Jooshaki, Mohammad; Abbaspour, Ali; Fotuhi-Firuzabad, Mahmud; Moeini-Aghtaie, Moein; Lehtonen, Matti

# MILP Model of Electricity Distribution System Expansion Planning Considering Incentive Reliability Regulations

*Published in:*
IEEE Transactions on Power Systems

*DOI:*
10.1109/TPWRS.2019.2914516

Published: 01/11/2019

*Document Version*
Peer-reviewed accepted author manuscript, also known as Final accepted manuscript or Post-print

*Please cite the original version:*
Jooshaki, M., Abbaspour, A., Fotuhi-Firuzabad, M., Moeini-Aghtaie, M., & Lehtonen, M. (2019). MILP Model of Electricity Distribution System Expansion Planning Considering Incentive Reliability Regulations. *IEEE Transactions on Power Systems*, *34*(6), 4300-4316. Article 8704888.
https://doi.org/10.1109/TPWRS.2019.2914516

# MILP Model of Electricity Distribution System Expansion Planning Considering Incentive Reliability Regulations

Mohammad Jooshaki, Ali Abbaspour, Mahmud Fotuhi-Firuzabad, *Fellow, IEEE,* Moein Moeini-Aghtaie, *Member, IEEE*, and Matti Lehtonen

***Abstract*—This paper aims at proposing a mixed-integer linear formulation to incorporate reliability oriented costs into the expansion planning model of electricity distribution networks. In this respect, revenue lost associated with the undelivered energy caused by network interruptions, as well as costs incurred by the widely-used reward-penalty regulations are considered as the major reliability related costs from distribution companies point of view. A set of mixed-integer linear equations is proposed to calculate the most common distribution system reliability indices, i.e. EENS, SAIFI, and SAIDI. It is found that these equations can also facilitate the formulation of radiality constraint in the presence of DG units. Moreover, application of the proposed method is investigated through various case studies performed on two test distribution networks with 24 and 54 nodes.**

***Index Terms*—Distribution system expansion planning, incentive reliability regulations, mixed-integer linear programming (MILP), reliability, reward-penalty scheme.**

## NOMENCLATURE

*Indices*

| | |
|---|---|
| $b$ | Index for feeder sections. |
| $k$ | Index for alternatives for investing on a network asset. |
| $ll$ | Load level. |
| $lp$ | Load points. |
| $ri$ | Index for reliability index. |
| $s$ | Index for substations. |
| $t$ | Index for time stages of planning horizon. |

*Sets*

| | |
|---|---|
| $\Gamma_b^{Fe}$ | Set of candidate alternatives for reinforcement of the existing feeder in the path of branch $b$. |
| $\Lambda_{lp}^{DG}$ | Set of candidate DG units for installation at bus $lp$. |
| $\Pi$ | Set of distribution network branches. $\Pi=\{\Pi^c, \Pi^f, \Pi^r\}$, where $\Pi^c$, $\Pi^f$, $\Pi^r$ are sets of candidate branches for construction of new feeders, branches with fixed feeders, and candidate branches for reinforcement of existing feeders, respectively. |
| $\Theta$ | Set of substations. |
| $\Upsilon_b^{Fe}$ | Set of candidate alternatives for construction of a new feeder in the path of branch $b$. |
| $\Upsilon_s^{STr}$ | Candidate transformer alternatives for substation $s$. |
| $\Omega$ | Set of distribution network nodes. $\Omega=\{\Omega^D, \Omega^S\}$, where $\Omega^D$, $\Omega^S$ are sets of demand and substation nodes. |
| $\Omega^{DG}$ | Set of candidate nodes for installation of DG units. |
| $\Xi$ | Set of reliability indices. |
| $\Psi^{lp}$ | Set of branches connected to load point $lp$. |

*Parameters*

| | |
|---|---|
| $\mathrm{Cap}_k^{\mathrm{DG}}$, $\mathrm{Cap}_{s,k}^{\mathrm{STr}}$ | Power capacity of DG units and substation transformers. |
| $\mathrm{CC}_{b,k}^{\mathrm{Fe}}$, $\mathrm{RC}_{b,k}^{\mathrm{Fe}}$ | Construction/reinforcement costs of feeders. |
| $\mathrm{CEC}_s^{\mathrm{Sub}}$ | Construction cost of new substations or expansion cost of existing ones. |
| $\mathrm{D}_{lp,t,ll}$, $\mathrm{Du}_{t,ll}$ | Electricity demand, and demand duration. |
| $\mathrm{ERev}_{t,ll}$ | Expected revenue from delivering energy to customers ($/MWh) |
| $\mathrm{I}_b^{\max}$, $\mathrm{I}_{b,k}^{\max}$ | Maximum current capacity of feeders. |
| $\mathrm{IC}_{s,k}^{\mathrm{STr}}$, $\mathrm{IC}_{lp,k}^{\mathrm{DG}}$ | Investment cost of various alternatives for substation transformers and DG units. |
| $\mathrm{InCa}_s^{\mathrm{Sub}}$ | Initial capacity of substations. |
| $\mathrm{IPR}_{ri,t}$, $\mathrm{IRR}_{ri,t}$ | Incentive rates of penalty and reward zones. |
| M | A big number. |
| $\mathrm{MC}_{s,k}^{\mathrm{STr}}$, $\mathrm{MC}_k^{\mathrm{DG}}$ | Maintenance cost of substation transformers, and DG units. |
| $\mathrm{N_L}$ | Number of load levels. |
| $\mathrm{N}_{lp,t}$ | Number of customers at load point $lp$. |
| $\mathrm{OC}_{(.)}^{(.)}$, $\mathrm{OC}_{(.),(.)}^{(.)}$ | Operating cost. |
| $\mathrm{PC}_{k,t,ll}^{\mathrm{DG}}$ | Production cost of DG units. |
| $\mathrm{PCap}_{ri,t}$, $\mathrm{RCap}_{ri,t}$ | Penalty and reward caps. |
| pf | Power factor. |
| $\mathrm{PL^{DG}}$ | Penetration limit of DGs in the network. |
| $\mathrm{PP}_{ri,t}$, $\mathrm{RR}_{ri,t}$ | Penalty point and reward point. |
| $\mathrm{Pr}_{t,ll}$ | Grid electricity price ($/kWh) |
| $\mathrm{PVF}_t^{\mathrm{Inv}}$, $\mathrm{PVF}_t^{\mathrm{Op}}$ | Present value factors for investment and operating costs. |
| r | Interest rate. |
| $\mathrm{rr}_k^{(.)}$, $\mathrm{UL}_k^{(.)}$ | Capital recovery rate of investment and useful lifetime of various assets. |
| $\mathrm{V^{min}}$, $\mathrm{V^{max}}$, $\mathrm{V^r}$ | Lower bound, upper bound, and rated values of nodal voltages. |
| $\chi_{lp,b}$ | Element of node-branch incidence matrix of distribution network, which is -1 or +1 if branch $b$ is connected to load point $lp$ and the predetermined current or flow direction is toward or away from node $lp$, respectively, and is 0, otherwise. It should be noted that the feeders and nodes which are not initially existent (i.e. are candidate for adding to the network) must also be included in the incidence matrix. Hence, elements of this matrix are initially determined and do not change during the optimization process. |
| $\lambda_{b,(.)}^{(.),(.)}$, $\mathrm{r}_{b,(.)}^{(.),(.)}$ | Failure rate and repair time of branches. |
| $\mu_{t,ll}^{\mathrm{Dem}}$ | Average demand of network customers. |

| | |
|---|---|
| $\xi_{lp,s}$ | A binary parameter which is 1 if substation $s$ is at load point $lp$, being 0 otherwise. |
| $\theta_s$ | A binary parameter which is 1 if substation $s$ is initially existent, being 0 otherwise. |

*Variables*

| | |
|---|---|
| $BENS_{b,t,ll}$, $BSAIFI_{b,t}$, $BSAIDI_{b,t}$ | Positive variables related to the portion of EENS, SAIFI, and SAIDI indices caused by outage of branch $b$. |
| $CCN_{b,t}$ | Total number of curtailed customers due to outage of branch $b$. |
| $CP_{b,t,ll}$ | Curtailed power due to outage of branch $b$. |
| $CRR_t$ | Cost associated with reliability regulations. |
| $DCN_{b,t}$, $DD_{b,t,ll}$, $DDGC_{b,t}$ | Sum of customer number, power demand, and installed DG capacity of the load points downstream of branch $b$. |
| $DCN_{b,t}^{+/-}$, $DD_{b,t,ll}^{+/-}$, $DDGC_{b,t}^{+/-}$ | Auxiliary positive variables used for calculation of $DCN_{b,t}$, $DD_{b,t,ll}$, and $DDGC_{b,t}$. |
| $EP_{lp,t,ll}^{Cur}$ | Curtailed power demand at load point $lp$. |
| $g_{lp,k,ll}^{DG}$, $g_{s,t,ll}^{Sub}$ | Electricity power injected by DG units and substations. |
| $GCN_{s,t}^{Sub}$, $GDD_{s,t,ll}^{Sub}$, $GDDGC_{s,t}^{Sub}$ | Sum of the power demand, customer number, and DG capacity of the load points connected to substation $s$. |
| $I_{b,t,ll}$, $V_{b,t,ll}$ | Magnitudes of branch currents and nodal voltages. |
| $Inv_t$, $Op_t$ | Investment and operating costs at stage $t$. |
| $LPM_{lp,t}$ | A binary variable associated with the load point mode, which is 1 when load point $lp$ is in-service at stage $t$, being 0 otherwise. |
| $Pen_{ri,t}$, $Rev_{ri,t}$ | Penalty and reward related to the incentive scheme on reliability index $ri$. |
| $RI_{ri,t}$ | Reliability index. |
| $RLUE_t$ | Revenue lost due to undelivered energy. |
| $RRC_t$ | Reliability related costs at stage $t$. |
| $UPen_{ri,t}$, $URew_{ri,t}$ | Auxiliary variables employed for calculation of penalty and rewards. |
| $\varphi_{(.),t}^{Fe,(.)}$, $\varphi_{(.),k,t}^{Fe,(.)}$ | Binary utilization variables for feeders, which take value 1 if associated feeder is in-use. |
| $v_{lp}$, $\delta_{lp}$ | Average number and duration of yearly interruptions of load point $lp$. |
| $\vartheta_{ri,t}^{p}$, $\vartheta_{ri,t}^{r}$, $\vartheta_{ri,t}^{pc}$, $\vartheta_{ri,t}^{rc}$ | Binary variables which take value 1 if the reliability index $ri$ is in the penalty zone, reward zone, penalty cap region, and reward cap region, respectively, being 0 otherwise. |
| $\rho_{b,t}^{(.)}$, $\rho_{b,t,ll}^{(.)}$ | Auxiliary binary variables. |
| $\sigma_{lp,k,t}^{DG}$ | Binary investment variables for installation of DG units. |
| $\sigma_{b,k,t}^{Fe,Ne}$, $\sigma_{b,k,t}^{Fe,Re}$ | Binary investment variables for construction of new feeders, or reinforcement of existing ones. |
| $\sigma_{s,k,t}^{STr}$ | Binary investment variables for installation of new substation transformers. |
| $\sigma_{s,t}^{Sub}$ | Binary investment variables for construction of new substations or expansion of existing ones. |

## I. Introduction

NOWADAYS, electricity power plays such an accentuated role in human's life that even small interruptions cannot be tolerated. Therefore, power system reliability has become an inevitable factor in power system studies. More specifically, since majority of customer interruptions are originated from distribution network failures, reliability considerations in this system has been being of great interest [1]. In this respect, reliability incentive regulations in the form of reward-penalty schemes (RPSs) and minimum quality standards have been implemented in many countries [2, 3]. According to a recent report by the Council of European Energy Regulators (CEER), incentive reliability regulations have been introduced in electricity distribution section of 16 out of 26 reviewed countries [2]. Moreover, most of the other European countries have plans or intentions to implement such regulations [2].

Importance of reliability considerations has also been reflected in the research publications, as well. In this regard, continuity of electricity supply is usually included as an unavoidable term in distribution system planning and operation studies. Radial distribution system reliability calculations can be typically performed through some linear equations as long as the network topology is definite. As an example, in [4] a mixed integer linear programming (MILP) model is proposed for optimal maintenance scheduling of electricity distribution companies considering the reliability constraints, and interruption costs. However, this is not the case in distribution network expansion planning (DNEP) problem for which the network topology is initially unknown. In other words, in such studies, the optimal network topology is the final solution of the problem which causes complexities and nonlinearities in formulation of reliability indices. Hence, in the existing power distribution planning literature, the incorporation of reliability related costs is mostly found in nonlinear models, for which obtaining the global optimal solution cannot be guaranteed. For instance, in [5] a multi-objective multi-stage model is developed for DNEP. In this model, which is in form of a mixed-integer nonlinear programming (MINLP) problem, not-served energy cost is calculated through nonlinear equations, and considered as one of the objectives. In [6], a novel framework is proposed for planning a distribution network considering automation capabilities. In this framework a new efficient method is proposed for calculation of reliability indices. However, this nonlinear model can only be solved by heuristic methods. Authors in [7], have developed a multistage model for network expansion. In this paper, not only the interruption cost is taken into account, but also maximum limits are considered for system average interruption duration index (SAIDI), and average energy not supplied (AENS) indices. Nonetheless, this model is also formed as a MINLP problem which has been solved by genetic algorithm (GA). Influence of load-point reliability indices and customer choices on reliability (CCOR) on DNEP has been conducted in [8], where a stochastic-programming-based model is proposed to obtain the optimal expansion plan for primary distribution network.

