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Homogenization of Multiscale Eddy Current Problem by Localized Orthogonal Decomposition Method

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A homogenization approach for the solution of multiscale eddy current problem is proposed. The method is based on subspace decomposition and it involves a coarse space and a nested fine space. The homogenized problem is posed in the coarse space with the help of a projection operator acting between the coarse space and a space of rapidly oscillating functions. A Helmholtz decomposition is applied to treat the null-space of the curl-operator so that the projection can be calculated locally. The results are illustrated in a two-dimensional numerical example.

***Index Terms*—homogenization, orthogonal projection, quasi-static Maxwell's equations.**

I. Introduction

ELECTRIC devices made of laminated iron cores or soft magnetic composites (SMC) [1] have multiple scales. Direct computation of quasi-static Maxwell's equations (eddy current problem [2]) is very expensive due to extremely large systems of equations. Homogenization is widely studied [3], [4] and proves a promising approach for multiscale eddy current problems [5], [6].

For lamination or SMC, there is a high parameter contrast between magnetic material and the insulation. A two-scale finite element method is proposed in [7] to deal with eddy current problem in lamination. This method homogenized the vector potential with a micro-shape function by exploiting the structure periodicity. A comprehensive study on eddy current problem based on heterogeneous multiscale method is developed in [8]. This magnetic induction conforming approach works on material with periodic structure and scale separation is necessary. In this article, we propose a general method that can be applied to solve problems involving materials with random electromagnetic properties.

In this paper, we study the extension of localized orthogonal decomposition (LOD) method, introduced in [9] for the Poisson's equation, to the eddy current problem. We decompose the solution of the eddy current problem over lamination or SMC into components from coarse finite element mesh and from the space of rapidly oscillating functions. In LOD, one numerically finds a problem for the coarse solution component without assuming periodicity of the material or scale separation. The challenge in application of LOD to the eddy current problem is in the design of the space of rapidly oscillating functions. As discussed later, direct extension of approach used in [9] does not have desirable properties. Our strategy is to apply the discrete Helmholtz decomposition and to treat gradient fields and their orthocomplement separately. A different construction for the space of rapidly oscillating functions is studied theoretically in [10], without numerical experiments.

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II. Eddy current problem

Consider a simply connected domain $\Omega \subset \mathbb{R}^d, d = 2, 3$ with sufficiently smooth boundary $\partial\Omega$. Denote the outer normal vector as $\mathbf{n}$. The eddy current boundary value problem with vector potential $\mathbf{A}$ in the time domain reads as

$$\begin{aligned} \mathbf{curl}\, \mu^{-1} \mathbf{curl}\, \mathbf{A} + \sigma \frac{\partial \mathbf{A}}{\partial t} &= \mathbf{0} \quad \text{in } \Omega \\ \mathbf{A} \times \mathbf{n} &= \alpha \quad \text{on } \partial\Omega \end{aligned} \tag{1}$$

where α is a given time-dependent function. The material parameters are the magnetic permeability μ, which is nonlinear for iron but is considered piecewise constant in this paper, and the electric conductivity σ, which is zero in air. Unique solution is obtained by choosing σ to be a small positive constant also in the non-conductive parts of Ω, see [11].

Using the backward Euler method to carry out the time integration, the problem: Find $\mathbf{A}^n$ such that

$$\begin{aligned} \mathbf{curl}\, \mu^{-1} \mathbf{curl}\, \mathbf{A}^n + \sigma \frac{\mathbf{A}^n}{\Delta t} &= \sigma \frac{\mathbf{A}^{n-1}}{\Delta t} \quad \text{in } \Omega \\ \mathbf{A}^n \times \mathbf{n} &= \alpha^n \quad \text{on } \partial\Omega \end{aligned}$$

is solved on each time step. Let $\mathbf{A}_D^n$ be an extension of the boundary data α^n at time instance n and denote $\mathbf{A}_0^n = \mathbf{A}^n - \mathbf{A}_D^n$, see [12]. There holds that $\mathbf{n} \times \mathbf{A}_0^n = 0$ on $\partial\Omega$. In the weak form $\mathbf{A}_0^n$ is a solution to: Find $\mathbf{A}_0^n \in H_0(\mathbf{curl})$ such that

$$a(\mathbf{A}_0^n, \mathbf{v}_0) = L_n(\mathbf{v}_0) \quad \forall \mathbf{v}_0 \in H_0(\mathbf{curl}) \tag{2}$$

