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A Mixed Integer Conic Model for Distribution Expansion Planning: Matheuristic Approach

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***Abstract*—This paper presents a mixed-integer conic programming model (MICP) and a hybrid solution approach based on classical and heuristic optimization techniques, namely matheuristic, to handle long-term distribution systems expansion planning (DSEP) problems. The model considers conventional planning actions as well as sizing and allocation of dispatchable/renewable distributed generation (DG) and energy storage devices (ESD). The existing uncertainties in the behavior of renewable sources and demands are characterized by grouping the historical data via the *k*-means. Since the resulting stochastic MICP is a convex-based formulation, finding the global solution of the problem using a commercial solver is guaranteed while the computational efficiency in simulating the planning problem of medium- or large-scale systems might not be satisfactory. To tackle this issue, the subproblems of the proposed mathematical model are solved iteratively via a specialized optimization technique based on variable neighborhood descent (VND) algorithm. To show the effectiveness of the proposed model and solution technique, the 24-node distribution system is profoundly analyzed, while the applicability of the model is tested on a 182-node distribution system. The results reveal the essential requirement of developing specialized solution techniques for large-scale systems where classical optimization techniques are no longer an alternative to solve such planning problems.**

***Index Terms*—Distribution systems expansion planning, energy storage devices, VND-based matheuristic algorithm, mixed-integer conic programming, stochastic programming.**

I. Introduction

DISTRIBUTION system expansion planning (DSEP) is a classical problem that requires in-depth development of specialized mathematical models and solution techniques to cope with the existing challenges. The challenges are mainly due to technological advances in the areas of generation, power storage, control, automation, and load modeling [1].

Traditionally the DSEP is considered as a static or multi-period deterministic mixed-integer non-linear programming (MINLP) model [2]–[4]. However, such models no longer meet the planning requirements of the current distribution systems. Nowadays, distribution systems consider distributed generation (DG) allocation, and consequently, mixed-integer linear programming (MILP) models in some cases have been proved to be more efficient [2]–[5]. Depending on the size of the system or model types, the DSEP can be an intractable problem. High computational times and memory limitations result in difficulties to solve this problem via classical optimization. In this regard, a relaxed PNL model and heuristic-based techniques have been developed and applied to handle this problem in [6]. Using such algorithms, although do not guarantee the optimal global solution, high-quality solutions are provided while the computational efficiency is higher than the classic optimization techniques [7]–[10].

Nowadays, due to the need of allocating and analyzing the operation of renewable- and non-renewable-based generation sources, considering the existing uncertainties in the behavior of the variables that define the operational state of the network in the DSEP problem is essential. Consequently, more realistic stochastic or robust programming models have been developed to represent the behaviors of the demand and energy sources [11]–[19]. In [11], a multi-period model for optimal allocation of renewable energy sources (RES) and reinforcement of substations was presented while taking into account the system losses. To cope with the uncertainties in demand, wind speed, and solar radiation, a two-stage stochastic MILP model was utilized. A MILP model was proposed in [12] to optimally install RES aiming at determining a set of solutions for the network expansion and later calculating the reliability indexes of each expansion proposal. In [13], a k-means clustering technique was used to consider the uncertainties in demand, RES, and the loading of electric vehicles while an MILP model was used to handle the DSEP problem. A stochastic MILP model was proposed in [14] and [15] to solve the DSEP problem considering the allocation of RES, ESD, and demand response. In order to consider the behavior of the demand and charging of electric vehicles in the DSEP problem, a robust MILP model was proposed in [16]. In [17], a robust chance-constrained programming model was proposed to solve the DSEP problem with uncertainties in demand and RES, using a linearized model. Besides MILP models, convex programming models are among the most efficient approaches to be used for the optimal location of renewable-based DGs in the distribution network [18], [19]. In [18], a conic-based model was proposed to handle the optimal allocation and sizing of photovoltaic (PV) and wind, and dispatchable DG units with the inclusion of batteries. In [19], a conic programming model was

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presented for the optimal simultaneous allocation of wind and gas turbines with the reinforcement of the existing network through the exchange of conductors gauges and the repowering of substations.