A pioneer effort for considering the reliability alongside the linearized DNEP model is done in [9]. In this model, a pool of solutions, in addition to the global optimal expansion plan, is obtained at the first stage. Then, the reliability related costs are calculated for all these solutions. This allows decision maker to analyze and choose the preferred solution from both expansion costs and reliability point of views. This method has further been adopted in [10], where a mixed-integer linear programming (MILP) model is proposed to carry out stochastic-programming-based distribution system planning. Nonetheless, this method does not necessarily obtain the global optimal expansion plan.

A ground breaking research on this subject can be found in [11], where some linear equations are derived to calculate distribution network reliability indices. This model is further employed in [12] to incorporate reliability related costs into the MILP model of DNEP. However, this model can only be applied to passive networks, i.e. effects of DG units on reliability indices are disregarded. Furthermore, reward-penalty schemes which are considered as a widely used incentive signal for motivating distribution companies to pay attention to their service reliability have not been modeled in DNEP literature.

Motivated by these points, this paper proposes a novel MILP model for incorporation of reliability indices into the DNEP studies. In this respect, taking into account the integration of DG units, some linear equations are derived to calculate the most common reliability indicators in distribution level: i.e. SAIDI, system average interruption frequency index (SAIFI), and energy not supplied (ENS). Furthermore, in order to capture the effect of reliability indices on the optimal planning problem, a novel mixed-integer linear formulation is proposed to model costs associated with reward-penalty schemes. The proposed multistage model takes into account costs of installation and reinforcement of substations and feeders, as well as investment and operating costs of distributed generation (DG) units, revenue lost due to undelivered energy, and rewards or penalties associated with the implementation of incentive reliability regulations. Accordingly, main contributions of this paper are as follows:

1) Proposing a novel optimization-based MILP formulation for reliability evaluation of radially-operated meshed-designed distribution networks considering integration of DG units.
2) Deriving an efficient MILP model for reward-penalty scheme.
3) Developing a new reliability-based DNEP model taking into account various reliability related costs, including incentives from reward-penalty regulations as well as revenue lost due to undelivered energy.

## II. Methodology

In this paper, a new linearized model is presented for expansion planning studies of distribution systems. Objective function of this model to be minimized is comprised of investment and operating costs of three major assets, i.e. substations, feeders, and DG units, as well as reliability related costs, as follows (It should be noted that in all the equations of this paper, variables are in italic, while non-italic letters represent the parameters):

$$\min OF = \sum_{t=1}^{T}\left(\mathrm{PVF}_t^{\mathrm{Inv}} Inv_t + \mathrm{PVF}_t^{\mathrm{Op}}\left(Op_t + RRC_t\right)\right) \quad (1)$$

$$\mathrm{PVF}_t^{\mathrm{Inv}} = \frac{1}{\mathrm{r}\left(1+\mathrm{r}\right)};\ \mathrm{PVF}_t^{\mathrm{Op}} = \begin{cases} \left(1+\mathrm{r}\right)^{-t} & : \forall t \in \{1,\dots,T-1\} \\ \left(1+\mathrm{r}\right)^{-(t-1)}/\mathrm{r} & : t = T \end{cases} \quad (1a)$$

As in [9, 10, 13], present value factors are calculated based on a perpetual or infinite planning horizon. In other words, it is assumed that each asset will be replaced by the same one after reaching the end of its lifetime, and the operating and interruption costs of the last time stage, $T$, will be repeated in the following years. In the rest of this section, various components of the objective function will be explained in detail.

### A. Investment Cost

As expressed in (2), this term includes cost of reinforcing the existing, and adding new feeders and substations, as well as installing new DG units. For the sake of clarity, various network assets considered in the model are illustrated in Fig. 1. The gray shaded items indicate the assets on which investment can be made. Moreover, abbreviated form of each item, which is used as superscript of the parameters and variables, is mentioned in the parenthesis.

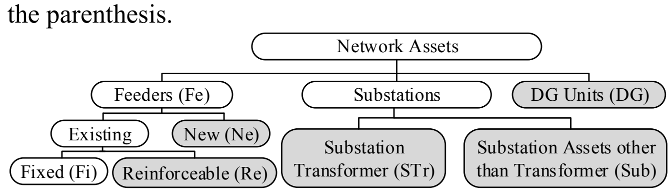


Fig. 1. Network assets considered in the DSEP model of this paper.

As can be inferred from (2), various alternatives are considered for each network asset. Take a feeder section $b$, which is candidate for construction, i.e., $b \in \Pi^c$, as an example: a set of alternatives, $\Upsilon_b^{Fe}$, is assumed for adding this new feeder so that these alternatives can have various technical and economical characteristics such as investment cost, current flow capacity, impedance, and failure rate.

$$\begin{aligned} Inv_t = & \sum_{b\in\Pi^c}\sum_{k\in\Upsilon_b^{Fe}} \mathrm{rr}_k^{\mathrm{Fe}}\mathrm{CC}_{b,k}^{\mathrm{Fe}}\sigma_{b,k,t}^{Fe,Ne} + \sum_{b\in\Pi^r}\sum_{k\in\Gamma_b^{Fe}} \mathrm{rr}_k^{\mathrm{Fe}}\mathrm{RC}_{b,k}^{\mathrm{Fe}}\sigma_{b,k,t}^{Fe,Re} \\ & + \sum_{s\in\Theta}\sum_{k\in\Upsilon_s^{STr}} \mathrm{rr}_k^{\mathrm{STr}}\mathrm{IC}_{s,k}^{\mathrm{STr}}\sigma_{s,k,t}^{STr} + \sum_{s\in\Theta}\mathrm{rr}_k^{\mathrm{Sub}}\mathrm{CEC}_s^{\mathrm{Sub}}\sigma_{s,t}^{Sub} \\ & + \sum_{lp\in\Omega^{DG}}\sum_{k\in\Lambda_{lp}^{DG}} \mathrm{rr}_k^{\mathrm{DG}}\mathrm{IC}_{lp,k}^{\mathrm{DG}}\sigma_{lp,k,t}^{DG} \end{aligned} \quad (2)$$

$$\mathrm{rr}_k^{(.)} = \frac{\mathrm{r}\left(1+\mathrm{r}\right)^{\mathrm{UL}_k^{(.)}}}{\left(1+\mathrm{r}\right)^{\mathrm{UL}_k^{(.)}} - 1} \quad (2a)$$

As in [9, 10, 13, 14], assuming that during the planning horizon only one investment is allowed for each network asset, the problem is subject to the following constraints:

$$\sum_{t=1}^{T}\sum_{k\in\Upsilon_b^{Fe}} \sigma_{b,k,t}^{Fe,Ne} \le 1; \quad \forall b \in \Pi^c \quad (3a)$$

$$\sum_{t=1}^{T}\sum_{k\in\Gamma_b^{Fe}} \sigma_{b,k,t}^{Fe,Re} \le 1; \quad \forall b \in \Pi^r \quad (3b)$$

$$\sum_{t=1}^{T}\sum_{k\in\Upsilon_s^{STr}} \sigma_{s,k,t}^{STr} \le 1; \quad \forall s \in \Theta \quad (3c)$$

$$\sum_{t=1}^{T} \sigma_{s,t}^{Sub} \le 1; \quad \forall s \in \Theta \quad (3d)$$

$$\sum_{t=1}^{T} \sigma_{lp,k,t}^{DG} \le 1; \quad \forall lp \in \Omega^{DG}, \forall k \in \Lambda_{lp}^{DG}. \quad (3e)$$

Moreover, installation of a substation transformer is only possible after construction or expansion of associated substation:

$$\sigma_{s,k,t}^{STr} \le \sum_{\tau=1}^{t} \sigma_{s,\tau}^{Sub}; \quad \forall s \in \Theta,\ \forall k \in \Upsilon_s^{STr},\ \forall t \in \{1,...,T\}. \tag{3f}$$

## B. Operating Cost

Total cost of distribution system operation is comprised of operating cost of in-use feeders, maintenance cost of substations, cost of purchasing energy from the power grid, as well as maintenance and production costs of DG units, which is formulated in (4).

$$\begin{aligned} Op_t = & \sum_{b\in\Pi^f} \mathrm{OC}_b^{\mathrm{Fe}} \varphi_{b,t}^{Fe,Fi} + \sum_{b\in\Pi^c} \sum_{k\in\Upsilon_b^{Fe}} \mathrm{OC}_{b,k}^{\mathrm{Fe}} \varphi_{b,k,t}^{Fe,Ne} \\ & + \sum_{b\in\Pi^r} \left( \mathrm{OC}_b^{\mathrm{Fe}} \varphi_{b,t}^{Fe,Re} + \sum_{k\in\Gamma_b^{Fe}} \mathrm{OC}_{b,k}^{\mathrm{Fe}} \varphi_{b,k,t}^{Fe,Re} \right) \\ & + \sum_{s\in\Theta} \sum_{k\in\Upsilon_s^{STr}} \left( \mathrm{MC}_{s,k}^{\mathrm{STr}} \sum_{\tau=1}^{t} \sigma_{s,k,\tau}^{STr} \right) + \sum_{s\in\Theta} \sum_{ll=1}^{N_L} \left( g_{s,t,ll}^{Sub} \mathrm{Du}_{t,ll} \mathrm{Pr}_{t,ll} \right) \\ & + \sum_{lp\in\Omega^{DG}} \sum_{k\in\Lambda_{lp}^{DG}} \sum_{ll=1}^{N_L} \left( g_{lp,k,t,ll}^{DG} \mathrm{Du}_{t,ll} \mathrm{PC}_{k,t,ll}^{\mathrm{DG}} \right) + \sum_{lp\in\Omega^{DG}} \sum_{k\in\Lambda_{lp}^{DG}} \left( \mathrm{MC}_k^{\mathrm{DG}} \sum_{\tau=1}^{t} \sigma_{lp,k,\tau}^{DG} \right) \end{aligned} \tag{4}$$

Network operation is also subject to a variety of constraints. The first set of constraints is related to the feeder utilization: a feeder can only be utilized after its related investment is performed:

$$\varphi_{b,t}^{Fe,Re} \le 1 - \sum_{\tau=1}^{t} \sum_{k\in\Gamma_b^{Fe}} \sigma_{b,k,\tau}^{Fe,Re}; \quad \forall b \in \Pi^r,\ \forall t \in \{1,...,T\} \tag{4a}$$

$$\varphi_{b,k,t}^{Fe,Re} \le \sum_{\tau=1}^{t} \sigma_{b,k,\tau}^{Fe,Re}; \quad \forall b \in \Pi^r,\ k \in \Gamma_b^{Fe}, \forall t \in \{1,...,T\} \tag{4b}$$

$$\varphi_{b,k,t}^{Fe,Ne} \le \sum_{\tau=1}^{t} \sigma_{b,k,\tau}^{Fe,Ne}; \quad \forall b \in \Pi^c,\ \forall k \in \Upsilon_b^{Fe},\ \forall t \in \{1,...,T\}. \tag{4c}$$

Constraint (4a) expresses that the base condition of a reinforceable feeder, can only be used before its reinforcement. Moreover, expressions (4b) and (4c) imply that a given candidate alternative is available just after making the associated investment. Limits on capacity of substations is also formulated in (4d). Note that the initial capacities, $\mathrm{InCa}_s^{\mathrm{Sub}}$, of substations that are candidate for construction are zero.

$$\frac{g_{s,t,ll}^{Sub}}{\mathrm{pf}} \le \mathrm{InCa}_s^{\mathrm{Sub}} + \sum_{k\in\Upsilon_s^{STr}} \sum_{\tau=1}^{t} \mathrm{Cap}_{s,k}^{\mathrm{STr}} \sigma_{s,k,\tau}^{STr}; \quad \forall s \in \Theta, \forall t \in \{1,...,T\}, \forall ll \in \{1,...,N_l\} \tag{4d}$$

Power generation restrictions of DG units with respect to their installed capacity and penetration limits are written as (4e) and (4f), respectively.

$$\frac{g_{lp,k,t,ll}^{DG}}{\mathrm{pf}} \le \sum_{\tau=1}^{t} \mathrm{Cap}_k^{\mathrm{DG}} \sigma_{lp,k,\tau}^{DG}; \quad \forall lp \in \Omega^{DG},\ \forall k \in \Lambda_{lp}^{DG},\ \forall t \in \{1,...,T\}, \forall ll \in \{1,...,N_l\} \tag{4e}$$

$$\sum_{lp\in\Omega^{DG}} \sum_{k\in\Lambda_{lp}^{DG}} g_{lp,k,t,ll}^{DG} \le \mathrm{PL}^{\mathrm{DG}} \sum_{lp\in\Omega^{D}} \mathrm{D}_{lp,t,ll}; \forall t \in \{1,...,T\}, \forall ll \in \{1,...,N_l\} \tag{4f}$$