where $a(\mathbf{u}, \mathbf{v}) = (\mu^{-1} \mathbf{curl}\, \mathbf{u}, \mathbf{curl}\, \mathbf{v}) + (\Delta t)^{-1}(\sigma \mathbf{u}, \mathbf{v})$ and $L_n(\mathbf{v}) = (\Delta t)^{-1}(\sigma \mathbf{A}^{n-1}, \mathbf{v}) - a(\mathbf{A}_D^n, \mathbf{v})$. The task in the following is to find an approximation to $\mathbf{A}_0^n$, when the permeability and conductivity are oscillating rapidly and are non-periodic.

A. Subspace Decomposition in $H(\mathbf{curl})$

Let $\mathcal{T}_H$ and $\mathcal{T}_h$ be two nested triangular partitions of the domain Ω, called coarse and fine grid, respectively. In addition, let $\gamma = h, H$ and $\mathcal{N}_\gamma \subset H_0(\mathbf{curl})$ be the space of lowest-order Nédélec elements related to the partition $\mathcal{T}_\gamma$ with basis functions φ_i^γ and dimension $|\mathcal{N}_\gamma|$.

The aim in the following is to approximately solve the problem: Find $\mathbf{A}_{0,h}^n \in \mathcal{N}_h$ such that

$$a(\mathbf{A}_{0,h}^n, \mathbf{v}_0) = L_n(\mathbf{v}_0) \quad \forall \mathbf{v}_0 \in \mathcal{N}_h. \tag{3}$$

We assume in the following that the extension of the boundary data $\mathbf{A}_D^n \in \mathcal{N}_H$ for every time instance. For notational simplicity, we use $\mathbf{A}_0$ instead of $\mathbf{A}_{0,h}^n$ and denote $\sigma = \sigma(\Delta t)^{-1}$ as well as $L = L_n$, when appropriate.

An approximate solution to (3) is obtained using a decomposition of the solution space as $\mathcal{N}_h = W_f \oplus \mathcal{N}_H$, where W_f is the space of *rapidly-oscillating* functions. Decompose solution $\mathbf{A}_0$ as

$$\mathbf{A}_0 = \mathbf{A}_{0,H} + \mathbf{A}_{0,f} \tag{4}$$

where $\mathbf{A}_{0,H} \in \mathcal{N}_H$ and $\mathbf{A}_{0,f} \in W_f$. To pose a problem only for $\mathbf{A}_{0,H}$, define an a-orthogonal projection operator $P : \mathcal{N}_h \to W_f$: For given $\mathbf{u} \in \mathcal{N}_h$, find $P\mathbf{u} \in W_f$ such that

$$a(P\mathbf{u}, \mathbf{v}) = a(\mathbf{u}, \mathbf{v}) \quad \forall \mathbf{v} \in W_f. \tag{5}$$

The projection operator satisfies the *orthogonality relation* $a((I-P)\mathbf{u}, \mathbf{v}) = 0 \quad \forall \mathbf{v} \in W_f$. Choosing the test function in (3) as $(I-P)\mathbf{v}_H$ for $\mathbf{v}_H \in \mathcal{N}_H$ and applying the orthogonality relation, we obtain a problem for the slow solution component: Find $\mathbf{A}_{0,H} \in \mathcal{N}_H$ such that

$$a(\mathbf{A}_{0,H}, (I-P)\mathbf{v}_H) = L((I-P)\mathbf{v}_H) \quad \forall \mathbf{v}_H \in \mathcal{N}_H. \tag{6}$$

The *full-field* solution can be recovered once $\mathbf{A}_{0,H}$ is known. For this purpose, let $\mathbf{w}^{n-1} \in W_f$ be such that $a(\mathbf{w}^{n-1}, \mathbf{v}) = (\sigma \mathbf{A}_h^{n-1}, \mathbf{v}) \quad \forall \mathbf{v} \in W_f$. With the help of the function $\mathbf{w}^{n-1}$, the load functional can be written as $L_n(\mathbf{v}) = a(\mathbf{w}^{n-1} - \mathbf{A}_D^n, \mathbf{v})$. Choosing the test function in (3) as $\mathbf{v} \in W_f$ gives

$$a(\mathbf{A}_{0,H}^n + \mathbf{A}_{0,f}^n, \mathbf{v}) = a(\mathbf{w}^{n-1} - \mathbf{A}_D^n, \mathbf{v}) \quad \forall \mathbf{v}_f \in W_f.$$

