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# Revisiting Two-Port Network Analysis for Wireless Power Transfer (WPT) Systems

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**Abstract**— This paper discusses the applicability of the two-port network model for the analysis of wireless power transfer (WPT) systems. The two-port network model has been extensively used to analyze microwave circuits, and identical principles have been applied to WPT. The standard gain parameters defined for the two-port networks are critically analyzed with regards to WPT systems. The usefulness and the practical limitations of the gain parameters are highlighted. In addition to power transfer efficiency (PTE), an alternative gain parameter, square voltage gain is defined to estimate the power transfer capability of the WPT system. The proposed performance indices are then computed in terms of generalized two-port network parameters along with a generalized optimization methodology. A case study is presented for the optimization of a WPT system with a repeater.

**Keywords**—Wireless power transfer, Equivalent circuit, Two-port network model

## I. INTRODUCTION

In recent years, wireless power transfer (WPT) technology has become an important topic in industry and academia due to its advantages to many emerging applications [1]-[5]. The specifications and characteristics differ significantly for diverse WPT applications. For example, the power level can be few microwatts in integrated circuit applications [1], or, few watts in consumer electronics and biomedical applications [2], [3], or even hundreds of kilowatts in electric vehicle and industrial applications [4], [5]. Similarly, operating frequency can vary from a few kHz up to MHz range [1]-[5]. Therefore, constraints, performance goals and design methodology for each WPT application are very different from each other. However, a unified two-port network model can be used to define the different performance criteria so that a generalized framework can be developed.

The network model representation of electrical circuits has been well established in microwave engineering. However, the theories developed in microwave engineering cannot be directly applied to the analysis of WPT systems. For example, three main gain parameters: power gain, available gain and transducer gain have been introduced for the analysis of WPT [6], [7]. However, some of these gain parameters have minimal use for practical WPT applications. Therefore, typical gain parameters and the impedance matching principles defined in microwave engineering need to be refined with new metrics related to WPT. This paper discusses the limitations of the classical two-port network analysis as applicable to WPT and

presents a set of modified gain parameters which can apply to the analysis of a practical WPT system.

This paper is organized as follows. Section II introduces the two-port network analysis for WPT systems and highlights the limitations of the gain parameters defined in the literature. An optimization procedure is proposed by considering a set of modified gain parameters in Section III. In Section IV, the results of a case study for a WPT system with a repeater are presented.

## II. THE TWO-PORT NETWORK ANALYSIS

### A. Generalized two-port network

A typical WPT system includes a transmitter (Tx) connected to a power source through a primary compensation network, and a receiver (Rx) connected to the load through a secondary compensation network. Optionally, there can be multiple coils in-between Tx and Rx as repeaters. Primary and secondary compensation networks can be simple series or parallel connected capacitors [8], or complex networks such as LCL, LCCL, and CCL [9], [10]. These compensation networks have also been termed as impedance matching networks [9] or load transformation networks [3] in the literature. Regardless of the complexity of the WPT system, any single-Tx – single-Rx WPT system can be represented as a two-port network. Two-port representation of a generalized WPT scheme is illustrated in Fig. 1.

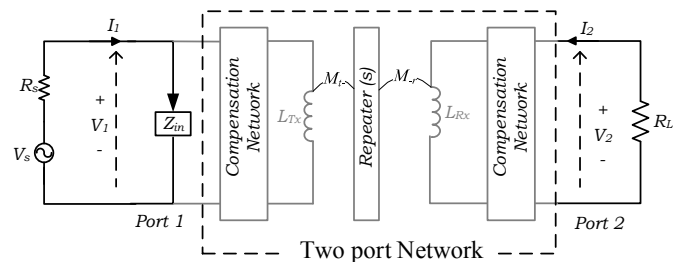


Fig. 1. The generalized equivalent circuit of a WPT system as a two-port network

The two-port network can be modeled using the impedance parameter matrix as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad (1)$$

where  $Z_{11}$ ,  $Z_{12}$ ,  $Z_{21}$ , and  $Z_{22}$  are impedance parameters,  $V_1$  and  $V_2$  are voltages at port 1 and port 2, respectively, and  $I_1$  and  $I_2$  are currents flowing into port 1 and port 2, respectively. If all the parameters of the WPT system are known, then the impedance matrix can be calculated. It should be highlighted that the use of S-parameters may not be ideal for the analysis of WPT systems because the characteristic impedance does not have a direct physical interpretation for WPT systems.

