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Can a dielectric sphere emulate the behavior of a surface impedance sphere?

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Abstract

We theoretically investigate whether a dielectric sphere can mimic the behavior of an equivalent surface impedance sphere and we discuss some of the of emergent properties, such as cloaking (minimum scattering) and maximum absorption conditions.

1 Introduction

Undoubtedly, electromagnetic scattering by a sphere is an archetypical testbed for physics [1], and exotic engineering conceptualization [2]. In this work, we attempt the merge between two characteristic scattering problems, i.e., scattering by a penetrable dielectric sphere and by a impenetrable surface impedance sphere. Both cases have a long history; dielectric spheres have been studied in the context of Lorentz–Mie theory since the end of 19th century (Lorentz 1890, Mie 1909, see [3]), while the concept of surface impedance has been introduced around 1940's (see [4] and the references therein).

Despite their long historical significance, the aforementioned fundamental problems are still encountered to the metasurfaces trend, a current research topic that drives a new round of studies on the topic of the artificially engineered surfaces for unprecedented control of the electromagnetic energy, from microwaves to optical wavelengths [5]. Inspired by these advancements, we revisit the problem of scattering by a sphere described by an impedance boundary condition (IBC). This particular type of sphere materializes a certain ratio between the (tangential) electric and magnetic field via the surface impedance

$$\mathbf{E}_{\mathrm{t}} = Z_{\mathrm{s}}\mathbf{n} \times (\eta_0 \mathbf{H}_{\mathrm{t}}) \tag{1}$$

Here, we assume the simplest IBC, that of an a homogeneous, linear, and isotropic impedance, described by a single scalar quantity Z_{s} and normalized by the free space impedance η_0.

The central question is whether an IBC sphere is able to mimic the behavior of a plasmonic/dielectric, graphene–coated, or any other kind of spherical inclusion through its

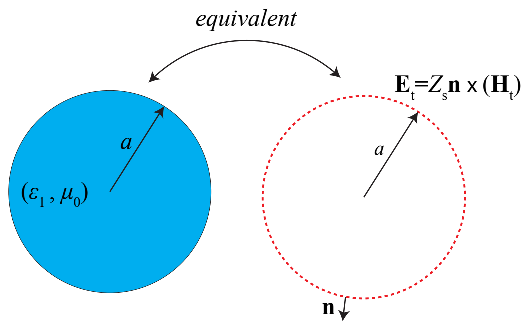

Figure 1. Conceptual illustration: can we find an equivalence between a dielectric sphere (left) and and impenetrable, surface impedance sphere? In this study we assume that both spheres have the same radius a.

main feature, the surface impedance Z_s (Fig. 1). First, we expose some exact and approximative properties for small IBC spheres (Section 2). The postulated problem is rigorously addressed by deriving a particular surface impedance for a given dielectric sphere, and it is discussed in Section 3. The paper concludes with a small discussion section (Section 4) where we highlight some of the most prominent features of the proposed idea.

2 Scattering by IBC spheres

Let us assume a sphere of radius a. The general type of electromagnetic boundaries for which the impedance boundary conditions are decomposed into TE and TM components with respect to the radial unit vector $\mathbf{u}_r$ read

$$\mathbf{E}_{\mathrm{TE}} = Z_{\mathrm{TE}}\mathbf{u}_r \times \mathbf{H}_{\mathrm{TE}} \tag{2}$$

$$\mathbf{E}_{\mathrm{TM}} = Z_{\mathrm{TM}}\mathbf{u}_r \times \mathbf{H}_{\mathrm{TM}} \tag{3}$$

where Z_{TM} and Z_{TE} are the normalized over η_0 partial impedances. This "mixed impedance" boundary condition approach reduces to the simple isotropic case when $Z_{\mathrm{TE}} = Z_{\mathrm{TM}} = Z_{\mathrm{s}} = R_{\mathrm{s}} - iX_{\mathrm{s}}$, where the $e^{-i\omega t}$ convention is used. Following the IBC arguments as presented in [6, 7], the external scattering coefficients for the IBC sphere read

$$a(n)_{\mathrm{IBC}} = \frac{ixZ_{\mathrm{TM}}j_n(x)+xj_{n-1}(x)-nj_n(x)}{ixZ_{\mathrm{TM}}h_n^{(1)}(x)+xh_{n-1}^{(1)}(x)-nh_n^{(1)}(x)} \quad (4)$$

and

$$b(n)_{\mathrm{IBC}} = \frac{i\frac{x}{Z_{\mathrm{TE}}}j_n(x)+xj_{n-1}(x)-nj_n(x)}{i\frac{x}{Z_{\mathrm{TE}}}h_n^{(1)}(x)+xh_{n-1}^{(1)}(x)-nh_n^{(1)}(x)} \quad (5)$$

with $x = ka$ being the size parameter and k the host medium wavenumber (here vacuum).