Hence, $\mathbf{A}_{0,f}^n = -P(\mathbf{A}_{0,H}^n + \mathbf{A}_D^n) + P\mathbf{w}^{n-1}$. The full field solution $\mathbf{A}_h^n = \mathbf{A}_D^n + \mathbf{A}_{0,H}^n + \mathbf{A}_{0,f}^n$ is then obtained as $\mathbf{A}_h^n = (I-P)(\mathbf{A}_{0,s}^n + \mathbf{A}_D^n) + P\mathbf{w}^{n-1}$. Motivated by [13], we approximate $\mathbf{A}_h^n \approx (I-P)(\mathbf{A}_{0,s}^n + \mathbf{A}_D^n)$. This expression is used to evaluate the load functional L_{n+1}.

The proposed method is an extension of the local orthogonal decomposition (LOD) method introduced in [9] to solve the Poisson's equation with rapidly oscillating random material parameters. Application of LOD method to the solution of parabolic problems is studied in [13].

To solve problem (6), one has to assembly the corresponding linear system $B_H \mathbf{x}_H = \mathbf{b}_H$, where $B_H \in \mathbb{R}^{|\mathcal{N}_H| \times |\mathcal{N}_H|}$ and $\mathbf{b}_H \in \mathbb{R}^{|\mathcal{N}_H|}$. The entries of B_H and $\mathbf{b}_H$ are obtained as

$$(B_H)_{ij} = a(\varphi_j^H, (I-P)\varphi_i^H) \tag{7}$$

$$(\mathbf{b}_H)_i = L((I-P)\varphi_i^H). \tag{8}$$

The linear system $B_H \mathbf{x}_H = \mathbf{b}_H$ has much smaller dimension in comparison to the original FE-problem.

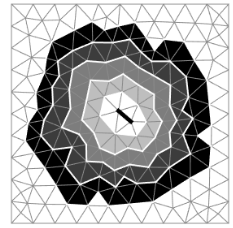

Fig. 1. Sets $\omega_k(e)$ for $k = 1, \ldots, 5$. The edge e is visualized with a black thick line.

Evaluation of $P\varphi_i^H$, required in the assembly of the matrix B_H, using (5) is as expensive as the solution of the original problem. When the subspace W_f is appropriately chosen, the projection operator has *a localization property*, i.e., $P\varphi_H^i$ can be accurately approximated on a local subdomain with diameter kH, where $k \in \mathbb{N}$ is independent on h, but dependent on σ and μ. Such property is proven in [9] for the application of the LOD method to the Poisson's equation.

Define the k-neighborhood of an edge e, denoted by $\omega_k(e)$, as follows: $\omega_0(e) = e$ and for $k > 0$

$$\omega_k(e) := \{\ K_H \in \mathcal{T}_H \mid \text{node of } \omega_{k-1}(e) \text{ is node of } K_H\}.$$

Example of $\omega_k(e)$ is given in Fig. 1. A local approximation of $P\varphi_i^H$, denoted by $P^{loc,k}\varphi_i^H$, is obtained by solving the problem (5) by using only those basis functions that are supported on the set $\omega_k(e)$. Here, e is the edge related to the basis function φ_i^H. The localization property states that the error $\|P\varphi_i^H - P^{loc,k}\varphi_i^H\|_{H(\mathbf{curl})}$ decays exponentially as a function of k.

B. Construction of W_f

The localization property is critical for the computational cost required to approximate $P\varphi_i^H$. A direct extension of the approach used in [9] is to choose

$$W_f := \{\mathbf{u} \in \mathcal{N}_h \mid (\mathbf{u}, \mathbf{v}_H) = 0 \quad \forall v_H \in \mathcal{N}_H\ \}.$$

Based on our numerical experiments, the projection operator P related to this decomposition cannot be approximated locally. Hence, this choice of W_f does not lead to a feasible numerical homogenization method. One possible explanation for this is the large null-space of the $\mathbf{curl}$-operator.

