability index tends to converge with the increase in the number of elements.

#### 4.2. Example Two

A continuous beam with a cantilever is subjected to a concentrated load,  $F = 1 \times 10^{-2}$  kN, as shown in Figure 7. The flexural stiffness, EI = k(x), of the continuous beam is simulated as a Gauss random field with a mean value of  $E_k = 100$  kN·m<sup>2</sup>, standard deviation of  $\sigma_k = 10$  kN·m<sup>2</sup>, and correlation length of L = 8 m. The correlation models contain four commonly used models: Triangular (T), second-order AR (AR), exponential (Exp), and Gaussian (G). The limit state functions of the deflection and bending moment are described as follows:

$$G_1 = 1.2\overline{\delta} - \delta_0,\tag{17}$$

$$G_2 = 1.1\overline{M} - M_0,\tag{18}$$

where  $\overline{\delta}$  and  $\overline{M}$  represent the deflection at the free end and the moment at the fixed end of the beam, respectively, when the mean values of the random variables are taken; and  $\delta_0$  and  $M_0$  indicate the deflection at the free end and the moment at the fixed end of beam, respectively.



Figure 7. Continuous beam subjected to a concentrated load.

The effect of the correlation model on structural reliability is discussed in this example. The mesh discretization of the random field is consistent with that of the FE model of the beam. For the four

correlation models, the deflection and bending moment reliabilities are calculated by the proposed method. The results are compared with those obtained from the PSFE method and the Monte-Carlo (MC) method [34], as shown in Figure 8.



Figure 8. Reliability of the continuous beam vs. different correlation models: (a) Deflection reliability; (b) bending moment reliability.

Under different correlation models of a random field, the deflection and bending moment reliability indexes obtained by the proposed method are consistent with the results of the PSFE method and the MC method (10,000 samples), as shown in Figure 8. The MC method requires a large number of samples to estimate the structural reliability, especially for large and complex bridges with a small failure probability. The computation time using the MC method, PSFE method, and proposed method is 2736, 79, and 45 ms, respectively. Compared with the MC method, the other two methods have higher computational efficiency. However, the estimation accuracy of the PSFE method cannot be guaranteed when the random variability of the structural parameters is large. Compared with the PSFE method, the proposed method improves the computational accuracy because its predicted results are not limited by the variability in the random variables. Figure 8 shows that the correlation models have some influence on the structural reliability. The reliability index calculated by the EXP correlation function is larger than those of the other three functions because the coefficient of variance reduction is small, as shown in Figure 2. In other words, the discreteness of the random variables obtained by the LAM is relatively lower.

# 4.3. Example Three

As shown in Figure 9, an elastic rectangular plate is subjected to a distributed load (q = 1 kN/m<sup>2</sup>) on one side, whose side length and plate thickness are 10 and 1 m, respectively, and Poisson's ratio is 0.2. The elastic modulus, *E*, of the plate is modeled as a 2D uniform random field expressed as  $E = \overline{E}(1 + k\lambda_e(x, y))$ , where  $\overline{E} = 1 \times 10^4$  kN/m<sup>2</sup> and k = 0.1. The correlation function is EXP, and the correlation length is described as  $L = d\sqrt{\pi}$ , where d = 1, 2, 5. The limit state function of deflection is defined as follows.

$$G = 1.1 \times \overline{\delta}_y - \delta_y,\tag{19}$$

where  $\delta_y$  denotes the displacement in the Y direction at the A point when the mean values of the random variables are taken; and  $\delta_y$  denotes the displacement in the Y direction at this point.





Figure 9. Rectangular plate subjected to a distributed load.

The purpose of this example is to investigate the effect of the correlation length on structural reliability. The mesh discretization of the random field of the elastic modulus is the same as that of the FE model of the rectangular plate. The proposed method is adopted to calculate the reliability index of displacement at point A under a distributed load, and the results are compared with those obtained from the PSFE method and the MC method, as shown in Figure 10 [34].

