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Robust trajectory planning of autonomous vehicles at intersections with communication impairments

Neha Chohan, Mohammad A. Nazari, Henk Wymeersch, and Themistoklis Charalambous

Abstract—In this paper, we consider the trajectory planning of an autonomous vehicle to cross an intersection within a given time interval. The vehicle communicates its sensor data to a central coordinator which then computes the trajectory for the given time horizon and sends it back to the vehicle. We consider a realistic scenario in which the communication links are unreliable, the evolution of the state has noise (e.g., due to the model simplification and environmental disturbances), and the observation is noisy (e.g., due to noisy sensing and/or delayed information). The intersection crossing is modeled as a chance constraint problem and the stochastic noise evolution is restricted by a terminal constraint. The communication impairments are modeled as packet drop probabilities and Kalman estimation techniques are used for predicting the states in the presence of state and observation noises. A robust sub-optimal solution is obtained using convex optimization methods which ensures that the intersection is crossed by the vehicle in the given time interval with very low chance of failure.

Index Terms—Intersection crossing; robust trajectory planning; unreliable communications.

I. INTRODUCTION

The expected advent of autonomous vehicles in the near future opens up a number of possibilities for efficient driving through communication and coordination. This, among others, can help mitigate the issues of road safety and traffic congestion as well as boost fuel efficiency. Road safety should be a high priority issue, since about 1.24 million people die and 50 million are hurt in road accidents each year [1]. In addition, drivers and passengers waste around 90 billion hours in traffic jams per year. Traffic conditions in major cities have become an important issue, since congestion results in time delays, carbon dioxide emissions, higher energy expenditure and higher accident risks (see, for example, [2] and references therein). To enhance the performance of the current transportation systems, a multitude of transportation components need to be improved through new, disruptive solutions.

A significant proportion of accidents take place near road intersections, which are among the most complex regulated traffic subsystems. The modeling of autonomous vehicles crossing at intersections is a crucial problem in this area. With efficient and effective algorithms in place, it is possible to completely do away with traffic signals and fixed predefined schedules for vehicle crossings. Indeed, by developing

intelligent algorithms where crossing vehicles exchange information and mutually decide on a safe schedule, one can improve the traffic flow, increase safety and save fuel [3]–[5].

This problem is usually broken down into three parts: 1) determining an optimal priority list for the concerned vehicles around the intersection, 2) finding optimal time intervals for each of these vehicles to cross the intersections, and 3) determining an optimal trajectory for each vehicle to cross the intersection in its appointed time interval.

Various methods have been suggested for dealing with the first part of the problem including approximation methods [6], heuristics-based methods [7] and other algorithms [8]. Similarly, a good amount of research has been carried out where the first part is assumed to be known and the second part of the problem is addressed [9] and [10]. In [11] and [12], a combination of the aforementioned problems is addressed simultaneously. However, in most of these papers two assumptions are common: perfect communication and a deterministic system model. Both aforementioned assumptions are far from reality and, hence, it is necessary to develop a framework where these assumptions are lifted in order to model a more realistic scenario.

Okamoto *et al.* in [13] provide a theoretical framework for optimal control of the covariance of stochastic finite discrete-time linear systems subject to chance constraints. This is cast as an MPC problem and solved using convex optimization techniques with the optimization variable being the control gain matrix. The usually separable problems of mean and covariance problem are, in this case, linked together by chance constraints. However, the observation is assumed to be noiseless, and no communication impairments are considered. The articles [14] and [15] give an elaborate description of how packet losses in network controlled systems can be modeled and corresponding stable control strategies can be derived. Nazari *et al.* in [16] take a step in the direction of a stochastic system with imperfect communication links for a single vehicle crossing the intersection. The authors use a chance-constrained MPC model and give the control input matrix using a Gaussian approximation. However, the process covariance is allowed to evolve unrestricted as per the system dynamics and the observation covariance is zero (perfectly observable state).

In this paper, we consider a stochastic, finite-horizon discrete-time linear vehicle system with both process and observation noises, modeled as Gaussian distributions. Moreover, we model communication impairments between the vehicle and central coordinator using packet loss probability

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distribution for channels. Here, the packet drops from vehicle sensor to central coordinator are taken into consideration. In case of packet drops from controller to vehicle actuator, the previously communicated control input sequence from the model predictive controller would be implemented. Hence, these packet drops are not separately addressed. Kalman estimation method is used to predict the evolution of the state taking into account the noise and packet drops. The objective of crossing the intersection is modeled as a chance constraint. In addition, we have a limit on the terminal covariance value in order to ensure efficient control input requirements. While we follow the basic framework from [13] for model dynamics and terminal covariance constraints, these are modified to account for communication impairments and observation noises. In particular, we extend the current model to account for estimating the state and also change the covariance propagation model to allow for communication losses.

The rest of this paper is organized as follows. In Section II, we introduce the notation used throughout the paper. In Section III we provide the system model and some preliminaries, necessary for the development of this work. The main results are presented in Section IV. The performance of our proposed approach is evaluated in Section V. Finally, in Section VI, we draw conclusions and discuss possible future directions.

II. NOTATION

The sets of natural numbers are denoted by \mathbb{N} and real numbers by \mathbb{R} . In addition, we denote an augmented set by $\mathbb{N}_0^T \triangleq \{0, 1, \dots, T\}$. Vectors are denoted by small letters whereas matrices are denoted by capital letters. The transpose of matrix A is denoted by A^T . For a square matrix $A \in \mathbb{R}^{n \times n}$, we denote by $A \succ 0$ the positive-definite matrix A , and by $\text{diag}\{A\}$ the matrix having entries on its diagonal and zero elsewhere. Note that when the diagonal entries are matrices, then we obtain a block diagonal matrix. $\mathbb{E}\{\cdot\}$ represents the expectation of its argument, $\text{tr}\{\cdot\}$ denotes the trace, and $\text{cov}(\cdot)$ denotes the covariance of a matrix. $\|\cdot\|$ of a vector denotes the 2-norm of its argument. By $\mathbf{1}$ we denote the all-ones vector, by $\mathbf{0}$ we denote the all-zeros vector and by I we denote the identity matrix (of appropriate dimensions).