We propose to construct the space W_f by first applying the discrete Helmholtz decomposition. This leads to two orthogonal spaces, null-space of the $\mathbf{curl}$-operator and it's orthocomplement that can be treated separately. Denote the space of first order Lagrange basis functions with homogeneous Dirichlet boundary condition as $\mathcal{V}_\gamma$ and define

$$X_{0,\gamma} := \{\mathbf{u} \in \mathcal{N}_\gamma \mid (\sigma \mathbf{u}, \nabla p) = 0 \quad \forall p \in \mathcal{V}_\gamma\ \}.$$

The spaces $X_{0,\gamma}$ and $\nabla \mathcal{V}_\gamma$ form an orthogonal decomposition of the space $\mathcal{N}_\gamma$ in the $(\sigma \cdot, \cdot)$-inner product. Define the fast space as $W_f = W_{f,0} \oplus \nabla W_{f,\nabla}$, where

$$W_{f,0} := \{\mathbf{u} \in X_{0,h} \mid (\sigma \mathbf{u}, \mathbf{v}_H) = 0 \quad \forall \mathbf{v}_H \in X_{0,H}\ \} \tag{9}$$

$$W_{f,\nabla} := \{p \mid p \in \mathcal{V}_h \text{ and } (p,q) = 0 \quad \forall q \in \mathcal{V}_H\ \}. \tag{10}$$

The corresponding projection operators $P_0 : \mathcal{N}_h \to W_{f,0}$ and $P_\nabla : \mathcal{N}_h \to W_{f,\nabla}$ are defined as

$$\begin{aligned} a(P_0\mathbf{u}, \mathbf{v}) &= a(\mathbf{u}, \mathbf{v}) \quad \forall \mathbf{v} \in W_{f,0} \\ a(\nabla P_\nabla \mathbf{u}, \nabla p) &= a(\mathbf{u}, \nabla p) \quad \forall p \in W_{f,\nabla}. \end{aligned} \tag{11}$$

Using the orthogonality properties of the spaces $X_{0,h}$ and $\nabla\mathcal{V}_h$, the definition of P_∇ reduces to

$$(\sigma\nabla P_\nabla \mathbf{u}, \nabla p) = (\sigma\mathbf{u}, \nabla p) \quad \forall p \in W_{f,\nabla}.$$

The projection operator P_∇ corresponds to one defined for the Poisson's equation in [9]. It is proven that this operator has a localization property. The accuracy of the local approximation depends on the size of the local subproblem as well as the contrast of the coefficient function σ, i.e., the ratio $\sigma_{max}\sigma_{min}^{-1}$. For high-contrast material one has to solve a larger local problem. Some recent improvement for high-contrast coefficients has been presented in [14] by modification of the operator P_∇.

The a-orthogonal projection operator $P : \mathcal{N}_h \to W_f$ is obtained as

$$P = P_0 + \nabla P_\nabla.$$

C. Computation of Projection Operators

The remaining task is to compute $P\varphi_i^H$ by solving (11). As finding a basis for $W_{f,0}$ and $W_{f,\nabla}$ is very costly, $P\varphi_i^H$ is computed by the method of Lagrange multipliers. The function $P_\nabla\varphi_i^H$ is obtained by solving the problem: Find $(\nabla P_\nabla\varphi_i^H, \lambda) \in \mathcal{V}_h \times \mathcal{V}_H$ such that

$$\begin{aligned} (\sigma\nabla P_\nabla\varphi_i^H, \nabla v) + (\lambda, v) &= (\sigma\varphi_i^H, \nabla v) \qquad &\forall v \in \mathcal{V}_h \\ (P_\nabla\varphi_i^H, v_H) &= 0 \qquad &\forall v_H \in \mathcal{V}_H \end{aligned}$$

A detailed discussion on computing P_∇ using Lagrange multiplier approach can be found from [15].

The function $P_0\varphi_i^H$ is computed by solving the problem: Find $(P_0\varphi_i^H, \lambda_1, \lambda_2, \lambda_3) \in \mathcal{N}_h \times \mathcal{V}_h \times \mathcal{N}_H \times \mathcal{V}_H$ such that

$$\begin{aligned} a(P_0\varphi_i^H, v) + (\sigma\nabla\lambda_1, v) + (\sigma\lambda_2, v) &= a(\varphi_i^H, v) \qquad &\forall v \in \mathcal{N}_h \\ (\sigma P_0\varphi_i^H, \nabla p_h) &= 0 \qquad &\forall p_h \in \mathcal{V}_h \\ (\sigma P_0\varphi_i^H, v_H) + (\sigma\nabla\lambda_3, v_H) &= 0 \qquad &\forall v_H \in \mathcal{N}_H \\ (\sigma\lambda_2, \nabla q_H) &= 0 \qquad &\forall q_H \in \mathcal{V}_H \end{aligned}$$

