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Sokolov, Maksim; Gruber, Wolfgang; Saarakkala, Seppo E.; Hinkkanen, Marko Modeling of a bearingless synchronous reluctance motor with combined windings

Published in: Proceedings of the 11th IEEE Energy Conversion Congress and Exposition, ECCE 2019

DOI: 10.1109/ECCE.2019.8913000

Published: 29/09/2019

Document Version Peer-reviewed accepted author manuscript, also known as Final accepted manuscript or Post-print

Please cite the original version:

Sokolov, M., Gruber, W., Saarakkala, S. E., & Hinkkanen, M. (2019). Modeling of a bearingless synchronous reluctance motor with combined windings. In *Proceedings of the 11th IEEE Energy Conversion Congress and Exposition, ECCE 2019* (pp. 7084-7090). (IEEE Energy Conversion Congress and Exposition). IEEE. https://doi.org/10.1109/ECCE.2019.8913000

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Modeling of a Bearingless Synchronous Reluctance Motor With Combined Windings

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Abstract—This paper deals with modeling of bearingless synchronous reluctance motors with a combined winding. A method to link an existing model used for the separated windings structure to the combined winding structure is proposed. A dynamic model applicable for the purposes of time-domain simulation, model-based control design, and realtime control is presented. The finite-element method (FEM) is used to validate the proposed model and to show the feasibility of the considered slice motor type, including passive stability of axial movement and tilting, force and torque production, and ripple. Applicability of the developed model in control design is demonstrated. The model is validated by means of experiments.

Index Terms—Control, levitation, model, slice motor.

I. INTRODUCTION

In magnetically levitated systems, bearingless motors are the next step in integrating the functions of an electric motor and an active magnetic bearing (AMB) in one unit. Different motor types have been used as bearingless motors. A conventional synchronous reluctance machine (SyRM) may be used as an alternative to an induction motor in industrial and automotive applications that require a simple rotor structure, cheap materials and manufacturing, and comparatively high torque density [1]. Bearingless versions of the conventional SyRMs have been presented in the literature [2]–[4]. However, a so-called slice motor variant of bearingless SyRMs has only recently been introduced in [5]. Fig. 1 shows an example of a bearingless slice SyRM.

In slice motors, the length of the rotor stack is small compared to its diameter. When magnetization is applied, such a disk-shaped rotor gives inherent passive stabilization for axial displacement and tilting degrees of freedom. Together with active control of radial displacement, slice motors can provide completely contactless operation with just one motor unit. For this reason, slice motors are often used in specialized fluid processing applications where the rotor can be placed inside and the stator outside the containment shell. This property is used, e.g., in bioreactor mixing, semiconductor manufacturing, and blood pumps [6].

Typically, slice motors utilize permanent magnets (PMs) to create a magnetic field in the airgap and produce passive stabilizing forces [7]. Bearingless slice SyRMs have the following advantages over the structures that include PMs: the ability to adjust magnetization level and turn off the field completely, easier levitation start-up, cheaper rotor construction, and higher temperature tolerance. The challenges



Fig. 1. Bearingless slice SyRM with a combined concentrated winding.

with this motor type are in the constant need for active magnetization which increases copper losses, requirement for smaller airgap, and potentially lower passive stabilization stiffness. In [5], a slice SyRM was proven feasible for bearingless operation and showed comparable performance to the motor types with PMs.

To be able to generate both driving torque and radial suspension force, bearingless motors require more degrees of freedom than conventional motors. This is commonly achieved with two separate winding sets: one for torque production and another for force production [8]. Alternatively, a multiphase winding can be used that carries both torque and force generating currents [9]. An important feature of this topology is a more efficient use of the stator slot space. A term "combined winding" is used due to combining the force and torque producing functionalities in the same winding [7]. Approaches to superimpose the torque and force current components into the combined winding are discussed in [10] and [11]. However, a dynamic model for such bearingless motors with a combined winding, to the best of the authors knowledge, has not been presented in literature before.

Modeling of bearingless SyRMs with a combined winding is the focus of this paper. The main contributions can be summarized as follows:

1) Proposing a way to link an established model of the

bearingless SyRM with separated windings [12] to the structures with a combined winding.