Moreover, network operation is subject to some technical constraints associated with power flow equations. In this respect, the Kirchhoff's current and voltage laws, as linearized by the method described in [10, 13, 14], can be written as follows:

$$\sum_{b\in\Psi^{lp}} \chi_{lp,b} I_{b,t,ll} = \frac{\mathrm{D}_{lp,t,ll}}{\sqrt{3}\mathrm{V}^{\mathrm{r}}}; \quad \forall lp \in \Omega^{D} / \Omega^{DG}, \forall t \in \{1,...,T\}, \forall ll \in \{1,...,N_l\} \tag{4g}$$

$$\sum_{b\in\Psi^{lp}} \chi_{lp,b} I_{b,t,ll} = \frac{\mathrm{D}_{lp,t,ll} - \sum_{k\in\Lambda_{lp}^{DG}} g_{lp,k,t,ll}^{DG}}{\sqrt{3}\mathrm{V}^{\mathrm{r}}}; \quad \forall lp \in \Omega^{DG}, \forall t \in \{1,...,T\}, \forall ll \in \{1,...,N_l\} \tag{4h}$$

$$\sum_{b\in\Psi^{lp}} \chi_{lp,b} I_{b,t,ll} = \frac{\mathrm{D}_{lp,t,ll} - \sum_{s\in\Theta} \xi_{lp,s} g_{s,t,ll}^{Sub}}{\sqrt{3}\mathrm{V}^{\mathrm{r}}}; \quad \forall lp \in \Omega^{S}, \forall t \in \{1,...,T\}, \forall ll \in \{1,...,N_l\} \tag{4i}$$

$$\left| \mathrm{Z}_b I_{b,t,ll} - \sum_{lp\in\Omega} \chi_{lp,b} V_{lp,t,ll} \right| \le \mathrm{M}(1 - \varphi_{b,t}^{Fe,Fi}); \quad \forall b \in \Pi^f, \forall t \in \{1,...,T\}, \forall ll \in \{1,...,N_l\} \tag{4j}$$

$$\left| \mathrm{Z}_b I_{b,t,ll} - \sum_{lp\in\Omega} \chi_{lp,b} V_{lp,t,ll} \right| \le \mathrm{M}\left(1 - \varphi_{b,t}^{Fe,Re}\right); \quad \forall b \in \Pi^r, \forall t \in \{1,...,T\}, \forall ll \in \{1,...,N_l\} \tag{4k}$$

$$\left| \mathrm{Z}_{b,k} I_{b,t,ll} - \sum_{lp\in\Omega} \chi_{lp,b} V_{lp,t,ll} \right| \le \mathrm{M}\left(1 - \varphi_{b,k,t}^{Fe,Re}\right); \quad \forall b \in \Pi^r, \forall k \in \Gamma_b^{Fe}, \forall t \in \{1,...,T\}, \forall ll \in \{1,...,N_l\} \tag{4l}$$

$$\left| \mathrm{Z}_{b,k} I_{b,t,ll} - \sum_{lp\in\Omega} \chi_{lp,b} V_{lp,t,ll} \right| \le \mathrm{M}\left(1 - \varphi_{b,k,t}^{Fe,Ne}\right); \quad \forall b \in \Pi^c, \forall k \in \Upsilon_b^{Fe}, \forall t \in \{1,...,T\}, \forall ll \in \{1,...,N_l\}. \tag{4m}$$

Constraints (4g)−(4i) demonstrate the current flow balance at demand, DG, and substation nodes, respectively. Voltage drop across various feeder sections are also modeled by (4j)−(4m). It is worth noting that when a given feeder is not in-service, its associated utilization variable, $\varphi$, is 0, and the related voltage drop constraint is relaxed. In addition, line currents and nodal voltages are bounded according to the following equations:

$$\left| I_{b,t,ll} \right| \le \mathrm{I}_b^{\max} \varphi_{b,t}^{Fe,Fi}; \quad \forall b \in \Pi^f, \forall t \in \{1,...,T\}, \forall ll \in \{1,...,N_l\} \tag{4n}$$

$$\left| I_{b,t,ll} \right| \le \mathrm{I}_b^{\max} \varphi_{b,t}^{Fe,Re} + \sum_{k\in\Gamma_b^{Fe}} \mathrm{I}_{b,k}^{\max} \varphi_{b,k,t}^{Fe,Re}; \quad b \in \Pi^r, \forall t \in \{1,...,T\}, \forall ll \in \{1,...,N_l\} \tag{4o}$$

$$\left| I_{b,t,ll} \right| \le \sum_{k\in\Upsilon_b^{Fe}} \mathrm{I}_{b,k}^{\max} \varphi_{b,k,t}^{Fe,Ne}; \forall b \in \Pi^c, \forall t \in \{1,...,T\}, \forall ll \in \{1,...,N_l\} \tag{4p}$$

$$\mathrm{V}^{\min} \le V_{lp,t,ll} \le \mathrm{V}^{\max}; \forall lp \in \Omega, \forall t \in \{1,...,T\}, \forall ll \in \{1,...,N_l\}. \tag{4q}$$

It should be noted that the abovementioned linear power flow equations are based on a DC approximate model proposed in [14] for expansion planning purposes. Detailed discussion about the accuracy of this approximate model can be found in [12]. However, depending on the required level of accuracy, other convex load flow models, such as conic programming based model [15], and linear AC models [16], [17] can also be utilized.

Finally, radial operation of distribution network should also be considered in the model. A detailed discussion on this constraint is delayed until Section IV.

## C. Reliability Related Costs

From the electricity distribution company's viewpoint, cost associated with the power interruptions is comprised of two terms: revenue lost due to undelivered energy, and cost incurred by reliability regulations, as stated in (5).

$$RRC_t = RLUE_t + CRR_t; \quad \forall t \in \{1,...,T\} \tag{5}$$

Amount of the revenue lost associated with unserved energy can be estimated using (6).

$$RLUE_t = \sum_{ll=1}^{N_L} \mathrm{ERev}_{t,ll} EENS_{t,ll}; \quad \forall t \in \{1,...,T\} \tag{6}$$

In order to calculate the cost incurred by service reliability regulation, it is assumed that the distribution company is under a reward-penalty regime. In this regime, companies which cannot provide an acceptable reliability level for their customers will be penalized. In contrast, if service reliability exceeds a specific value, the distribution company will get bonus. General shape of reward-penalty scheme is illustrated in Fig. 2 [18-20]. Benchmark value implies acceptable value of the reliability index. There is a zone around the benchmark value, known as dead-band, in which neither reward nor penalty exists. Beyond this zone, the value of reward or penalty changes according to the incentive rate, i.e. slope of the lines between reward point (or penalty point) and reward cap point (or penalty cap point) [21]. Incentive rate can take different values in reward and penalty zones [22]. In practice, this graph is usually capped at specific values to avoid excessive amount of reward or penalty. It should be noted that some features of the designated reward-penalty graph might be eliminated in practical implementations. For instance, the incentive schemes of Denmark and Hungary are just based on the penalties, i.e. they do not have reward zone [2]. As another example, the dead band is omitted in the scheme adopted in Great Britain [2]. Nonetheless, almost all these schemes can be obtained with slight modifications in the general reward-penalty graph depicted in Fig. 2. Therefore, in the following the mathematical model is developed based on this graph.

In addition, reward-penalty scheme is usually applied to more than one reliability index, typically two indicators with respect to the duration and frequency of interruptions [2, 19, 21, 23]. Hence, considering the implementation of a number of such schemes correspond to a set of reliability indices, cost of reliability regulations would be the sum of penalty or reward values associated with these schemes:

$$CRR_t = \sum_{ri\in\Xi}\left(Pen_{ri,t} - Rew_{ri,t}\right);\quad \forall t\in\{1,\ldots,T\}. \tag{7}$$

Since these equations are written in terms of cost values, unlike Fig. 2, penalties are considered as positive numbers. In order to derive a linear formulation for the penalty zone, according to Fig. 3, penalty value can be initially expressed as follows:

$$Pen_{ri,t} = \max\left\{0, \min\left\{\left(RI_{ri,t} - \mathrm{PP}_{ri,t}\right)\mathrm{IPR}_{ri,t}, \mathrm{PCap}_{ri,t}\right\}\right\}; \forall t\in\{1,\ldots,T\} \tag{8}$$

Minimum value calculation in (8) can be further formulated by some linear constraints as (9a)−(9d). In order to describe this

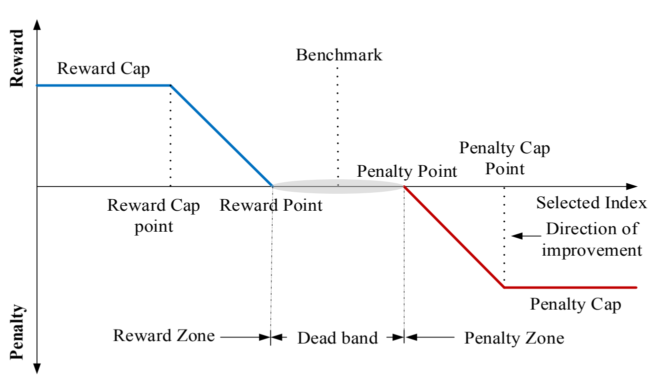


Fig. 2. General graph of reward-penalty scheme.

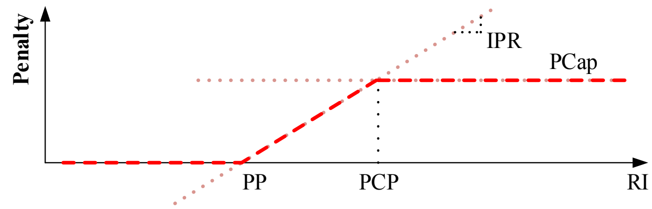


Fig. 3. Penalty zone model.

formulation, assume that the reliability index is less than PCP. In this case, (9b) sets the maximum value of $UPen_{ri,t}$. Consequently, equations (9c)−(9d) can only be satisfied when $\vartheta^{pc}_{ri,t}$, which is a binary variable, is 0. Accordingly, $UPen_{ri,t}$ is then forced to $(RI_{ri,t}\text{-}\mathrm{PP}_{ri,t})\mathrm{IPR}_{ri,t}$. On the contrary, while reliability index is more than PCP, $\vartheta^{pc}_{ri,t}$ becomes 1, and (9a) together with (9c) force $UPen_{ri,t}$ to the penalty cap. Similarly, the maximum value calculation in (8) can also be formulated as (9e)−(9h).

$$UPen_{ri,t} \le \mathrm{PCap}_{ri,t};\qquad \forall ri\in\Xi, \forall t\in\{1,\ldots,T\} \tag{9a}$$

$$UPen_{ri,t} \le (RI_{ri,t} - \mathrm{PP}_{ri,t})\mathrm{IPR}_{ri,t}; \forall ri\in\Xi, \forall t\in\{1,\ldots,T\} \tag{9b}$$

$$UPen_{ri,t} \ge \mathrm{PCap}_{ri,t} - \mathrm{M}(1-\vartheta^{pc}_{ri,t}); \forall ri\in\Xi, \forall t\in\{1,\ldots,T\} \tag{9c}$$

$$UPen_{ri,t} \ge (RI_{ri,t} - \mathrm{PP}_{ri,t})\mathrm{IPR}_{ri,t} - \mathrm{M}\vartheta^{pc}_{ri,t}; \forall ri\in\Xi, \forall t\in\{1,\ldots,T\} \tag{9d}$$

$$Pen_{ri,t} \ge UPen_{ri,t};\qquad \forall ri\in\Xi, \forall t\in\{1,\ldots,T\} \tag{9e}$$

$$Pen_{ri,t} \ge 0;\qquad \forall ri\in\Xi, \forall t\in\{1,\ldots,T\} \tag{9f}$$

$$Pen_{ri,t} \le \mathrm{M}\vartheta^{p}_{ri,t};\qquad \forall ri\in\Xi, \forall t\in\{1,\ldots,T\} \tag{9g}$$

$$Pen_{ri,t} \le UPen_{ri,t} + \mathrm{M}(1-\vartheta^{p}_{ri,t});\quad \forall ri\in\Xi, \forall t\in\{1,\ldots,T\} \tag{9h}$$

By the same manner, reward value is obtained as below:

$$URew_{ri,t} \le \mathrm{RCap}_{ri,t};\qquad \forall ri\in\Xi, \forall t\in\{1,\ldots,T\} \tag{10a}$$

$$\mathrm{RCap}_{ri,t} - \mathrm{M}(1-\vartheta^{rc}_{ri,t}) \le URew_{ri,t} \le (\mathrm{RP}_{ri,t} - RI_{ri,t})\mathrm{IRR}_{ri,t} \quad \forall ri\in\Xi, \forall t\in\{1,\ldots,T\} \tag{10b}$$

$$URew_{ri,t} \ge (\mathrm{RP}_{ri,t} - RI_{ri,t})\mathrm{IRR}_{ri,t} - \mathrm{M}\vartheta^{rc}_{ri,t}; \forall ri\in\Xi, \forall t\in\{1,\ldots,T\} \tag{10c}$$

$$Rew_{ri,t} \ge URew_{ri,t};\qquad \forall ri\in\Xi, \forall t\in\{1,\ldots,T\} \tag{10d}$$

$$Rew_{ri,t} \ge 0;\qquad \forall ri\in\Xi, \forall t\in\{1,\ldots,T\} \tag{10e}$$

$$Rew_{ri,t} \le \mathrm{M}\vartheta^{r}_{ri,t};\qquad \forall ri\in\Xi, \forall t\in\{1,\ldots,T\} \tag{10f}$$

$$Rew_{ri,t} \le URew_{ri,t} + \mathrm{M}(1-\vartheta^{r}_{ri,t});\qquad \forall ri\in\Xi, \forall t\in\{1,\ldots,T\}. \tag{10g}$$

## III. Formulation of Reliability Indices

In order to quantify continuity of electricity distribution supply, a wide range of reliability indicators have been introduced [1]. However, based on the practical experiences, rewards-penalty schemes are typically applied to the average system reliability indices, more specifically SAIFI and SAIDI [2]. In addition, in order to estimate the revenue lost caused by distribution network interruptions, we need to calculate EENS index according to (6). Therefore, in this section a mixed-integer linear formulation is derived for calculation of EENS, SAIFI, and SAIDI indices.