The second and the fourth equation impose that $P\varphi_i^H \in X_{0,h}$ and $\lambda_2 \in X_{0,H}$, respectively. Choosing $v_H \in X_{0,H}$ in the third equation leads to $P\varphi_i^H \in W_{f,0}$. Setting $v_H \in \nabla\mathcal{V}_H$ in the third equation gives $\lambda_3 = 0$. As $\lambda_2 \in X_{0,H}$, choosing $v = v_0 \in X_{0,h}$ in the first equation gives

$$a(P_0\varphi_i^H, v_0) = a(\varphi_i^H, v_0) \quad \forall v_0 \in X_{0,h}.$$

Hence, $P_0\varphi_i^H$ satisfies (11). The Lagrange multiplier λ_1 is the unique solution to: Find $\lambda_1 \in \mathcal{V}_h$.

$$(\sigma\nabla\lambda_1, \nabla p_h) = a(\varphi_i^H, \nabla p_h) \quad \forall p_h \in \nabla\mathcal{V}_h.$$

By these justifications, the above problem has a unique solution. The orthogonality in the definition of $W_{f,0}$ is chosen in the inner product $(\sigma\cdot, \cdot)$ to make the above construction possible.

TABLE I
ON LEFT: THE NUMBER OF DEGREES OF FREEDOM FOR THE COARSE AND FINE SPACE WHEN Ω_c IS COMPOSED OF $n \times n$-CELLS. ON RIGHT: THE NUMBER OF DEGREES OF FREEDOM IN THE LOCALIZED PROBLEM FOR VARYING SIZE OF THE NEIGHBORHOOD FOR DOMAIN WITH 24×24-CELLS.

n	$\lvert\mathcal{N}_H\rvert$	$\lvert\mathcal{N}_h\rvert$
4	280	89740
9	645	202110
14	1160	359480
19	1825	561850
24	2640	809220

k	$\lvert\omega_k\rvert$	$\lvert\omega_k\rvert\lvert\mathcal{N}_h\rvert^{-1}$
2	19026	2 %
3	39388	5 %
4	66974	8 %
5	101784	13 %
6	143818	18 %

D. Implementation

Computing $P_0\varphi_i^H$ and $P_\nabla\varphi_i^H$ requires the assembly of the corresponding matrix equation. Both Lagrange multiplier problems include terms related to finite element spaces on different partitions that require calculation of integrals between basis functions from coarse and fine grid. In [15], such integrals are avoided by constructing a prolongation matrix mapping coarse grid functions to fine grid functions. Due to orientation of the edge element, construction of the prolongation matrix for Nédélec elements is rather complicated. Hence, we compute these integrals directly and avoid the construction of the prolongation matrix.

III. Numerical Examples

In this section, we illustrate the performance of the proposed method in SMC shown in Fig. 2. The computational domain is composed of two subdomains, $\Omega_c = (0,1)^2$ and $\Omega_{air} = (-0.25, 1.25)^2 \setminus \Omega_c$, corresponding to the SMC enclosed in air. The SMC consists of $n \times n$-copies of a scaled unit cell as shown in Fig. 2 together with coarse and fine computational grids, $\mathcal{T}_H$ and $\mathcal{T}_h$. For simplicity, the fine grid for the air region is constructed by copying the same scaled unit cell. Dirichlet boundary condition $\mathbf{n} \times \mathbf{A} = 1$ is imposed on $\partial\Omega$.

First, we demonstrate the localization properties of the operator P_0. For this purpose, a fixed domain with SMC composed of 24×24 cells is used. We study the behavior of $P_0\varphi_i^H$ where the basis function φ_i^H is related to the edge whose center point is closest to $(0.5, 0.5)$. The function $P_0\varphi_i^H$ is approximated using neighborhoods $k = 2, 3, \ldots, 8$. To study the effect of contrast, the artificial conductivity in air, denoted by σ_{air}, is varied between 10 and $1 \cdot 10^{-4}\,\mathrm{S/m}$. The results are presented in Fig. 3. One can observe that the error $\|P_0\varphi_i - P_{\omega,0}\varphi_i\|_{H(\mathbf{curl})}$ decays exponentially as the neighborhood size increases. It indicates that the operator P_0 has a localization property. However, the rate of the decay is dependent on σ_{air}.