Interestingly, the minimum scattering condition (cloaking) for every electric and magnetic multipole reads

$$Z_{\mathrm{TE}} = Z_{\mathrm{TM}} = -i\left(n - \frac{xj_{n-1}(x)}{j_n(x)}\right) \quad (6)$$

which is a generalization of the proposed mantle cloaking conditions presented in [8].

The corresponding extinction, scattering and absorption efficiencies are

$$Q_{\mathrm{ext}} = \frac{2}{x^2}\sum_n^{\infty}(2n+1)\left(\Re\{a_{\mathrm{IBC}}(n)\}+\Re\{b_{\mathrm{IBC}}(n)\}\right) \quad (7)$$

$$Q_{\mathrm{sca}} = \frac{2}{x^2}\sum_n^{\infty}(2n+1)\left(|a_{\mathrm{IBC}}(n)|^2+|b_{\mathrm{IBC}}(n)|^2\right) \quad (8)$$

$$Q_{\mathrm{abs}} = Q_{\mathrm{ext}} - Q_{\mathrm{sca}} \quad (9)$$

We observe that a small size parameter expansion ($Q_{\mathrm{ext}} \approx Q_{\mathrm{abs}}$) can give maximum absorption conditions as a function of the surface impedances. Namely, when we assume the simple isotropic surface impedance $Z_{\mathrm{s}} = R_{\mathrm{s}} - iX_{\mathrm{s}}$, the extinction efficiency reads (for $R_{\mathrm{s}} \neq 0$)

$$Q_{\mathrm{ext}} = 6x^2\left(R_{\mathrm{s}} + \frac{R_{\mathrm{s}}}{R_{\mathrm{s}}^2+X_{\mathrm{s}}^2}\right) \quad (10)$$

keeping only the first electric and magnetic dipole coefficients. This expression is maximum when $X_{\mathrm{s}} = 0$ and $R_{\mathrm{s}} = 1$ implying that a maximum absorption occurs at the point where the reactive part is zero and the real part (losses) of both magnetic and electric dipole is matched [7].

The basic question of this study is formulated as follows: is there any kind of conceptual or practical realization of an IBC sphere that implements, fully or partially, the aforementioned IBC properties? Here, we postulate that a particular IBC sphere already exists; any dielectric sphere exhibits electric and magnetic characteristics, hence emulates the behavior of an IBC sphere to a certain extent. A secondary goal is to explore the properties for the equivalent surface impedances, proposing an alternative mathematical description of a dielectric sphere and discuss some of the main features of this perspective.

3 Dielectric sphere and its IBC equivalence

The standard Mie coefficients (Bohren–Huffman [9]) read

$$a(n)_{\mathrm{Mie}} = \frac{\mu m^2 j_n(mx)\,(xj_n(x))' - \mu_1 j_n(x)\,(mxj_n(mx))'}{\mu m^2 j_n(mx)\left(xh_n^{(1)}(x)\right)' - \mu_1 h_n^{(1)}(x)\,(xj_n(mx))'} \quad (11)$$

and

$$b(n)_{\mathrm{Mie}} = \frac{\mu_1 j_n(mx)\,(xj_n(x))' - \mu j_n(x)\,(mxj_n(mx))'}{\mu_1 j_n(mx)\left(xh_n^{(1)}(x)\right)' - \mu h_n^{(1)}(x)\,(mxj_n(mx))'} \quad (12)$$

where μ, ε are the host material parameters, μ_1, ε_1 are the sphere parameters, and $m = \frac{\sqrt{\mu_1\varepsilon_1}}{\mu\varepsilon}$ is the relative refractive index parameter.

Since both IBC and Mie analyses utilize exactly the same spherical wave decomposition, an IBC sphere emulates the behavior of a dielectric sphere if both scattering coefficients reproduce the same scattering phenomena. Equations (4) and (5) contain three parameters, Z_{TM}, Z_{TE}, and the radius a that can be utilized for this matching. For simplicity we assume that both IBC and dielectric spheres have the same radius, although allowing different radii we can extract some interesting scaling features, i.e., emulating a large/small dielectric sphere with an arbitrary sized IBC sphere. This topic is beyond the scopes of this short analysis.