### A. Expected Energy not Served (EENS)

This reliability measure is usually stated by (11) [1]. The most important issue with this formula is the calculation of average frequency and duration of load point interruptions, i.e. $v_{lp}$, $\delta_{lp}$, respectively. This is because of the fact that these load point reliability indicators are functions of network topology, which in turn is the outcome of the optimal expansion planning problem. Moreover, given that the island operation of DG units is allowable during contingencies, the expected value of curtailed power, $EP^{Cur}_{lp,t,ll}$, is also a complex function of network topology and available DG units.

$$EENS_{t,ll} = \sum_{lp\in\Omega^D} v_{lp}\delta_{lp}\frac{\mathrm{Du}_{t,ll}}{8760}EP^{Cur}_{lp,t,ll};\forall t\in\{1,...,T\},\forall ll\in\{1,...,N_l\} \quad (11)$$

Hence, this equation cannot be directly employed in the model, owing to its high nonlinearity. However, by taking the advantage of radial operation of distribution networks, this problem can be addressed. In this respect, assuming that each feeder has a fully reliable disconnecting device (e.g. fuse, circuit breaker, or sectionalizer), one can say that the outage of each feeder can only result in islanding of the downstream network. Moreover, in order to make the problem tractable, it is assumed, as in [11, 12], that operation of normally open switches during the outages is negligible. Hence, the main assumptions of the proposed reliability evaluation technique are as follows:

1) Only permanent failures of feeder sections are considered.
2) A faulty branch is immediately isolated from the network, so that the outage of upstream customers are negligible.
3) Operation of normally open switches are not taken into account. Hence, the downstream section remains isolated until the completion of the repair action.

Therefore, equation (11) can be expressed as (12), where the EENS during load level $ll$ of time stage $t$ is calculated as the summation of the amount of energy curtailment caused by the outage of each feeder section $b$, i.e., $BENS_{b,t,ll}$. For fixed branches, $BENS_{b,t,ll}$ is simply attained using (12a) as the failure rate and repair time of such branches are unique. In contrast, reinforceable and new branches whose failure rates and repair times depend on the selected alternative, $BENS_{b,t,ll}$ is also a function of binary utilization variables. Accordingly, if the initial state of a reinforceable branch is chosen, $\varphi^{Fe,Re}_{b,t}$ equals to one and constraint (12b) becomes active, while (12c), (12d) are relaxed since according to logical constraints (4a), (4b) only one of the binary utilization variables of branch $b$ can take a non-zero value at stage $t$. On the other hand, if for a given reinforcable branch $b$, a $\varphi^{F,Re}_{b,k,t}$ becomes 1, then it enforces (12c) and relaxes (12b), (12d). Finally, if the branch is not in-service, i.e. all utilization variables are 0, then (12d) forces $BENS_{b,t,ll}$ to 0. Hence, depending on the selected alternative for a reinforceable branch $b$, the proper value of $BENS_{b,t,ll}$ is attained through (12b)−(12d). The same method are adopted to obtain $BENS_{b,t,ll}$ for new branches in (12e) and (12f).

$$EENS_{t,ll} = \sum\nolimits_{b\in\Pi} BENS_{b,t,ll};\ \ \forall t\in\{1,...,T\},\forall ll\in\{1,...,N_l\} \quad (12)$$

$$BENS_{b,t,ll} = \lambda^{\mathrm{Fe,Fi}}_{b}\mathrm{r}^{\mathrm{Fe,Fi}}_{b}CP_{b,t,ll};\quad \forall b\in\Pi^f,\forall t\in\{1,...,T\},\forall ll\in\{1,...,N_l\} \quad (12a)$$

$$\left|BENS_{b,t,ll} - \lambda^{\mathrm{Fe,Re}}_{b}\mathrm{r}^{\mathrm{Fe,Re}}_{b}CP_{b,t,ll}\right| \le \mathrm{M}\left(1-\varphi^{Fe,Re}_{b,t}\right);\quad \forall b\in\Pi^r,\forall t\in\{1,...,T\},\forall ll\in\{1,...,N_l\} \quad (12b)$$

$$\left|BENS_{b,t,ll} - \lambda^{\mathrm{Fe,Re}}_{b,k}\mathrm{r}^{\mathrm{Fe,Re}}_{b,k}CP_{b,t,ll}\right| \le \mathrm{M}\left(1-\varphi^{Fe,Re}_{b,k,t}\right);\quad \forall b\in\Pi^r,\forall t\in\{1,...,T\},\forall ll\in\{1,...,N_l\} \quad (12c)$$

$$BENS_{b,t,ll} \le \mathrm{M}\left(\varphi^{Fe,Re}_{b,t} + \sum\nolimits_{k\in\Gamma^{Fe}_b}\varphi^{Fe,Re}_{b,k,t}\right);\quad \forall b\in\Pi^r,\forall t\in\{1,...,T\},\forall ll\in\{1,...,N_l\} \quad (12d)$$

$$\left|BENS_{b,t,ll} - \lambda^{\mathrm{Fe,Ne}}_{b,k}\mathrm{r}^{\mathrm{Fe,Ne}}_{b,k}CP_{b,t,ll}\right| \le \mathrm{M}\left(1-\varphi^{Fe,Ne}_{b,k,t}\right);\quad \forall b\in\Pi^c,\forall t\in\{1,...,T\},\forall ll\in\{1,...,N_l\} \quad (12e)$$

$$BENS_{b,t,ll} \le \mathrm{M}\sum_{k\in\Gamma^{Fe}_b}\varphi^{Fe,Ne}_{b,k,t};\forall b\in\Pi^c,\forall t\in\{1,...,T\},\forall ll\in\{1,...,N_l\} \quad (12f)$$

Now, we need to calculate the value of curtailed power owing to the outage of branch $b$, $CP_{b,t,ll}$. Based on the aforementioned assumption of island operation of DG units under contingencies, $CP_{b,t,ll}$, can also be obtained as follows:

$$CP_{b,t,ll} \ge DD_{b,t,ll} - DDGC_{b,t};\quad \forall b\in\Pi,\forall t\in\{1,...,T\},\forall ll\in\{1,...,N_l\} \quad (13a)$$

$$CP_{b,t,ll} \ge 0;\quad \forall b\in\Pi,\forall t\in\{1,...,T\},\forall ll\in\{1,...,N_l\} \quad (13b)$$

$$CP_{b,t,ll} \le DD_{b,t,ll} - DDGC_{b,t} + \mathrm{M}(1-\rho^{CP}_{b,t,ll});\quad \forall b\in\Pi,\forall t\in\{1,...,T\},\forall ll\in\{1,...,N_l\} \quad (13c)$$

$$CP_{b,t,ll} \le \mathrm{M}\rho^{CP}_{b,t,ll};\quad \forall b\in\Pi,\forall t\in\{1,...,T\},\forall ll\in\{1,...,N_l\} \quad (13d)$$

As can be inferred, if $DD_{b,t,ll}$ is greater than $DDGC_{b,t}$, the minimum value of $CP_{b,t,ll}$ is set to a positive value by (13a), which enforces the auxiliary variable $\rho^{CP}_{b,t,ll}$ to 1 and relaxes (13d), because otherwise the equations would have no possible solution. Consequently, the minimum and maximum of $CP_{b,t,ll}$, as determined by (13a) and (13c), are set to the same value, i.e. $DD_{b,t,ll}-DDGC_{b,t}$. By contrast, when $DD_{b,t,ll}$ is lower than $DDGC_{b,t}$, auxiliary variable $\rho^{CP}_{b,t,ll}$ becomes 0 and (13b) together with (13d) set the $CP_{b,t,ll}$ to 0. It is worth noting that because the objective function is a strictly increasing function of $EENS_{t,ll}$ (see (1), (5), and (6)), and $EENS_{t,ll}$ is monotonically increasing with $CP_{b,t,ll}$, the optimization algorithm selects the minimum possible value of $CP_{b,t,ll}$. Hence, constraints (13c) and (13d) are redundant in practice and can be omitted. Therefore, this set of equations implies that the curtailed power due to outage of a given branch is equal to the difference between total downstream demand and DG capacity, in case that the DG capacity in the islanded region is not quite enough to supply its whole demand, and is 0, otherwise. It is worth noting that this assumption is valid as long as the optimal planning solution provides enough feeder capacity for full-capacity operation of DG units during the normal operating conditions, which is typically the case in practice. Hence, considering the radial topology of distribution network, no feeder capacity violation would occur in the islanded section. Subsequently, in order to obtain $DD_{b,t,ll}$, a fictitious KCL based on the nodal demands can be performed as formulated in (14a)−(14i). In other words, considering the radial operation of distribution network, if we solve KCL equations for a fictitious network in which all DG units are eliminated, the flow of a given branch $b$ would be equal to the total demand downstream to this branch.

$$DD_{b,t,ll} = DD^{+}_{b,t,ll} + DD^{-}_{b,t,ll};\forall b\in\Pi,\forall t\in\{1,...,T\},\forall ll\in\{1,...,N_l\} \quad (14a)$$

$$\sum_{b\in\Psi^{lp}} \chi_{lp,b}\left(DD^{+}_{b,t,ll} - DD^{-}_{b,t,ll}\right) = D_{lp,t,ll}\,;$$
$$\forall lp \in \Omega^{D}, \forall t \in \{1,...,T\}, \forall ll \in \{1,...,N_l\} \quad (14b)$$

$$\sum_{b\in\Psi^{lp}} \chi_{lp,b}\left(DD^{+}_{b,t,ll} - DD^{-}_{b,t,ll}\right) = D_{lp,t,ll} - \sum_{s\in\Theta} \xi_{lp,s} GDD^{Sub}_{s,t,ll}\,;$$
$$\forall lp \in \Omega^{S}, \forall t \in \{1,...,T\}, \forall ll \in \{1,...,N_l\} \quad (14c)$$

$$DD_{b,t,ll} \le \mathrm{M}\varphi^{Fe,Fi}_{b,t}; \quad \forall b \in \Pi^{f}, \forall t \in \{1,...,T\}, \forall ll \in \{1,...,N_l\} \quad (14d)$$

$$DD_{b,t,ll} \le \mathrm{M}\left(\varphi^{Fe,Re}_{b,t} + \sum_{k\in\Gamma^{Fe}_{b}} \varphi^{Fe,Re}_{b,k,t}\right);$$
$$\forall b \in \Pi^{r}, \forall t \in \{1,...,T\}, \forall ll \in \{1,...,N_l\} \quad (14e)$$

$$DD_{b,t,ll} \le \mathrm{M}\sum_{k\in\Gamma^{Fe}_{b}} \varphi^{Fe,Ne}_{b,k,t}; \forall b \in \Pi^{c}, \forall t \in \{1,...,T\}, \forall ll \in \{1,...,N_l\} \quad (14f)$$

$$DD^{+}_{b,t,ll} \le \mathrm{M}\rho^{DD}_{b,t,ll}; \quad \forall b \in \Pi, \forall t \in \{1,...,T\}, \forall ll \in \{1,...,N_l\} \quad (14g)$$

$$DD^{-}_{b,t,ll} \le \mathrm{M}(1-\rho^{DD}_{b,t,ll}); \forall b \in \Pi, \forall t \in \{1,...,T\}, \forall ll \in \{1,...,N_l\} \quad (14h)$$

$$GDD^{Sub}_{s,t,ll} \le \mathrm{M}\left(\theta_s + \sum_{k\in\Upsilon^{STr}_{s}} \sum_{\tau=1}^{t} \sigma^{STr}_{s,k,\tau}\right);$$
$$\forall s \in \Theta, \forall t \in \{1,...,T\}, \forall ll \in \{1,...,N_l\} \quad (14i)$$

In these equations, in order to avoid negative values of $DD_{b,t,ll}$, it is divided to two positive variables as positive and negative parts, denoted by $DD^{+}_{b,t,ll}$ and $DD^{-}_{b,t,ll}$, respectively. Constraints (14b) and (14c) indicate the fictitious nodal balance of customer demands. By the use of (14d)–(14f), downstream demand of not utilized branches are forced to 0. In addition, (14g) and (14h) are to ensure that only one of the two positive variables, $DD^{+}_{b,t,ll}$ and $DD^{-}_{b,t,ll}$ can have a non-zero value. Nevertheless, since the objective function is strictly increasing with respect to $DD_{b,t,ll}=DD^{+}_{b,t,ll} + DD^{-}_{b,t,ll}$, these constraints are redundant and can be eliminated. Finally, constraint (14i) is to force $GDD^{Sub}_{s,t,ll}$ of unavailable substations to 0. Similarly, $DDGC_{b,t}$ can be calculated using the following fictitious KCL:

$$DDGC_{b,t} = DDGC^{+}_{b,t} + DDGC^{-}_{b,t}; \quad \forall b \in \Pi, \forall t \in \{1,...,T\} \quad (15a)$$

$$\sum_{b\in\Psi^{lp}} \chi_{lp,b}(DDGC^{+}_{b,t} - DDGC^{-}_{b,t}) = \sum_{k\in\Lambda^{DG}_{lp}} \sum_{\tau=1}^{t} \mathrm{Cap}^{\mathrm{DG}}_{k} \sigma^{DG}_{lp,k,\tau};$$
$$\forall lp \in \Omega^{D}, \forall t \in \{1,...,T\} \quad (15b)$$

$$\sum_{b\in\Psi^{lp}} \chi_{lp,b}(DDGC^{-}_{b,t} - DDGC^{+}_{b,t}) = \sum_{s\in\Theta} \xi_{lp,s} GDDGC^{Sub}_{s,t};$$
$$\forall lp \in \Omega^{S}, \forall t \in \{1,...,T\} \quad (15c)$$

$$DDGC_{b,t} \le \mathrm{M}\varphi^{Fe,Fi}_{b,t}; \quad \forall b \in \Pi^{f}, \forall t \in \{1,...,T\} \quad (15d)$$

$$DDGC_{b,t} \le \mathrm{M}\left(\varphi^{Fe,Re}_{b,t} + \sum_{k\in\Gamma^{Fe}_{b}} \varphi^{Fe,Re}_{b,k,t}\right); \forall b \in \Pi^{r}, \forall t \in \{1,...,T\} \quad (15e)$$

$$DDGC_{b,t} \le \mathrm{M}\sum_{k\in\Gamma^{Fe}_{b}} \varphi^{Fe,Ne}_{b,k,t}; \quad \forall b \in \Pi^{c}, \forall t \in \{1,...,T\} \quad (15f)$$

$$DDGC^{+}_{b,t} \le \mathrm{M}\rho^{DDGC}_{b,t}; \quad \forall b \in \Pi, \forall t \in \{1,...,T\} \quad (15g)$$

$$DDGC^{-}_{b,t} \le \mathrm{M}(1-\rho^{DDGC}_{b,t}); \quad \forall b \in \Pi, \forall t \in \{1,...,T\} \quad (15h)$$

$$GDDGC^{Sub}_{s,t} \le \mathrm{M}\left(\theta_s + \sum_{k\in\Upsilon^{STr}_{s}} \sum_{\tau=1}^{t} \sigma^{STr}_{s,k,\tau}\right); \forall s \in \Theta, \forall t \in \{1,...,T\} \quad (15i)$$

The main idea behind this fictitious KCL is that considering the radial operation of the network, total DG capacity downstream to each branch would be equal to the fictitious flow of that branch if fictitious demand of each node *lp* is set to the total installed DG capacity at *lp*.