Once the impedance matrix and the load characteristics are known, the performance of the WPT system can be fully characterized. First, the input impedance,  $Z_{in}$ , seen at port 1, can be determined as

$$Z_{in} = \frac{V_1}{I_1} = Z_{11} - \frac{Z_{12}Z_{21}}{Z_{22} + R_L} \quad (2).$$

Afterwards, the input power ( $P_{in}$ ) flowing into the two-port network, and the output power ( $P_{out}$ ) received at the load are calculated using

$$P_{in} = |I_1|^2 \operatorname{Re}\{Z_{in}\} = \frac{|V_s|^2}{|Z_{in} + R_s|^2} \operatorname{Re}\{Z_{in}\} \quad (3)$$

and

$$P_{out} = |I_2|^2 R_L = \frac{|Z_{21}|^2 |V_s|^2}{|(Z_{in} + R_s)(Z_{22} + R_L)|^2} R_L \quad (4)$$

respectively. However, the powers  $P_{in}$  and  $P_{out}$  are dependent on the source voltage, therefore, they cannot be considered as normalized terms for a generalized comparison. Therefore, a number of gain parameters have been defined to represent normalized performance indices.

### B. Performance indices for the analysis of WPT systems

Three gain parameters are typically used in microwave engineering: power gain ( $G_P$ ), available gain ( $G_A$ ) and transducer gain ( $G_T$ ) as

$$G_P = \frac{P_{out}}{P_{in}} = PTE \quad (5)$$

$$G_A = \frac{P_{out-\max}}{P_{s-\max}} = \frac{P_{out-\max}}{|V_s|^2 / 8R_s} \quad (6)$$

$$G_T = \frac{P_{out}}{P_{s-\max}} = \frac{P_{out}}{|V_s|^2 / 8R_s} \quad (7)$$

[6], [7], and they have been applied recently for WPT. Here,  $G_P$  represents power transfer efficiency (PTE) of the WPT systems, therefore, it is a very useful term for the characterization of WPT systems. Gain terms,  $G_A$  and  $G_T$  represent the maximum available output power ( $P_{out-\max}$ ) and

output power ( $P_{out}$ ) at a given operating point normalized to the maximum available input power at the source ( $P_{s-\max}$ ), respectively. However, the use of  $G_A$  and  $G_T$  has practical limitations with regards to WPT because  $P_{s-\max}$  is not practically achievable in most real-world WPT power sources. Firstly, unlike in microwave power amplifiers where source impedance is usually designed to be  $50 \Omega$ , power converters used for the WPT applications exhibit very low source impedance ( $R_s$ ) which is usually considered as zero. Therefore, in practice, if we calculate  $P_{s-\max}$ , it will be very high value (most likely,  $P_{s-\max}$  is out of the rated power of the converter) resulting in low values for  $G_A$  and  $G_T$ . In fact, the source impedance of a practical power converter is not a fixed value as it represents the effective loss component of the converter; it is dependent on the system operating conditions. For example, if one attempts to maximize  $G_A$  by matching the input impedance to the source impedance (at a known operating condition), the converter will reach to its boundaries of the rated power. In such a scenario, the converter losses will be significantly higher which will result in a high equivalent source impedance.

In power electronic converter design, it is not a usual practice to match the impedance of the load to the converter impedance. Instead, voltage and current characteristics are considered. For instance, in case of a battery charging applications, constant voltage and constant current charging are the main design goals.

