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Adaptive Coded Modulation for Stabilization of Wireless Networked Control Systems over Binary Erasure Channels

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***Abstract*— This paper proposes adaptive coded modulation for stabilization of wireless networked control systems (WNCSs). We combine a well-known data rate theorem with adaptive coded modulation to guarantee stability and optimize the spectral efficiency in WNCSs. We believe that this is the first work in adaptive coded modulation for stabilization. In addition, we propose three schemes to optimize various objectives with given constraints. Our proposed schemes are as follows: maximizing throughput with energy constraint (MaxTEC), minimizing energy with throughput constraint (MinETC), and minimizing delay with energy constraint (MinDEC). The numerical results show that each scheme is able to select the optimal modulation to optimize objectives at given channel conditions and constraints.**

***Index Terms*— Wireless networked control system; stability; data rate theorem; adaptive modulation; binary erasure channels.**

I. Introduction

The way to control processes that involve communications is revolutionized, because of the recent development of smart devices with advanced wireless communication, sensing, and computing. The Internet of Things (IoT) [1], Tactile Internet [2], and Cyber-Physical Systems (CPS) [3] are emerging technologies that build on the advanced capabilities of such devices. Typically, the overall system is spatially distributed and communication between smart devices (being sensors, actuators or controllers) is mainly supported by a (wireless) communication network. Such spatially distributed systems wherein the control loops occur through a (possibly shared) wireless channel are known as wireless networked control systems (WNCSs).

A conventional feedback control system is based on the assumption of a perfect communication feedback channel and instantaneous sensing. However, the wireless channels have limited resource and are inherently less reliable. From a control perspective, such a channel can lead to message dropouts and/or delays that can cause degradation of the system performance or even instability. As a consequence, WNCSs require a novel design that considers the strict delays and reliability constraints imposed by the communication impairments. Furthermore, in several occasions WNCSs may consist of battery-powered devices, thus adding the energy consumption as yet another challenge [4]. With WNCSs it is necessary to ensure that the closed-loop system satisfies various objectives while handling massage dropout, delay, and attenuate disturbances. Among those objectives, stability is the most important. The stabilization in the presence of message dropout and rate constraints has been extensively studied during the last two decades; see surveys [5], [6] and references therein.

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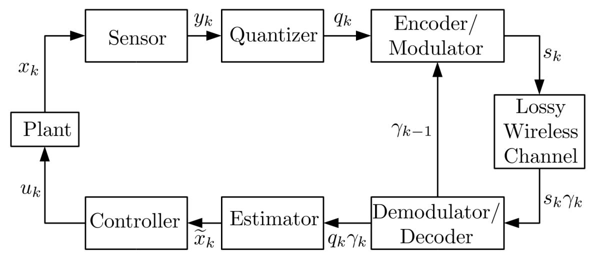


Fig. 1: Network Configuration.

To transmit a sensor reading signal over a wireless channel, first, the signal needs to be quantized, encoded, and modulated into a sequence of symbols. Then, the signal is transmitted over a lossy wireless channel (see Fig. 1). However, there is a possibility that the demodulator/decoder is not able to successfully receive/decode the sensor reading. As a result, the controller fails to generate an updated feedback control signal and stabilize the plant. Ideally, having a high data rate r and high signal-to-noise ratio (SNR) that guarantees the successful reception of the signals can solve the problem. However, such a scenario requires a considerable amount of energy/power, which may not be available in low-power devices.

In this paper, we combine the results for minimum data rates with adaptive coded modulation techniques for wireless channels in order to improve the performance of the WNCS. Adaptive coded modulation is a powerful mechanism for optimizing the spectral efficiency of a wireless channel [7]. This is the first work, as far as the authors are concerned, that uses adaptive coded modulation for stabilization and performance of wireless networked control systems. Our proposed schemes are the following:

1) *Maximizing Throughput with Energy Constraint* (MaxTEC). The MaxTEC scheme selects optimal modulation to maximize achievable throughput in case of limited consumable energy. This scheme is useful when there are uncertainties in the system and/or in the channel parameters, such that it is unknown what is the true minimum data rate required to stabilize the system.

2) *Minimizing Energy with Throughput Constraint* (MinETC). The MinETC scheme selects optimal modulation to minimize required energy if we have a throughput requirement to enhance control performance. For systems which we know the minimum data rate required and performance in not an issue, while there are battery-operated devices and energy is a limited resource, one may wish to minimize the energy comsumption for a throughput constraint that meets the stabilizability conditions.
3) *Minimizing Delay with Energy Constraint* (MinDEC).The MinDEC scheme selects optimal modulation to minimize transmission delay when energy consumption is limited. Minimizing the delay allows for smaller sampling times, which in turn result into a system that requires a lower data rate for stabilization.

The remainder of this paper is organized as follows. In Section II, the system model and preliminaries are presented in detail. In Section III, the proposed optimization schemes are elaborated. The numerical analysis of the proposed schemes is presented and discussed in Section IV. Finally, Section V presents the conclusions.