Solving the conventional DSEP problems is a critical task due to the model complexity, and considering the uncertainties significantly increases the size of the problem and its complexity. Therefore, finding a solution for large-scale systems via classical optimization techniques is almost impossible or practically unviable, due to the high computational time which is required for finding a high-quality or even a feasible solution. Viable methodologies that can handle the medium- and large-scale DSEP problems with high computational efficiency can be categorized into heuristic-based algorithms and joint heuristic-based and classical techniques [20]–[25]. The authors in [20], [21] proposed an evolutionary PSO to find the optimal investment in new equipment for the network while the uncertainties in demand and energy costs are treated with Monte Carlo simulation (MCS). A two-stage PSO algorithm was proposed in [22] for solving the DSEP problem where at the first stage, the reinforcement of the existing circuits and the allocation of DG were determined, and at the second stage, the demand growth was determined through probability curves and MCS. In [23], a genetic algorithm (GA) was proposed to determine the network reinforcement and the allocation of dispatchable- and wind-based DGs, allowing the system to operate isolated. In this work, the uncertainties in wind speed behavior were considered with Rayleigh probability curves. In [24], the model corresponds with the reinforcement of the existing network, and installation of RES and intelligent meters considering the demand response program and CO_2 reduction was handled via a joint GA and interior point method. A tabu search algorithm was used in [25] to solve the network expansion problem with the installation of wind-based DG while the uncertainties were addressed via a scenario-based stochastic programming model, while the operational state of the network was determined via a conic programming model.

Since an appropriate DSEP model must consider the stochastic nature of the decision variables associated with the planning and operation of the systems, the complexity is an undeniable part of the model and finding a remedy for this issue is a challenging topic. In this sense, having a tool that offers a high-quality solution within a reasonable computational time is essential for planners. Due to the fact that the commercial solvers suffer from solving a highly complicated large-scale mixed-integer-based problems, it is deduced that an important line of research in the area of DSEP is to combine heuristic-based algorithms with classical optimization techniques to handle the practical models more efficiently.

Table I summarizes the main differences between this paper and the state of the art of the DSEP problem considering topological changes and uncertainty. In this table, symbols "✓" and "-" respectively indicate whether a particular aspect has been considered or not. According to the presented state of the art, the contributions of this paper are as follows.

1) Proposing a stochastic multi-period MICP formulation for the DSEP problem that includes distributed generation facility with dispatchable and renewable technologies, energy storage devices (ESD), construction and reinforcement of substations, conductor replacement, and topological changes.
2) Development of a new specialized hybrid metaheuristic-classical (matheuristic) optimization technique to solve the proposed mathematical model. The DSEP problem is approached through the optimization of derived subproblems from the proposed MICP model via commercial solver. The obtained solutions by these subproblems are taken as the neighborhood structures of the VND metaheuristic approach.
3) Proposing the optimal neighborhood structures for the DSEP problem; this structure is a novel framework that is quite beneficial for operation-planning problems.

TABLE I
COMPARISON OF THE DISTRIBUTION SYSTEMS EXPANSION PLANNING APPROACHES WITH TOPOLOGICAL CHANGES AND UNCERTAINTY

Ref.	Mathematical model	Metaheuristic Approach	Matheuristic Approach	RES	ESD	CO_2	Node systems
[12]	MILP	-	-	✓	-	-	54, 86, 138
[13]	MILP	-	-	✓	✓	-	54
[14], [15]	MILP	-	-	✓	✓	-	26
[16]	MILP	-	-	-	-	-	18-54
[17]	MILP	-	-	✓	-	-	24-138
[22]	-	✓	-	-	-	-	9, 72
[23]	-	✓	-	✓	-	-	54, 101
[25]	MICP	✓	-	✓	-	-	24
Proposed approach	MICP	✓	✓	✓	✓	✓	24-182

The rest of this paper is organized as follows. Section II describes the proposed formulation for the DSEP problem including the uncertainty approach and optimization model. The specialized matheuristic solution technique is presented in Section III. Section IV presents the case study and compare the results obtained by a commercial solver and the proposed matheuristic technique. Finally, in Section V, the conclusions are drawn.