It can be seen from Figure 10 that the reliability index estimated by the proposed method is in agreement with the results obtained by the PSFE method and the MC method (10,000 samples) [34], which shows the validity and correctness of this method. The computation time by using the MC method, PSFE method, and proposed method is 12,208, 428, and 289 ms, respectively, and it can be seen that the proposed method has the highest computational efficiency. It can also be found that the correlation length has a significant effect on the reliability of the plate, and the reliability index decreases rapidly with increasing correlation length. This is because the correlation between two points in the distribution space of the random field will cause a wider range of variations. When the correlation length tends to be infinite, the random field will degenerate into a random variable. For example, the local variation in the elastic modulus will cause random fluctuations in the whole structure, which reduces the structural reliability. It is the main reason that when the structural parameters are considered as random variables, the calculated failure probability is greater than that obtained by considering the spatial randomness of the parameters.



**Figure 10.** Reliability of the rectangular plate vs. the correlation length: (**a**)  $3 \times 3$  (discrete elements); (**b**)  $5 \times 5$  (discrete elements).

# 5. Time-Dependent Reliability Analysis of the PSC Box-Girder Bridge

# 5.1. Bridge Description.

The Jinghang Canal Bridge, with a span of 150 m, located in Suzhou, China, is a three-span PSC continuous box-girder bridge. It consists of two identical but independent single-cell box girders that carry vehicles travelling in two traffic directions. Figure 11a,b illustrate the cantilever casting process and completed elevation of the bridge, respectively, and the segment division of the box girder is shown in Figure 11c. Following a quadratic parabolic curve, the height of the box girder gradually decreases from 9.0 m at the piers to 3.3 m at the midspan. Similarly, the thickness of the bottom slab and webs varies from 0.9 m at the piers to 0.32 and 0.5 m at the midspan, respectively. The top plate of the girder has the same thickness (0.28 m) along the traffic direction.



**Figure 11.** Profile of the Jinghang Canal Bridge: (**a**) Cantilever construction; (**b**) completed elevation; (**c**) segment division.

# 5.2. Finite Element Model

In order to maintain a balance between computational efficiency and accuracy, the composite degenerated shell element in the FE program DIANA [35] is adopted to simulate the mechanical behaviors of a thin-walled box-girder structure, as shown in Figure 12. This shell element is easily capable of modeling non-prestressed reinforcement and prestressed tendons. Non-prestressed reinforcement is modeled by a steel grid, which is embedded in the shell element, see Figure 12a. The reinforcement ratios in two perpendicular directions are represented by two equivalent thicknesses,  $t_{eq}$  (the area of cross section per unit length) of the grid, see Figure 12b. The parameter z identifies the location of each reinforcement grid, which determines the distance from this grid to the shell midsurface, as shown in Figure 12c. The area of the steel grid is divided into several segments, as shown in Figure 12d, which add stiffness to the shell element. Prestressed tendons can be established easily using an automatic tendon generation scheme, which defines tendons by several key position points in shell elements and shape functions (i.e., straight, quadratic, or cubic curve), as shown in Figure 12e. *DIANA* automatically searches the shell elements for the intersections of the tendons with its boundaries. These intersections serve as location points, which divide the tendons into segments. For each tendon segment, the numerical integration is carried out separately. The embedded tendons add stiffness to the FE model and their strains are calculated from the displacement field of the surrounding shell elements based on deformation coordination.



**Figure 12.** Composite degenerated shell element: (**a**) Composite shell element; (**b**) modeling of steel bars; (**c**) position of the steel grid in the shell element; (**d**) segment of the steel bar; (**e**) automatic generation of the tendon.

Meanwhile, the shell element can be further integrated with some time-dependent constitutive models, to simulate the time-varying responses of PSC box-girder bridges, such as creep and shrinkage and steel relaxation. At present, dozens of creep and shrinkage models have been proposed, including the commonly used CEB-FIP model [36], the B3 model [37], the EC2 model [38], the ACI model [39], and the GL2000 model [40]. According to the research results by Goel [41], it is concluded that the CEB-FIP90, B3, and GL2000 models can predict creep deformation well. In addition, the CEB-FIP90 and GL2000 models require fewer input parameters than other creep models, and in this study, the CEB-FIP 90 model is used to describe the concrete creep and shrinkage. As one important source of long-term prestress loss, the steel relaxation is modeled by the relaxation function and the generalized Maxwell model [12]. Moreover, the shell element can automatically consider the effect of shear lag on the long-term deflection of PSC box-girder bridges. The CEB-FIP 90 and steel relaxation models are integrated into the FE program *DIANA*, and users can adjust the parameters according to their needs. In this way, the influences of concrete creep and shrinkage, steel relaxation, and shear lag on the long-term deformation of PSC box-girder bridges can be computed automatically through FE analyses.