Next, the accuracy of the homogenized full-field solution, $\mathbf{u}_{LOD}^k$, obtained using approximate projection computed from a neighborhood of size k is studied. Define the relative error χ in the energy norm,

$$\chi = 100 \cdot \left(\frac{a(\mathbf{u} - \mathbf{u}_{LOD}^k, \mathbf{u} - \mathbf{u}_{LOD}^k)}{a(\mathbf{u}, \mathbf{u})} \right)^{1/2} \% \tag{12}$$

where $\mathbf{u}$ is the exact finite element solution. The error χ is visualized in Fig. 4. This figure depicts that the relative error

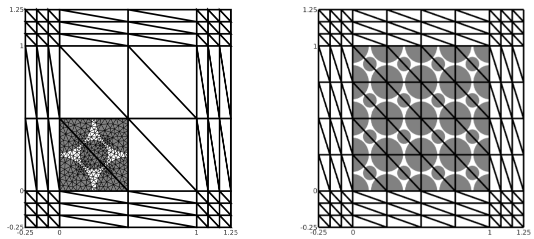

Fig. 2. On left: computational domain with 2×2 cells. Coarse mesh $\mathcal{T}_h$ of the whole domain is drawn together with the fine mesh for the cell $[0, 0.5]^2$. On Right: computational domain with 4×4 SMC conductor. Dark gray denotes conductor with material parameters μ_c, σ_c. Rest of the domain is air with material parameters μ_{air}, σ_{air}.

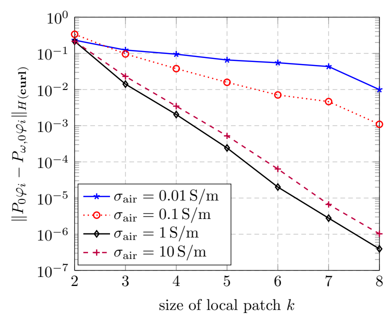

Fig. 3. The error $\|P_0\varphi_i - P_{\omega,0}\varphi_i\|_{H(\mathbf{curl})}$ for $\mu_c = 50$, $\mu_{air} = 1$, $\sigma_c = 100\,\mathrm{S/m}$ and varying σ_{air} as a function of the neighborhood size.

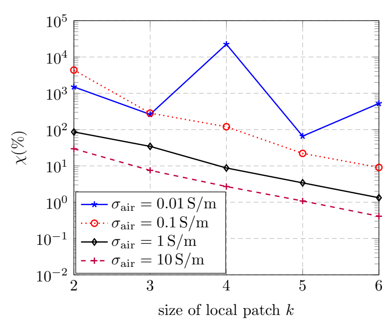

Fig. 4. The relative error χ as a function of the neighborhood size. The same material parameters as in Fig. 3.

χ decays exponentially as the neighborhood size increases, but the decay rate depends on conductivity contrast. It also shows that the error oscillates at $\sigma_{air} = 0.01\,\mathrm{S/m}$, which indicates the projector operator is not sufficiently accurately approximated for given neighbourhood sizes, when the conductivity contrast is high. The application of LOD to Poisson equation with high-contrast material parameter has been studied in [14].

The number of degrees of freedom corresponding to the local problem is given in Table I. It took in average 1.66 seconds to compute $P_0\varphi_i^H$ for $k = 5$. One should note that the size of the local problem remains fixed independent of the number of cells, allowing one in theory to tackle very large problems. However, the size of the matrix B_H is dependent on $|\mathcal{N}_H|$ where as the time required to compute single $P_0\varphi_i^H$ is independent on $|\mathcal{N}_H|$.

IV. Conclusion

A numerical homogenization method for the eddy current problem that allows one to compute an accurate approximation to the full-field solution by using a small number of degrees of freedom is presented. The homogenized problem is obtained by using orthogonal projection operator. A construction of the projection operator, which can be approximated in a local patch, is given. This localization property is numerically verified in SMC material along with the convergence of the full-field solution in the energy norm. The error grows when the contrast in conductivity increases.

The presented approach can be applied to solve three dimensional problems. The main disadvantage of LOD is its high computational cost: time required to compute single $P_0\varphi_i^H$ remains fixed and $|\mathcal{N}_H|$ local problems have to be solved. The computation can be sped up easily by solving the local problems in parallel. This cost is acceptable when several time steps have to be computed, or when the problem is too large to be solved using a single desktop computer.

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