Therefore, we propose to define square voltage gain ( $G_{V^2}$ ) as an alternative to typical power gains migrated from microwave engineering applications to WPT. If the source impedance is negligibly small (i.e., a nearly ideal voltage source),  $G_{V^2}$  represents the ratio between the output power of the WPT system and the output power when the load is directly connected to the source. Therefore, it compares the power capability of the WPT system with the direct wired connection with the source. Therefore, the source impedance is neglected for the subsequent analysis in this paper.

$$PTE = \frac{P_{out}}{P_{in}} = \frac{|Z_{21}|^2 R_L}{|Z_{22} + R_L|^2 (R_{in} + R_s)} \quad (8)$$

$$G_{V^2} = \frac{|V_2|^2}{|V_1|^2} = \frac{|Z_{21}|^2 R_L^2}{|(Z_{in} + R_s)(Z_{22} + R_L)|^2} \quad (9)$$

The following section discusses the optimization of a WPT system by considering PTE and  $G_{V^2}$  as the performance indices.

### III. OPTIMIZATION OF WPT SYSTEMS

In order to analyze the performance characteristics with respect to the impedance parameters, the equivalent impedance parameters are written as in

$$Z_{11} = r_{11} + jx_{11}, Z_{22} = r_{22} + jx_{22} \quad (10)$$

$$Z_{12} = Z_{21} = r_{12} + jx_{12}$$

where  $r_{ij}$  and  $x_{ij}$  are the real and imaginary parts, respectively. Here, the off-diagonal terms are identical ( $Z_{12}=Z_{21}$ ) because the network is reciprocal.

First, the optimum value of  $x_{22}$  which gives maximum PTE,  $x_{22,PTE,max}$  is computed by equating the derivative to zero, as

$$\frac{\partial(PTE)}{\partial x_{22}} = 0 \rightarrow x_{22,PTE,max} = \frac{r_{12}x_{12}}{r_{11}} \quad (11)$$

and the respective maximum PTE is found using

$$PTE = \frac{r_{11}(x_{12}^2 + r_{12}^2)R_L}{(r_{11}(R_L + r_{22}) + x_{12}^2)(r_{11}(R_L + r_{22}) - r_{12}^2)} \quad (12)$$

For a classical 2-coil WPT,  $x_{22}$  represents the reactive impedance of the Rx coil. According to (11), for the classical 2-coil WPT,  $x_{22,PTE,max}$  is zero, since the value of  $r_{12}$  is zero which coincides with the well-known relation of maximum PTE at the resonance frequency of Rx. It is interesting to note here that the optimal PTE in (12) is not dependent on  $x_{11}$ . However,  $x_{11}$  determines the reactive component of the input impedance. Therefore, the value of  $x_{11}$  can be chosen to make  $Z_{in}$  purely real, as

$$\Im(Z_{in}) = 0 \rightarrow x_{11,\Im(Z_{in})=0} = \frac{x_{22}(x_{12}^2 - r_{12}^2) + 2r_{12}x_{12}(R_L + r_{22})}{(R_L + r_{22})^2 + x_{22}^2} \quad (13)$$

With the optimal values of  $x_{11}$  and  $x_{22}$ ,  $G_{v^2}$  reduces to

$$G_{v^2} = \frac{(r_{12}^2 + x_{12}^2)(r_{12}^2x_{12}^2 + r_{11}^2(R_L + r_{22})^2)R_L^2}{(x_{12}^2 + r_{11}(R_L + r_{22}))^2(r_{11}(R_L + r_{22}) - r_{12}^2)^2} \quad (14)$$

Now, the load characteristics of PTE and  $G_{v^2}$  can be analyzed. The optimal load which gives maximum efficiency can be derived as

$$R_{22,PTE,max} = \frac{\sqrt{(r_{11}r_{22} + x_{12}^2)(r_{11}r_{22} - r_{12}^2)}}{r_{11}} \quad (15)$$

If the load is assumed to be at its optimal value, the maximum efficiency ( $PTE_{max}$ ) can be derived as

$$PTE_{max} = 1 - \frac{2}{1 + \sqrt{1 + \left(\frac{x_{12}^2 + r_{12}^2}{r_{11}r_{22} - r_{12}^2}\right)}} \quad (16)$$

It can be seen that higher  $r_{12}$  is always better to obtain higher efficiency. The upper limit for  $r_{12}$  is  $\sqrt{r_{11}r_{22}}$ .