- 2) Using FEM analysis to validate the proposed transformations and to analyze the feasibility of the considered bearingless slice SyRM structure.
- 3) Demonstrating the applicability of the developed model in control design.
- 4) Validating the model by means of experiments.

The resulting dynamic model can be applied, e.g., in timedomain simulations, stability analysis, model-based control design, and in real-time control algorithms.

II. COMBINED WINDING MOTOR MODEL

In order to develop a model-based control system, a mathematical model of the machine is required. For successful levitation control, it is especially important to have a model that is able to predict the radial forces in all possible operating points. A well-known model, presented in [12], has been widely used to describe the behavior of bearingless motors, and it was shown to be appropriate in the case of bearingless SyRMs [2]. However, this model is only directly applicable to the machines with separated windings, in which torque and force are produced by two separate sets of three-phase windings with p and $p \pm 1$ number of pole pairs, respectively. The main and suspension windings each occupy part of the stator slots and can be supplied by two three-phase inverters to control torque and radial force. A combined winding topology is another option that can be realized with multiphase machines, in which every phase contributes to the production of both torque and force. The combined winding features a better utilization of the stator slot space, as all copper can be simultaneously used for both torque and force production. One limitation of this winding type is that the rotation-induced back-EMF can adversely affect the voltage margin available for the production of radial force [12].

A. Combined Winding Transformation

This paper considers a six-phase winding structure. Two three-phase windings are arranged into independent star connections, which allows to supply the machine with two general-purpose three-phase inverters. The proposition is that the separated windings model from [12] can be used for the combined winding structure by superimposing the main and suspension winding currents into the combined winding.

Only four-pole flux is generated when the condition $[i_{A1}, i_{B1}, i_{C1}] = [i_{A2}, i_{B2}, i_{C2}]$ is fulfilled. This case can be visualized by an equivalent four-pole winding connection in Fig. 2(a) and the corresponding field solution in Fig. 3(a). The four-pole current component of phase A can be defined as $i_{tA} = (i_{A1} + i_{A2})/2$, which is equal to the mean current value between the opposite phases. Corresponding definitions apply to the components i_{tB} and i_{tC} .

Only two-pole flux is generated when $[i_{A1}, i_{B1}, i_{C1}] = [-i_{A2}, -i_{B2}, -i_{C2}]$ This case is visualized by an equivalent

two-pole winding connection in Fig. 2(b) and the corresponding field solution in Fig. 3(b). The two-pole current component of phase A can be defined as $i_{fA} = (i_{A1} - i_{A2})/2$, which is equal to half of the difference between the current values in the opposite phases. Corresponding definitions apply to the components i_{fB} and i_{fC} .

Both of these connections can be realized at the same time by using a six-phase winding connection in Fig. 2(c). When both the two-pole flux and the four-pole flux are produced, a radial force is generated due to the unbalanced flux distribution in the airgap. This case is demonstrated in Fig. 3(c).

The four-pole flux is responsible for generating the driving torque and the two-pole flux generates the unbalanced flux distribution that results in radial force. Hence, the currents producing the four-pole and the two-pole fluxes are referred to as torque and force producing current components, respectively. Although note that the radial force on a centric rotor is produced only when both the two-pole and the four-pole fluxes are present. Four-pole magnetization plays the same role as the bias flux in AMBs.

Superimposing the torque and force producing currents results in

$$\begin{bmatrix} i_{A1} \\ i_{B1} \\ i_{C1} \end{bmatrix} = \begin{bmatrix} i_{tA} + i_{fA} \\ i_{tB} + i_{fB} \\ i_{tC} + i_{fC} \end{bmatrix}, \begin{bmatrix} i_{A2} \\ i_{B2} \\ i_{C2} \end{bmatrix} = \begin{bmatrix} i_{tA} - i_{fA} \\ i_{tB} - i_{fB} \\ i_{tC} - i_{fC} \end{bmatrix}$$
(1)

and the corresponding inverse transformation is given as

$$i_{\rm ta} = \frac{i_{\rm A1} + i_{\rm A2}}{2}, \ i_{\rm tb} = \frac{i_{\rm B1} + i_{\rm B2}}{2}, \ i_{\rm tc} = \frac{i_{\rm C1} + i_{\rm C2}}{2},$$

$$i_{\rm fa} = \frac{i_{\rm A1} - i_{\rm A2}}{2}, \ i_{\rm fb} = \frac{i_{\rm B1} - i_{\rm B2}}{2}, \ i_{\rm fc} = \frac{i_{\rm C1} - i_{\rm C2}}{2}.$$
 (2)