For instance, take simple radial network depicted in Fig. 4(a) as an example. In order to attain total demand downstream to each branch, we can form the fictitious network depicted in Fig. 4(b) in which the fictitious demand ($FD_{lp}$) of load points are set to their actual load and DG units are eliminated. Hence, solving the KCL equations for this fictitious network the $DD_b$ values are obtained, as illustrated in the figure. In the same manner, solving the KCL equations for the fictitious network provided in Fig. 4(c), gives the $DDGC_b$ values.

Finally, it should be noted that the equations derived in this part are based on the validity of the assumption made about full-capacity operation of DG units during the normal operation. Thus, capacity limits of feeder sections during the islanded states are disregarded. However, in order to ensure that feeder capacity violation does not occur in the islanded sections, regardless of the validity of the aforementioned assumption, the modified formulation explained in the Appendix can be employed.

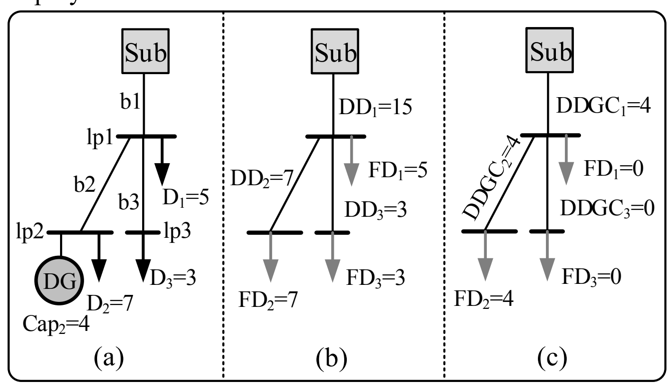


Fig. 4. A sample distribution network. (a) Network diagram; (b) Fictitious network for calculation of downstream demand for each branch ($DD_b$); (c) Fictitious network for attaining downstream DG capacity ($DDGC_b$).

## *B. System Average Interruption Frequency Index (SAIFI)*

This reliability index is generally defined as (16) [1].

$$SAIFI_t = \sum_{lp\in\Omega^{D}} \nu_{lp} \mathrm{N}_{lp,t} \Big/ \sum_{lp\in\Omega^{D}} \mathrm{N}_{lp,t}\,; \forall t \in \{1,...,T\} \quad (16)$$

Again, due to the issues regarding the calculation of $\nu_{lp}$, it is not possible to directly incorporate this formula into the MILP model of distribution system planning. However, under the same assumptions as those mentioned for EENS calculation, this index can also be obtained as follows:

$$SAIFI_t = \sum_{b\in\Pi} BSAIFI_{b,t} \Big/ \sum_{lp\in\Omega^{D}} \mathrm{N}_{lp,t}\,; \forall t \in \{1,...,T\} \quad (17)$$

$$BSAIFI_{b,t} = \lambda^{\mathrm{Fe,Fi}}_{b} CCN_{b,t}; \forall b \in \Pi^{f}, \forall t \in \{1,...,T\} \quad (17a)$$

$$\left|BSAIFI_{b,t} - \lambda^{\mathrm{Fe,Re}}_{b} CCN_{b,t}\right| \le \mathrm{M}\left(1-\varphi^{Fe,Re}_{b,t}\right); \forall b \in \Pi^{r}, \forall t \in \{1,...,T\} \quad (17b)$$

$$\left|BSAIFI_{b,t} - \lambda^{\mathrm{Fe,Re}}_{b,k} CCN_{b,t}\right| \le \mathrm{M}\left(1-\varphi^{Fe,Re}_{b,k,t}\right);$$
$$\forall b \in \Pi^{r}, \forall k \in \Gamma^{Fe}_{b}, \forall t \in \{1,...,T\} \quad (17c)$$

$$BSAIFI_{b,t} \le \mathrm{M}\left(\varphi^{Fe,Re}_{b,t} + \sum_{k\in\Gamma^{Fe}_{b}} \varphi^{Fe,Re}_{b,k,t}\right);$$
$$\forall b \in \Pi^{r}, \forall k \in \Gamma^{Fe}_{b}, \forall t \in \{1,...,T\} \quad (17d)$$

$$\left|BSAIFI_{b,t} - \lambda^{\mathrm{Fe,Ne}}_{b,k} CCN_{b,t}\right| \le \mathrm{M}\left(1-\varphi^{Fe,Ne}_{b,k,t}\right);$$
$$\forall b \in \Pi^{c}, \forall k \in \Upsilon^{Fe}_{b}, \forall t \in \{1,...,\mathrm{T}\} \quad (17e)$$

$$BSAIFI_{b,t} \le \mathrm{M}\sum_{k\in\Gamma^{Fe}_{b}} \varphi^{Fe,Ne}_{b,k,t}; \forall b \in \Pi^{c}, \forall k \in \Upsilon^{Fe}_{b}, \forall t \in \{1,...,\mathrm{T}\}. \quad (17f)$$

Equation (17) expresses the SAIFI as a function of number of annual customer outages caused by the failure of each line section, $BSAIFI_{b,t}$. Moreover, $BSAIFI_{b,t}$ is calculated using (17a)−(17f), which are structurally identical to (12a)−(12f). Number of customers affected by the outage of branch $b$, $CCN_{b,t}$, is further estimated in the same way as (13a)−(13d), as below:

$$CCN_{b,t} = \sum_{ll=1}^{N_L}\left( CCN_{b,t,ll}\frac{\mathrm{Du}_{t,ll}}{8760}\right); \quad \forall b\in\Pi, \forall t\in\{1,...,T\} \tag{18}$$

$$CCN_{b,t,ll} \geq DCN_{b,t} - \frac{DDGC_{b,t}}{\mu^{\mathrm{Dem}}_{t,ll}}; \quad \forall b\in\Pi, \forall t\in\{1,...,T\}, \forall ll\in\{1,...,N_l\} \tag{18a}$$

$$CCN_{b,t,ll} \geq 0; \quad \forall b\in\Pi, \forall t\in\{1,...,T\}, \forall ll\in\{1,...,N_l\} \tag{18b}$$

$$CCN_{b,t,ll} \leq DCN_{b,t} - \frac{DDGC_{b,t}}{\mu^{\mathrm{Dem}}_{t,ll}} + \mathrm{M}(1-\rho^{CCN}_{b,t,ll}) \quad \forall b\in\Pi, \forall t\in\{1,...,T\}, \forall ll\in\{1,...,N_l\} \tag{18c}$$

$$CCN_{b,t,ll} \leq \mathrm{M}\rho^{CCN}_{b,t,ll}; \quad \forall b\in\Pi, \forall t\in\{1,...,T\}, \forall ll\in\{1,...,N_l\}. \tag{18d}$$

Equation (18) expresses the number of affected customers in each stage $t$ as a function of the number of affected customers during various load levels of that stage. This is because in these equations, the number of customers served by DG units in the islanded section downstream of a given branch $b$ is estimated as the total downstream DG capacity $DDGC_{b,t}$ divided by the average power demand of the customers $\mu^{\mathrm{Dem}}_{t,ll}$, as can be seen in (18a) and (18c). Thus, the number of affected customers in a given islanded section varies for different load levels, since $\mu^{\mathrm{Dem}}_{t,ll}$ is a function of the power demand of load level $ll$.

In case the DG capacity in the islanded section is not adequate for meeting all the demand, the right-hand-side of (18a) becomes positive and sets the lower bound of $CCN_{b,t,ll}$ to the number of not served costumers. As a consequence, binary variable $\rho^{CCN}_{b,t,ll}$ becomes 1 to make the equations feasible by relaxing (18d). This, in turn, sets the upper bound of $CCN_{b,t,ll}$, determined by (18c), equal to its lover bound enforced by (18a). On the other hand, if the total DG capacity in the islanded section is higher than the demand, the right-hand side of (18a) becomes negative. Thus, determined by (18b), the lower limit of $CCN_{b,t,ll}$ is 0, and $\rho^{CCN}_{b,t,ll}$ becomes 0, which relaxes (18c) and sets the upper bound of $CCN_{b,t,ll}$ to 0. Therefore, the $CCN_{b,t,ll}$ becomes 0 as desired. Nonetheless, in case the objective function is monotonically increasing with respect to SAIFI, $CCN_{b,t,ll}$ is set to its lower bound by the optimization algorithm. Thus, constraints (18c) and (18d) can be eliminated.

Subsequently, number of customers supplied through each branch, $DCN_{b,t}$, can be calculated by performing a fictitious KCL based on the number of customers at each load point, using (19a)−(19i). In other words, a fictitious network is formed here in which the fictitious demand of each load point is equal to the number of customers connected to that node. Hence, the fictitious flow of each branch is equal to the total number of customers served through it, based on the similar concept explained in Fig. 4, with $\mathrm{N}_{lp,t}$ playing the role of total DG capacity at each node.

Equation (19a) expresses $DCN_{b,t}$ in terms of two non-negative variables to avoid negative values caused by inconsistency of the predetermined direction of branches with their actual direction in a given radial topology. Flow balance at load points and substation nodes of the fictitious network are formulated as (19b) and (19c), respectively. Constraints (19d)−(19f) are to set the $DCN_{b,t}$ of the switched-off branches to 0. Expressions (19g) and (19h) denote the logic that only one of the two auxiliary variables $DCN^{+}_{b,t}$ and $DCN^{-}_{b,t}$ can take a non-zero value at a time. Finally, equation (19i) shows that non-existent substations cannot serve the demand.

$$DCN_{b,t} = DCN^{+}_{b,t} + DCN^{-}_{b,t}; \quad \forall b\in\Pi, \forall t\in\{1,...,T\} \tag{19a}$$

$$\sum_{b\in\Psi^{lp}} \chi_{lp,b}(DCN^{+}_{b,t} - DCN^{-}_{b,t}) = \mathrm{N}_{lp,t}; \forall lp\in\Omega^{D}, \forall t\in\{1,...,\mathrm{T}\} \tag{19b}$$

$$\sum_{b\in\Psi^{lp}} \chi_{lp,b}\left(DCN^{-}_{b,t} - DCN^{+}_{b,t}\right) = \sum_{s\in\Theta} \xi_{lp,s} GCN^{Sub}_{s,t}; \quad \forall lp\in\Omega^{S}, \forall t\in\{1,...,T\} \tag{19c}$$

$$DCN_{b,t} \leq \mathrm{M}\varphi^{Fe,Fi}_{b,t}; \quad \forall b\in\Pi^{f}, \forall t\in\{1,...,T\} \tag{19d}$$

$$DCN_{b,t} \leq \mathrm{M}\left(\varphi^{Fe,Re}_{b,t} + \sum_{k\in\Gamma^{Fe}_{b}} \varphi^{Fe,Re}_{b,k,t}\right); \ \forall b\in\Pi^{r}, \forall t\in\{1,...,T\} \tag{19e}$$

$$DCN_{b,t} \leq \mathrm{M}\sum_{k\in\Gamma^{Fe}_{b}} \varphi^{Fe,Ne}_{b,k,t}; \quad \forall b\in\Pi^{c}, \forall t\in\{1,...,T\} \tag{19f}$$

$$DCN^{+}_{b,t} \leq \mathrm{M}\rho^{DCN}_{b,t}; \quad \forall b\in\Pi, \forall t\in\{1,...,T\} \tag{19g}$$

$$DCN^{-}_{b,t} \leq \mathrm{M}\left(1-\rho^{DCN}_{b,t}\right); \quad \forall b\in\Pi, \forall t\in\{1,...,T\} \tag{19h}$$

$$GCN^{Sub}_{s,t} \leq \mathrm{M}\left(\theta_{s} + \sum_{k\in\Upsilon^{STr}_{s}} \sum_{\tau=1}^{t} \sigma^{STr}_{s,k,\tau}\right); \forall s\in\Theta, \forall t\in\{1,...,T\} \tag{19i}$$

*C. System Average Interruption Duration Index (SAIDI)*

SAIDI index is typically expressed as (20) [1].

$$SAIDI_{t} = \sum_{lp\in\Omega^{D}} \nu_{lp}\delta_{lp}\mathrm{N}_{lp,t} \Big/ \sum_{lp\in\Omega^{D}} \mathrm{N}_{lp,t}; \quad \forall t\in\{1,...,T\} \tag{20}$$