III. SYSTEM MODEL AND PRELIMINARIES

A. Dynamic Model

The stochastic, finite-horizon, discrete, linear dynamics of the vehicle is modeled as follows:

$$x_{k+1} = A_k x_k + B_k u_k + D_k w_k, \quad (1a)$$

$$y_{k+1} = C_{k+1} x_{k+1} + G_{k+1} r_{k+1}, \quad k \in \mathbb{N}_0^{N-1}, \quad (1b)$$

where $x_k \in \mathbb{R}^{n_x}$, in this case $n_x = 3$, since x_k is a three dimensional vector representing the position, velocity and acceleration of the vehicle, *i.e.*, $x_k = [s_k \ v_k \ a_k]^T$, $u_k \in \mathbb{R}$ is the control input, $y_k \in \mathbb{R}^{n_y}$ is the output observation of the system, and $w_k, r_k \in \mathbb{R}^{n_x}$ are zero mean white Gaussian process and observation noise, respectively,

with unit covariance. The time step k ranges from 0 to $N-1$, where N is the total number of time steps considered. Matrices A_k and B_k are defined as in [17] as

$$A_k = A = \begin{bmatrix} 1 & h & 0 \\ 0 & 1 & h \\ 0 & 0 & 1 - \frac{h}{\tau} \end{bmatrix}, B_k = B = \begin{bmatrix} 0 \\ h \\ \frac{h}{\tau} \end{bmatrix},$$

where h is the discrete-time interval and τ is a vehicle-specific parameter called the acceleration-to-deceleration ratio. In this model, we assume all output states are measurable *i.e.* $C_k = C = I$ for all k . Process noise matrix D_k and observation noise matrix G_k can be modeled according to the given scenario properties. Note that, currently, we assume the position state to be just a one-dimensional value along the length of the road. The lateral position is assumed to be constant.

Our cost function to be minimized is given by:

$$J(x_0, \dots, x_{N-1}, u_0, \dots, u_{N-1}) = \mathbb{E} \left[\sum_{k=0}^{N-1} (x_k^T Q_k x_k + u_k^T R_k u_k) \right], \quad (2)$$

where $x_k, k \in \mathbb{N}_0^{N-1}$, can be seen as the deviation from the optimal trajectory, $Q_k \succeq 0$, and $R_k \succ 0$. There is no terminal cost since the end state is constrained, as explained in the subsequent sections.

The initial state x_0 is a random vector of Gaussian distribution given by a known mean vector and a known covariance matrix, *i.e.*, $x_0 \sim \mathcal{N}(\mu_0, \Sigma_0)$. We assume that the end state can be approximated by a Gaussian distribution of given mean and covariance matrices (μ_N, Σ_N) , *i.e.*, $x_N \sim \mathcal{N}(\mu_N, \Sigma_N)$.

B. Communication Model

The output observation of the system, y_{k+1} , is communicated to the central coordinator subject to a packet drop probability p_d . If $p_d = 0$, there are no packet drops and all observations are communicated from the vehicle to the central coordinator. If $p_d = 1$, then all packets get dropped and there is no observation being communicated by the vehicle. We introduce the success indicator term, δ_k , which is 1 if the observation data has been communicated to the vehicle, and 0 otherwise. The value of δ_k is obtained according to the packet drop probability p_d . We consider several different packet drop probabilities between and including 0 and 1. A sample of $\delta = [\delta_1, \delta_2, \dots, \delta_N]$ is generated based on the packet drop probability p_d .

C. State Estimation Using Kalman Filtering

The Kalman estimator is used to get the best estimation of the state taking into account process noise, observation noise and packet drops. Let the Kalman gain be denoted by F_k . The covariance prediction and update using Kalman estimation based on [18] is calculated as follows:

The *a priori* covariance estimation is

$$\Sigma_{k+1|k} = A \Sigma_{k|k} A + D_k D_k^T. \quad (3)$$

The Kalman gain update:

$$F_{k+1} = \Sigma_{k+1|k} (G_{k+1} G_{k+1}^T + \Sigma_{k+1|k})^{-1}.$$

The *a posteriori* covariance update is given by

$$\Sigma_{k+1|k+1} = (I - \gamma_{k+1} F_{k+1}) \Sigma_{k+1|k}, \quad (4)$$

where $\gamma_k = \delta_k$ in case of sampled distribution and $\gamma_k = p_d$ in case of direct probability distribution. The *a priori* and *a posteriori* estimates of the state are given by

$$\hat{x}_{k+1|k} = A\hat{x}_{k|k} + Bu_k, \quad k \in \mathbb{N}_0^{N-1}, \quad (5a)$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + F_{k+1}(y_{k+1} - C\hat{x}_{k+1|k}), \quad (5b)$$

respectively.

Let $e_k = x_k - \hat{x}_{k|k}$, then y_{k+1} from (1b), after algebraic manipulations, becomes

$$\begin{aligned} y_{k+1} &= C(Ax_k + Bu_k + D_k w_k) + G_{k+1} r_{k+1} \\ &= CAx_k + CBu_k + CD_k w_k + G_{k+1} r_{k+1} \\ &= CA\hat{x}_{k|k} + CBu_k + CD_k w_k + G_{k+1} r_{k+1} + CAe_k. \end{aligned}$$

When no observation is received, y_{k+1} uses the prediction $\hat{x}_{k+1|k}$ and the innovation term $y_{k+1} - C\hat{x}_{k+1|k}$ is zero, i.e.,

$$y_{k+1} = C\hat{x}_{k+1|k} = CA\hat{x}_{k|k} + CBu_k,$$

Thus, both these cases can be written compactly as

$$\begin{aligned} y_{k+1} &= CA\hat{x}_{k|k} + CBu_k \\ &\quad + \gamma_{k+1}(CD_k w_k + G_{k+1} r_{k+1} + CAe_k) \\ &= CA\hat{x}_{k|k} + CBu_k + \gamma_{k+1} H_k \xi_k \end{aligned}$$

where, $H_k = [CD_k \ G_{k+1} \ CA]$ and $\xi_k = [w_k^T \ r_{k+1}^T \ e_k^T]^T$. Substituting for y_{k+1} in the *a posteriori* state equation, we get,

$$\begin{aligned} \hat{x}_{k+1|k+1} &= \hat{x}_{k+1|k} + F_{k+1}(y_{k+1} - C\hat{x}_{k+1|k}) \\ &= \hat{x}_{k+1|k} + \gamma_{k+1} F_{k+1} H_k \xi_k \\ &= A\hat{x}_{k|k} + Bu_k + \Delta_k \xi_k, \end{aligned}$$

where, $\Delta_k = \gamma_{k+1} F_{k+1} H_k$. Thus, the evolution of the *a posteriori* state estimate is given by

$$\hat{x}_{k+1|k+1} = A\hat{x}_{k|k} + Bu_k + \Delta_k \xi_k. \quad (6)$$

For a given intersection to be crossed by a certain time horizon, say at position s_{exit} within N time steps, a vehicle, modeled by (1a), needs to plan its trajectory accordingly. Therefore, the decision making is done at the beginning of the horizon. Since the *a posteriori* state estimate, which evolves according to (6) and is a reminiscent of (1a), takes into account the communication impairments (packet losses) one can effectively organize the trajectory according to the best known estimate that we have of the state (i.e., $\hat{x}_{k|k}$) instead of the state equation (1a).