In view of the symmetry of structure and load, only half of the bridge was modeled using *DIANA* software to reduce the computation cost, as shown in Figure 13. The aforementioned composite degenerated shell element was used to simulate the thin-walled box-girder, and there were 1768 elements in total. The distributed reinforcements were simulated by using a series of steel grids, with two layers of steel mesh for each slab (i.e., top plates, bottom plates, and webs) and the prestressed tendons were established using the automatic tendon generation scheme [35]. Moreover, the unique phase analysis in the *DIANA* program was used to consider the influence of the cantilever construction process on the bridge. Specifically, based on the function of the "set" in the software, the sets of shell elements, reinforcement elements, external loads, and boundaries in the same construction stage were established. According to the actual construction progress and closure sequence of the box girder, the corresponding sets in each phase were activated or passivated. The Newton modified tangent method was selected as the iterative method, and the displacement convergence criterion was 0.01.



Figure 13. FE model of the bridge and the mesh of random field.

# 5.3. Statistical Characteristics of Random Parameters

The probabilistic models of the structural parameters contain random variables and random fields in this time-dependent reliability analysis. The uncertainties of shrinkage and creep models controlling prestressing stress and ambient humidity are considered as random variables, which are shown in Table 2. The coefficients of  $\psi_1$  and  $\psi_2$  are uncertainty factors of the creep and shrinkage models, respectively. For the CEB-FIP90 model, they approximately follow the normal distribution with the mean value of 1, and the coefficients of variation (COVs) are 0.339 for creep and 0.451 for shrinkage, respectively [42].The density of concrete (DC) is regarded as a random variable, which obeys a normal distribution, with a mean value 25.5 kN/m<sup>3</sup> and a COV of 0.046 [43]. The control prestressing stress ( $\sigma_{con}$ ) is a normally distributed variable with a mean value of 1395 MPa, a COV of 0.04, and a truncated upper bound at 1488 MPa [44]. According to the tests, the concrete strength (f<sub>cm28</sub>) followed a normal distribution with a mean value of 63.9 MPa and a COV of 0.089 [45]. The distribution of annual relative humidity (RH) at the bridge site was obtained from the statistical results of monitoring data provided by the China Meteorological Administration. The initial loading age (t<sub>0</sub>) was assumed to follow the uniform distribution with a mean value of 7 days and a COV of 0.19, which ranged from 5 to 9 days during the construction of this bridge.

Table 2. Statistical properties of random variables.

Random Variables	Unit	Mean	COV	Distribution	Data Sources	
Creep uncertain coefficient $\psi_1$	_	1	0.339	Normal (truncated at 0)	Reference [42]	
Shrinkage uncertain coefficient $\psi_2$	—	1	0.450	Normal (truncated at 0)	Reference [42]	
Density of concrete DC	kN/m <sup>3</sup>	25.5	0.046	Normal (truncated at 0)	Reference [43]	
Control prestressing stress $\sigma_{con}$	MPa	1395	0.040	Normal (truncated at 1488)	Reference [44]	
Concrete strength f <sub>cm28</sub>	MPa	63.9	0.089	Normal (truncated at 40)	Concrete strength test	
Relative humidity RH	%	67.3	0.191	Normal (truncated at 100)	China Meteorological Administration	
Loading age t <sub>0</sub>	d	7	0.110	Uniform	Design assumption	