Since the star points are not connected, the zero-sequence currents cannot flow. Hence, an equivalent two-phase $\alpha\beta$ model can be used

$$\boldsymbol{i}_{t}^{s} = \begin{bmatrix} i_{t\alpha} \\ i_{t\beta} \end{bmatrix} = \underbrace{\frac{2}{3} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix}}_{\boldsymbol{T}_{t}} \begin{bmatrix} i_{tA} \\ i_{tB} \\ i_{tC} \end{bmatrix}$$
$$\boldsymbol{i}_{f}^{s} = \begin{bmatrix} i_{f\alpha} \\ i_{f\beta} \end{bmatrix} = \underbrace{\frac{2}{3} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & -\sqrt{3}/2 & \sqrt{3}/2 \end{bmatrix}}_{\boldsymbol{T}_{f}} \begin{bmatrix} i_{fA} \\ i_{fB} \\ i_{fC} \end{bmatrix}}$$
(3)

Notice, that the phase sequence is reversed in the transformation of the current vector $i_{\rm f}^{\rm s}$ since for the two-pole counter-clockwise phase sequence is ACB, while for the four-pole it is ABC.

The resulting space vectors can be transformed into electrical rotor (dq) coordinates

$$\boldsymbol{i}_{\mathrm{t}} = \begin{bmatrix} i_{\mathrm{td}} \\ i_{\mathrm{tq}} \end{bmatrix} = \mathrm{e}^{-2\vartheta_{\mathrm{m}}} \boldsymbol{J} \boldsymbol{i}_{\mathrm{t}}^{\mathrm{s}}$$
 (4a)

$$\dot{\boldsymbol{i}}_{\mathrm{f}} = \begin{bmatrix} i_{\mathrm{fd}} \\ i_{\mathrm{fq}} \end{bmatrix} = \mathrm{e}^{-\vartheta_{\mathrm{m}} \boldsymbol{J}} \boldsymbol{i}_{\mathrm{f}}^{\mathrm{s}}$$
 (4b)



Fig. 2. (a) Three-phase connection that produces four-pole field; (b) Three-phase connection that produces two-pole field; (c) Six-phase combined winding arranged in two three-phase star connected windings $[A_1, B_1, C_1]$ and $[A_2, B_2, C_2]$.



Fig. 3. FEM geometry and magnetic field solutions with color gradient representing the flux density in a range from 0 to 1.6 T: (a) Four-pole flux is produced with the winding connection in Fig. 2(a) and currents $[i_{tA}, i_{tB}, i_{tC}] = [4, -2, -2]$ A; (b) Two-pole flux is produced with the winding connection in Fig. 2(b) and currents $[i_{fA}, i_{fB}, i_{fC}] = [4, -2, -2]$ A; (c) Both four-pole and two-pole flux components are produced with the six-phase combined winding in Fig. 2(c) with phase currents $[i_{A1}, i_{B1}, i_{C1}, i_{A2}, i_{B2}, i_{C2}] = [5, -2.5, -2.5, 3, -1.5, -1.5]$ A or $i_{td} = 4$ A, $i_{fd} = 1$ A and $i_{tq} = i_{fq} = 0$.

where $J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and ϑ_m is the mechanical angle of the shaft. Notice that the electrical angle is used in the transformations and is equal to $2\vartheta_m$ for the torque component and ϑ_m for the force component.

A magnetic field solution in Fig. 3(c) demonstrates a case when both magnetization and positive x-axis force are requested with $i_{td} = 4$ A, $i_{fd} = 1$ A and $i_{tq} = i_{fq} = 0$.