Nonetheless, analogous to the model proposed for SAIFI formulation, this index can be readily calculated as below:

$$SAIDI_{t} = \sum_{b\in\Pi} BSAIDI_{b,t} \Big/ \sum_{lp\in\Omega^{D}} \mathrm{N}_{lp,t}; \ \forall t\in\{1,...,T\} \tag{21}$$

$$BSAIDI_{b,t} = \lambda^{\mathrm{Fe,Fi}}_{b} \mathrm{r}^{\mathrm{Fe,Fi}}_{b} CCN_{b,t}; \ \forall b\in\Pi^{f}, \forall t\in\{1,...,T\} \tag{21a}$$

$$\left| BSAIDI_{b,t} - \lambda^{\mathrm{Fe,Re}}_{b} \mathrm{r}^{\mathrm{Fe,Re}}_{b} CCN_{b,t} \right| \leq \mathrm{M}\left(1-\varphi^{Fe,Re}_{b,t}\right); \quad \forall b\in\Pi^{r}, \forall t\in\{1,...,T\} \tag{21b}$$

$$\left| BSAIDI_{b,t} - \lambda^{\mathrm{Fe,Re}}_{b,k} \mathrm{r}^{\mathrm{Fe,Re}}_{b,k} CCN_{b,t} \right| \leq \mathrm{M}\left(1-\varphi^{Fe,Re}_{b,k,t}\right); \quad \forall b\in\Pi^{r}, \forall k\in\Gamma^{Fe}_{b}, \forall t\in\{1,...,T\} \tag{21c}$$

$$BSAIDI_{b,t} \leq \mathrm{M}\left(\varphi^{Fe,Re}_{b,t} + \sum_{k\in\Gamma^{Fe}_{b}} \varphi^{Fe,Re}_{b,k,t}\right); \quad \forall b\in\Pi^{r}, \forall k\in\Gamma^{Fe}_{b}, \forall t\in\{1,...,T\} \tag{21d}$$

$$\left| BSAIDI_{b,t} - \lambda^{\mathrm{Fe,Ne}}_{b,k} \mathrm{r}^{\mathrm{Fe,Ne}}_{b,k} CCN_{b,t} \right| \leq \mathrm{M}\left(1-\varphi^{Fe,Ne}_{b,k,t}\right); \quad \forall b\in\Pi^{c}, \forall k\in\Upsilon^{Fe}_{b}, \forall t\in\{1,...,T\} \tag{21e}$$

$$BSAIDI_{b,t} \leq \mathrm{M}\sum_{k\in\Gamma^{Fe}_{b}} \varphi^{Fe,Ne}_{b,k,t}; \forall b\in\Pi^{c}, \forall k\in\Upsilon^{Fe}_{b}, \forall t\in\{1,...,T\}. \tag{21f}$$

This set of equations is structurally identical to that of SAIFI, i.e., (17)−(17f), whereas parameters associated with the repair time of the feeder sections, namely $\mathrm{r}^{\mathrm{Fe,Fi}}_{b}$, $\mathrm{r}^{\mathrm{Fe,Re}}_{b}$, $\mathrm{r}^{\mathrm{Fe,Re}}_{b,k}$, and $\mathrm{r}^{\mathrm{Fe,Ne}}_{b,k}$, are considered in the related expressions (21a), (21b), (21c), and (21e), respectively.

## IV. Discussion on Radiality Constraint Formulation

Radiality of distribution network simply means that during the normal operation, there must be only a unique path between each load point and one of the substations. Note that the island operation of DG units is considered to be prohibited during the normal operation.

In general, constraints associated with the radial operation of distribution network are deduced based on a well-known graph theory stating that the number of nodes of a forest (i.e., a disjoint union of trees) is equal to the number of branches plus the number of trees which are within the forest [24]. In order to apply this theory to the proposed model, two steps should be considered:

1) It has to be initially ensured that the island operation of DGs is not possible, i.e. there is at least one path between each in-service load point and one of the substations.
2) The radiality constraints based on the abovementioned theory should be incorporated into the model.

Indeed, the first one is satisfied by the fictitious KCL equations derived for calculation of reliability indices. Take fictitious KCL of equations set (19) for instance: this set of equations is equivalent to power flow balance in a fictitious network whose loads are customer numbers, $\mathrm{N}_{lp,t}$, which of course are positive numbers. Hence, in order to satisfy balance in the fictitious network, each in-service load point whose $\mathrm{N}_{lp,t}$ is non-zero, has to at least be connected through a path to a source bus, which according to (19c) can only be a substation bus. This claim can be justified using *indirect proof*. In this context, assuming that the opposite of the proposition is true, it can be stated that islanding of some load nodes are possible. If we consider $\Omega^{Isl}$ as the set of load points within the islanded section, then according to (19b) the nodal balance at these load points would be:

$$\sum\nolimits_{b\in\Psi^{lp}} \chi_{lp,b}\left(DCN^{+}_{b,t}-DCN^{-}_{b,t}\right)=\mathrm{N}_{lp,t}\,;\forall lp\in\Omega^{Isl},\forall t\in\{1,...,T\} \tag{22a}$$

Then after, the summation of nodal balance equations of all these load points can be written as below:

$$\sum_{lp\in\Omega^{Isl}}\sum_{b\in\Psi^{lp}} \chi_{lp,b}\left(DCN^{+}_{b,t}-DCN^{-}_{b,t}\right)=\sum_{lp\in\Omega^{Isl}}\mathrm{N}_{lp,t}\,;\forall t\in\{1,...,T\} \tag{22b}$$

It can be proven that the result of left-hand-side equation is zero, since the nodes at both ends of a given branch $b$, which is in the islanded zone, are in the $\Omega^{Isl}$. As an example, assuming a branch $b_1$ between nodes $lp_1, lp_2 \in \Omega^{Isl}$ with a predetermined direction from $lp_1$ to $lp_2$, and considering the definition of $\chi_{lp,b}$ (see Nomenclature Section), $\chi_{lp1,b1}$ and $\chi_{lp2,b1}$ would be +1 and -1, respectively. Therefore, the summation over this branch would be 0. Nonetheless, the right-hand-side summation is a positive number since all $\mathrm{N}_{lp,t}$ values are positive, which is a contradiction. Therefore, it can be concluded that the islanded operation is avoided through the considered fictitious KCL formulated in equation set (19). Note that this is not a case for normal KCL equations, since referring to (4g)−(4i) it can be concluded that DG nodes can also serve as source buses, as well. In other words, in the right side of (22b) we might have negative terms, i.e., $-\sum\nolimits_{k\in\Lambda_{lp}^{DG}} g^{DG}_{lp,k,t,ll}$ (see (4h)).

Now, as the second step, the radiality constraint can be formulated as follows:

$$\begin{aligned}LPM_{lp,t} \le 0.5 &+ \sum\nolimits_{b\in\Pi^{f}\cap\Psi^{lp}} \varphi^{Fe,Fi}_{b,t} + \sum\nolimits_{b\in\Pi^{c}\cap\Psi^{lp}} \sum\nolimits_{k\in\Upsilon^{Fe}_{b}} \varphi^{Fe,Ne}_{b,k,t}\\ &+ \sum\nolimits_{b\in\Pi^{r}\cap\Psi^{lp}}\left(\varphi^{Fe,Re}_{b,t}+\sum\nolimits_{k\in\Gamma^{Fe}_{b}}\varphi^{Fe,Re}_{b,k,t}\right);\forall lp\in\Omega,\forall t\in\{1,...,T\}\end{aligned} \tag{23a}$$

$$\begin{aligned}LPM_{lp,t} \ge \Bigg(&\sum_{b\in\Pi^{f}\cap\Psi^{lp}} \varphi^{Fe,Fi}_{b,t} + \sum_{b\in\Pi^{c}\cap\Psi^{lp}}\sum_{k\in\Upsilon^{Fe}_{b}} \varphi^{Fe,Ne}_{b,k,t}\\ &+\sum_{b\in\Pi^{r}\cap\Psi^{lp}}\left(\varphi^{Fe,Re}_{b,t}+\sum_{k\in\Gamma^{Fe}_{b}}\varphi^{Fe,Re}_{b,k,t}\right)\Bigg)\Bigg/\sum_{b\in\Psi^{lp}}1;\forall lp\in\Omega,\forall t\in\{1,...,T\}\end{aligned} \tag{23b}$$

$$\begin{aligned}\sum_{lp\in\Omega} LPM_{lp,t} = &\sum\nolimits_{b\in\Pi^{f}} \varphi^{Fe,Fi}_{b,t} + \sum\nolimits_{b\in\Pi^{c}} \sum\nolimits_{k\in\Upsilon^{Fe}_{b}} \varphi^{Fe,Ne}_{b,k,t}\\ &+\sum\nolimits_{b\in\Pi^{r}}\left(\varphi^{Fe,Re}_{b,t}+\sum\nolimits_{k\in\Gamma^{Fe}_{b}}\varphi^{Fe,Re}_{b,k,t}\right)+\sum\nolimits_{lp\in\Omega^{S}} LPM_{lp,t};\forall t\in\{1,...,T\}\end{aligned} \tag{23c}$$

$$GCN^{Sub}_{s,t} \le \mathrm{M}\left(\mathrm{InCa}^{\mathrm{Sub}}_{s}+\sum_{k\in\Upsilon^{STr}_{s}}\sum_{\tau=1}^{t}\mathrm{Cap}^{\mathrm{STr}}_{s,k}\sigma^{STr}_{s,k,\tau}\right);\forall s\in\Theta,\forall t\in\{1,...,T\} \tag{23d}$$

$$\varphi^{Fe,Fi}_{b,t},\varphi^{Fe,Re}_{b,t},\varphi^{Fe,Re}_{b,k,t},\varphi^{Fe,Ne}_{b,k,t} \le DCN_{b,t};\quad \forall b\in\Pi,\forall t\in\{1,...,T\}. \tag{23e}$$

The first two constraints indicate whether a load point is in-service, or not. In this respect, it is worth mentioning that a load point is considered to be in-service in case it is connected to at least an in-service feeder. Constraint (23c) is inspired by the aforementioned theory: the number of active load points is equal to the number of in-service branches plus the number of substations (note that the number of substations are equal to the number of separate parts of the network) [6]. Equation (23d) is to make sure that only available substations can serve as source buses of the fictitious network. Finally, the last constraint, (23e), is to avoid the utilization of unloaded feeders. Note that this constraint is necessary to guarantee the radiality of the network. In this respect, consider the simple network depicted in Fig. 5. This network is not a radial one, since load point 4 is connected to more than one substation. However, this topology satisfies all the radiality equations, except the last one.

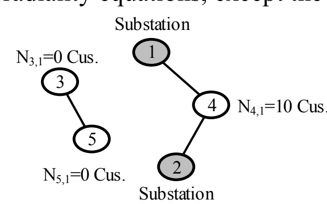


Fig. 5. An example of the issue caused by neglecting constraint (23e).

## V. Case Study

The proposed multi-stage MILP model of distribution system planning is applied to the 24-node and 54-node distribution networks described in [13], [12] with slight modification. All data associated with the test systems can be downloaded from [25]. The proposed model has been implemented on GAMS environment and solved by the CPLEX 12.6.3 solver on a Fujitsu Celsius W530 POWER with a Quad 3.30 GHz Intel Xeon E3-1230 processor and 16 GB of RAM. Maximum threshold of optimality gap is also set to 1 percent.

### A. 24-node Test Distribution System

One-line diagram of this network is illustrated in Fig. 6. Various alternatives for adding new feeder sections, DG units, and substation transformers are also presented in Tables I−III, respectively. Regarding the reliability regulations, two reward-penalty schemes based on SAIFI and SAIDI indices are taken into account.

In order to better investigate the effect of DGs, two sets of cases are considered: one in the absence and another in the presence of DG investment option, denoted as (I) and (II), respectively. Moreover, in each set, two different strategies are of interest: (a) without and (b) with incorporating the reward-penalty costs in objective function. Outcomes associated with these cases are presented in Table IV. Comparing these cases, it can be inferred that installation of DG units can considerably decrease total cost by cutting back the investment and operation costs of feeders and substations. Another interesting point is that the monetary value of revenue lost due to undelivered energy, $RLUE_t$, is too much lower than the other terms of the cost function. Hence, it can be expected that this value may not provide suitable motivation for distribution companies to increase their service reliability. This can be translated to importance of incentive reliability regulations for reaching the regulatory goal of reliability enhancement. In this respect, compared with Cases I(a) and II(a) for which reward-penalty costs are eliminated in the objective function, the other two cases, i.e. I(b) and II(b), in which the cost of incentive regulation is taken into account, have better reliability levels. Next to be noted is the importance of incorporation of reward-penalty costs into the planning studies. Take cases I(a) and I(b) as an example, the former in which the company neglected the incentive reliability costs into the planning study results in a high total cost due to the penalties from reward-penalty schemes. This is because in Cases I(a) and II(a), the objective function of the optimization problem is almost insensitive to the network reliability level. In contrast, outcomes of Case I(b), which is completely similar to Case I(a) except for the incorporation of the reward-penalty costs into the objective function, show a reduced total cost owing to the bonuses gained from incentive schemes.

In order to more accurately compare the results, optimal network topology of all cases are represented in Fig. 7. Comparing the first two cases, i.e. I(a), I(b), it can be observed that although the topologies are somehow similar, in the second case where network reliability is linked to the companies revenue through the reward-penalty scheme, more reliable alternatives are selected for construction of the feeder sections. In other words, in Case I(b) most of the feeders were added based on alternatives $k3$, $k4$ (see Table I) with lower failure rates.

A glance at the optimal topologies for Cases II(a), II(b) reveals that in the latter case, DG units are installed in more nodes. Moreover, total installed capacity of DGs are also increased. Even though these result in higher investment cost owing to the more required network capacity, system reliability is markedly enhanced in this case. In addition, similar to Case I(b), majority of the new feeders are chosen from alternatives $k3$, $k4$.