Let us assume a magnetically transparent ($\mu_1 = 1$) dielectric sphere. The IBC sphere emulate precisely the external scattering behavior of the magnetic multipole Mie term (4) when simultaneously

$$a(n)_{\mathrm{IBC}} = a(n)_{\mathrm{Mie}} \quad (13)$$

and

$$b(n)_{\mathrm{IBC}} = b(n)_{\mathrm{Mie}} \quad (14)$$

The expressions of the impedances that satisfying the above conditions read

$$Z_{\mathrm{TM}}(n) = \frac{-i}{4\varepsilon_1 x}\left(2\varepsilon_1(2n+1) - 4(n+1) + W_1 + W_2\right) \quad (15)$$

$$Z_{\mathrm{TE}}(n) = \frac{i4x}{W_1 + W_2 - 2} \quad (16)$$

where

$$W_1 = \frac{\pi\varepsilon_1 x^2}{\cos(n\pi)}\left(J_{-n-\frac{3}{2}}(x)J_{n-\frac{1}{2}}(x) - J_{-n+\frac{1}{2}}(x)J_{n+\frac{3}{2}}(x)\right) \quad (17)$$

and

$$W_2 = 4\frac{\sqrt{\varepsilon_1}xJ_{n+\frac{3}{2}}(\sqrt{\varepsilon_1}x)}{J_{n+\frac{1}{2}}(\sqrt{\varepsilon_1}x)} \quad (18)$$

and $J_n(x)$ is the Bessel function of the first kind.

As an illustrative example we use the value $n = 1$ and we get the small size ($x \to 0$) expansion expressions to be

$$Z_{\mathrm{TM}}^{\mathrm{T}}(1) = \frac{2i}{\varepsilon_1 x} - \frac{i}{5}x - i\frac{\varepsilon_1}{175}x^3 + \ldots \quad (19)$$

and apparently

$$Z_{\mathrm{TE}}^{\mathrm{T}}(1) = -\frac{i}{2}x - i\frac{1}{20}\varepsilon_1 x^3 + \dots \tag{20}$$

It is remarkable to notice that these very simple values restore the physics of the small dielectric spheres in the following way. In particular, by applying the condition of Eq. (19) to Eq. (4) and take the series expansion we get

$$a(1)_{\mathrm{IBC}} \approx -i\frac{2}{3}\frac{\varepsilon_1 - 1}{\varepsilon_1 + 2}x^3 \tag{21}$$

which is exactly the Taylor expansion of a small dielectric sphere ($a_{\mathrm{Mie}}(1)$) revealing the plasmonic resonance condition ($\varepsilon_1 = -2$) [10].

4 Discussion

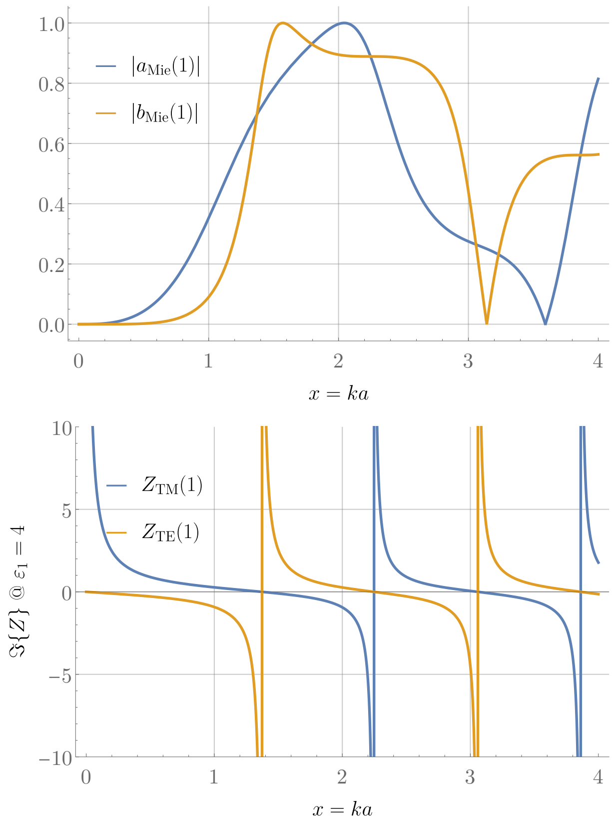

Figure 2. TOP: The absolute value of the first electric (blue line) and magnetic (orange line) Mie coefficients for $\varepsilon_1 = 4$ as a function of the size parameter. BOTTOM: The normalized impedances Z_{TM} and Z_{TE} of the same dipoles and the same condition as comparison. Note that the first Kerker condition ($a(1)_{\mathrm{Mie}} = b(1)_{\mathrm{Mie}}$) correspond to the first zero crossing of the Z_{TM}. The amplitude of the impedances spans between $(-\infty, +\infty)$, and here is suppressed for better visibility.