The resulting current vectors i_t and i_f can be used independently to create four-pole and two-pole rotating magnetic fields and are essentially equivalent to having physically separate four-pole main and two-pole suspension windings placed in the stator slots. Therefore, control algorithms that are used for the machines with separated windings can be directly applied to the systems with a combined winding.

B. Dynamic Model

In this subsection, an existing modeling approach from [12] is directly applied to the machine with a combined winding. The torque and force components are equivalent to the main-winding and suspension-winding components, which are used in separated windings machines. The voltages, flux linkages, and currents can be transformed into the dq coordinates using (2), (3) and (4). The flux linkages are

$$\begin{split} \boldsymbol{\psi}_{t} &= \boldsymbol{L}_{t} \boldsymbol{i}_{t} + \boldsymbol{M} \boldsymbol{i}_{f} \\ \boldsymbol{\psi}_{f} &= \boldsymbol{L}_{f} \boldsymbol{i}_{f} + \boldsymbol{M}^{\mathrm{T}} \boldsymbol{i}_{t} \end{split} \qquad \boldsymbol{L}_{t} = \begin{bmatrix} \boldsymbol{L}_{\mathrm{d}} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{L}_{\mathrm{q}} \end{bmatrix} \tag{5} \\ \boldsymbol{L}_{\mathrm{f}} &= \begin{bmatrix} \boldsymbol{L}_{\mathrm{f}} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{L}_{\mathrm{f}} \end{bmatrix} \qquad \boldsymbol{M} = \begin{bmatrix} \boldsymbol{M}_{\mathrm{d}}^{\prime} \boldsymbol{i} & -\boldsymbol{M}_{\mathrm{d}}^{\prime} \boldsymbol{j} \\ \boldsymbol{M}_{\mathrm{q}}^{\prime} \boldsymbol{j} & \boldsymbol{M}_{\mathrm{q}}^{\prime} \boldsymbol{i} \end{bmatrix} \end{split}$$

where $L_{\rm d}$, $L_{\rm q}$, $L_{\rm f}$ are inductances and $M'_{\rm d}$, $M'_{\rm q}$ are radialforce constants (H/m). The rotor displacement in mechanical rotor coordinates is denoted with *i* and *j*.

The voltage equations of the fictitious torque and force windings are

$$\frac{\mathrm{d}\boldsymbol{\psi}_{\mathrm{t}}}{\mathrm{d}t} = \boldsymbol{u}_{\mathrm{t}} - R\boldsymbol{i}_{\mathrm{t}} - 2\omega_{\mathrm{m}}\boldsymbol{J}\boldsymbol{\psi}_{\mathrm{t}}$$

$$\frac{\mathrm{d}\boldsymbol{\psi}_{\mathrm{f}}}{\mathrm{d}t} = \boldsymbol{u}_{\mathrm{f}} - R\boldsymbol{i}_{\mathrm{f}} - \omega_{\mathrm{m}}\boldsymbol{J}\boldsymbol{\psi}_{\mathrm{f}}$$
(6)

where R is the winding resistance and $\omega_{\rm m} = \mathrm{d}\vartheta_{\rm m}/\mathrm{d}t$ is the rotational speed of the shaft.

Assuming small deviations from the centric rotor position, the radial-force vector in stationary mechanical xycoordinates is



Fig. 4. Control system structure of a combined winding bearingless SyRM, based on the developed model and transformations.



Fig. 5. (a) Current transformations; (b) Voltage transformations.

$$\boldsymbol{F}^{\mathrm{s}} = \begin{bmatrix} F_{\mathrm{x}} \\ F_{\mathrm{y}} \end{bmatrix} = \mathbf{e}^{\vartheta_{\mathrm{m}}\boldsymbol{J}} \begin{bmatrix} M'_{\mathrm{d}}i_{\mathrm{td}} & M'_{\mathrm{q}}i_{\mathrm{tq}} \\ M'_{\mathrm{q}}i_{\mathrm{tq}} & -M'_{\mathrm{d}}i_{\mathrm{td}} \end{bmatrix} \boldsymbol{i}_{\mathrm{f}}$$
(7)

and the electromagnetic torque is

$$T_{\rm M} = 3(\psi_{\rm td} i_{\rm tq} - \psi_{\rm tq} i_{\rm td}) = 3(L_{\rm d} - L_{\rm q})i_{\rm td} i_{\rm tq} \qquad (8)$$

Torque generation is additionally influenced by the two-pole flux, but this effect is neglected.

