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Soliton mode-locked large-mode-area Tm-doped fiber oscillator

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We report a passively mode-locked fiber laser based on thulium-doped large-mode-area fiber. Self-starting mode-locking soliton operation can be readily achieved after the suppression of the higher-order modes by using mode field adapters in combination with coiling fiber method. The laser delivers 600-fs level pulses at 2 μm with an average power of up to 243 mW and a repetition rate of ~ 53.7 MHz. The output corresponds to a maximum pulse energy of 4.5 nJ, >5 times larger than that of a similar thulium-doped soliton oscillator fully based on single mode fibers. To the best of our knowledge, this is the first demonstration of a soliton mode-locked large-mode-area fiber laser at 2 μm, opening up new possibilities for high performance ultrafast fiber sources at mid-infrared region.

High power ultrafast lasers emitting at 2 μm have attracted growing attention over the past decades due to an increasing number of potential applications. For example, the strong H_2O and NO_2/CO_2 absorption lines in this spectral region facilitate several medical and gas sensing applications [1, 2]. They can also be used as efficient pump sources of non-oxide crystals due to the low two-photon absorption to access the mid-infrared (mid-IR) region of up to 6 μm via nonlinear frequency conversion [3]. Such mid-IR ultrafast pulse sources are highly sought after for spectroscopy applications since many molecular absorption features appear in the 2-6 μm spectral region [4]. Furthermore, they are of particular interest in the field of transparent material processing and even high field physics (e.g. high harmonic generation [5]).

The well-known benefits of fiber-based sources include compact footprint, flexibility and environmental reliability, making them an ideal option for many out-of-lab applications. Thulium (Tm) -doped fibers (TDFs) are one of the most successful gain media in the fiber laser family that have a wide gain bandwidth of about 300 nm centered at ~1.9 μm, capable of supporting sub-100 fs pulse generation and amplification. However, due to the inherently large negative material dispersion of silica fiber at 2 μm, most of ultrafast Tm-doped fiber oscillators operate in the soliton regime, with pulse energy typically limited to < 1 nJ [6-9]. Intra-cavity dispersion engineering can be utilized to break through this barrier and boost the energy to few nJ level [10], in which the output typically requires complex pulse compression technique to get bandwidth-limited pulses. Beyond this, higher energy is eventually limited by excessive optical nonlinearity due to strong light confinement in conventional single mode (SM) silica fibers (typically in ~9 μm core diameter). To increase the pulse energy further while keeping the duration at fs level, several techniques have been exploited, including well-known chirped pulse amplification [11], Raman-induced soliton self-frequency shift [12] and higher order soliton generation [13-15]. However, they generally require external amplification or nonlinear optical processing, significantly increasing the system complexity and may introduce extra undesirable amplitude and phase noise. These limitations motivate the ongoing research on fiber oscillators that can directly generate ultrafast pulse with high energy.

An alternative strategy is to utilize large mode area (LMA) fibers in the oscillator which can significantly reduce the nonlinear phase accumulation and thus support higher peak power compared to traditional SM fibers. Indeed, the past decade has seen great

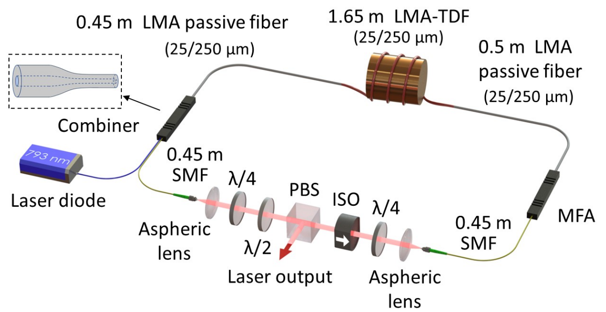

Fig. 1. Schematic setup of the LMA Tm fiber oscillator. LMA-TDF, large-mode-area Tm-doped fiber; MFA, mode field adaptor; λ/4, quarter-wave plate; λ/2, half-wave plate; ISO, isolator; PBS, polarization beam splitter.

progresses in developing fiber oscillators based on LMA fibers [16-20]. A LMA Yb-doped fiber with core diameter of 20 μm was employed to generate 115 fs pulses with energies of up to 21 nJ based on nonlinear polarization evolution (NPE) technique [18]. B. Ortaç *et al.* used a Yb-doped LMA photonic crystal fiber with 70 μm core diameter as gain fiber to construct an all normal dispersion fiber oscillator, which can deliver 400 fs compressed pulses with 265 nJ pulse energies [16]. The energy was later scaled up to as high as 927 nJ by using a fiber with large core diameter of up to 80 μm [17]. However, most of the previous reports focused on the 1 μm region. Development of the LMA-based oscillators in other spectral regions (e.g. 1.5 μm [21] and 2 μm [22]) have lagged far behind that of the Yb fiber lasers.