Figures 2 and 3 depict the imaginary part for a lossless sphere of both $Z_{\mathrm{TM}}(1)$ and $Z_{\mathrm{TE}}(1)$ for fixed ε_1 and size parameter x, respectively. It is evident that a common dielectric sphere exhibits a particular form of a "modulated" surface impedance. This modulation happens both as a function of the permittivity and the size parameter. The zero crossings of the Z_{TM} impedance correspond to infinite values of Z_{TE}, and vice versa; a manifestation of the electric-magnetic duality of the impedances. Interestingly, this combination resembles the so-called DB–D'B' impedance boundary conditions [6]. It is therefore evident that a simple dielectric sphere can be used as a building block materializing this exotic impedance condition.

Turning to the details, Fig. 2 depicts the connection of the equivalent surface impedance and the characteristic resonant points. The first Kerker condition (maximum forward scattering [10]) occurs when $Z_{\mathrm{TM}} = 0$ and $Z_{\mathrm{TE}} \to \infty$. Likewise, Fig. 3 exposes the corresponding behavior as a function of the permittivity.

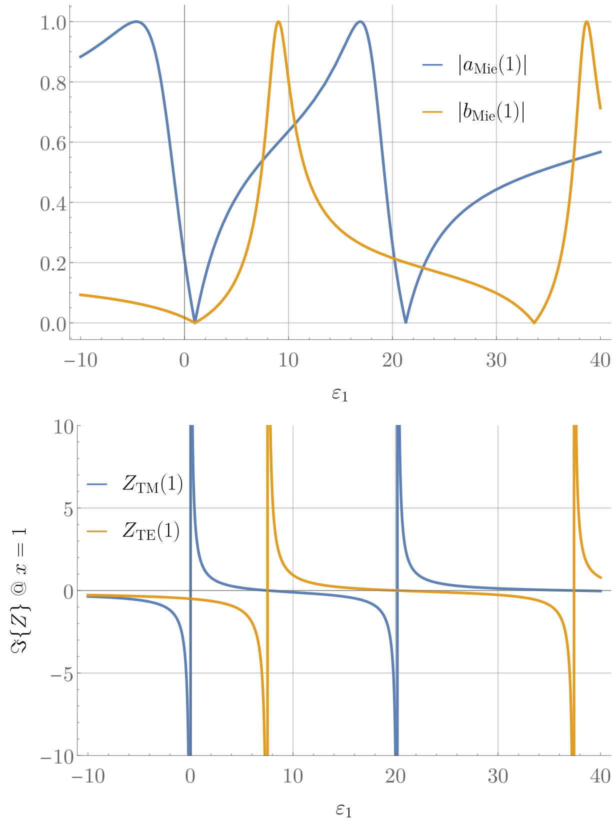

Figure 3. Similar to Fig. 2. TOP: The absolute value of the first electric (blue line) and magnetic (orange line) Mie coefficients for $x = 1$ as a function of the permittivity. BOTTOM: The normalized impedances Z_{TM} and Z_{TE} of the same dipoles and the same condition as comparison.

Similar results have been utilized for an approximate derivation of physical bounds related to the minimization of the scattering coefficients, see [11]. There, the notion of partial reactance has been used for every mode based on an circuital analogy derived by the Mie coefficients. However, the exact reactance derived there has not been utilized since it represents a non-causal system, therefore the Bode–Fano reactance theorem cannot immediately be applied. The authors of [11] circumvent this restriction by introducing an approximative effective partial reactance that restores causality. In our case, the introduced impedances presented in Figs. 2 and 3, are monotonous, hence can be readily used for a more precise derivation of cloaking bounds.

Deviating beyond the dielectric behavior, where the impedances are functions of both ε_1 and x, the aforementioned equivalent IBC model can be immediately used for the design of general inductive/capacitive resonances offering an alternative approach to the resonant particle design [7].

Concluding, the presented ideas can be analogously followed for connecting the IBC sphere with other cases. For example, a graphene coated sphere is an excellent candidate for implementing the particular surface impedance properties at the THz regime for both inductive/capacitive type of resonances. Finally, the concept of the IBC spheres can be used for composite metasurfaces comprised by surface impedance spheres, creating a "metasurface-in-metasurface" scheme, enabling the design of applications with advanced and exotic light-matter functionalities.

Acknowledgments

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