Equations (5), (6), (7) and (8) describe the dynamic model of a bearingless SyRM. Using transformations (2), (3), and (4) the model can be applied to the combined winding machines for purposes of simulations or real-time control.

III. MODEL-BASED CONTROL SYSTEM

This section describes how the developed model can be applied in control design. Fig. 4 shows the proposed overall control structure applicable to combined winding machines. The cascaded control system includes the inner and outer control loops, a reference calculation block between them, and necessary transformations for the voltages and currents. The outer control loop consists of levitation and rotation controllers designed using model-based control principles [13]. The levitation controller drives the measured radial positions x and y to zero using the radial-force references $F_{\rm x,ref}$, $F_{\rm y,ref}$. The rotation controller provides the torque reference $T_{\rm M,ref}$. Due to cascaded control structure, the poles of the levitation control loop and the rotation control loop should be placed at least one decade slower than the poles of the current control loop.

As can be seen from (7), when the rotor is rotating and a constant radial force \mathbf{F}^{s} is produced, \mathbf{i}_{f} (similarly as \mathbf{u}_{f} and ψ_{f}) varies sinusoidally, with the angular frequency ω_{m} . Thus, the electrical-angular frequencies of both windings are the same. In order to control the radial force without steady-state errors, the two-pole winding currents in (4b) are further transformed to synchronous-coordinate system using $\mathbf{i}'_{f} = e^{-\vartheta_{m}J}\mathbf{i}_{f}$. These coordinate transformations are depicted in Fig. 5(a). In addition to the coordinate transformations, Fig. 5(a) shows the space-vector transformations (3) as well as the proposed transformation (2) required to map the measured combined winding phase currents to the phase currents of the separated windings.

The current references can be calculated using the same approach as for the machines with separated windings. A minimum value of $i_{\rm td,ref} > 0$ is selected based on required axial and tilting stiffness. When the requested torque $T_{\rm M,ref}$ is known, $i_{\rm tq,ref}$ can be solved from (8)

$$i_{\rm tq,ref} = \frac{T_{\rm M,ref}}{3(L_{\rm d} - L_{\rm q})i_{\rm td,ref}}$$
(9)



Fig. 6. Model of the bearingless slice SyRM used in 3D FEM analysis.

In synchronous coordinates, the force expression (7) is

$$\mathbf{F}^{s} = \begin{bmatrix} F_{x} \\ F_{y} \end{bmatrix} = \mathbf{e}^{\vartheta_{m} \boldsymbol{J}} \begin{bmatrix} M'_{d} i_{td} & M'_{q} i_{tq} \\ M'_{q} i_{tq} & -M'_{d} i_{td} \end{bmatrix} \mathbf{e}^{\vartheta_{m} \boldsymbol{J}} \boldsymbol{i}'_{f} \\
= \begin{bmatrix} M'_{d} i_{td} & M'_{q} i_{tq} \\ M'_{q} i_{tq} & -M'_{d} i_{td} \end{bmatrix} \boldsymbol{i}'_{f}$$
(10)

When $i_{td,ref}$, $i_{tq,ref}$, $F_{x,ref}$, and $F_{y,ref}$ are known, $i'_{f,ref}$ can be calculated using (10)

$$\boldsymbol{i}_{\rm f,ref}' = \begin{bmatrix} M_{\rm d}' i_{\rm td,ref} & M_{\rm q}' i_{\rm tq,ref} \\ M_{\rm q}' i_{\rm tq,ref} & -M_{\rm d}' i_{\rm td,ref} \end{bmatrix}^{-1} \begin{bmatrix} F_{\rm x,ref} \\ F_{\rm y,ref} \end{bmatrix}$$
(11)

To realize the requested currents, a PI-type current controller is implemented in synchronous coordinates [14].