D. Preliminaries

The compact system model containing the equations at every time instant, from time step 0 until time step N , stacked together or the *state* can be written as:

$$\chi = \mathcal{A}x_0 + \mathcal{B}v + \mathcal{D}\omega, \quad (7)$$

where

$$\begin{aligned} \chi &= [x_0^T \ x_{1|1}^T \ \dots \ x_{N|N}^T]^T \in \mathbb{R}^{(N+1)n_x}, \\ v &= [u_0^T \ u_1^T \ \dots \ u_{N-1}^T]^T \in \mathbb{R}^{Nn_u}, \\ \omega &= [w_0^T \ w_1^T \ \dots \ w_{N-1}^T]^T \in \mathbb{R}^{Nn_w}, \end{aligned}$$

and the matrices $\mathcal{A} \in \mathbb{R}^{(N+1)n_x \times n_x}$, $\mathcal{B} \in \mathbb{R}^{(N+1)n_x \times Nn_u}$, $\mathcal{D} \in \mathbb{R}^{(N+1)n_x \times Nn_w}$ are defined as $\mathcal{A} = [I \ A \ A^2 \ \dots \ A^N]^T$,

$$\mathcal{B} = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ B & 0 & 0 & \dots & 0 \\ AB & B & 0 & \dots & 0 \\ A^2 B & AB & B & \dots & 0 \\ \vdots & & & \ddots & \\ A^{N-1} B & A^{N-2} B & A^{N-3} B & \dots & B \end{bmatrix},$$

and

$$\mathcal{D} = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ D_0 & 0 & 0 & \dots & 0 \\ AD_0 & D_1 & 0 & \dots & 0 \\ A^2 D_0 & AD_1 & D_2 & \dots & 0 \\ \vdots & & & \ddots & \\ A^{N-1} D_0 & A^{N-2} D_1 & A^{N-3} D_2 & \dots & D_{N-1} \end{bmatrix}.$$

The compact system model containing the equations at every time instant, from time step 0 until time step N , stacked together for the *state estimate* can be written as:

$$\hat{\chi} = \mathcal{A}x_0 + \mathcal{B}v + \bar{\mathcal{D}}\xi, \quad (8)$$

where

$$\begin{aligned} \hat{\chi} &= [x_0^T \ \hat{x}_{1|1}^T \ \dots \ \hat{x}_{N|N}^T]^T \in \mathbb{R}^{(N+1)n_x}, \\ \xi &= [\xi_0^T \ \xi_1^T \ \dots \ \xi_{N-1}^T]^T \in \mathbb{R}^{Nn_\xi}, \end{aligned}$$

and the matrices $\mathcal{A} \in \mathbb{R}^{(N+1)n_x \times n_x}$, $\mathcal{B} \in \mathbb{R}^{(N+1)n_x \times Nn_u}$ are as defined before. $\bar{\mathcal{D}} \in \mathbb{R}^{(N+1)n_x \times Nn_w}$ is defined as

$$\bar{\mathcal{D}} = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ \Delta_0 & 0 & 0 & \dots & 0 \\ A\Delta_0 & \Delta_1 & 0 & \dots & 0 \\ A^2\Delta_0 & A\Delta_1 & \Delta_2 & \dots & 0 \\ \vdots & & & \ddots & \\ A^{N-1}\Delta_0 & A^{N-2}\Delta_1 & A^{N-3}\Delta_2 & \dots & \Delta_{N-1} \end{bmatrix}.$$

Objective function (2) can be rewritten as

$$\begin{aligned} J(\chi, v) &= \mathbb{E} [\chi^T \bar{Q} \chi + v^T \bar{R} v] \\ &= \text{tr} (\bar{Q} \mathbb{E} [(\chi - \bar{\chi})(\chi - \bar{\chi})^T]) + \bar{\chi}^T \bar{Q} \bar{\chi} + \mathbb{E} [v^T \bar{R} v] \\ &= \text{tr} (\bar{Q} (\mathbb{E} [(\chi - \hat{\chi})(\chi - \hat{\chi})^T] + \mathbb{E} [(\hat{\chi} - \bar{\chi})(\hat{\chi} - \bar{\chi})^T])) \\ &\quad + \bar{\chi}^T \bar{Q} \bar{\chi} + \mathbb{E} [v^T \bar{R} v] \\ &= \text{tr} (\bar{Q} \Sigma_\chi) + \text{tr} (\bar{Q} \Sigma_{\hat{\chi}}) + \bar{\chi}^T \bar{Q} \bar{\chi} + \mathbb{E} [v^T \bar{R} v] \end{aligned} \quad (9)$$

where $\bar{\chi} \triangleq \mathbf{A}\mu_0 + \mathbf{B}v$, $\bar{Q} \triangleq \text{blkdiag}(Q_1, Q_2, \dots, Q_{N-1}, 0)$, $\bar{R} \triangleq \text{blkdiag}(R_1, R_2, \dots, R_{N-1})$, $\Sigma_{\chi} \triangleq \mathbb{E}[(\chi - \hat{\chi})(\chi - \hat{\chi})^T]$, and $\Sigma_{\bar{\chi}} = \mathbb{E}[(\hat{\chi} - \bar{\chi})(\hat{\chi} - \bar{\chi})^T]$. Note that $\text{tr}(\bar{Q}\Sigma_{\chi})$ is independent of the control input v and $\bar{\chi}^T \bar{Q} \bar{\chi} + \mathbb{E}[v^T \bar{R} v]$ can be written as a quadratic cost over v . Therefore, without loss of generality of our problem, we can use the following cost:

$$J(\hat{\chi}, v) = \mathbb{E}[(\hat{\chi} - \bar{\chi})^T \bar{Q} (\hat{\chi} - \bar{\chi}) + v^T \bar{R} v], \quad (10)$$

where \bar{R} is of similar structure as \bar{R} .