On the other hand, it can be observed from (14) that  $G_{v^2}$  is monotonically increasing with the increase of load resistance. Therefore, in terms of high  $G_{v^2}$ , higher load resistance is

always better. However, in practice, it is not required to maximize the voltage ratio. Instead, it is only required to keep the voltage ratio at a realistic value close to unity.

One cannot maximize both PTE and the voltage ratio at the same time. The maximum efficiency condition may result in a lower voltage ratio. Therefore, a balance between PTE and  $G_{v^2}$  should be considered. Probably the best approach is to fix the target voltage ratio and optimize the remaining impedance parameters to obtain maximum efficiency. As an example, the following section presents an optimization of a WPT system with a repeater.

#### IV. A CASE STUDY: WPT WITH A REPEATER

The general two-port analysis is applied to a WPT system with a repeater where an intermediate repeater coil is introduced in-between Tx and Rx as illustrated in Fig. 2 (a). Tx, Rx, and repeater are made of spiral coils as illustrated in Fig. 2 (b) with its design parameters given in Table 1. Transfer distance ( $z_{rx}$ ) is considered as the unobstructed axial separation between the repeater and Rx. The axial distance between Tx and repeater is denoted as  $z_x$ . The equivalent circuit of the WPT system is shown in Fig. 3.

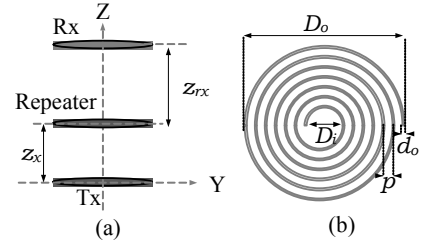


Fig. 2. WPT system with a single repeater as a relay element; (a) – coil arrangement, and (b) – coil design parameters

The system equation is defined in (17) using the equivalent circuit theory.

$$\begin{bmatrix} V_1 \\ 0 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_t & j\omega M_{t,x} & j\omega M_{t,r} \\ j\omega M_{t,x} & Z_x & j\omega M_{x,r} \\ j\omega M_{t,r} & j\omega M_{x,r} & Z_r \end{bmatrix} \begin{bmatrix} I_t \\ I_x \\ I_r \end{bmatrix} \quad (17)$$

$$\text{where } Z_k = R_k + jX_k, \quad X_k = \omega L_k - \frac{1}{\omega C_k}$$

$$V_1 = V_s, \text{ and } V_r = I_r R_L$$

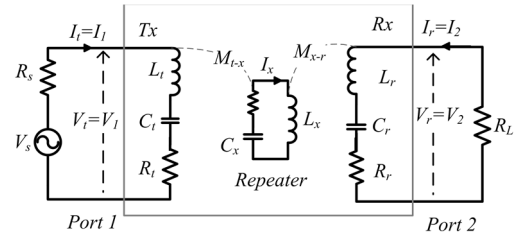


Fig. 3. The equivalent circuit and two-port model illustration

The diagonal elements of the impedance matrix,  $Z_k$  ( $k=t, x, r$ ) are the self-impedances of Tx, Rx and repeater coils, while  $L_k$ ,  $C_k$ , and  $R_k$  are the inductance, capacitance, and

parasitic resistance, respectively. All three coils are connected with a series capacitor to provide respective reactive impedance,  $X_k$  ( $k=t, x$ , and  $r$ ) at the operating frequency ( $f_s$ ).  $j\omega M_{k,l} = j\omega M_{l,k}$  ( $k, l=t, x$ , and  $r$ ), where  $M_{k,l} = M_{l,k}$  is the mutual inductance between  $k^{\text{th}}$  coil and  $l^{\text{th}}$  coil, and  $\omega$  is the operating angular frequency of the source.  $R_L$  is the equivalent load resistance. Once the system equation is defined, currents in each coil ( $I_k$ ;  $k=t, x$ , and  $r$ ) can be determined. Subscripts  $t, x$ , and  $r$  represent Tx, repeater, and Rx, respectively.