The current controller voltage commands are also in dq coordinates and have to be transformed into the phase voltage commands. Fig. 5(b) shows the necessary inverse transformations to transform the synchronous frame voltage references $u_{t,ref}$ and $u_{f,ref}$ to the corresponding combined winding phase-voltage references, which are finally fed to the inverters.

IV. FINITE-ELEMENT ANALYSIS

In this section, the FEM has been used to validate the proposed model and to analyze feasibility of the designed bearingless slice SyRM with concentrated windings.

A. Comparison of the Proposed Model and FEM Results

The studied machine has six stator teeth with doublelayer concentrated windings and a four-pole transverselaminated rotor with flux barriers. The rotor geometry is similar to [15], in which a concentrated winding SyRM was optimized for reduced torque ripple. The rotor stack length is 10 mm. The nominal airgap is 1 mm. The geometry of the machine is shown in Figs. 3 and 6. FEM analysis was carried out in FEMM software using 2D simulations.

For the following comparisons, the model is parametrized with $L_{\rm d} = 18$ mH, $L_{\rm q} = 6.5$ mH, $L_{\rm f} = 16$ mH, $M_{\rm d} = 13.2$ H/m, $M_{\rm q} = 2$ H/m, which were identified using the FEM.



Fig. 7. Comparison between the FEM results and the developed model: (a) Torque as a function of $\vartheta_{\rm m}$ at the nominal operating point $i_{\rm td} = i_{\rm tq} = 6$ A; (b) Forces during constant $i_{\rm td} = 2$ A and $\vartheta_{\rm m}$ varying from 0 to $\pi/2$.

Fig. 7(a) shows the torque as a function of the rotor angle. Based on the FEM results at the nominal operating point, the torque varies from 1.1 to 1.5 Nm with an average value of 1.32 Nm. Calculating the torque from (8) of the model gives a comparable result.

Fig. 7(b) shows a comparison between the forces based on the FEM results and the force expression (7) as $\vartheta_{\rm m}$ varies from 0 to $\pi/2$ with $i_{\rm td} = 2$ A and $i_{\rm tq} = i_{\rm fq} = 0$. The force ripple due to the concentrated winding structure is visible. The model is able to predict the average force amplitude at given currents with accuracy sufficient for use in real-time control.

B. Feasibility of a Slice SyRM With Concentrated Windings

3D FEM simulations were used to investigate passive stabilization of axial displacement and tilting degrees of freedom. The axial passive stabilizing force $F_{\rm stab}$ is mainly a function of the magnetization current $i_{\rm td}$. According to the FEM results, at the axial displacement of 1 mm, $F_{\rm stab} \approx 2.5 \cdot i_{\rm td}$ N/A, which means that 1 A of the



Fig. 8. Radial force production of the bearingless SyRM with concentrated windings: (a) Radial forces at no load $i_{td} = 2$ A, $i_{tq} = 0$; (b) Increase in the radial force ripple when the torque producing current is added $i_{td} = i_{tq} = 2$ A; (c) Force orbits corresponding to (a) and (b), showing ripple and force vector angle error due to cross-coupling between the x and y axes.

magnetizing current is enough to compensate for the weight of the rotor. A higher magnetizing current will further reduce the axial displacement. The tilting stabilizing torque $T_{\rm stab}$ at the rotor tilt angle of 1° is $T_{\rm stab} \approx 0.05 \cdot i_{\rm td}$ Nm/A, which is comparable to other bearingless slice motors of a similar size. The corresponding tilting stiffness is 2.86 Nm/rad at $i_{\rm td} = 1$ A.

2D FEM simulation results show a good correspondence with 3D results in terms of the driving torque and radial force production. Thus, in the following paragraphs, the force ripple is investigated using static 2D simulations.