II. System Model and Preliminaries

A. System Model

We consider a continuous-time dynamical system which is discretized with sampling period T_s, and it is given by

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + w_k, \\ y_k &= Hx_k + v_k, \end{aligned} \tag{1}$$

where $x_k \in \mathbb{R}^{\alpha}$ is the state of the system, $A \in \mathbb{R}^{\alpha\times\alpha}$ is the state matrix, $u_k \in \mathbb{R}^{\beta}$ represents the control input, $w_k \in \mathbb{R}^{\alpha}$ is the process noise with Gaussian distribution $w_k \sim \mathcal{N}(0, W)$, $y_k \in \mathbb{R}^{\kappa}$ is the output of the system observed by a sensor, H is the output matrix, and $v_k \in \mathbb{R}^{\kappa}$ is the measurement noise with Gaussian distribution $v_k \sim \mathcal{N}(0, V)$. Furthermore, A is assumed to be unstable, i.e., $\exists\ \lambda_i > 1$ where $\{\lambda_1, \lambda_2, ..., \lambda_n\}$ denote the eigenvalues of A. Therefore, the dynamic system is unstable in open loop (i.e., $x_k \to \infty$ as $k \to \infty$). We further assume (A, B) is stabilizable, and (A, H) is observable.

Given that there is a limited amount of information bits that can be transmitted via a communication channel, the continuous state y_k must be quantized. As shown in Fig. 1, a quantizer $Q : \mathbb{R}^{\kappa} \to Q_k$, which maps states y_k to an element of a countable set Q_k at time k, where $|Q_k| = 2^r$ is the cardinality of set, when transmitting r bits in each transmission. Subsequently, present quantized signal q_k is encoded into block of n data symbol (complex numbers) s_k. Afterward, the encoded signal is transmitted through a communication channel. Note that the encoded signal s_k is transmitted via a lossy wireless communication channel. The packet reception/dropout is represented by a binary random variable γ_k, with $\gamma_k = 1$ indicating that the packet has been successfully delivered to the decoder; otherwise, $\gamma_k = 0$. The packet loss process $\{\gamma_k\}_k \geq 0$ is assumed to be an IID process with the probability of a packet being lost is given by the packet error rate (PER), herein denoted by ρ, i.e.,

$$\rho = \mathbb{P}\{\gamma_k = 0\}. \tag{2}$$

Meanwhile, the decoder acquires the output of the communication channel. We assume that the feedback link from the estimator to the sensor node is noiseless. Afterward, the estimator estimates the decoded state and the controller generates an input signal u_k from the estimated state $\hat{x}_k$ for the plant. In addition, for the decoder to inform the encoder about a successful reception of a packet through an acknowledgment/negative-acknowledgment (ACK/NACK) mechanism, in which short-length error-free packets are broadcasted by the receiver over a separate narrow-band channel, informing the transmitter on whether or not, the transmission of the packet was successful [8]. It should be noted that when the packet is transmitted over a binary erasure channel (BEC), the coded system is able to detect decoding failures in practice [9].

We further assume that to transmit r-bit requires a delay d, with $d \leq T_s$ to ensure that the packet is received before a new packet is generated. The required block length n for transmission is determined by [10]

$$n = dW, \tag{3}$$

where W is channel bandwidth.

B. Packet Error Rate Analysis

In the coded transmission, the r-bit information messages are encoded into block of n data symbol through the coded channel. Based on Shannon limit theorem [11], the rate of a block code which is defined as the ratio between its message length and its block length needs to be lower than the channel capacity $\frac{r}{n} = R < C$. Assuming we have AWGN channel, Polyanskiy *et al.* [12] state that the PER for modulation-i of wireless coded transmission is determined by

$$\rho^{(i)} = Q\left(\sqrt{\frac{n}{V}}\,(C - R(i))\right), \tag{4}$$

where $Q(\cdot)$ is the Q-function, and

$$V = \frac{\delta\ (\delta + 2)}{(\delta + 1)^2}(\log_2 e)^2, \tag{5}$$

$$C = \log_2(1 + \delta), \tag{6}$$

with V and C denoting the dispersion and channel capacity, respectively. In addition, the signal to noise ratio (SNR) is denoted by δ and defined by

$$\delta = \frac{E_s}{N_0}, \tag{7}$$

where E_s is the energy per symbol in joules and N_0 is the noise spectral density, the noise power in a 1 Hz bandwidth, measured in joules. Furthermore, the rate $R(i)$ function is defined as

$$R(i) = r_i,\ \ \forall i \in \{1, 2, ..., N_{mod}\} \subseteq \mathbb{N}, \tag{8}$$

where $r_i < r_j$ when $i < j$, $\forall i, j \in \{1, 2, ..., N_{mod}\} \subseteq \mathbb{N}$. In (8), N_{mod} denotes number of modulation schemes that are considered for a communication channel.

To satisfy Shannon limit theorem, the rate R should be smaller than the capacity ($C > R$) and, hence, due to (6),

$$\delta > 2^R - 1. \tag{9}$$

As deduced by (9), a lower bound on δ for each modulation scheme is $2^R - 1$. Although high-order modulation results in higher rates, based on (4) and (9), faster modulation has higher ρ in low δ scenario which leads to lower throughput and higher lower-bound of δ. It is trivial that consumed energy is increased when δ is increased due to (7).