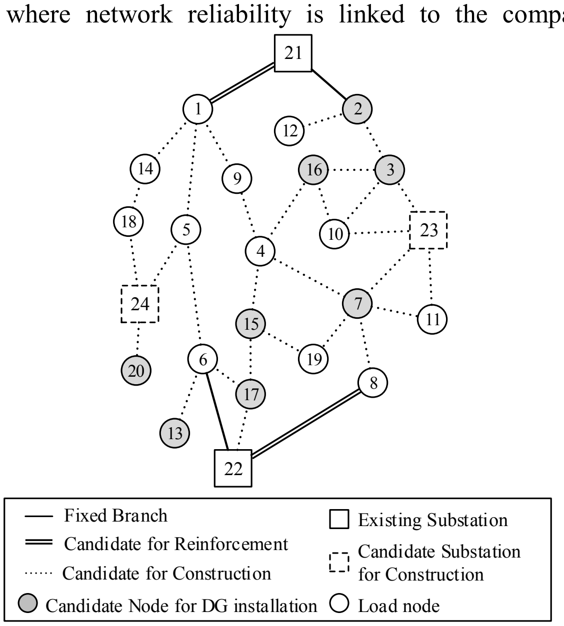


Fig. 6. One-line diagram of the 24-node test distribution network.

TABLE I
ALTERNATIVES FOR CONSTRUCTION OF NEW FEEDER SECTIONS

| ALTERNATIVE | $k1$ | $k2$ | $k3$ | $k4$ |
|---|---|---|---|---|
| INVESTMENT COST (K$/KM) | 15.02 | 25.03 | 17.02 | 27.33 |
| OPERATING COST ($/KM) | 450 | 450 | 450 | 450 |
| FAILURE RATE (FAILURE/KM/YEAR) | 0.3 | 0.3 | 0.2 | 0.2 |
| REPAIR TIME (HOURS/REPAIR) | 0.5 | 0.5 | 0.5 | 0.5 |
| CAPACITY (MVA) | 3.94 | 6.28 | 3.94 | 6.28 |

TABLE II
ALTERNATIVES FOR INSTALLATION OF DG UNITS

| ALTERNATIVE | $k1$ | $k2$ |
|---|---|---|
| INVESTMENT COST (K$/MVA) | 490 | 500 |
| PRODUCTION COST ($/MWH) | 47 | 45 |
| CAPACITY (MVA) | 1 | 2 |

TABLE III
ALTERNATIVES FOR NEW SUBSTATION TRANSFORMERS

| ALTERNATIVE | $k1$ | $k2$ |
|---|---|---|
| INVESTMENT COST (K$) | 750 | 950 |
| MAINTENANCE COST ($) | 2000 | 3000 |
| CAPACITY (MVA) | 12 | 15 |

TABLE IV
SIMULATION RESULTS FOR THE 24-NODE SYSTEM (ALL COSTS ARE IN M$)

| | | Cases | | | |
|---|---|---|---|---|---|
| | | Without DG installation | | With DG installation | |
| Reward-penalty costs | | Neglected | Included | Neglected | Included |
| | Year | I(a) | I(b) | II(a) | II(b) |
| Investment cost of substations and feeders | t=1 | 2.963 | 3.368 | 3.021 | 3.299 |
| | t=2 | 1.822 | 1.595 | 0.299 | 0.311 |
| | t=3 | 1.525 | 1.392 | 1.561 | 0.222 |
| Operating cost of substations and feeders | t=1 | 7.508 | 7.510 | 6.009 | 6.010 |
| | t=2 | 12.443 | 12.444 | 9.955 | 9.956 |
| | t=3 | 21.132 | 21.132 | 16.906 | 16.905 |
| Investment cost of DGs | t=1 | 0 | 0 | 3.966 | 3.966 |
| | t=2 | 0 | 0 | 2.626 | 3.296 |
| | t=3 | 0 | 0 | 0 | 0 |
| Operating cost of DGs | t=1 | 0 | 0 | 1.192 | 1.189 |
| | t=2 | 0 | 0 | 1.957 | 2.002 |
| | t=3 | 0 | 0 | 2.728 | 2.727 |
| EENS (MWh) | t=1 | 21.828 | 5.881 | 4.924 | 6.089 |
| | t=2 | 27.33 | 14.877 | 8.203 | 6.699 |
| | t=3 | 30.434 | 25.65 | 24.703 | 15.577 |
| SAIFI (Int./Cust./Year) | t=1 | 1.608 | 0.662 | 0.653 | 0.595 |
| | t=2 | 1.318 | 0.794 | 0.499 | 0.396 |
| | t=3 | 0.891 | 0.768 | 0.64 | 0.456 |
| SAIDI (h/Cust./Year) | t=1 | 1.659 | 0.482 | 0.488 | 0.572 |
| | t=2 | 1.305 | 0.732 | 0.42 | 0.343 |
| | t=3 | 0.843 | 0.736 | 0.598 | 0.436 |
| **INV+OP** | | **282.96** | **283.004** | **268.563** | **268.32** |
| ***RLUE*** | | **0.032** | **0.026** | **0.024** | **0.016** |
| $Pen_{SAIFI}$-$Rew_{SAIFI}$ | | **0.541** | **-0.078** | **-0.627** | **-1.344** |
| $Pen_{SAIDI}$-$Rew_{SAIDI}$ | | **0.425** | **-2.274** | **-4.231** | **-6.21** |
| **Total cost** | | **283.958** | **280.678** | **263.729** | **260.782** |
| Simulation time (hour) | | 0.2080 | 30.3901 | 0.2998 | 13.2202 |

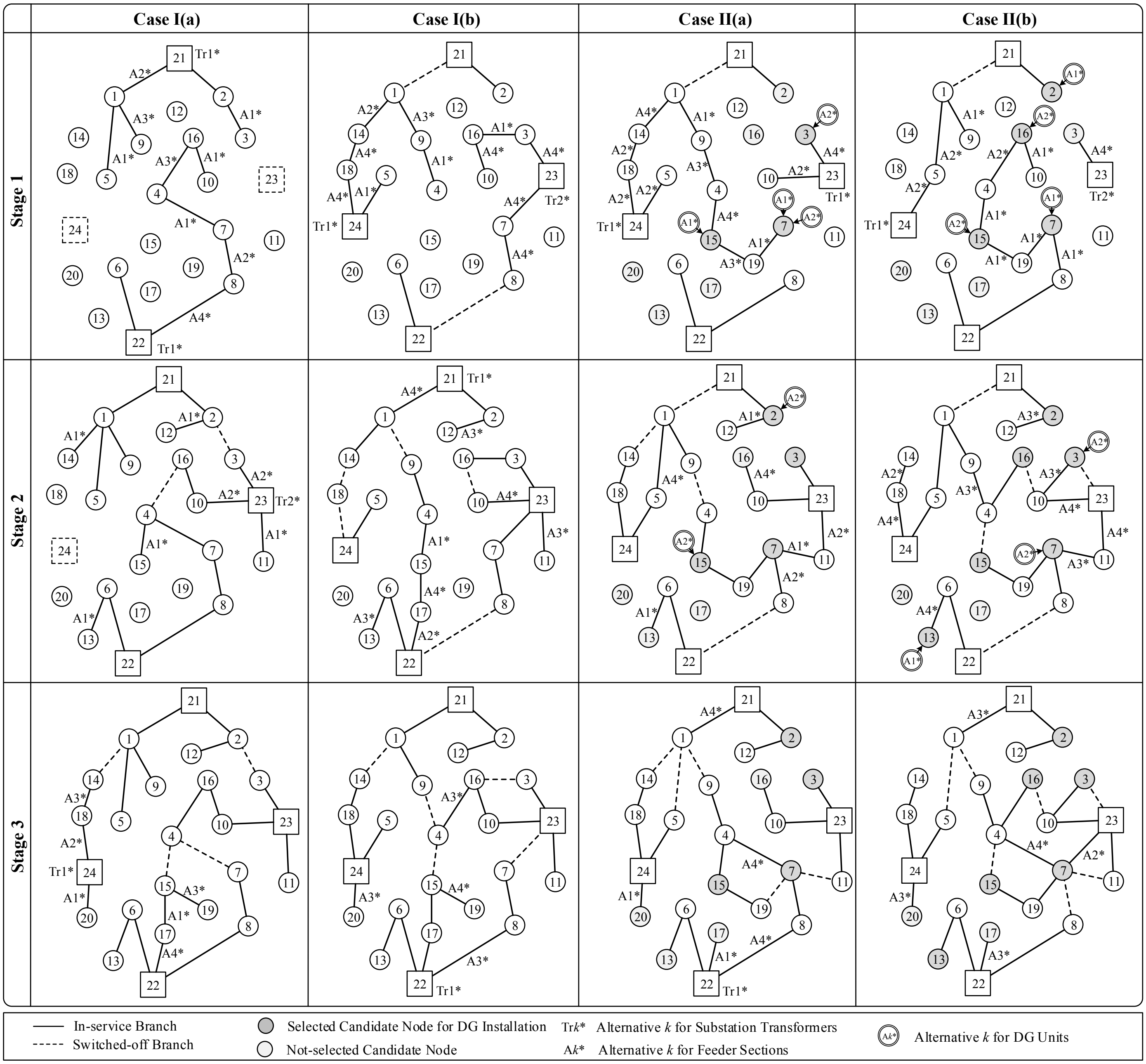


Fig. 7. Optimal network topology of the 24-node test system for investigated cases.

Furthermore, as can be inferred from Table IV, a significant share of total network expansion cost is related to the operating expenses. Hence, improving the network reliability using this extra investment cost does not considerably increase the total expansion cost.

Under the aforementioned stop criteria, the execution time for various cases is also presented in Table IV. As can be seen, simulation time for cases including the reward-penalty costs are considerably more than the other two cases. Nonetheless, it should be noted that the proposed formulation for modeling reward-penalty scheme has a better performance in comparison with the only available MILP model which can be found in the literature [23]. This is most likely due to the existence of some equations which put sharp boundaries on binary variables in [23]. In fact, utilizing the proposed formulation of [23] in our model, CPLEX was unable to reduce the optimality gap of Case I(b) under 3.01 percent after about 145 hours. Another interesting point is the case dependency of CPLEX algorithm: comparing I(a) to II(a), it can be concluded that by introducing the DG related decision variables to the planning model, the simulation time would be increased. However, execution time of the other two cases, I(b) and II(b), show a completely different trend. Hence increasing the number of decision variables do not necessarily increase simulation time.

In addition, it is worth nothing that the arbitrary value of big numbers, $M$, have a considerable effect on performance of the CPLEX algorithm. In this respect, it should be noted that choosing different values of big numbers for each equation is strongly recommended. We found that selection of the lowest possible value of $M$ for some equations can reduce the simulation time. However, it was not a case for most of the equations. In general, no specific trend was found between values of big numbers, *M*, and the efficiency of CPLEX solver. Nonetheless, we tried to find the best values of these parameters for Case I(a) based on a trial and error method and then we used the same values for the other cases.

It should be noted that full-capacity operation of DG units is possible during all load levels. Thus, the assumption made for considering the DG impact on reliability indices is valid and there is no need to use the model presented in the Appendix.

The complete set of results associated with the abovementioned cases, including topology of network during each stage can also be downloaded from [25].

*B. 54-node Test Distribution System*

This test system is comprised of 50 load points, 4 substations, and 63 branches including 9 fixed, and 8 reinforceable feeder sections, as well as 46 candidate branches for adding to the network. The planning horizon is 10 years, which is divided to one-year stages. In order to assess the effect of reliability considerations on the expansion planning solution, four cases are investigated herein. In Case I, only revenue lost due to undelivered energy, *RLUE*, is included in the objective function, while in Case II, value of customer interruption costs are also considered using the traditional technique based on the value of lost load (VoLL). In this case, reliability related costs $RRC_t$ in (1) is equal to the multiplication of the VoLL and the *EENS* index. Moreover, VoLL is set to 11200$ per MWh of not served electricity [12]. In Cases III and IV, reliability related costs are calculated using two reward-penalty schemes based on SAIFI and SAIDI indices. Although investment in DG units is disregarded in the first three cases, it is included in Case IV.

Summary of the obtained results are provided in Table V. A glance at this table reveals that, the *RLUE* is relatively insignificant and cannot satisfactorily motivate the company to invest on reliability.

Comparing Cases I and II, it can be observed that in the latter, investment and operating costs of the system rise in exchange for the reliability enhancement. However, since the reduction of the customer interruption cost is dramatically higher, the total cost for the second case is markedly lower than that of Case I. Moreover, as mentioned in the table, the simulation time is considerably higher when the customer interruption cost is included in the objective function.

Under the incentive reliability regulation regime considered in Cases III and IV, the company has higher motivation for enhancing the reliability level, which is reflected in lower values of *RLUE*. As a consequence, the investment and operating costs of the network increases in Case III, compared to the first two cases. Nonetheless, these costs are lower for Case IV due to the impact of DGs on investment costs of feeders and substations as well as cost of supplying demands. Again, incorporating the reward-penalty schemes into the model increases the simulation time. In addition, the simulation time for the case with DG investment option, i.e., Case IV, is lower than that of Case III in which DGs are disregarded.

TABLE V
SIMULATION RESULTS FOR THE 54-NODE SYSTEM (ALL COSTS ARE IN M$)

| **YEAR** | **Case I** | **Case II** | **Case III** | **Case IV** |
|---|---|---|---|---|
| **INV+OP** | **29.223** | **30.030** | **30.422** | **28.547** |
| *RLUE* | 0.037 | 0.025 | 0.024 | 0.015 |
| Interruption cost | 9.655 | 6.438 | - | - |
| ***$Pen_{SAIFI}$-$Rew_{SAIFI}$*** | - | - | -0.008 | -0.728 |
| ***$Pen_{SAIDI}$-$Rew_{SAIDI}$*** | - | - | -3.289 | -12.334 |
| **TOTAL COST** | **38.915** | **36.493** | **27.149** | **15.500** |
| Simulation time (Hour) | 2.59 | 9.90 | 54.87 | 33.78 |

Reliability indices of the test system over the planning horizon are depicted in Fig. 8.