We define a state feedback controller of the form

$$v_k = l_k \left[\mathbf{1}^T, x_0^T, \hat{x}_{1|1}^T, \dots, \hat{x}_{k|k}^T \right]^T,$$

where $l_k \in \mathbb{R}^{n_u \times n_x(k+2)}$. Thus, the relationship between $\hat{\chi}$ and v can be written as

$$v = L\hat{\chi},$$

where $\hat{\chi} = [\mathbf{1}^T, \hat{\chi}^T]^T \in \mathbb{R}^{n_x(N+2)}$ is the augmented state sequence until step N , $v \in \mathbb{R}^{Nn_u}$ is the control matrix until step $N-1$ and $L \in \mathbb{R}^{Nn_u \times (N+2)n_x}$ is the control gain matrix. L matrix is required to be causal and hence takes the form $L = [L_1, L_{\hat{\chi}}]$, where $L_1 \in \mathbb{R}^{Nn_u \times n_x}$ and $L_{\hat{\chi}} \in \mathbb{R}^{Nn_u \times (N+1)n_x}$ is a lower triangular block matrix. From (8), we can write the new state equation as:

$$\hat{\chi} = \begin{bmatrix} I & \mathbf{0} \\ \mathbf{0} & \mathcal{A} \end{bmatrix} \chi_0 + \begin{bmatrix} \mathbf{0} \\ \mathcal{B} \end{bmatrix} L\hat{\chi} + \begin{bmatrix} \mathbf{0} \\ \bar{\mathcal{D}} \end{bmatrix} \xi, \quad (11)$$

where $\chi_0 = [\mathbf{1}^T, x_0^T]^T$ and $\xi = [0^T, \xi^T]^T$. Rewriting (11) we get,

$$\hat{\chi} = (I - \mathbf{B}L)^{-1}(\mathbf{A}\chi_0 + \mathbf{D}\xi), \quad (12)$$

where $\mathbf{A} = \text{blkdiag}(I, \mathcal{A})$, $\mathbf{B} = [0, \mathcal{B}]^T$ and $\mathbf{D} = [0, \bar{\mathcal{D}}]^T$.

We define a new variable $K \triangleq L(I - \mathbf{B}L)^{-1}$, where $K = [K_1, K_{\hat{\chi}}]$, with $K_1 \in \mathbb{R}^{Nn_u \times n_x}$ and $K_{\hat{\chi}} \in \mathbb{R}^{Nn_u \times (N+1)n_x}$, is a lower triangular block matrix. Substituting K into (12), we get the following equations for state and control matrices:

$$\hat{\chi} = (I + \mathbf{B}K)(\mathbf{A}\chi_0 + \mathbf{D}\xi), \quad (13)$$

$$v = K(\mathbf{A}\chi_0 + \mathbf{D}\xi). \quad (14)$$

The variable to be optimized and tuned in this problem is K or equivalently, L .

IV. MAIN RESULTS

A. Cost function

Redefining the initial state mean and covariance as

$$\mu_0 = \begin{bmatrix} \mathbf{1} \\ \mu_0 \end{bmatrix}, \quad \Sigma_0 = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \Sigma_0 \end{bmatrix}, \quad (15)$$

respectively, we can now define the desired trajectory (i.e., the trajectory that is followed when there are no uncertainties and the initial state is given by the initial state mean), denoted by $\bar{\chi}$ as,

$$\bar{\chi} = (I + \mathbf{B}K)\mathbf{A}\mu_0. \quad (16)$$

Substituting (13), (14) and (16) in objective function (9) and knowing that $\mathbb{E}[\chi_0] = \mu_0$, $\mathbb{E}[\chi_0 \chi_0^T] = \mu_0 \mu_0^T + \Sigma_0$ and $\mathbb{E}[\chi_0 \omega^T] = 0$, the objective function can be re-written as

$$J(K) = \text{tr}(((I + \mathbf{B}K)^T \bar{Q} (I + \mathbf{B}K) + K^T \bar{R} K) (\mathbf{A}\Sigma_0 \mathbf{A}^T + \mathbf{D}\mathbf{D}^T) + K^T \bar{R} K \mathbf{A}\mu_0 \mu_0^T \mathbf{A}^T).$$

This is a quadratic expression in K .

B. Covariance Matrix

We define the following covariance matrices,

$$\Sigma_{\chi} \triangleq \mathbb{E}[(\chi - \hat{\chi})(\chi - \hat{\chi})^T], \quad (17)$$

$$\Sigma_{\bar{\chi}} \triangleq \mathbb{E}[(\chi - \bar{\chi})(\chi - \bar{\chi})^T], \quad (18)$$

$$\Sigma_{\hat{\chi}} \triangleq \mathbb{E}[(\hat{\chi} - \bar{\chi})(\hat{\chi} - \bar{\chi})^T]. \quad (19)$$

It can be easily deduced that $\Sigma_{\bar{\chi}} = \Sigma_{\chi} + \Sigma_{\hat{\chi}}$. Since, Σ_{χ} in independent of K , without loss of generality, we can consider only $\Sigma_{\hat{\chi}}$ in our optimization. Substituting (13) and (16) into (19), we get

$$\Sigma_{\hat{\chi}} = \mathbb{E}[(I + \mathbf{B}K)(\mathbf{A}(\chi_0 - \mu_0) + \mathbf{D}\xi) ((I + \mathbf{B}K)(\mathbf{A}(\chi_0 - \mu_0) + \mathbf{D}\xi))^T],$$

which after algebraic manipulations it becomes

$$\Sigma_{\hat{\chi}} = (I + \mathbf{B}K)(\mathbf{A}\Sigma_0 \mathbf{A}^T + \mathbf{D}\mathbf{D}^T)(I + \mathbf{B}K)^T. \quad (20)$$

Let $\Sigma_{op} \triangleq \mathbf{A}\Sigma_0 \mathbf{A}^T + \mathbf{D}\mathbf{D}^T$, be the matrix denoting open loop covariance dynamics. Hence,

$$\Sigma_{op} = \begin{bmatrix} I \\ \bar{A}_1 \\ \bar{A}_2 \\ \vdots \\ \bar{A}_N \end{bmatrix} \Sigma_0 \begin{bmatrix} I & \bar{A}_1^T & \bar{A}_2^T & \dots & \bar{A}_N^T \end{bmatrix} + \mathbf{D}\mathbf{D}^T$$

$$= \begin{bmatrix} \Sigma_0 & \Sigma_0 \bar{A}_1^T & \Sigma_0 \bar{A}_2^T & \Sigma_0 \bar{A}_3^T & \dots & \Sigma_0 \bar{A}_N^T \\ \bar{A}_1 \Sigma_0 & \bar{A}_1 \Sigma_0 \bar{A}_1^T & \bar{A}_1 \Sigma_0 \bar{A}_2^T & \bar{A}_1 \Sigma_0 \bar{A}_3^T & \dots & \bar{A}_1 \Sigma_0 \bar{A}_N^T \\ \bar{A}_2 \Sigma_0 & \bar{A}_2 \Sigma_0 \bar{A}_1^T & \bar{A}_2 \Sigma_0 \bar{A}_2^T & \bar{A}_2 \Sigma_0 \bar{A}_3^T & \dots & \bar{A}_2 \Sigma_0 \bar{A}_N^T \\ \bar{A}_3 \Sigma_0 & \bar{A}_3 \Sigma_0 \bar{A}_1^T & \bar{A}_3 \Sigma_0 \bar{A}_2^T & \bar{A}_3 \Sigma_0 \bar{A}_3^T & \dots & \bar{A}_3 \Sigma_0 \bar{A}_N^T \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \bar{A}_N \Sigma_0 & \bar{A}_N \Sigma_0 \bar{A}_1^T & \bar{A}_N \Sigma_0 \bar{A}_2^T & \bar{A}_N \Sigma_0 \bar{A}_3^T & \dots & \bar{A}_N \Sigma_0 \bar{A}_N^T \end{bmatrix} + \mathbf{D}\mathbf{D}^T.$$