C. Data Rate Theorem for Stabilization over Binary Erasure Channels

In the process of networked control systems, the quantizer discretizes a continuous state. Thus, it introduces loss of information due quantization error ($x_k - \hat{x}_k > 0$). In addition, there is a probability ρ that the decoder does not receive the output of the encoder s_k since the packet is transmitted through a lossy wireless communication channel. This condition induces more loss of information. Intuitively, the fewer bits used for quantization, the larger quantization error. However, if quantization error is too large, the controller is not able to generate a stabilizing control input. Nevertheless, having a faster rate requires a higher amount of energy to achieve the required throughput. Therefore, the problem of interest in this paper is determining the required minimum data rate r_{min} to asymptotically stabilize the system in the mean square sense, given the system dynamics and channel conditions.

Definition 1. System (1) is asymptotically stabilizable, in the mean square sense, if $\sup_k E[||x_k^2||] < \infty$, where $||\cdot||$ is the L^2-norm in $\mathbb{R}^\alpha$ and expectation $E[\cdot]$ is determined with respect to the packet loss process $\{\gamma_k\}_k \geq 0$.

Theorem 1. *[13] Consider the dynamical system in* (1) *with the network configuration shown in Fig. 1, where state transition matrix A is unstable, but* (A, B) *is a stabilizable, and the network is transmitted through IID lossy wireless communication channel with* $\rho \in (0, 1)$. *Further, the system is asymptotically stabilizable in the mean square sense if and only if,*

- *The packet error rate* ρ *is less than the upper-bound* ρ_{max} *determined by*

$$\rho < \rho_{max} = \frac{1}{M(A)^2}, \tag{10}$$

where $M(A)$ *is the Mahler measure of a square matrix* $A \in \mathbb{R}^{\alpha\times\alpha}$ *in the dynamical system* (1). *The Mahler measure is determined by its characteristic polynomial:*

$$M(A) = \prod_i \max\{|\lambda_i|, 1\} = 2^{H_T(A)}, \tag{11}$$

where $H_T(A)$ *is topological entropy of an LTI system with open loop matrix A. Furthermore, it is defined as*

$$H_T(A) = \sum_i \max\{\log_2 |\lambda_i|, 0\}, \tag{12}$$

where $\lambda_1, ..., \lambda_n$ *denote all the eigenvalues of A.*

- *The data rate r is greater that lower-bound given by*

$$r > r_{min} = H_T(A) + \frac{1}{2}\log_2 \frac{1-\rho}{1-\rho\ M(A)^2}. \tag{13}$$

In (12), $H_T(A)$ quantifies the uncertainty growth rate of system (1). The data rate determines how fast to process the information received to reduce the uncertainty. The rate must be greater than the growth rate $H_T(A)$ to guarantee the system is asymptotically stabilizable. However, due to the probability of packet loss, we require extra bits to address the impact of packet loss on the stabilizability. As shown in (13), these additional bits are explicitly quantified by

$$f(\rho, A) = \frac{1}{2}\log_2 \frac{1-\rho}{1-\rho\ M(A)^2}, \tag{14}$$

which is a function of ρ and the system dynamics. If inequality (10) is satisfied, because of

$$\frac{\partial f(\rho, A)}{\partial \rho} = \frac{M(A)^2 - 1}{2(1-\rho)(1 - M(A)^2\rho)\log(2)} > 0, \tag{15}$$

function (14) is a strictly increasing function with respect to ρ. In addition, for perfect channel ($\rho = 0$), it is defined that $f(0, A) = 0$. Thus, the data rate only need be greater than the growth rate $H_T(A)$. In addition, when the channel bandwidth is very large ($r \to \infty$), inequality (13) is always satisfied and the only condition need to be satisfied is (10).

III. Optimization Schemes

The wireless networked control systems heavily depend on the guarantees on the bounded service times for messages. Particularly, in industrial control the real-time requirement, such as throughput, is considered the most important. However, it requires high rate and low PER to achieve high throughput, for which more energy is required. Therefore, the trade-off between energy and throughput must be investigated. In this paper, we offer two optimization schemes for this trade-off, while guaranteeing that the system is stabilizable. Firstly, if we have limited energy to be consumed, we can achieve maximum achievable throughput by selecting the optimal modulation through maximizing throughput with an energy constraint scheme. Secondly, if we have a minimum throughput requirement to enhance control performance, we can minimize the energy by selecting the optimal modulation by minimizing energy with a throughput constraint scheme.

Besides, minimizing transmission delay is also important. The increase in delay can heavily degrade system performance. In a wireless network, the delay depends on the amount of data and the rate. If we increase the rate, the delay decreases; while causing more interference, decreasing the reliability (packet success rate), where the reliability of a communication channel is essential to keep the system stable. In this situation, we can increase the energy to enhance the reliability of the communication channel. Therefore, to investigate this trade-off, the last scheme is introduced. If we have limited energy to be consumed, we can minimize delay and keep the system stable by selecting the optimal modulation.

For simplicity we assume that the noise spectral density $N_0 = 1$, therefore, $\delta = E_s$. Thus, the function of ρ in (4) is function of E_s instead of δ and defined as

$$\rho^{(i)}(E_s) = Q\left((E_s + 1)\log\left(\frac{E_s + 1}{2^{R(i)}}\right)\sqrt{\frac{n}{E_s(E_s + 2)}}\right), \quad (16)$$

where $\rho^{(i)}(E_s)$ is the PER with modulation-i when E_s joule energy per symbol is consumed. Therefore, to satisfy Shannon's limit theorem, the lower-bound for energy is determined by

$$E_s > 2^R - 1. \quad (17)$$

It should be noted that in general WNCSs use low-power devices. To reflect this, we set the bandwidth $W = 100$ kHz in the following. For Sections III-A and III-B, we set the delay ($d = 1$ ms). Hence, the block length is fixed ($n = 100$ symbols). However, the block length n and delay d become objectives which need to be minimized in Section III-C.