II. MATHEMATICAL MODEL

The DSEP problem is formulated through a stochastic optimization model involving both binary and continuous variables and uncertain parameters. The modeling of this problem is accomplished via a scenario-based two-stage stochastic framework. In this two-stage approach, the investment actions are decided at the first stage, while the expected value of the operating costs is calculated at the second stage when considering the realization of the uncertainties [26]. In this way, the DSEP problem is formulated as a multi-period stochastic mixed-integer conic programming model where the objective function determines the investment costs of new equipment, reinforcement of the existing network, and the expected value of the CO_2 emissions and the expected system operating cost over a predefined planning horizon. The model is subject to the sets of technical and operational constraints that must be satisfied for each operational scenario and period of the planning horizon.

The following indexes will be used to represent the mathematical model with the corresponding sets: a for conductor; b for blocks of time; c for scenarios; i for nodes; ij for branches; s for substation nodes; t for periods; e, p, w, g for candidate nodes to install ESD, PV, wind, and dispatchable DG, respectively; Ω_c^b represents the set of scenarios c in time block b; the sets $\Omega_k^{PV,W,GS,ES}$ represent the types of PV, wind, dispatchable DGs, and ESD, respectively; the set Ω_k^{TR} stands for the types of available transformer.

A. Uncertainty Model

Uncertainties in the behavior of RES and demand have a great impact on the operating costs, and consequently, on the network expansion decisions. In the proposed model, the behaviors of these uncertainties are considered through the historical data of electrical demand, wind speed, and solar irradiation that are divided into time blocks b. The *k-means* clustering technique is applied to group the historical data to represent the load and the generation capacity of the RES [18].

B. Objective Function

The set of equations (1)-(8) represents the objective function (OF) of the model and its terms for the dynamic planning in each period $t \in \Omega_t$.

$$\text{minimize } OF = \sum_{t\in\Omega_t} \frac{1}{\tau(1+\tau)^{(t-1)\lambda}} \left(C_t^L + \sum_E C_t^E \right) + \sum_{t\in\Omega_t} \frac{1}{(1+\tau)^{(t-1)\lambda}} \left(C_t^M + OC_t^{SS} + OC_t^{DG} + CT_t + C_t^{LS} \right) \quad (1)$$

$$C_t^L = \sum_{ij} \sum_a y_{ij,a,t}^L C_{a_0,a}^{I,L} \frac{\tau(1+\tau)^{\varsigma_L}}{(1+\tau)^{\varsigma_L} - 1} \quad (2)$$

$$C_t^E = \sum_i \sum_{k\in\Omega_k^E} y_{i,k,t}^E C_k^{I,E} \frac{\tau(1+\tau)^{\varsigma_E}}{(1+\tau)^{\varsigma_E} - 1} \qquad \forall(E) \quad (3)$$

$$C_t^M = F(\tau,\lambda) \sum_E \left(\sum_i \sum_{k\in\Omega_k^E} \sum_{h=1}^{t} C^{M,E} y_{i,k,h}^E \right) \quad (4)$$

$$OC_t^{SS} = F(\tau,\lambda) \sum_c T_c \sum_s C_{c,t}^{SS} P_{s,c,t}^{SS} \quad (5)$$