For no communication losses, this can be written in terms of covariance evolution at each step given as defined in equation (3) by the new variable,

$$\Sigma_{op}^{nl} = \begin{bmatrix} \Sigma_0 & \Sigma_0 \bar{A}_1^T & \Sigma_0 \bar{A}_2^T & \Sigma_0 \bar{A}_3^T & \dots & \Sigma_0 \bar{A}_N^T \\ \bar{A}_1 \Sigma_0 & \Sigma_1 & \Sigma_1 \bar{A}_1^T & \Sigma_1 \bar{A}_2^T & \dots & \Sigma_1 \bar{A}_N^T \\ \bar{A}_2 \Sigma_0 & \bar{A}_2 \Sigma_1 & \Sigma_2 & \Sigma_2 \bar{A}_1^T & \dots & \Sigma_2 \bar{A}_N^T \\ \bar{A}_3 \Sigma_0 & \bar{A}_3 \Sigma_1 & \bar{A}_3 \Sigma_2 & \Sigma_3 & \dots & \Sigma_3 \bar{A}_N^T \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \bar{A}_N \Sigma_0 & \bar{A}_N \Sigma_1 & \bar{A}_N \Sigma_2 & \bar{A}_N \Sigma_3 & \dots & \Sigma_N \end{bmatrix}.$$

Now consider the case with communication packet losses. The value of each of the Σ_k in the above equation changes to that given by equation (4). Thus, we can define a new variable, Σ_{op}^l , where the Σ_k values are given by (4).

Thus, we can write the covariance of state matrix $\hat{\chi}$ in the presence of packet losses as:

$$\Sigma_{\hat{\chi}} \triangleq (I + \mathbf{B}K)\Sigma_{op}^l(I + \mathbf{B}K)^T. \quad (21)$$

C. Chance constraints for intersection crossing

Due to the stochastic nature of the system, putting hard constraints on the intersection crossing within a given time might result in infeasible solutions. Hence we model this as a chance constraint with very low probability of failure. The required chance constraint is given by

$$Pr(s_N \leq s_{\text{exit}}) \leq p_f,$$

where p_f is a predefined low probability of failure. Rewriting this constraint in terms of $\hat{\chi}$ and using appropriate multiplicative matrices α and β , we get,

$$Pr(\alpha^T \hat{\chi} \leq \beta) = \Phi \left(\frac{\beta - \alpha^T \hat{\chi}}{\sqrt{\alpha^T \Sigma_{\hat{\chi}} \alpha}} \right) \leq p_f, \quad (22)$$

where Φ is the cumulative distribution function of the standard Gaussian distribution. Now, let

$$\Sigma_p = \alpha^T \Sigma_{\hat{\chi}} \alpha = \alpha^T (I + \mathbf{BK}) \Sigma_{op}^l (I + \mathbf{BK})^T \alpha.$$

By construction, Σ_{op}^l is symmetric, although not necessarily positive definite. We define M such that $M \triangleq (\Sigma_{op}^l)^{\frac{1}{2}}$. If Σ_{op}^l is positive definite, M is real, otherwise M is complex. However in both cases, M is symmetric. Thus, we can rewrite,

$$\Sigma_p = \alpha^T \Sigma_{\hat{\chi}} \alpha = \alpha^T (I + \mathbf{BK}) M^T M (I + \mathbf{BK})^T \alpha.$$

Now, $M(I + \mathbf{BK})^T \alpha$ is a vector. Let us denote it by \bar{u} . Thus, $\Sigma_p = \bar{u}^T \bar{u}$. Rewriting (22), we get,

$$\beta - \alpha^T \hat{\chi} - \sqrt{\Sigma_p} \Phi^{-1}(p_f) \leq 0.$$

Substituting for Σ_p ,

$$\beta - \alpha^T \hat{\chi} - \sqrt{\bar{u}^T \bar{u}} \Phi^{-1}(p_f) \leq 0.$$

Now, $\bar{u}^T \bar{u} = \lambda_{\max}(\bar{u}^T \bar{u})$ since $\bar{u}^T \bar{u}$ is a 1×1 matrix. Hence,

$$\beta - \alpha^T \hat{\chi} - \sqrt{\lambda_{\max}(\bar{u}^T \bar{u})} \Phi^{-1}(p_f) \leq 0.$$

Now, $\lambda_{\max}(\bar{u}^T \bar{u}) \leq \lambda_{\max}(\bar{u}^H \bar{u})$ where the equality holds for real matrices. Since $\Phi^{-1}(p_f) < 0$ for $p_f < 0.5$, the following inequality holds:

$$\begin{aligned} \beta - \alpha^T \hat{\chi} - \sqrt{\lambda_{\max} \bar{u}^T \bar{u}} \Phi^{-1}(p_f) &\leq \\ \beta - \alpha^T \hat{\chi} - \sqrt{\lambda_{\max} \bar{u}^H \bar{u}} \Phi^{-1}(p_f) &\leq 0. \end{aligned} \quad (23)$$

By definition of 2-norm for matrices,

$$\beta - \alpha^T \hat{\chi} - \|\bar{u}\| \Phi^{-1}(p_f) \leq 0.$$

Resubstituting for \bar{u} , the final form of the chance constraint is,

$$\beta - \alpha^T (I + \mathbf{BK}) \mathbf{A} \mu_0 - \|M(I + \mathbf{BK})^T \alpha\| \Phi^{-1}(p_f) \leq 0. \quad (24)$$