A. MaxTEC Scheme

In this scheme, by setting the limit of total consumable energy, the achievable throughput is maximized by selecting optimal modulation. With the consumed energy per symbol E_s, the achievable throughput in bits per second (bps) with selected modulation-i is determined by $(1 - \rho^{(i)}(E_s))\frac{r(i)}{T_s}$, where function $r(i)$ denotes the length of the transmitted message in bits given by $r(i) = nR(i)$.

Maximizing throughput with an energy constraint by selecting optimal modulation can be formalized as an integer programming problem with respect to binary variables θ_i as follows :

$$\max_{\theta_i, \forall i} \quad \sum_i \theta_i (1 - \rho^{(i)}(E_s))\frac{r(i)}{T_s}, \quad (18)$$

$$\text{s.t.} \quad E_s > 2^{R(i)} - 1, \quad (19)$$

$$\sum_i \theta_i \rho^{(i)}(E_s) < \rho_{max}(A), \quad (20)$$

$$\sum_i \theta_i r(i) > \sum_i \theta_i r_{min}(A, \rho^{(i)}(E_s)), \quad (21)$$

$$nE_s \leq \epsilon, \quad (22)$$

$$\sum_i \theta_i = 1 \quad \text{with} \quad \forall i \in \{1 ... N_{mod}\}, \ \theta_i \in \{0, 1\}, \quad (23)$$

where (18) represents achievable throughput in bits per second (bps). In addition, $\rho_{max}(A)$ and r_{min} from (20) and (21) are determined by (10) and (13), respectively. Further, the energy constraint is stated in (22). It is obvious that the value of (18) is directly proportional to consumed energy E_s. Thus, to maximize (18), E_s must be maximized, i.e., $E_s = \epsilon/n$. Since E_s is fixed, the optimization problem becomes linear.

It should be noted that constraint (23) forms the objective of this scheme which is to select optimal modulation-i^* among N_{mod} modulations which maximizes throughput with energy constraint and guarantee the system is stabilizable.

To assure the selected modulation guarantees the dynamical system to be stabilizable; first, the selected modulation-i^* must satisfy the Shannon limit theorem, which is stated in (19). Furthermore, the data rate theorem that is stated in (20) and (21) must be satisfied with the selected modulation. Finally, among modulations that satisfy the whole constraint, the modulation that generates maximum throughput is selected.

The overview of the scheme is elaborated in Algorithm 1. In Algorithm 1, $sMod$ and $MaxT$ denote the selected modulation and the achievable throughput that is generated by selected modulation, respectively.

Algorithm 1: MaxTEC

$sMod \leftarrow \emptyset$
$MaxT \leftarrow 0$
for $i \leftarrow 1$ ***to*** N_{mod} **do**
    if $\rho^{(i)}(E_s) < \rho_{max}(A)$ **then**
        if $r(i) > r_{min}(A, \rho^{(i)}(E_s))$ **then**
            if $MaxT < (1 - \rho^{(i)}(E_s))\frac{r(i)}{T_s}$ **then**
                $MaxT \leftarrow (1 - \rho^{(i)}(E_s))\frac{r(i)}{T_s}$
                $sMod \leftarrow i$
end
return $sMod$, $MaxT$

B. MinETC Scheme

On contrary to Subsection III-A, the problem of interest in this scheme is determining required minimum energy when we have the lower bound of achievable throughput. Minimizing energy with a achievable throughput constraint scheme by selecting optimal modulation can be formalized as an integer programming problem with respect to binary variables θ_i as follows:

$$\min_{\theta_i, \forall i} \quad \sum_i \theta_i n E_s, \quad (24)$$

$$\text{s.t.} \quad E_s > 2^{R(i)} - 1,, \quad (25)$$

$$\sum_i \theta_i \rho^{(i)}(E_s) < \rho_{max}(A), \quad (26)$$

$$\sum_i \theta_i r(i) > \sum_i \theta_i r_{min}(A, \rho^{(i)}(E_s)), \quad (27)$$

$$\sum_i \theta_i (1 - \rho^{(i)}(E_s))\frac{r(i)}{T_s} \geq \zeta, \quad (28)$$

$$\sum_i \theta_i = 1 \quad \text{with} \quad \forall i \in \{1 ... N_{mod}\}, \ \theta_i \in \{0, 1\}, \quad (29)$$

where (24) represents total consumed energy. Furthermore, the achievable throughput constraint is stated in (28). Similar to previous case, it is obvious that to minimize (24), the throughput must be minimized, i.e.,

$$\sum_i \theta_i (1 - \rho^{(i)}(E_s))\frac{r(i)}{T_s} = \zeta. \quad (30)$$

Assume that there exists modulation-i^* that satisfy all constraints. Therefore, we have