$$OC_t^{DG} = F(\tau,\lambda) \sum_c T_c \left(\sum_p C^{PV} P_{p,c,t}^{PV} + \sum_w C^W P_{w,c,t}^W + \sum_g C^{GS} P_{g,c,t}^{GS} + \sum_e C^{ESC} P_{e,c,t}^{ESC} + \sum_e C^{ESD} P_{e,c,t}^{ESD} \right) \quad (6)$$

$$CT_t = F(\tau,\lambda) \sum_c T_c C_t^C \left(\sum_s e^{SS} P_{s,c,t}^{SS} + T_c \sum_g e^{GS} P_{g,k,c,t}^{GS} \right) \quad (7)$$

$$C_t^{LS} = F(\tau,\lambda) \sum_c T_c \left(C^{PLS} \sum_i P_{i,c,t}^{LS} + C^{QLS} \sum_i Q_{i,c,t}^{LS} \right) \quad (8)$$

where (2) determines the investment costs in branches, C_t^L, thus, the investment binary decision variable $y_{ij,a,t}^L$ represents the construction/reinforcement of circuit ij changing the conductor type a_0 by a type a, with the investment cost $C_{a_0,a}^{I,L}$, life cycle ς_L, and interest rate τ; similarly, the total investment cost of all the new equipment, C_t^E, is determined by (3), where $E = \{PV, W, GS, ES, SS\}$ is a set of technologies, and the binary decision variable $y_{i,k,t}^E$ stands for the investments associated with the technology E at node i, and type k, with the corresponding investment cost $C_k^{I,E}$ and life cycle ς_E; similarly, maintenance costs C_t^M are presented in (4), where $C^{M,E}$ is the annual maintenance cost for technology E; the cost of energy purchased from the substations, OC_t^{SS}, is determined in (5) as a function of the active power supply by the substations $P_{s,c,t}^{SS}$, the energy cost $C_{c,t}^{SS}$, and the operating time T_c; the operational cost by DG and ESD, OC_t^{DG}, is presented in (6) as a function of the active power injected by PV, $P_{p,c,t}^{PV}$, wind, $P_{w,c,t}^W$, dispatchable, $P_{g,c,t}^{GS}$, charge and discharge of ESD, $P_{e,c,t}^{ESC}$ and $P_{e,c,t}^{ESD}$, respectively with operational costs C^{PV}, C^W, C^{GS}, C^{ESC}, and C^{ESD}; the CO_2 emission taxation, CT_t, is presented in (7) where e^{SS} and e^{GS} are the emission factors by substations and dispatchable DG, respectively, and C_t^C is the carbon tax; (8) stands for the costs related to the load shedding as a function of the active and reactive fictitious power injections, $P_{i,c,t}^{LS}$, and $Q_{i,c,t}^{LS}$, with penalty factors C^{PLS} and C^{QLS}, respectively. The annualized present value of operational is calculated by the function $F(\tau,\lambda) = \left(1 - (1+\tau)^{-\lambda}\right)/\tau$ where the parameter λ represents the number of years in each period.

C. Constraints

The set of equations (9)-(47) represents the economic, physical, and operational constraints for the distribution system. Constraint (9) defines the gauges of conductors that can be allocated in branch ij using the binary decision variable $z_{ij,a,t}^L$, which determines the operational state of this branch, and the existent conductor type is defined by the parameter $z_{ij,a,0}^L$; throughout the horizon planning, constraints (10) and (11) guarantee that only the investment on one conductor and one technology E can be made for each branch and node, respectively; the limit on the investment cost is presented in (12) for each period where parameter $\overline{INV}$ is the investment budget of the expansion project.

$$z_{ij,a,t}^L \le z_{ij,a,0}^L + \sum_{h=1}^{t} y_{ij,a,h}^L \qquad \forall(ij,a,t) \quad (9)$$

$$\sum_t \sum_a y_{ij,a,t}^L \le 1 \qquad \forall(ij) \quad (10)$$

$$\sum_t \sum_{k\in\Omega_k^E} y_{i,k,t}^E \le 1 \qquad \forall(i) \quad (11)$$

$$\sum_{ij} \sum_a y_{ij,a,t}^L C_{a0,a}^{I,L} + \sum_E \sum_i \sum_{k\Omega_k^E} y_{i,k,t}^E C_k^{I,E} \le \overline{INV} \qquad \forall(t) \quad (12)$$