Remark 1: Inequality (23) holds under the assumption that the failure probability, $p_f < 0.5$, i.e., the term $\Phi^{-1}(p_f)$ is negative. At $p_f = 0.5$, $\Phi^{-1}(p_f) = 0$, which eliminates the nonlinear part (i.e., the covariance term) from the equation

and only the mean value is left. For $p_f > 0.5$, $\Phi^{-1}(p_f)$ becomes positive and this inequality no longer holds. However, in real life scenarios, the system is designed such that the failure probability is much smaller than 0.5, and, hence, our assumption is reasonable. \lrcorner

D. Constraint on terminal covariance

We constrain the evolution of the process covariance to be less than a predefined value Σ_{lim} . It provides a narrow distribution of the end state and thus the predicted states are closer to the mean μ_N . This ensures that the μ_N required for the success of the chance constraint is not too far away from the required s_{exit} . The constraint is modeled as follows:

$$E_N \Sigma_{\hat{\chi}} E_N^T \leq \Sigma_{\text{lim}},$$

where $E_N \triangleq [0, 0, 0, \dots, I] \in \mathbb{R}^{n_x \times (N+2)n_x}$ is the appropriate multiplying matrix to get the N^{th} time step covariance. Hence the constraint becomes,

$$E_N (I + \mathbf{BK}) \Sigma_{op}^l (I + \mathbf{BK})^T E_N^T \leq \Sigma_{\text{lim}}.$$

Since by assumption, $\Sigma_{\text{lim}} \geq 0$, the above inequality can be rewritten as:

$$I - (\Sigma_{\text{lim}}^{-1/2})^T E_N (I + \mathbf{BK}) \Sigma_{op}^l (I + \mathbf{BK})^T E_N^T \Sigma_{\text{lim}}^{-1/2} \geq 0.$$

Substituting for Σ_{op}^l ,

$$I - (\Sigma_{\text{lim}}^{-1/2})^T E_N (I + \mathbf{BK}) M^T M (I + \mathbf{BK})^T E_N^T \Sigma_{\text{lim}}^{-1/2} \geq 0.$$

Being symmetric, matrix

$$\Xi \triangleq (\Sigma_{\text{lim}}^{-1/2})^T E_N (I + \mathbf{BK}) M^T M (I + \mathbf{BK})^T E_N^T \Sigma_{\text{lim}}^{-1/2}$$

is diagonalizable via the orthogonal matrix $S \in \mathbb{R}^{n_x \times n_x}$. Thus, $S(I - \text{diag}(\lambda_1, \dots, \lambda_{n_x})) S^T \geq 0$ where $\lambda_1, \dots, \lambda_{n_x}$ are the eigenvalues of Ξ . The last inequality is implied by $1 - \lambda_{\max}(\Xi) \geq 0$. Again, since $\lambda_{\max}(M^T M) \leq \lambda_{\max}(M^H M)$, we get,

$$\begin{aligned} \lambda_{\max}(\Xi) &\leq \\ \lambda_{\max}((\Sigma_{\text{lim}}^{-1/2})^T E_N (I + \mathbf{BK}) M^H M (I + \mathbf{BK})^T E_N^T \Sigma_{\text{lim}}^{-1/2}). \end{aligned}$$

Hence a stronger constraint on the covariance is:

$$\begin{aligned} 1 - \lambda_{\max}((\Sigma_{\text{lim}}^{-1/2})^T E_N (I + \mathbf{BK}) \\ M^H M (I + \mathbf{BK})^T E_N^T \Sigma_{\text{lim}}^{-1/2}) &\geq 0. \end{aligned} \quad (25)$$

By definition of 2-norm, we can rewrite constraint (25) as:

$$1 - \left\| M(I + \mathbf{BK})^T E_N^T \Sigma_{\text{lim}}^{-1/2} \right\|^2 \geq 0. \quad (26)$$

This is the final form of the constraint on the terminal covariance of the state.

E. Input constraints

In any real vehicle scenario, the power input from the engine and consequently, the available acceleration to the vehicle is limited. This needs to be accounted while designing the solution. To this end, in this paper, we restrict the available input acceleration to the vehicle based on generic car acceleration limits as follows:

$$u_{\min} \leq u \leq u_{\max}. \quad (27)$$

F. Optimization problem

Based on the system model and all the constraint considerations, the final optimization problem can be defined as follows:

$$\begin{aligned} \min_K \quad & J(K) \\ \text{s.t.} \quad & (24), (26), (27). \end{aligned} \quad (28)$$

The result of this optimization gives a robust control input for each step in the given horizon to cross the intersection successfully in the presence of all the aforementioned constraints. The control input of the first step can be implemented by the vehicle and the entire process can be repeated at the next time step, as per the MPC methodology. This control input is communicated by the central coordinator back to the vehicle. In case of communication loss in this process or infeasibility at any step, the previously computed input scheme can be implemented till a new version is communicated. For the proof of convexity of the above optimization problem, see [13].

V. NUMERICAL EXAMPLES

In this section, we demonstrate the performance of the proposed algorithm through numerical examples and, thereby, quantify its advantages over previous methods. Problem (28) is coded in MATLAB and run using the CVX toolset for disciplined convex optimization.

A horizon of $N = 20$ time steps with time period, $h = 0.2s$ is considered. A vehicle starts from $s_0 = 0m$ with velocity $v = 5m/s$ and acceleration $a = 0m/s^2$. This is taken as the initial mean of the state $u_0 = [0 \ 5 \ 0]$. It is required that it crosses the intersection given by $s_{\text{exit}} = 30m$ in the given time horizon. Since $s_{\text{exit}} = 30m$, the vehicle needs to accelerate from its current speed to a higher value to cross the intersection in the given horizon. The initial covariance of the state is assumed to be $\Sigma_0 = \text{diag}\{1 \ 0.1 \ 0.1\}$. The terminal covariance limit is taken as $\Sigma_{\text{lim}} = \text{diag}\{3 \ 0.1 \ 0.1\}$. The noise matrices for state, is taken as $D = \text{diag}\{0.5 \ 0.05 \ 0.05\}$ and for observer noise, $G = 0.5D$. Vehicle specific parameters are taken as $\tau = 10$ and input acceleration limits are $[-5, \ 3]m/s^2$. The cost matrices for state and input costs are considered are $Q = \text{diag}\{1 \ 0.01 \ 5\}$ and $R = 5$, respectively.

In what follows, the trajectory is predicted for $N = 20$ time steps by the convex optimization problem (28) and the corresponding control sequence is generated (open-loop control).

First, the packet drop probability is kept constant at $p_d = 0.5$ and the probability of failure is varied from $p_f = [0.49, 0.05, 0.005, 0.0005]$. From the plot of vehicle position (see Fig. 1), we can see that, at $p_f = 0.49$ (blue curve), the position mean is very close to $s = 30m$ and the covariance bar is distributed such that there is approximately 50% chance that the vehicle crosses the intersection.