$$(1-\rho^{(i^*)}(E_s)) = \frac{T_s}{r(i^*)}\zeta, \tag{31}$$

$$\rho^{(i^*)}(E_s) = 1 - \frac{T_s}{r(i^*)}\zeta. \tag{32}$$

Constraint (27) must be satisfied. Therefore, we have

$$r_{min}(A, \rho^{(i)}(E_s)) < r(i^*), \tag{33}$$

$$H_T(A) + \frac{1}{2}\log_2 \frac{1-\rho^{(i^*)}(E_s)}{1-\rho^{(i^*)}(E_s)\ M(A)^2} < r(i^*), \tag{34}$$

$$\frac{1-2^{\psi(i^*)} - \rho^{(i^*)}(E_s)(1-2^{\psi(i^*)}\ M(A)^2)}{1-\rho^{(i^*)}(E_s)\ M(A)^2} < 0, \tag{35}$$

where $\psi(i) = 2(r(i) - H_T(A))$. To satisfy constraint (26),

$$1-\rho^{(i^*)}(E_s)\ M(A)^2 > 0. \tag{36}$$

Therefore,

$$\rho^{(i^*)}(E_s) > \frac{1-2^{\psi(i^*)}}{1-2^{\psi(i^*)}\ M(A)^2}. \tag{37}$$

Because we assume that the state transition matrix A is unstable, $M(A) > 1$. Thus, we have

$$\frac{1}{M(A)^2} > \frac{1-2^{\psi(i^*)}}{1-2^{\psi(i^*)}\ M(A)^2}. \tag{38}$$

To satisfy every constraints

$$\frac{1-2^{\psi(i^*)}}{1-2^{\psi(i^*)}\ M(A)^2} < \rho^{(i^*)}(E_s) < \frac{1}{M(A)^2} \tag{39}$$

must be satisfied. By substituting Eq. (32) into (39), we have

$$\begin{aligned} &\frac{1-2^{\psi(i^*)}}{1-2^{\psi(i^*)}\ M(A)^2} < 1 - \frac{T_s}{r(i^*)}\zeta < \frac{1}{M(A)^2} \\ &\frac{2^{\psi(i^*)}M(A)^2 - 1}{2^{\psi(i^*)}(M(A)^2-1)}T_s\zeta < r(i^*) < \frac{M(A)^2}{M(A)^2-1}T_s\zeta. \end{aligned} \tag{40}$$

A set $\mathbb{V}$ is called valid if every element of set $\mathbb{V}$ is modulation that satisfy (40) where $\mathbb{V} \subseteq \{1, 2, ..., N_{mod}\}$. To minimize the energy, the selected modulation- $i^* = \min(\mathbb{V})$.

Because solving E_s from nonlinear equation (32) is difficult, approximating approach is used. Therefore, we assume E_s is a discrete such that

$$E_s = \{x : x = x_0 + \Delta j, j \in \{0, 1, 2, 3, .., N_e\}\}, \tag{41}$$

and

$$E_s(j) = x_0 + \Delta j, \tag{42}$$

where Δ is a small number for the space between element, N_e is the number of element in the set, $\min(E_s) = x_0$, and $\max(E_s) = x_0 + \Delta N_e$. In this scheme, the solution is the smallest element from discrete set E_s that satisfy constraint (28) with selected modulation-i^*.

The overview of the scheme is elaborated in Algorithm 2. In Algorithm 2, $sMod$ and $MinE$ denote the selected modulation, and the minimum required energy that is generated by selected modulation, respectively.

Algorithm 2: MinETC Scheme

$sMod \leftarrow \emptyset$
$MinE \leftarrow \infty$
$j \leftarrow 0$
for $i \leftarrow 1$ ***to*** N_{mod} **do**
  if $r(i) > \frac{2^{\psi(i)}M(A)^2-1}{2^{\psi(i)}(M(A)^2-1)}T_s\zeta$ **then**
    if $r(i) < \frac{M(A)^2}{M(A)^2-1}T_s$ **then**
      while $\left(\rho^{(i)}(E_s(j)) < 1 - \frac{T_s}{r(i)}\zeta\right)$ & $(j \leq N_e)$ **do**
        $j \leftarrow j + 1$
      end
      if $(MinE > E_s(j))$ & $\left(\rho^{(i)}(E_s(j)) \geq 1 - \frac{T_s}{r(i)}\zeta\right)$ **then**
        $MinE \leftarrow E_s(j)$
        $sMod \leftarrow i$
end
return $sMod$, $MinE$

C. MinDEC Scheme

As defined in (3), to minimize delay d, we have to minimize the block length n. Minimizing block length with energy constraint scheme is formalized as an integer programming problem with binary variables as follows:

$$\min_{\theta_i, \forall i} \quad \sum_i \theta_i n, \tag{43}$$

$$\text{s.t.} \quad E_s > 2^{R(i)} - 1, \tag{44}$$

$$\sum_i \theta_i \rho^{(i)}(E_s) < \rho_{max}(A), \tag{45}$$

$$\sum_i \theta_i r(i) > \sum_i \theta_i r_{min}(A, \rho^{(i)}(E_s)), \tag{46}$$

$$d \leq T_s \tag{47}$$

$$E_s \leq \eta. \tag{48}$$

$$\sum_i \theta_i = 1 \quad \text{with} \quad \forall i \in \{1...N_{mod}\},\ \theta_i \in \{0, 1\}, \tag{49}$$

where (43) represents block length. Furthermore, the energy constraint is stated in (48). Constraint (47) is required to bound the delay based on the assumption from Section II. It is obvious that the value of (43) is inversely proportional to consumed energy E_s. Thus, to minimize (43), E_s must be maximized, i.e., $E_s = \eta$. Therefore, if (44) is satisfied and with the fact that Q function is monotone decreasing function, we have

$$\begin{aligned} &\rho^{(i)}_{coded} < \rho_{max} \\ &\sqrt{\frac{n}{V}}\,(C - R(i)) > Q^{-1}(\rho_{max}) \\ &n > \frac{(\log_2 e)^2 Q^{-1}(\rho_{max})^2(\eta)(\eta+2)}{(\eta+1)^2 \log_2((\eta+1)/2^{R(i)})}. \end{aligned} \tag{50}$$