The physical and operational constraints of the DSEP problem are presented by (13)-(47). The active and reactive power flow balances for each node i, scenario c and period t are presented in (13) and (14), respectively, where the variables $P_{ij,c,t}$ and $Q_{ij,c,t}$ are the active and reactive power flow on branch ij, $N(i)$ is the set of nodes connected to the node i by a branch, variables $Q_{i,c,t}^{SS}$, $Q_{i,c,t}^{PV}$, $Q_{i,c,t}^{W}$ and $Q_{i,c,t}^{GS}$ are the reactive power injected by substation, PV, wind, and dispatchable generation, respectively, and the parameters $P_{i,c,t}^{D}$ and $Q_{i,c,t}^{D}$ stand for the active and reactive power demand.

$$\sum_{j\in N(i)} P_{ij,c,t} = P_{i,c,t}^{SS} + P_{i,c,t}^{PV} + P_{i,c,t}^{W} + P_{i,c,t}^{GS} + P_{i,c,t}^{ESD} - P_{i,c,t}^{ESC} + P_{i,c,t}^{LS} - P_{i,c,t}^{D} \tag{13}$$

$$\sum_{j\in N(i)} Q_{ij,c,t} = Q_{i,c,t}^{SS} + Q_{i,c,t}^{PV} + Q_{i,c,t}^{W} + Q_{i,c,t}^{GS} + Q_{i,c,t}^{LS} - Q_{i,c,t}^{D} \quad \forall(i,c,t) \tag{14}$$

The power limits provided by the DG are determined by (15)-(21). The parameters $\overline{P_k^{PV}}$ and $\overline{P_k^{W}}$ stand for the nominal active power limit of PV and wind generators type k, while $\overline{S_k^{W}}$ and $\overline{S_k^{GS}}$ present the nominal apparent power limit of wind and dispatchable DG with capacity k, respectively; the parameters $f_{k,c}^{PV}$, and $f_{k,c}^{W}$ represent the output power level in the scenario c for PV and wind DGs with capacity k, respectively, while the inductive and capacitive power factor angle is determined by the parameters φ_{id}^{PV}, φ_{cp}^{PV}, φ_{id}^{W}, φ_{cp}^{W}, φ_{id}^{GS}, φ_{cp}^{GS} for each generation technology.

$$0 \le P_{p,c,t}^{PV} \le \sum_{h=1}^{t}\sum_{k\in\Omega_k^{PV}} y_{p,k,h}^{PV} f_{k,c}^{PV} \overline{P_k^{PV}} \tag{15}$$

$$-\tan\left(\varphi_{cp}^{PV}\right) P_{p,c,t}^{PV} \le Q_{p,c,t}^{PV} \le \tan\left(\varphi_{id}^{PV}\right) P_{p,c,t}^{PV} \quad \forall(p,c,t) \tag{16}$$

$$\left(P_{w,c,t}^{W}\right)^2 + \left(Q_{w,c,t}^{W}\right)^2 \le \sum_{h=1}^{t}\sum_{k\in\Omega_k^{W}} y_{w,k,h}^{W} \left(\overline{S_k^{W}}\right)^2 \tag{17}$$

$$0 \le P_{w,c,t}^{W} \le \sum_{h=1}^{t}\sum_{k\in\Omega_k^{W}} y_{w,k,h}^{W} f_{k,c}^{W} \overline{P_k^{W}} \tag{18}$$

$$-\tan\left(\varphi_{cp}^{W}\right) P_{w,c,t}^{W} \le Q_{w,c,t}^{W} \le \tan\left(\varphi_{id}^{W}\right) P_{w,c,t}^{W} \quad \forall(w,c,t) \tag{19}$$