Fig. 8(a) shows the radial forces as function of rotor angle $\vartheta_{\rm m}$. The currents $i_{\rm td} = 2$ A and $\begin{bmatrix} i_{\rm fd} & i_{\rm fq} \end{bmatrix}^{\rm T} = {\rm e}^{\vartheta_{\rm m}J} \begin{bmatrix} 0.5 & 0 \end{bmatrix}^{\rm T}$ A are supplied to create a constant force vector in the x direction. However, $F_{\rm x}$ changes due to the slotting effects and non-sinusoidal stator magnetomotive force (MMF). Coupling to the y-axis force is also visible. Fig. 8(b) shows that these adverse effects increase when the torque is generated with $i_{\rm tq} = 2$ A.

The error in the force vector angle in bearingless motors is a quantity defined as the angle difference between the requested and produced force vectors. As a common rule of thumb, the angle error of less than 15 degrees is acceptable [16]. This holds for the results in Fig. 8(a). However, when both torque and force are requested in Fig. 8(b), the angle error is larger, which can be seen in the force orbit plot in Fig. 8(c). Thus, for operating at high torque values, an additional compensation in the control system may be needed, e.g., using angle-dependent look-up tables.

V. EXPERIMENTAL RESULTS

In this section experimental results are presented to verify the developed modeling and control methods. The control approach presented in Section III was implemented in a real-time digital signal processor (DSP). Stable levitation and rotation were achieved with the prototype shown in Fig. 1. The winding of the motor is arranged in two threephase star connections and supplied by PWM-operated inverters. The radial and angular position of the rotor, six phase currents, and DC-link voltage are measured and fed back to the DSP to provide necessary feedback information for the controllers. Four eddy current sensors are used for measuring the radial position of the rotor. The rotor rotation angle is measured using Hall sensors, which read the magnetic field of a multipole PM ring attached to the bottom side of the rotor.

The nominal airgap of the motor is 1 mm. Back-up bearings are limiting the radial movement to ± 0.5 mm range. The mass of the rotor is 0.25 kg. The DC-link voltage is 60 V. The current controller is tuned to have a bandwidth of 6000 rad/s. The approximate bandwidth of the levitation control loop is 350 rad/s. The control system is running at the sampling frequency of 10 kHz.

Fig. 9 shows preliminary experimental results. The liftup of the rotor is demonstrated in Fig. 9(a). The upper subplot shows the rotor coordinates x and y as active levitation control is initiated. The bottom subplot shows the current components during the lift-up. The magnetizing current is kept at a constant level, while the force-producing current components are actively controlled to achieve radial position stability. After the active levitation is established, rotation can be started as shown in Fig. 9(b). At t = 0.05 s, the speed reference $\omega_{m,ref}$ is stepped to 30 rad/s. The upper subplot shows the speed response together with the measured rotor angle. The ripple in the speed signal is due to the inaccuracy of the rotor angle measurement in the experimental system. Contactless rotor angle measurement is a non-trivial challenge with a motor type that lacks PMs. However, stable levitation is maintained undeterred by the angle measurement inaccuracy. The phase currents are shown in the bottom subplot.

The tuning of the control system is preliminary with relatively slow dynamics. However, it suffices to demonstrate the bearingless motor in operation and validate the applicability of the developed model to real-time control.



Fig. 9. Preliminary experimental results, demonstrating the applicability of the developed model: (a) Lift-up test showing the rotor transition to centric position and current components during the transient; (b) Rotation test during active levitation control. A speed reference step of 30 rad/s is applied at t = 0.05 s.

VI. CONCLUSIONS

Transformations, developed in this paper, allow linking of the dynamic model of bearingless SyRMs with a combined winding to an existing model of the separated windings structure. The resulting model is applicable, e.g., in timedomain simulations, robustness analysis, control design, and real-time control algorithms. Results of the FEM analysis demonstrate the validity of the proposed model. Furthermore, the FEM was used to study the feasibility of the considered bearingless slice SyRM, including passive stability against axial displacement and tilting, as well as angle dependencies of radial force and torque. Successful levitation and rotation control of the prototype machine shows the applicability of the model in control design and real-time implementation.

ACKNOWLEDGMENT

This work has been supported by the "LCM – K2 Center for Symbiotic Mechatronics" within the framework of the Austrian COMET-K2 program.

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