In this letter, we demonstrate the first soliton mode-locked Tm-doped LMA fiber laser based on NPE that directly generates ~600-fs pulses with a fundamental repetition rate of ~53.7 MHz at 2 μm. Neither active cooling stage nor grating-based bulk compressor is needed in our laser. The maximum output power reaches 243 mW, corresponding to a pulse energy of ~4.5 nJ. This is >5 times larger than that of a homemade mode-locked Tm laser with the LMA fibers replaced by SM fibers, coinciding with theoretical prediction, and even much larger (>20 times) than that of previously reported NPE- and SM fibers-based soliton Tm oscillators [23, 24]. Higher laser performance could thus be expected by employing a larger LMA active fiber.

The laser setup is schematically illustrated in Fig. 1. A segment of 1.65 m Tm-doped LMA fiber (Nufern, LMA-TDF-25P/250-HE) with a core diameter of 25 μm serves as the gain medium. The numerical aperture (NA) is 0.09, yielding a V number of 3.53 at 2 μm which supports six degenerate waveguide modes. The mode field diameter (MFD) is calculated to be 22.4 μm [25]. To favor the fundamental mode excitation, a mode-filed adapter (MFA) and a multifunctional pump-signal combiner with an embedded MFA are used to suppress the higher-order modes (HOMs) excited in the LMA fiber. Both of them are fabricated by tapered fusion splicing a short piece of passive LMA fiber (25 μm core diameter, 0.09 NA, Nufern LMA-GDF-25/250-09M) to a standard single mode fiber (SMF-28e, Corning) (see the inset of Fig. 1). The pump light offered by a 793 nm multimode laser diode is guided into the cladding of the gain fiber through the combiner. Moreover, we coil the gain fiber on a 7 cm diameter copper drum, which is used not only for the

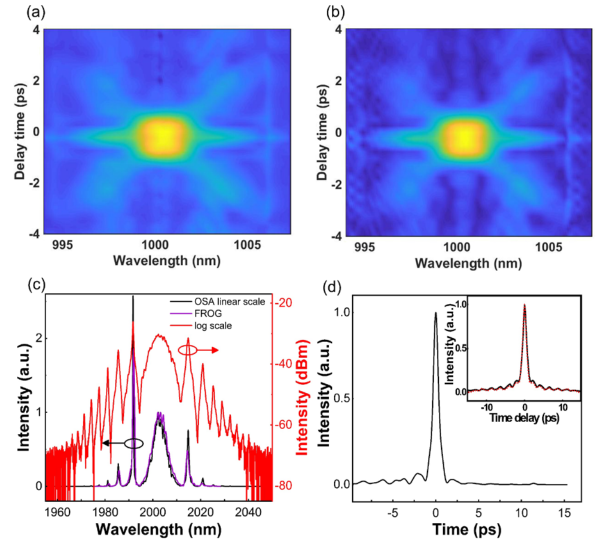

Fig. 2. (a) Measured and (b) retrieved SHG FROG traces. (c) The measured and retrieved spectra from OSA and FROG, respectively. (d) Retrieved intensity in the time domain. Inset, measured AC trace (black solid) from an autocorrelator and retrieved AC trace (red dotted) from FROG.

attenuation of HOMs [26], but also for passive heat dissipation. Without using the MFAs and fiber coiling method, the self-starting capability decreases significantly. Mode-locking is driven and stabilized by NPE, which is based on the manipulation of three waveplates and a polarization beam splitter (PBS). An isolator placed after the PBS ensures unidirectional laser propagation in the ring cavity. The rejected port of the PBS is used as the laser output port. The free-space path inside the cavity has a length of 45 cm and the total cavity length is ~3.95 m.

With adequate pump power and suitable waveplate setting, reliable self-starting of mode-locked operation at a repetition rate of 53.7 MHz can be achieved. In the experiment, the mode-locking threshold of pump power is 7.5 W. The high threshold could be attributed to the high intra-cavity loss caused by the high transmission loss of the MFA (~20% loss), the bending loss of the gain fiber, the fiber coupling loss, and as well as the overheating-induced loss resulting from thermal influences. As soon as mode-locking is established, the pump power can be continuously tuned between 7 W and 7.75 W without pulse vanishing or breaking. The average output power varies from 155.7 mW to 242.8 mW, corresponding to the pulse energy ranging from 2.9 nJ to 4.5 nJ.