The elastic modulus, secondary dead load, and live load are simulated as 2D random fields, and the statistical properties are summarized in Table 3. Similar to the FE model of the structure, the accuracy of discreteness is higher as the mesh density increases in the random field. The difference lies in that the mesh density of FE model is determined by the stress gradient, while for the random field, it mainly depends on the correlation length [22]. When the correlation length is longer, the correlation between two points in the distribution space will be stronger, and fewer discrete elements will be required to achieve the same accuracy. Given that the determination of the correlation length requires more measured data, and the paper mainly discusses the reliability estimation method of a PSC box-girder bridge based on random field theory, the longitudinal and transverse correlation lengths are both assumed to be 40 m. To balance the computational accuracy and efficiency, the size of the random field element is generally 1/3 to 1/2 of the correlation length. Meanwhile, considering the convenience of grid discretization, the element size of the random field for the elastic modulus at the top plate of the box-girder is set as 7.3 m  $\times$  20 m, and the transverse direction of the random field is symmetrically meshed along the central line of the bridge. The number of discretized elements is 32, as shown in Figure 12. The meshes of the random fields of the elastic modulus at the bottom plate or the webs are similar to those at the top plate, which are both discretized into 32 elements. The meshes of the random fields for the secondary dead load and live load are the same as those of the elastic modulus random field at the top plate, and the random fields of the structure are divided into 160 random variables. It should be noted that the grid number of the random fields should be consistent with the discrete random variables, whose properties are endued to the corresponding elements of the structural FE model.

Random Field	Mean	COV	Distribution	Correlation Model	Correlation Length
Elastic modulus (GPa)	36.8	0.09	Normal	Exp	40 m
Secondary dead load (kN/m)	60.3	0.11	Normal	Exp	40 m
Live load (kN/m)	21.0	0.19	Normal	Exp	40 m

Table 3. Random field parameters of the Jinghang Canal Bridge.

#### 5.4. Definition of Limit State

The serviceability and ultimate limit states are taken into account in the reliability analysis of this bridge. Under the serviceability limit state, the determination of whether the structure meets the requirements of serviceability is usually based on the fact of whether the maximum deflection of the main girder under load exceeds the limit value specified in the code. For the PSC bridges, the code for highway bridges in China [46] specifies that the maximum deflection of the bridge should not exceed *L*/600 under general operating conditions, where *L* denotes the length of the main span of the bridge. For the Jinghang Canal Bridge, with a main span of 150 m, the allowable deflection is [ $\mu$ ] = 0.25 m. Under the ultimate limit state, the failure state is judged by the maximum compressive stress of the concrete at the bottom of the main pier section of the box girder exceeding its allowable value. The code [46] also specifies that the allowable compressive stress is 0.5 f<sub>ck</sub> (f<sub>ck</sub> is the standard value of concrete used in this bridge. Thus, the well-known performance functions can be expressed as follows:

$$G_1(X) = [\mu] - [\mu]_{\max}(X),$$
(20)

$$G_2(X) = [\sigma] - [\sigma]_{\max}(X), \tag{21}$$

where *X* is a vector consisting of 167 random variables that affect the structural reliability, including the random variables in Table 2 and the discrete variables of the random fields in Table 3;  $\mu_{max}(X)$  represents the midspan deflection of the box-girder, and  $\sigma_{max}(X)$  denotes the compressive stress of the concrete at the bottom flange of the main pier section. Both are implicit functions of X and obtained by finite element analysis.

# 5.5. Results and Discussion.

Based on the proposed method considering random fields in this paper, the reliability evaluation of the Jinghang Canal Bridge was carried out under serviceability and ultimate limit states. The evolution of the reliability index was obtained, as shown in Figures 14 and 15.

As indicated in Figure 14, the concrete creep and shrinkage have a significant effect on the reliability index of the deflection at midspan of the box girder. When the structural parameters are considered as random variables, the reliability index is 6.5 at the time of completion of the bridge, as shown in Figure 14a, implying that the bridge has a high reliability and can satisfy the requirements of serviceability. However, with the development of shrinkage and creep and the increase in the long-term prestress loss, the deflection at the midspan of the box girder continues to increase, resulting in the reduction in the reliability index. In the early stage of bridge operation, the reliability index of the deflection decreases rapidly, which is consistent with the time-dependent development of creep and shrinkage. The reliability index decreases to the target reliability index of 1.5 after approximately 30 years of service. When the influence of a random field is considered, the reliability index falls below the target value of 1.5 after approximately 40 years of service. It can be concluded that the results of the reliability analysis are conservative when the structural parameters are treated as random variables. However, it does not mean that the failure probability of the structure will decrease when the randomness of the spatial distribution of the parameters is considered. In fact, the random field can describe the reliable state of the structure more truthfully. As shown in Figure 14a, the number of