$$\left(P_{g,c,t}^{GS}\right)^2 + \left(Q_{g,c,t}^{GS}\right)^2 \le \sum_{h=1}^{t}\sum_{k\in\Omega_k^{G}} y_{g,k,h}^{GS} \left(\overline{S_k^{GS}}\right)^2 \tag{20}$$

$$-\tan\left(\varphi_{cp}^{GS}\right) P_{g,c,t}^{GS} \le Q_{g,c,t}^{GS} \le \tan\left(\varphi_{id}^{GS}\right) P_{g,c,t}^{GS} \quad \forall(g,c,t) \tag{21}$$

The charging and discharging power of the ESD at node e, scenario c and period t are represented by (22) and (23), respectively. The parameter $\overline{P_k^{ES}}$ represents the nominal power limit of ESD type k, and the parameters η_{ESC} and η_{ESD} stand for the charging/discharging efficiencies, respectively. In long-term planning, considering an appropriate estimation of these processes is recommended, thus, for each node e, constraint (24) is used to determine the charging and discharging processes of the installed battery bank during each time block b [18]. To reduce the computational effort, this formulation does not consider binary variables, however, a simultaneous charging/discharging process is not reasonable due to the logic in the power flow balance and the operational costs of these processes in the objective function. Note that a simultaneous charging/discharging process increases the operational costs and may prevent injecting power into the power flow balance equations, and this is the main reason that there exist either the charging or discharging process at a time.

$$0 \le P_{e,c,t}^{ESC} \le \sum_{h=1}^{t}\sum_{k\in\Omega_k^{ES}} y_{e,k,h}^{ES} \overline{P_k^{ES}} \quad \forall(e,c,t) \tag{22}$$

$$0 \le P_{e,c,t}^{ESD} \le \sum_{h=1}^{t}\sum_{k\in\Omega_k^{ES}} y_{e,k,h}^{ES} \overline{P_k^{ES}} \quad \forall(e,c,t) \tag{23}$$

$$\sum_{c\in\Omega_c^{b}} T_c \left(\eta_{ESC} P_{e,c,t}^{ESC} - {}^{1}\!/_{\eta_{ESD}}\, P_{e,c,t}^{ESD}\right) = 0 \quad \forall(e,b,t) \tag{24}$$

The voltage magnitude limit of nodes is presented by (25)-(27), where parameters $\underline{V_i}$ and $\overline{V_i}$ are the minimum and maximum voltage limits at node i, respectively. Constraint (25) stands for the voltage limits through the auxiliary voltage variable $\delta_{i,c,t}$; in (26) and (27), variables $\delta_{i,a,c,t}^{l}$ and $\delta_{j,a,c,t}^{l}$ represent voltages at nodes i and j as a function of the operational state of the branch l determined by the binary variable $z_{ij,a,t}^{L}$.

$$\underline{V_i}^2 \le \sqrt{2}\delta_{i,c,t} \le \overline{V_i}^2 \quad \forall(i,c,t) \tag{25}$$

$$0 \le \sqrt{2}\delta_{i,a,c,t}^{l} \le \overline{V_i}^2 z_{ij,a,t}^{L} \quad \forall(l/ij,a,c,t) \tag{26}$$

$$0 \le \sqrt{2}\delta_{j,a,c,t}^{l} \le \overline{V_i}^2 z_{ij,a,t}^{L} \quad \forall(l/ij,a,c,t) \tag{27}$$

In case that the branch ij is not in operation $\left(z_{ij,a,t}^{L} = 0\right)$, the voltage variables $\delta_{i,a,c,t}^{l}$ and $\delta_{j,a,c,t}^{l}$ are equal to zero, and under this condition, the voltage magnitudes at nodes i and j, are determined by (28) and (29) that are used to determine the value of $\delta_{i,c,t}$ and $\delta_{j,c,t}$ as also to guarantee the feasibility of the problem.