Because $n \in \mathbb{N}$, thus,

$$n \geq MinPL = \left\lceil \frac{(\log_2 e)^2 Q^{-1}(\rho_{max})^2(\eta)(\eta+2)}{(\eta+1)^2 \log_2((\eta+1)/2^{R(i)})} \right\rceil + 1, \quad (51)$$

where $\lfloor \cdot \rfloor$ is the ceiling function. It is important to check whether constraint (46) is satisfied. If the minimum block length determined by (51) does not satisfy (46), the scheme will increment $MinPL$ by 1 and keep incrementing until the constraint is satisfied. Finally, the minimum delay is determined by (3), where $n = MinPL$.

It must be noted that when we have sufficiently fast sampling time T_s that generates $\rho_{max} \geq 1/2$ and if inequality (44) is satisfied then inequality (45) is always satisfied even when $n = 1$, and minimum delay to keep system stable is determined by $d = 1/W$ (see also example in Section IV-C).

The overview of the scheme is elaborated in Algorithm 3. In Algorithm 3, $sMod$, Min_n, and Min_d denote the selected modulation, the minimum required block length, the minimum required a delay, and that is generated by selected modulation, respectively.

Algorithm 3: MinDEC Scheme

```
a ← 0
sMod ← ∅
temp_n ← ∞
Min_n ← ∞
Min_d ← ∞
for i ← 1 to N_mod do
    if E_s > 2^{R(i)} − 1 then
        temp_n ← MinPL (from (51))
        r(i) ← R(i) × temp_n
        Determine ρ^{(i)}(E_s) with n = temp_n by (16)
        Determine r_min(A, ρ^{(i)}(E_s)) by (13)
        while r(i) ≤ r_min(A, ρ^{(i)}(E_s)) do
            temp_n ← temp_n + 1
            r(i) ← R(i) × temp_n
            Update ρ^{(i)}(E_s) with n = temp_n by (16)
            Update r_min(A, ρ^{(i)}(E_s)) by (13)
        end
        if Min_n > temp_n then
            Min_n ← temp_n
            sMod ← i
end
Min_d ← Min_n / W
return sMod, Min_d
```

IV. Numerical Results

In this section, we simulate the implementation of all three schemes over a simulated wireless channel. The chosen plant is the well-known inverted pendulums on a horizontal cart.

Although it is simple, the highly unstable dynamics of the inverted pendulum captures the general need of the control system when being controlled over a wireless channel. The control input is the force F that moves the cart horizontally, and the outputs are the angular position of the pendulum θ (angle from the vertical) and the horizontal position of the cart x. The linearized equations of motion of the system with $x(t) = [\dot{x}(t) \ x(t) \ \dot{\theta}(t) \ \theta(t)]^T$ are as follows:

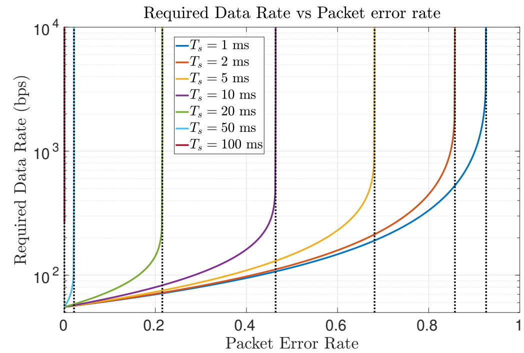


Fig. 2: Required Data Rate vs Packet error rate.

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-(I+ml^2)b}{I(M+n)+Mml^2} & \frac{m^2gl^2}{I(M+n)+Mml^2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-mlb}{I(M+n)+Mml^2} & \frac{mgl(M+m)}{I(M+n)+Mml^2} & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \frac{I+ml^2}{I(M+n)+Mml^2} \\ 0 \\ \frac{ml}{I(M+n)+Mml^2} \end{bmatrix} u(t) + w(t), \quad (52)$$

where we assume the following quantities: mass of the cart $M = 10$ kg, mass of the pendulum $m = 150$ kg, coefficient of friction for cart $b = 0.1$ N/m/sec, length to pendulum center of mass, $l = 0.3$ m, mass moment of inertia of the pendulum $I = 0.006$ kg m^2, coefficient of gravity $g = 9.81$ ms^{-2}. Afterward, (52) is discretized by applying a zeroth order hold (ZOH) with sampling period T_s seconds to form (1).

By using defined system, we simulate the required data rate over packet error rate as depicted in Fig. 2. The system in Fig. 2 is sampled with five different sampling period (T_s). Faster T_s corresponds to more stable system and vice versa. As shown in Fig. 2, the more stable system has wider stabilizable coverage. In addition, required data rate $\rightarrow \infty$ as $\rho \rightarrow \rho_{max}$.