As we keep decreasing the probability of failure, the mean and the corresponding covariance bar move further away from the s_{exit} position, thereby ensuring that the vehicle

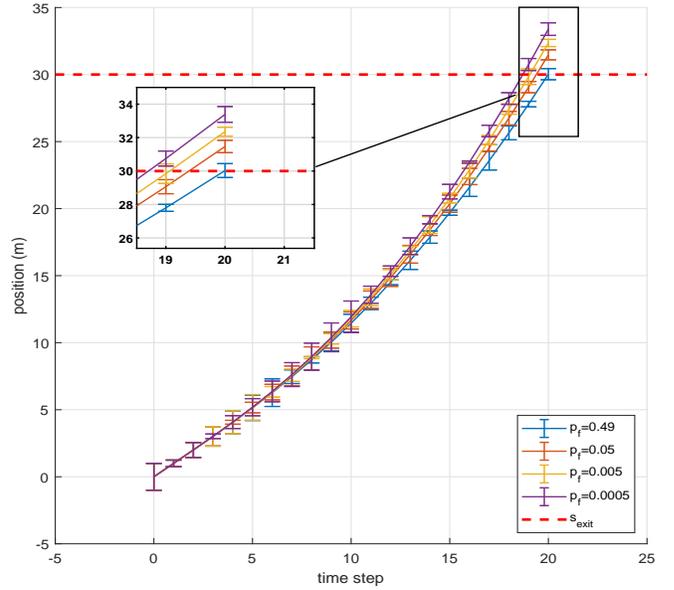


Fig. 1: Vehicle position evolution wrt time step k as predicted at $k = 0$ until $k = N$ with varying p_f and constant $p_d = 0.5$. The length of the errorbar above and below the mean, as depicted in the figure, is the standard deviation $1\sigma_k = \sqrt{\Sigma_k}$.

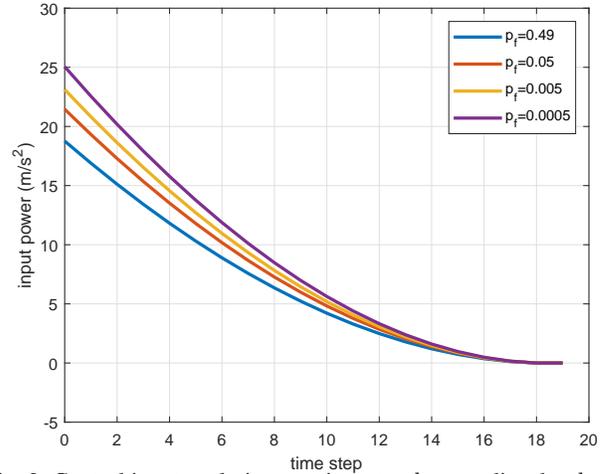


Fig. 2: Control input evolution wrt time step k as predicted at $k = 0$ until $k = N$ with varying p_f and constant $p_d = 0.5$.

crosses the intersection with more and more certainty. For all practical purposes, we will assume $p_f = 0.0005$, thereby ensuring that the vehicle crosses the intersection with a very high level of certainty. The control input requirement (see Fig. 2) increases as we decrease the probability of failure and this is expected, since control becomes increasingly conservative.

Next, the failure probability is kept constant at $p_f = 0.0005$ and packet drop probabilities considered are $p_d = [0.0, 0.2, 0.4, 0.6, 0.8]$. Beyond $p_d = 0.8$, the optimization becomes infeasible because of the high packet drops, since the covariance cannot be kept within the required limits. Consider Fig. 3 which gives the position of the vehicle. As the packet drop probability increases, the noise in the system increases and hence the covariance gets bigger.

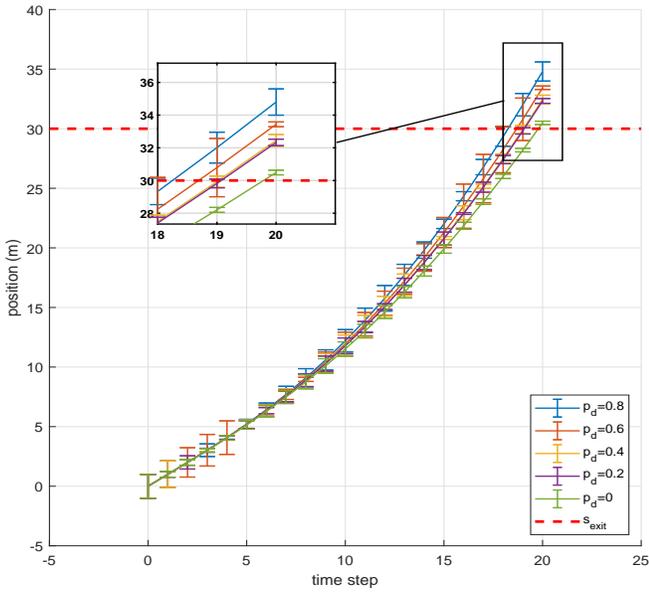


Fig. 3: Vehicle position evolution wrt time step k as predicted at $k = 0$ until $k = N$ with varying p_d and constant $p_f = 0.0005$.

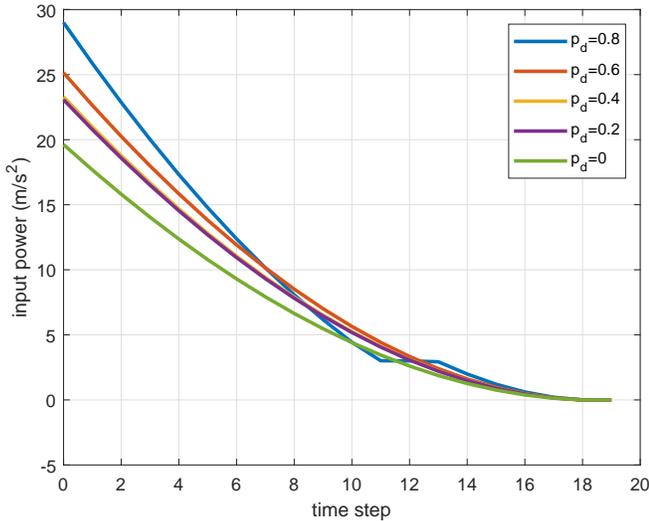


Fig. 4: Control input evolution wrt time step k as predicted at $k = 0$ until $k = N$ with varying p_d and constant $p_f = 0.0005$.