discrete elements in the random field has little influence on the reliability index of deflection, which is similar to the correlation functions of the random field, as shown in Figure 14b. This fact reveals that the reliability index is insensitive to the type of correlation function. The correlation length of the random field has a great influence on the reliability index of the deflection, which decreases with increasing of the correlation length, as shown in Figure 14c. Moreover, it is necessary to take corresponding maintenance and rehabilitation measures before the reliability index falls below the target value, such as tensioning back-up tendons, to ensure serviceability throughout the lifespan of the bridge.



**Figure 14.** Influence of parameters of the random field on the deflection reliability: (**a**) Discrete element; (**b**) correlation model; (**c**) correlation length.



**Figure 15.** Influence of parameters of the random field on the stress reliability: (**a**) Discrete element; (**b**) correlation model; (**c**) correlation length.

As shown in Figure 15, the concrete creep and shrinkage also have a significant effect on the reliability index of stress at the bottom flange of the main pier section. When the parameters affecting the structural reliability are considered as random variables, the reliability index is 8.7 at the time of completion, which demonstrates that the reliability of the bearing capacity is very high and can ensure the safety of the structure, as shown in Figure 15a. Due to the stress redistribution in the section caused by creep and shrinkage and the long-term loss of prestress force, the compressive stress at the bottom flange of the main pier section increases continuously, and the reliability index of the stress decreases continuously. During the 20 years after the completion of the bridge, the reliability index of the stress decreases relatively fast, and then the development of creep and shrinkage gradually slows down and reaches convergence, resulting in the reliability index tending to be stable. This reveals that the decay in the reliability index with time coincides with the time-varying process of shrinkage, creep, and steel relaxation. Compared with the number of discrete elements and the correlation model, the correlation length also has a great influence on the reliability index of the stress, as presented in Figure 15b,c. Meanwhile, it can be seen from Figure 15 that the reliability index is larger than the target value of 4.2 throughout the entire lifespan of the bridge, indicating that there will be no damage to the cross section of the box-girder caused by the crushing of the concrete, and it can satisfy the requirements of the bearing capacity of the bridge.

#### 6. Conclusions

According to the presented study, conclusions are drawn as follows.

In previous reliability analysis of PSC bridges, the parameters affecting structural reliability are usually simulated as uncorrelated random variables while the spatial distribution of structural parameters (e.g., concrete elastic modulus and distributed load, etc.) is neglected. In this paper, a method was proposed for time-dependent reliability evaluation of PSC bridges, in which the influences of random field can be taken into account. Based on the random field theory and IS method, the proposed method keeps the balance between accuracy and efficiency, as demonstrated through three numerical examples.

Concrete creep and shrinkage have a significant impact on the reliability index of the PSC box-girder bridge. Especially in the early stage after bridge completion, the reliability index decreases rapidly with time, which is consistent with the time-dependent development of shrinkage, creep, and steel relaxation. Before the reliability index of deflection falls below the target value of 1.5, some maintenance and rehabilitation measures (e.g., jacking back-up tendons) are required to restrain the long-term deflection and ensure the normal use of the bridge. During the whole service period, the reliability index of stress is larger than the target value of 4.2, indicating that the bridge satisfies the bearing capacity requirements.

The number of discrete elements in the random fields and correlated models has little effect on the deflection and stress reliability index of the PSC box-girder bridge. However, the correlation length has a great influence on the reliability of the structure, and the reliability index decreases with the increasing of the correlation length. It should be noted that the correlation length was used as the assumed values in the reliability analysis in this paper. However, to obtain more precise results for the reliability analysis, the correlation length should be determined by the measured data to describe the random field. Moreover, the local average method of a random field is only applicable to the mesh of a uniform random field, but the discretization of a nonuniform random field deserves further discussion.

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