For this simulation, we assume there are $N_{mod} = 5$ modulations, where $R \in \{0.5, 1, 1.5, 2, 3\}$. Fig. 3 depicts the required r bits data that need to be transmitted each transmission over consumed energy E_s. Kostina *et al.* [14] noted that in networked control system with noiseless channel, an unstable scalar LTI, $x_{k+1} = ax_k + bu_k + w_k$ can be stabilized with 1 bit feedback when $a \in [1, 2)$. Our result also shows that even in noisy channel, a LTI system can be stabilized with only 1 bit feedback if sampling time is sufficiently fast and consumable energy is sufficient.

A. Result of MaxTEC Scheme

In this scheme, maximum achievable throughput is determined with limited energy while keeping the system stable.

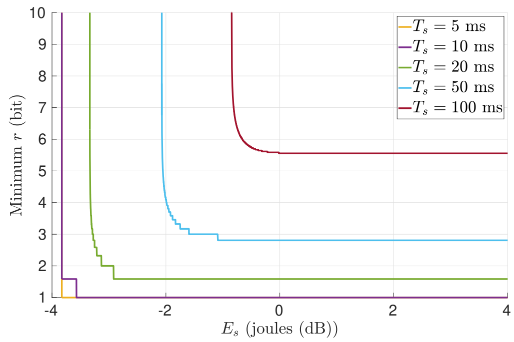
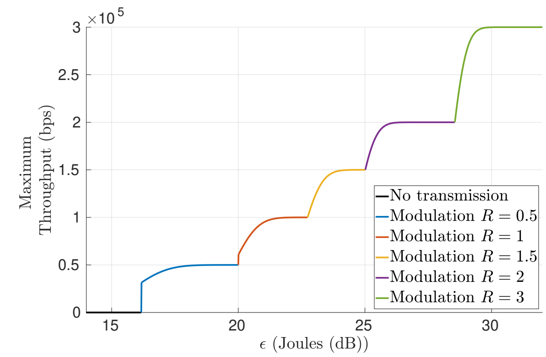
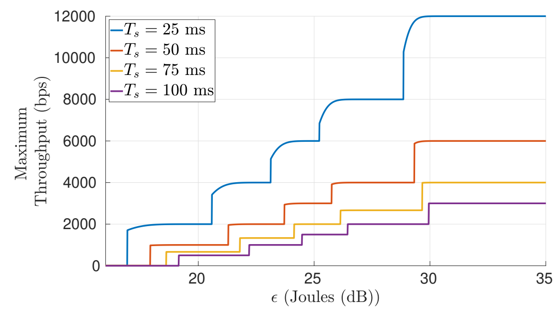


Fig. 3: Minimum required r bits vs Consumed Energy E_s.



(a) For $T_s = 1$ ms.



(b) For $T_s = \{25,\ 50,\ 75,\ 100\}$ ms.

Fig. 4: MaxTEC scheme.

As shown in Fig. 4a, the scheme chooses the optimal modulation based on channel condition. The scheme can choose not to transmit when the channel condition is severely bad due to there is not sufficient energy to generate transmission. When we have better channel condition, we can transmit the payload with the optimal modulation. When total energy $\epsilon \in (17, 20)$ Joules (dB), the biggest factor that leads the scheme to select modulation with coding rate $R = 0.5$ is the fact that the lowest modulation has lowest PER in lower channel condition. When the total consumed energy $\epsilon \approx 18$ Joules (dB), modulation with coding rate $R = 0.5$ reaches its the maximum capability by generating 50 kbps. Once $\epsilon \approx 20$ Joules (dB), modulation with coding rate $R = 1$ is selected by the scheme due to having higher achievable throughput. Fig. 4b shows how significant the sampling time to overall system performance. The achievable throughput increases gradually when faster sampling time is applied and the system with slower sampling time requires more energy to be able to transmit the payload.

B. Result of MinETC Scheme

In this scheme, we determine the minimum total consumed energy to generate required achievable throughput for enhancing system performance. As shown in Fig. 5, the schemes have similar characteristic with the previous scheme where the modulation changes to adapt condition and requirement. It should be noted that the scheme tends to select the lowest modulation that satisfies every constraints to minimize energy. However, each modulation has its maximum capability to transmit payload per transmission. Hence, the scheme will select faster modulation once slower modulation pasts its limit. In Fig. 5, we compare two MinETC schemes with different number of modulation configuration. The first MinETC scheme is generated with $N_{mod} = 5$ where $R = \{0.5, 1, 1.5, 2, 3\}$ and the second one is generated with $N_{mod} = 3$ where $R = \{0.5, 2, 3\}$. In both cases, 16 joules (dB) energy is required to transmit the payload even with slowest modulation ($R = 0.5$). The second MinETC decided to activate modulation with coding rate $R = 2$ at interval $\zeta \in [50, 200]$ kbps that required more energy than the first MinETC scheme. In this scenario, the amount of energy that can be saved by applying more modulation configuration is denoted by ΔE_s. The ΔE_s is represented as green shading in Fig. 5.