A customized frequency-resolved optical gating (FROG) based on second harmonic generation from MesaPhotonics LLC is employed to characterize the output pulse features both in time and spectral domain. The typical output features measured at a pump power of 7 W are shown in Fig. 2. An error of only 0.7% can be achieved by comparing the measured FROG trace (Fig. 2(a)) with the retrieved trace (Fig. 2(b)). An optical spectrum analyzer (OSA, Yokogawa AQ6375B) is used to measure the spectrum, which is centered at 2002.8 nm with a FWHM bandwidth of ~6 nm and clear Kelly sidebands resulting from soliton mode-locking regime. Strong agreement between the measured and retrieved spectra (see Fig. 3(c)) indicates highly reliable FROG measurement. Despite mode filtering and gain fiber coiling, the HOMs are still incompletely eliminated, leading to weak intermodal interference and thus slight spectral speckles observable over the spectrum. The retrieved pulse duration is 715 fs with a time bandwidth product (TBP) of 0.318, close to the transform limit value of 700 fs for a $sech^2$ soliton pulse, as shown in Fig. 4(d). By integrating the whole temporal profile, the peak power is estimated as 2.5 kW. Moreover, clear pedestals and satellite pulses observed from the pulse profile could result from the intense Kelly sidebands and the weak excitation of HOMs. It should be noted that the shape of pulse is a little bit different from a typical $sech^2$ shape, which is due to the contribution of pedestal energy to the main pulse. As a result, the laser produces slightly distorted solitons. An autocorrelator (APE PulseCheck) is also used to measure the autocorrelation (AC) trace, which shows good agreement with the retrieved AC trace from FROG (see the inset of Fig. 4(d)), further indicating the reliability of the retrieved pulse profile.

According to the positions of the sidebands, the total intra-cavity dispersion can be calculated by $GDD = -\lambda_0^4/\pi mc^2$ [27]. Here, λ_0 is the central wavelength, c is the light speed and m is the slope of a linear fitting curve of the relationship between squared wavelength offset $\Delta\lambda_N^2$ and sideband order N. In our experiment, m is estimated as 168.5 nm², as shown in Fig. 3(a). This gives a total cavity dispersion of -0.338 ps². Considering a 0.9 m total length of SMF-28e fiber with group velocity dispersion (GVD) value of -86 ps²/km at 2 μm, the average GVD of our LMA fiber is -112 ps²/km, slightly larger than that of the SM fiber.

We note that the Kelly sidebands in the optical spectrum which is similar to that of Ref. [28], are much stronger than previously reported soliton mode-locked fiber lasers [6-9]. This is due to the fact that the amplitude of soliton pulse circulating in this laser cavity experiences strong periodic perturbations induced by the significantly discrete losses mentioned above and rapidly variable gain across the bent active fiber [29]. Additional perturbations can result from the discrete nature of the dispersion and nonlinearity of the fibers and air in the cavity. These strong periodically occurring variations couple the soliton into the intense copropagating dispersive waves in the formation of Kelly sidebands on the spectrum. On the other hand, the Kelly sidebands become stronger when the pulse energy is increased due to the reshaping of soliton by shedding energy to dispersive waves. Therefore, the strong sidebands indicate the soliton energy reaching near the maximum possible value, resulting in the near minimum pulse duration according to the soliton theorem area (*i.e.* the pulse duration is inversely proportional to the pulse energy) [30]. This could be examined by comparing the optical spectra with different output powers (see Fig. 3. (b)). As the pump power increases, the sidebands grow significantly while the area of central spectrum peak remains almost unchanged, indicating the energy of main pulse (*i.e.* soliton) approaching near maximum value. In terms of experimental demonstration, the soliton energy can be precisely measured by combining a diffraction grating for spatially dispersing the different wavelengths with two sharp knife-edges for selectively blocking the sideband light. With the entire output average power increasing from 155.7 mW to 242.8 mW, the power of soliton varies from 116 mW to 121.3 mW (corresponding to the soliton energy from 2.2 nJ to 2.3 nJ), without a noticeable increase, indicating that the soliton energy reaches near the maximum value. The corresponding pulse duration slightly decreases from 715 fs to 623 fs. If the pump power is further increased, higher energy would be driven into the dispersive waves, making the pulse unstable.

To evaluate the stability of this LMA-based laser, a 12.5 GHz photodetector (Newport, 818-BB-51F) connected with a radio frequency (RF) spectrum analyzer (Keysight, N9010B) is used to measure the laser pulse train. The fundamental repetition rate recorded with a resolution bandwidth (RBW) of 100 Hz is 53.7 MHz (see Fig. 4(a)), with a signal-to-noise contrast of 72 dB, comparable to previously reported mode-locked fiber lasers [6-9]. This confirms the stability of our laser as an ultrafast source for various applications. The observable noise spikes near the central peak are mainly caused by the pump noise, the temperature fluctuation, as well as the acoustic noise [31]. Moreover, near flat structure over the wideband RF spectrum up to 3.6 GHz (corresponding to 67th order harmonics) indicates that the laser operates in good continuous wave mode-locking regime (see Fig. 4(b)).