The difference between the covariance when $p_d = 0$ (green bars) and $p_d = 0.8$ (blue bars) is starkly visible. As the covariance gets bigger, the mean position also moves further upward from s_{exit} so as to ensure that the probability of failure stays within the bounds. As it is shown in Fig. 4, the control input also increases as the packet drop probability increases. This is because, packet losses mean that the covariance evolves as an open loop without any feedback from the sensors. Hence the input has to be made increasingly conservative to account for this higher covariance.

Next, the advantage of having the terminal constraint on the covariance evolution is illustrated. This is done by comparing the results with and without the terminal constraint. Packet drop probability is taken as $p_d = 0.5$ and the failure

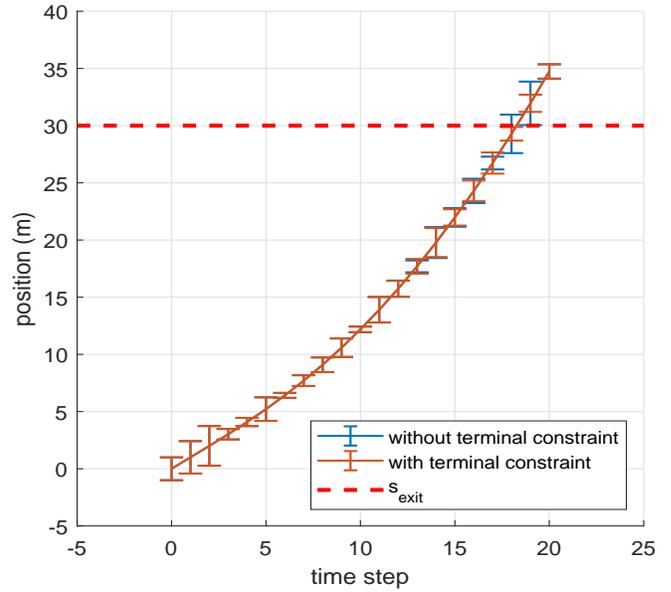


Fig. 5: Vehicle position evolution with and without the terminal covariance constraint with $p_d = 0.8$ and $p_f = 0.0005$.

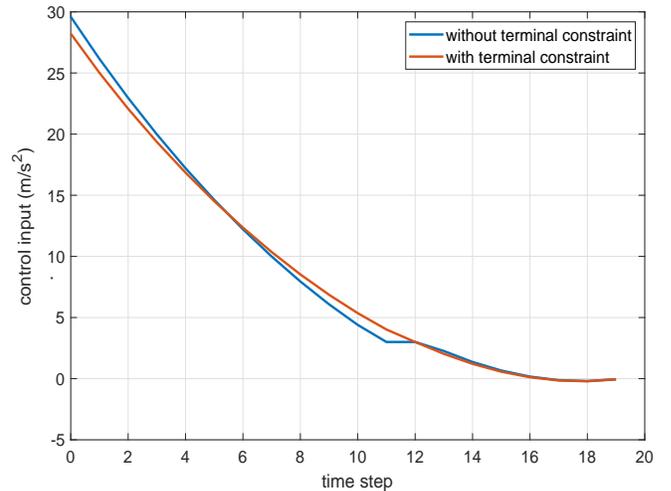


Fig. 6: Control input with and without the terminal covariance constraint with $p_d = 0.8$ and $p_f = 0.0005$.

probability is taken as $p_f = 0.0005$. We observe that in the case with no terminal constraint, the control input is high in the beginning and then it has to be cut off because the acceleration limit is reached. On the other hand, with terminal constraint in effect, the control input gets adjusted from the beginning in order to ensure a smooth input and, thus, retain the acceleration within the limits.

As aforementioned, results have been run with $N = 20$, *i.e.*, the trajectory is predicted for 20 time steps using optimization problem (28) and the corresponding control sequence is generated (open-loop control). Following the MPC method, however, the control at one time step is to be executed and the entire problem should be re-evaluated for the next step. Fig.7 shows this simulation. This is how the vehicle would run the algorithm in practice.

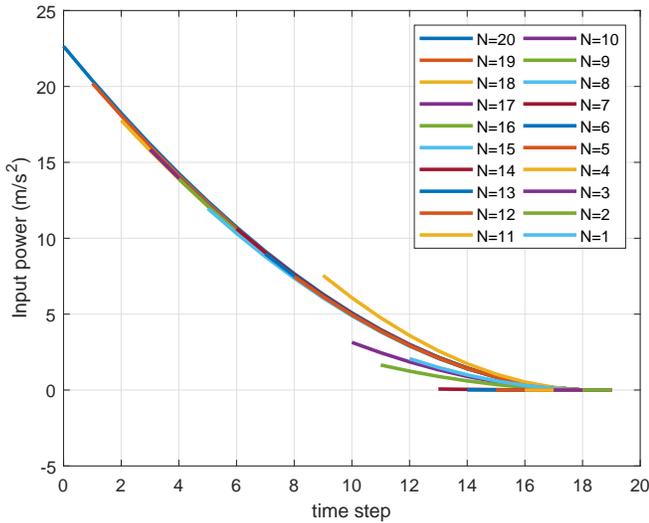


Fig. 7: Plot of control sequence generated at each time step of the receding horizon with $p_d = 0.5$ and $p_f = 0.0005$ with same ensemble for each loop.

VI. CONCLUSIONS AND FUTURE DIRECTIONS

A. Conclusions

This paper has considered the problem of determining a robust trajectory for vehicles to cross an intersection within a given time-slot in the presence of communication impairments. First, the problem was relaxed with the intersection crossing requirement as a chance constraint one. Additionally, a constraint on the terminal covariance was included. To be able to cast the optimization problem as a convex one, a more conservative version of the terminal constraint was incorporated. Since the constraints used are rather conservative, the final trajectory is a feasible one, but not (necessarily) optimal.

This paper gives a theoretical framework to include the real world scenarios of communication losses as well as noisy observations in the path planning for automated vehicles, which were in most cases so far, taken to be perfect. We obtain a solution which works with varying degrees of packet losses and noise levels to provide a feasible, sub-optimal solution with very low probability of failure in crossing the intersection within a given time frame.

B. Future Directions

It is envisioned to extend the solution to find the optimal path for multiple vehicles simultaneously. In this case, the communication resources are scarce making the intersection crossing problem even more challenging.

The prediction of packet losses can also be improved based on more intricate predictive algorithms.

A further step in this direction would be to consider the time-slots for vehicle as optimization variables and solve for the optimal crossing time for each vehicle, given a predetermined priority order. Once the above objectives have been achieved and a solid framework has been developed for them, the problem of determining the priority list for vehicles to cross the intersection can be addressed.

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