$$0 \le \delta_{i,c,t} - \sum_{a\in\Omega_a} \delta_{i,a,c,t}^{l} \le \frac{\overline{V_i}^2}{\sqrt{2}} \left(1 - \sum_{a\in\Omega_a} z_{ij,a,t}^{L}\right) \tag{28}$$

$$0 \le \delta_{j,c,t} - \sum_{a\in\Omega_a} \delta_{j,a,c,t}^{l} \le \frac{\overline{V_i}^2}{\sqrt{2}} \left(1 - \sum_{a\in\Omega_a} z_{ij,a,t}^{L}\right) \quad \forall(l/ij,c,t) \tag{29}$$

The limit of the power supplied by the substation s is defined in (30) where S_s^0 is the initial capacity of the substation and S_k is the power capacity of the transformer type k for the construction or reinforcement of substations.

$$\left(S_s^0\right)^2 + \sum_{h=1}^{t}\sum_{k\in\Omega_k^{TR}} y_{s,k,h}^{SS}\left(2S_s^0 S_k + S_k^2\right) \ge \left(P_{s,c,t}^{SS}\right)^2 + \left(Q_{s,c,t}^{SS}\right)^2 \quad \forall(s,c,t) \tag{30}$$

The conic constraint (31)-(39) and variables X, Y, and δ are used to calculate the active and reactive power flows represented in (40) and (41) respectively, which are the functions of the admittance and susceptance parameters g_{ij} and b_{ij}, [27], [28]; constraints (32)-(35) impose a reduction in the number of conical rotation constraints (31) through the auxiliary variables $\hat{\delta}^{l}_{i,c,t}$, $\hat{\delta}^{l}_{j,c,t}$, $\hat{X}^{2}_{ij,c,t}$ and $\hat{Y}^{2}_{ij,c,t}$; finally and the variables $X_{ij,a,c,t}$ and $Y_{ij,a,c,t}$ are limited by the product of the voltage limit (36)-(39).

$$2\hat{\delta}^{l}_{i,c,t}\hat{\delta}^{l}_{j,c,t} \geq \hat{X}^{2}_{ij,c,t}+\hat{Y}^{2}_{ij,c,t} \quad (31)$$

$$\hat{\delta}^{l}_{i,c,t} = \sum_{a} \delta^{l}_{i,a,c,t} \quad (32)$$

$$\hat{\delta}^{l}_{j,c,t} = \sum_{a} \delta^{l}_{j,a,c,t} \qquad \forall(l/ij,c,t) \quad (33)$$

$$\hat{X}_{ij,c,t} = \sum_{a} X_{ij,a,c,t} \quad (34)$$

$$\hat{Y}_{ij,c,t} = \sum_{a} Y_{ij,a,c,t} \qquad \forall(ij,c,t) \quad (35)$$

$$X_{ij,a,c,t} = X_{ji,a,c,t}\,, \quad X_{ij,a,c,t} \geq 0 \quad (36)$$

$$Y_{ij,a,c,t} = -Y_{ji,a,c,t} \quad (37)$$

$$X_{ij,a,c,t} \leq \overline{V_i}\,\overline{V_j} k_{l,a,t} \quad (38)$$

$$|Y_{ij,a,c,t}| \leq \overline{V_i}\,\overline{V_j} k_{l,a,t} \qquad \forall(l/ij,a,c,t) \quad (39)$$

$$P_{ij,c,t} = \sum_{a} \left(\sqrt{2}\delta^{l}_{i,a,c,t} g_{ij,a} - X_{ij,a,c,t} g_{ij,a} - Y_{ij,a,c,t} b_{ij,a}\right) \quad (40)$$

$$Q_{ij,c,t} = \sum_{a} \left(-\sqrt{2}\delta^{l}_{i,a,c,t} b_{ij,a} + X_{ij,a,c,t} b_{ij,a} - Y_{ij,a,c,t} g_{ij,a}\right) \quad (41)$$
$$\forall(ij,c,t).$$