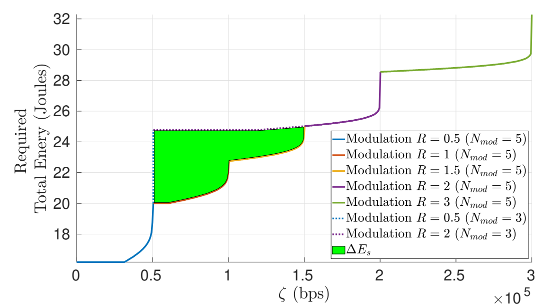
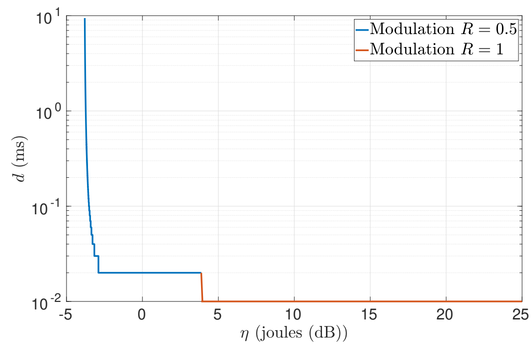


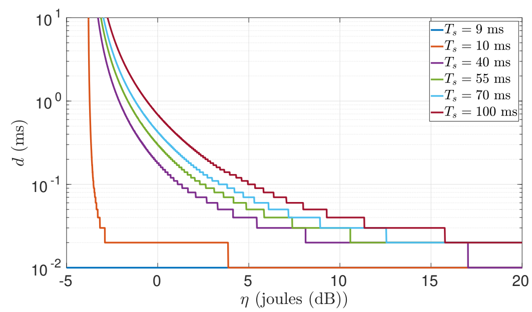
Fig. 5: MinETC scheme for $T_s = 1$ ms.

C. Result of MinDEC scheme

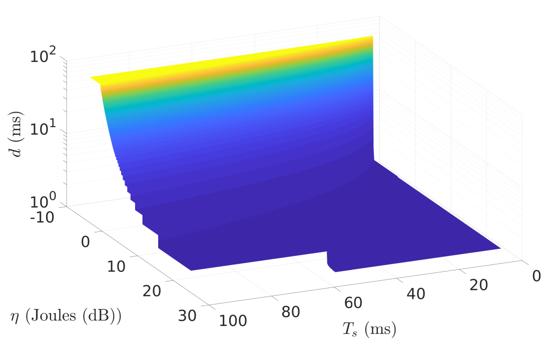
In this scheme, we determine the delay lower-bound to keep system stable. Fig. 6a depicts the delay lower-bound when the sampling period $T_s = 10$ ms. The scheme starts to be able to stabilize the system once $\eta \approx -3.7$ (joules (dB)) where the delay closes to its upper-bound ($T_s = 10$ ms). Because the consumable energy is limited, the scheme tends to selects the slowest possible modulation. However, once $\eta \approx 3.8$ (joules (dB)), there is enough energy to generate faster modulation to satisfy every constraints; the scheme selects the faster modulation to decrease the delay lower-bound into 10^{-2} ms. Due to $n \in \mathbb{N}$, the fastest possible delay is $1/W$ where in this case, 10^{-2} ms. Thus, the scheme keeps selecting modulation with coding rate $R = 1$ once $\eta > 3.8$ (joules (dB)) in Fig. 6a. However, there is possibility that the fastest delay lower-bound can not be achieved even $\eta \to \infty$. Both Fig. 6b and Fig. 6c show how sampling time T_s affects the delay lower-bound. Both results show that once $T_s >$ 55 ms, fastest delay lower-bound is no longer achievable. It should be noted that if there is sufficient energy to satisfy Shannon limit ($\eta > 2^R - 1$), the packet error rate $\rho^{(i)}(\eta) <$ 0.5, $\forall i$. Therefore, once the sampling period is fast enough to generates $\rho_{max} > 0.5$, one symbol ($MinPL = 1$) is sufficient to stabilize the system $\forall \eta > 2^R - 1$. This result is shown in Fig. 6c once the sampling period $T_s < 9$ ms.



(a) For $T_s = 10$ ms.



(b) For $T_s = \{9, 10, 40, 55, 70, 100\}$ ms.



(c) For $T_s \in [20, 100]$ ms.

Fig. 6: Minimum block length with energy constraint

V. Conclusion

A. Conclusions

In this paper, we proposed adaptive coded modulation for stabilization and performance optimization of WNCSs. Towards this end, we proposed three schemes with which we optimized various objectives under different constraints. The results show that each scheme is able to select the optimal modulation to optimize objectives at given various channel conditions and constraints. It should be noted that a scheme can decide not to transmit if stability cannot be guaranteed. Besides, it is trivial that sampling time T_s heavily impacts the performance. In extreme scenarios, such as when we have low sampling rate ($T_s = 100$ ms), the MaxTEC scheme is able to stabilize the system although it requires considerable amount of energy and produces small throughput. Another interesting point, shown in the results of MinETC scheme, is that in some scenarios having more options for modulation can save more energy. Furthermore, our study complements the findings of Kostina *et al.* [14] which state that 1-bit feedback is sufficient to stabilize scalar LTI system in noiseless channels. Additionally, our findings show that LTI system in noisy channel can be stabilized with 1-bit feedback, provided sampling rate is sufficiently fast and consumable energy is sufficiently large.

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