Finally, we construct another cavity based on SM fibers for comparison. The new cavity is constructed by replacing the LMA fiber with a 2 m long SM-TDF (9 μm core diameter, 10.4 μm MFD at 2 μm, Nufern SM-TSF-9/125) which is pumped by a home-made 1570 nm fiber laser through a SMF based wavelength division multiplexer, while keeping the free optical components unchanged. This SM-TDF laser has a similar cavity length to that of the LMA-TDF laser and operates at a repetition rate of ~51.8 MHz. Table 1

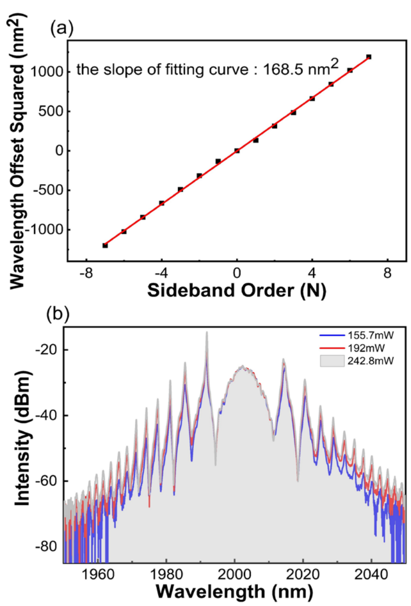

Fig. 3. (a) The square of wavelength offset of N^{th} order sideband as a function of N. (b) Evolution of the spectrum with different output powers.

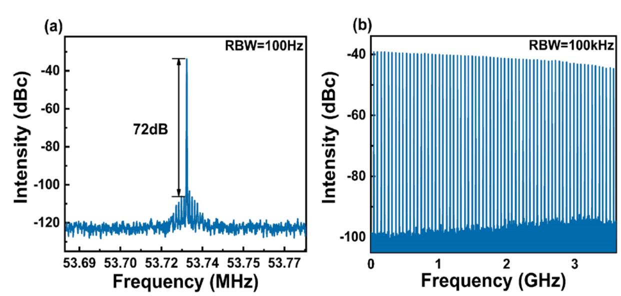

Fig. 4. (a) RF spectrum of the fundamental repetition rate centered at ~53.73 MHz. (b) Wide span RF spectrum up to 3.6 GHz.

summarizes the typical output performance comparison between these two lasers. Compared to the SM fibers-based laser, the LMA counterpart can generate pulses with more than 5.6- and 3-times improvement in terms of pulse energy and peak power, respectively, while with a slightly larger (~22 %) pulse duration. The pulse duration discrepancy is due to that the negative dispersion of the LMA fiber is larger than the SM fiber, as the soliton pulse duration is proportional to the square of total cavity dispersion [30]. On the other hand, the maximum energy or peak power is limited by the optical nonlinearity of fiber which scales inversely to the effective mode field area [32]. In our experiments, by scaling the MFD of 10.4 μm to 22.4 μm, a 4.5-fold improvement is thus predicted in pulse energy, coinciding with the experimental results. Building on this successful demonstration, much higher energy could be achieved in the future by employing a much larger LMA Tm fiber.

Table 1. Performance comparison of SM-TDF and LMA-TDF mode-locked lasers.

Gain fiber	Output power (mW)	Pulse energy (nJ)	Pulse duration (fs)*	Peak power (kW)
LMA-TDF	155.7~242.8	2.9~4.5	715~623	2.5~3.1
SM-TDF	28~41.5	0.54~0.8	579~509	0.79~1

*The pulse duration decreases with pulse energy due to its inverse proportion to the pulse energy.

In summary, we have demonstrated a soliton mode-locked LMA Tm fiber laser for the first time. The laser operates at 2 μm with a pulse duration of ~600-fs level. The self-starting mode-locking capabilities are strongly affected by intermodal interference, which can be suppressed by using the MFAs and fiber coiling method. The maximum pulse energy is >5 times larger than that of a similar SM-TDF laser, reaching 4.5 nJ (corresponding to 3.1 kW peak power). According to the experimental demonstration together with theoretical prediction, a larger mode-area Tm fiber could support higher pulse energy. Furthermore, if the intra-cavity dispersion engineering technique is employed, much higher laser performance can also be expected. The success of our LMA fiber laser opens new possibilities for high performance mid-IR ultrafast oscillators.

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