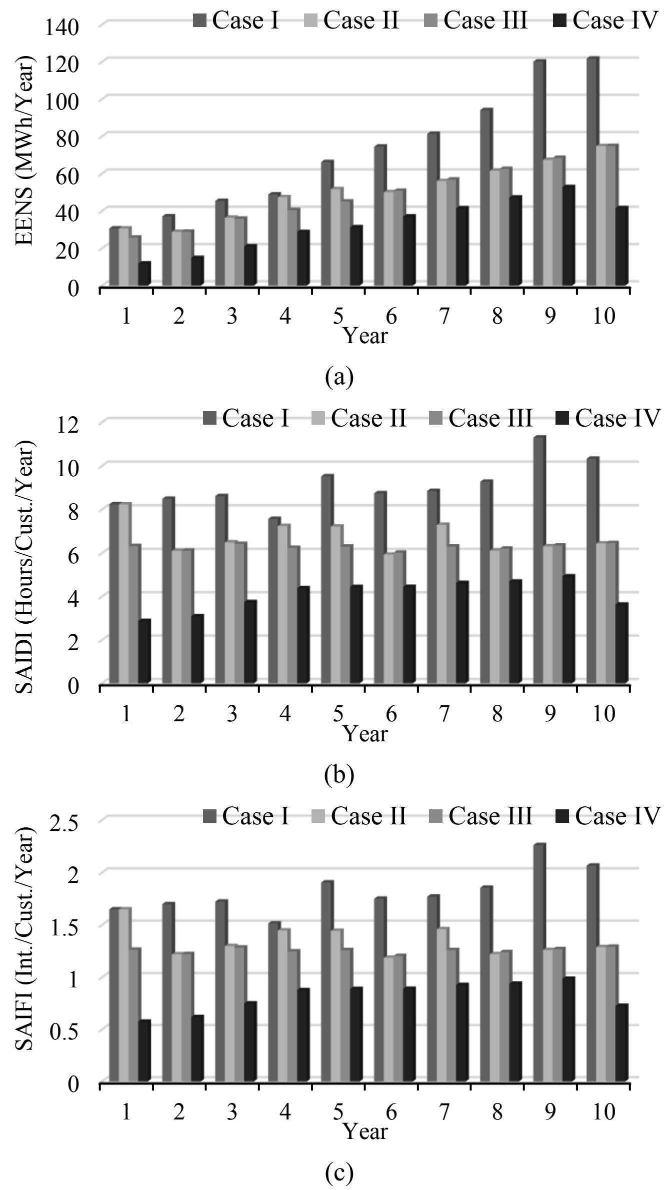


Fig. 8. Reliability indices for the 54-node test system. (a) EENS; (b) SAIDI; (c) SAIFI.

A glance at EENS graph reveals the importance of the reliability considerations in the distribution system planning. In fact, the gaps between EENS graph of Case I and the other three cases widen over time, since the reliability level of the network in later years are more dependent to the investment decisions made in network expansion. By contrast, network reliability in the former years is mostly influenced by the initial condition of the network. Another interesting point is the significant role of

DG units in network reliability enhancement.

Values of SAIFI and SAIDI indices are also depicted in Fig. 8(b) and (c), respectively. Although, there is a noteworthy correlation between these indices, their trends are different from the EENS. Nevertheless, the fourth case has higher reliability level even from the perspective of SAIFI, and SAIDI indicators.

Again, the optimal planning solution satisfies the assumption made for considering the DG impact on reliability indices and implementation of the model presented in the Appendix is not required. Detailed results of these cases are also available in [25].

## VI. Conclusion

A reliability-based framework for expansion planning problem of distribution networks has been studied in the paper. To reach this goal, at first linearized models of different reliability indices are introduced and then involved in MILP model of distribution network planning, for which convergence to the global optimal solution can be guaranteed. The proposed multistage model takes into account costs of installation and reinforcement of substations and feeders, as well as investment and operating costs of distributed generation (DG) units, revenue lost due to undelivered energy, and rewards or penalties associated with the implementation of incentive reliability regulations. The proposed method was implemented on 24-node and 54-node distribution test networks and different cases were defined. We observed that installation of DGs could effectively help system operators and decision makers in cutting back total network investment and operating costs as well as enhancing reliability level. Moreover, we discussed that how incentive-based regulation can motivate distribution companies in applying reliability-based expansion plans. Some useful points are also presented about the application of CPLEX for solving the proposed model.

## Appendix

In the model proposed in Section III, $DDGC_{b,t}$ is the key variable for taking into account the DG impact on reliability indices. However, for calculation of this variable using (15a)−(15i), feeder capacity limits are disregarded. Considering the radial operation of distribution systems, it can be simply proven that during the islanded operation, the DG power flowing down a feeder do not overload feeder sections. However, transferring surplus DG generation to the upstream nodes may result in feeder capacity limits violation. Since the amount of surplus DG generation depends on the load level, we first need to consider the $ll$ index for all the variables involved in the calculation of the downstream DG capacity. Thus, in all the equations of Section III, $DDGC_{b,t}$, $DDGC^{+}_{b,t}$, $DDGC^{-}_{b,t}$, and $GDDGC^{Sub}_{s,t}$ are replaced by $DDGC_{b,t,ll}$, $DDGC^{+}_{b,t,ll}$, $DDGC^{-}_{b,t,ll}$, and $GDDGC^{Sub}_{s,t,ll}$, respectively. In order to restrict the surplus DG power generated downstream of a feeder section during the islanded operation, following constraints have to be considered:

$$DDGC_{b,t,ll} - DD_{b,t,ll} \le \sqrt{3}\mathrm{V}^{\mathrm{r}}\mathrm{I}^{\max}_{b}\varphi^{Fe,Fi}_{b,t}; \quad \forall b \in \Pi^{f}, \forall t \in \{1,...,T\}, \forall ll \in \{1,...,N_l\} \tag{24}$$

$$DDGC_{b,t,ll} - DD_{b,t,ll} \le \sqrt{3}\mathrm{V}^{\mathrm{r}}\left(\mathrm{I}^{\max}_{b}\varphi^{Fe,Re}_{b,t} + \sum\nolimits_{k\in\Gamma^{Fe}_{b}}\mathrm{I}^{\max}_{b,k}\varphi^{Fe,Re}_{b,k,t}\right); \quad \forall b \in \Pi^{r}, \forall t \in \{1,...,T\}, \forall ll \in \{1,...,N_l\} \tag{25}$$

$$DDGC_{b,t,ll} - DD_{b,t,ll} \le \sqrt{3}\mathrm{V}^{\mathrm{r}}\left(\sum\nolimits_{k\in\Upsilon^{Fe}_{b}}\mathrm{I}^{\max}_{b,k}\varphi^{Fe,Ne}_{b,k,t}\right); \quad \forall b \in \Pi^{c}, \forall t \in \{1,...,T\}, \forall ll \in \{1,...,N_l\} \tag{26}$$

Moreover, equation (15b) must be modified to allow operating DG units under full capacity in the islanded states. In this regard, a new non-negative axillary variable $\kappa_{lp,t}$ is added to (15b) as follows:

$$\sum_{b\in\Psi^{lp}}\chi_{lp,b}(DDGC^{+}_{b,t,ll} - DDGC^{-}_{b,t,ll}) = \sum_{k\in\Lambda^{DG}_{lp}}\sum_{\tau=1}^{t}\mathrm{Cap}^{\mathrm{DG}}_{k}\sigma^{DG}_{lp,k,\tau} - \kappa_{lp,t}; \quad \forall lp \in \Omega^{D}, \forall t \in \{1,...,T\}, \forall ll \in \{1,...,N_l\} \tag{27}$$

These modifications guarantee that feeder sections are not overloaded during the islanding states.

## References


[1] R. Billinton and R. Allan, *Reliability Evaluation of Power Systems*. Berlin, Germany: Springer Science & Business Media, 2013.

[2] Council of European energy regulators (CEER), "6th CEER benchmarking report on the quality of electricity and gas supply," 2016.

[3] E. Fumagalli, L. Schiavo, and F. Delestre, *Service Quality Regulation in Electricity Distribution and Retail*. Berlin, Germany: Springer Science & Business Media, 2007.

[4] A. Abiri-Jahromi, M. Fotuhi-Firuzabad, and E. Abbasi, "An efficient mixed-integer linear formulation for long-term overhead lines maintenance scheduling in power distribution systems," *IEEE trans. Power Delivery,* vol. 24, no. 4, pp. 2043-2053, Oct. 2009.

[5] B. R. P. Junior, A. M. Cossi, J. Contreras, and J. R. S. Mantovani, "Multiobjective multistage distribution system planning using tabu search," *IET Gener. Transm. Distrib.,* vol. 8, no. 1, pp. 35-45, Jan. 2014.

[6] S. Heidari, M. Fotuhi-Firuzabad, and S. Kazemi, "Power distribution network expansion planning considering distribution automation," *IEEE Trans. Power Syst.,* vol. 30, no. 3, pp. 1261-1269, May 2015.

[7] S. Heidari, M. Fotuhi-Firuzabad, and M. Lehtonen, "Planning to Equip the Power Distribution Networks with Automation System," *IEEE Trans. Power Syst.,* vol. 32, no. 5, Sep. 2017.

[8] S. M. Mazhari, H. Monsef, and R. Romero, "A multi-objective distribution system expansion planning incorporating customer choices on reliability," *IEEE Trans. Power Syst.,* vol. 31, no. 2, pp. 1330-1340, Mar. 2016.

[9] R. C. Lotero and J. Contreras, "Distribution system planning with reliability," *IEEE Trans. Power Delivery,* vol. 26, no. 4, pp. 2552-2562, Oct. 2011.

[10] G. Muñoz-Delgado, J. Contreras, and J. M. Arroyo, "Multistage generation and network expansion planning in distribution systems considering uncertainty and reliability," *IEEE Trans. Power Syst.,* vol. 31, no. 5, pp. 3715-3728, Sep. 2016.

[11] G. Munoz-Delgado, J. Contreras, and J. Arroyo, "Reliability assessment for distribution optimization models: a non-simulation-based linear programming approach," *IEEE Trans. Smart Grid,* vol. 9, no. 4, Jul. 2018.

[12] G. Munoz-Delgado, J. Contreras, and J. M. Arroyo, "Distribution Network Expansion Planning With an Explicit Formulation for Reliability Assessment," *IEEE Trans. Power Syst.,* vol. 33, no. 3, pp. 2583-2596, May 2018.

[13] G. Muñoz-Delgado, J. Contreras, and J. M. Arroyo, "Joint expansion planning of distributed generation and distribution networks," *IEEE Trans. Power Syst.,* vol. 30, no. 5, pp. 2579-2590, Sep. 2015.

[14] S. Haffner, L. F. A. Pereira, L. A. Pereira, and L. S. Barreto, "Multistage model for distribution expansion planning with distributed generation—Part I: Problem formulation," *IEEE Trans. Power Delivery,* vol. 23, no. 2, pp. 915-923, Apr. 2008.

[15] R. A. Jabr, "Radial distribution load flow using conic programming," *IEEE trans. power syst.,* vol. 21, no. 3, pp. 1458-1459, Aug. 2006.

[16] P. A. Trodden, W. A. Bukhsh, A. Grothey, and K. I. McKinnon, "Optimization-based islanding of power networks using piecewise linear AC power flow," *IEEE Trans. Power Syst.,* vol. 29, no. 3, pp. 1212-1220, May 2014.

[17] R. S. Ferreira, C. L. T. Borges, and M. V. Pereira, “A flexible mixed-integer linear programming approach to the AC optimal power flow in distribution systems,” *IEEE Trans. Power Syst.,* vol. 29, no. 5, pp. 2447-2459, Sep. 2014.

[18] M. Ghasemi and R. Dashti, “A risk-based model for performance-based regulation of electric distribution companies,” *Utilities Policy,* vol. 45, pp. 36-44, Apr. 2017.

[19] R. Billinton and Z. Pan, “Historic performance-based distribution system risk assessment,” *IEEE Trans. Power Delivery,* vol. 19, no. 4, pp. 1759-1765, Oct. 2004.

[20] A. Moradkhani, M. R. Haghifam, and S. M. Abedi, “Risk-based maintenance scheduling in the presence of reward penalty scheme,” *Electric Power Systems Research,* vol. 121, pp. 126-133, Apr. 2015.

[21] K. Alvehag and K. Awodele, “Impact of reward and penalty scheme on the incentives for distribution system reliability,” *IEEE Trans. Power Syst.,* vol. 29, no. 1, pp. 386-394, Jan. 2014.

[22] M. Simab, K. Alvehag, L. Söder, and M.-R. Haghifam, “Designing reward and penalty scheme in performance-based regulation for electric distribution companies,” *IET gener. transm. distrib.,* vol. 6, no. 9, pp. 893-901, Sep. 2012.

[23] H. Mohammadnezhad-Shourkaei, A. Abiri-Jahromi, and M. Fotuhi-Firuzabad, “Incorporating service quality regulation in distribution system maintenance strategy,” *IEEE Trans. Power Delivery,* vol. 26, no. 4, pp. 2495-2504, Oct. 2011.

[24] B. Bollobás, *Modern Graph Theory, vol. 184*. Berlin, Germany: Springer Science & Business Media, 2013.

[25] https://drive.google.com/drive/folders/1ZKJn1ouh4XmU_WiIvqqHjCZn-_Rq9829?usp=sharing