Next, using the system equations (17) and (1), two-port network impedance parameters can be calculated as

$$\begin{aligned}
 r_{11} &= R_t + \frac{R_x (\omega M_{t,x})^2}{R_x^2 + X_x^2} \\
 r_{22} &= R_r + \frac{R_x (\omega M_{x,r})^2}{R_x^2 + X_x^2} \\
 r_{12} &= \frac{R_x \omega^2 M_{t,x} M_{x,r}}{R_x^2 + X_x^2} \\
 x_{11} &= X_t - \frac{X_x (\omega M_{t,x})^2}{R_x^2 + X_x^2} \\
 x_{22} &= X_r - \frac{X_x (\omega M_{x,r})^2}{R_x^2 + X_x^2} \\
 x_{12} &= \omega M_{t,r} - \frac{X_x \omega^2 M_{t,x} M_{x,r}}{R_x^2 + X_x^2}
 \end{aligned} \tag{18}$$

For simplicity, all the coils, Tx, Rx, and repeater are considered to be identical, and the coil specifications are detailed in Table 1. With the assumption of identical coils, coil resistances are kept identical; therefore,  $R_t = R_r = R_x = R$ . It can be seen from (18) that the maximum  $r_{12}$  is obtained when reactive impedance of the repeater,  $X_x$ , is zero. In other words, repeater resonance frequency should be tuned to the operating frequency. With these simplifications, (18) can be reduced to

$$\begin{aligned}
 r_{11} &= R + \frac{(\omega M_{t,x})^2}{R}, \quad r_{22} = R + \frac{(\omega M_{x,r})^2}{R} \\
 r_{12} &= \frac{(\omega M_{t,x})(\omega M_{x,r})}{R} \\
 x_{11} &= X_t, \quad x_{22} = X_r, \quad x_{12} = \omega M_{t,r}
 \end{aligned} \tag{19}$$

Then, the optimization procedure is presented for nominal transfer distance  $z_{rx}$  of 150 mm. First, as seen in (19), the optimal values of  $x_{11}$  and  $x_{22}$  can easily be achieved by tuning the reactive impedances of Tx and Rx, respectively. The values of  $r_{12}$ ,  $r_{11}$ ,  $r_{22}$ , and  $x_{12}$ , are determined by the mutual inductance between the coils. Therefore, the position of the repeater is chosen as the optimization variable for obtaining the best

possible performance. The mutual inductance variation of the selected coils with respect to the axial distance is shown in Fig. 4. The variation of PTE and  $G_{V^2}$  with respect to the repeater position ( $z_x$ ) and load resistance is shown in Fig. 5 and Fig. 6, respectively. It can be seen that PTE is less dependent on the position of the repeater, however,  $G_{V^2}$  is severely affected by the position of the repeater.

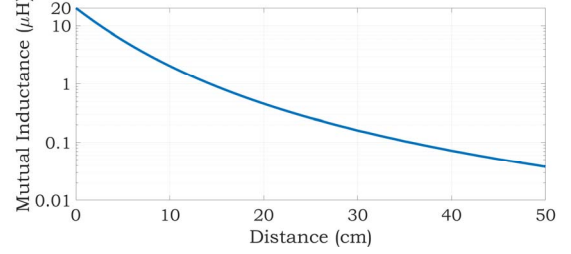


Fig. 4. Mutual inductance variation with respect to axial distance between coils

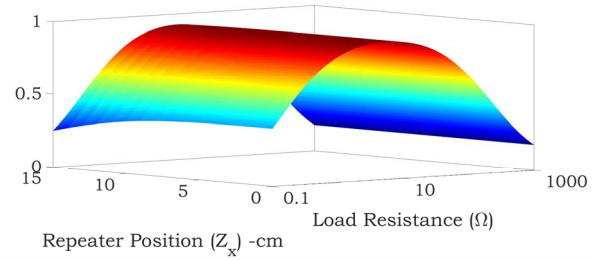


Fig. 5. PTE variation with respect to repeater position ( $z_x$ ) and load resistance ( $R_L$ )

For example, Fig. 7 shows PTE and  $G_{V^2}$  variation with respect to the position of the repeater for selected set values of the load. If the repeater position is optimized to obtain the maximum efficiency, then the voltage ratio can be very low, indicating very little practical use. On the other hand, if the design focus is only on the voltage ratio, efficiency can be low. Therefore, first, a threshold for the required voltage ratio should be chosen depending on the application criterion. For example, if the voltage ratio is chosen to be unity when  $z_x$  of 15 cm, with a load resistance of 10  $\Omega$ , PTE can be as high as 93% at 15 cm transfer distance. Such an operating point will have a reasonably good PTE above 90% and unity square voltage gain close to unity.

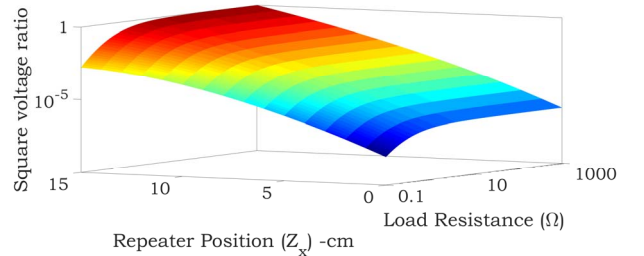


Fig. 6. Square voltage gain ( $G_{V^2}$ ) variation with respect to repeater position ( $z_x$ ) and load resistance ( $R_L$ )

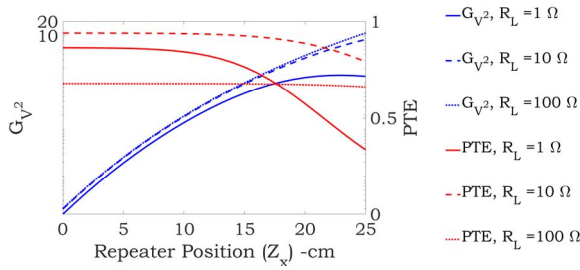


Fig. 7. PTE and  $G_{V2}$  variations with respect to the position of the repeater

Table 1. Design parameters of the WPT system and the coils

WPT System		WPT coils	
Parameter	Value	Parameter	Value
Operating frequency	1 MHz	Outer diameter ( $D_o$ )	180 mm
Transfer distance ( $z_{rx}$ )	150 mm	Turn pitch ( $p$ )	3 mm
Self-inductance	20 $\mu$ H	Wire diameter ( $d_o$ )	2 mm
AC resistance at 1MHz ( $R$ )	150 m $\Omega$	Number of turns ( $N$ )	14
Repeater position ( $z_x$ )	0 – 100 mm	Inner diameter ( $D_i$ )	50 mm

If  $z_x$  is chosen to be 15 cm, the values of  $r_{12}$ ,  $r_{11}$ ,  $r_{22}$ , and  $x_{12}$  can be calculated using (19). Therefore, the optimal values for the reactive impedances of Tx and Rx  $x_{11}$  and  $x_{22}$  can be calculated using (13) and (11). Finally, the self-resonance frequency of Tx and Rx can be calculated to provide respective reactive impedances. The calculated numerical values are summarized in Table 2.

Table 2. Optimized parameters of the WPT system

Parameter	Value
$r_{11}=r_{22}$	213.01 $\Omega$
$r_{22}$	212.86 $\Omega$
$x_{11}=X_f$	0.9987 $\Omega$
$x_{22}=X_f$	0.9980 $\Omega$
The self-resonance frequency of Tx and Rx	996 kHz

## V. CONCLUSION

This paper presented an analysis of the two-port network model applicable to wireless power transfer (WPT) systems, in general. The two-port network model can be used to analyze any complex variations of single-transmitter-single-receiver WPT systems, however, the performance indices should be carefully chosen for the system optimizations. We have shown that the classical gain parameters defined for two-port networks

in microwave engineering may not be suitable to describe and optimize the performance of a practical WPT system. We propose as crucial performance evaluation criteria for a WPT system, the power transfer efficiency (PTE) and square voltage gain, which can be analyzed using two-port network model. A case study is presented for the optimization of a WPT system with a repeater.

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