The thermal conductor limit is determined in (42) by using the current limit parameter $\overline{I_{ij,a}}$.

$$0 \leq I^{sqr}_{ij,a,c,t} \leq \overline{I_{ij,a}}^{2} z^{L}_{ij,a,t} \qquad \forall(ij,a,c,t) \quad (42)$$

Where the square of the current on the branch ij is determined by the variable $I^{sqr}_{ij,a,c,t}$ in (43)

$$I^{sqr}_{ij,a,c,t} = \sqrt{2}\delta^{l}_{i,a,c,t} g^{2}_{ij} + \left(b_{ij} + \frac{b^{sh}_{ij}}{2}\right)^{2} + \sqrt{2}\delta^{l}_{j,a,c,t}\left(g^{2}_{ij} + b^{2}_{ij}\right) - 2X_{ij,c,t}\left(g^{2}_{ij} + b_{ij}\left(b_{ij} + \frac{b^{sh}_{ij}}{2}\right)\right) + 2Y\left(\frac{g_{ij} b^{sh}_{ij}}{2}\right) \quad (43)$$
$$\forall(l/ij,a,c,t)$$

The set of constraints (44)-(47) is used to guarantee the radial operation of the network in each planning period. The set Ω^{*}_{l} stands for the set of branches with indexing ij and ji, and the sets Ω^{t}_{nc} and Ω^{t}_{np} represent the load nodes and the no load nodes (transfer nodes) in period t.

$$\beta_{ij,t} + \beta_{ji,t} = \sum_{a} z^{L}_{ij,a,t} \qquad \forall(ij,t) \quad (44)$$

$$\sum_{ij\in\Omega^{*}_{l}} \beta_{ij,t} = 1 \qquad \forall(i \in \Omega^{t}_{nc}, t) \quad (45)$$

$$\sum_{ij\in\Omega^{*}_{l}} \beta_{ij,t} \leq 1 \qquad \forall(i \in \Omega^{t}_{np}, t) \quad (46)$$

$$\beta_{sj,t} = 0 \qquad \forall(sj \in \Omega^{*}_{l}, t) \quad (47)$$

This set of constraints uses the substation nodes s as a reference point for the flow direction. Constraint (44) determines the flow direction of the branch ij. Thus, if the binary variable $\beta_{ij,t} = 1$, then in period t there is a connection between nodes i and j, in the direction $j \rightarrow i$, therefore $\beta_{ji,t} = 0$ and this connection is with conductor type a, which is determined by variable $z^{L}_{ij,a,t}$. Constraint (45) keeps all the load nodes connected to the system. Constraint (46) is used to prevent the unnecessary connection of transfer nodes. Finally, constraint (47) avoids the interconnection between substation nodes.

III. Solution Technique

Finding the optimal solution or a high-quality solution for large-scale and complicated problems is a challenging issue. The existing commercial solvers may fail in handling such problems and the existing heuristic-based algorithm cannot guarantee the optimality of the solution. In this regard, a joint optimization method that takes the advantages of both the mathematical programming and metaheuristic algorithms, namely matheuristic approaches, are strong tools to address the aforementioned issues. Since applying the commercial solvers to handle the proposed stochastic MICP model (1)-(47) presents difficulties in convergence and very low computational efficiency, this section proposes a matheuristic approach that contemplates the use of MICP mathematical models and the meta-heuristic VND algorithm to determine the optimal values of the decision variables of the problem. The meta-heuristic VND algorithm is an optimization algorithm that deeply explores the search space to find high-quality solutions via iterative process over a set of neighborhood structures. In this paper, these neighborhood structures are applied to MICP models, derived from (1)-(47) based on the physical and operational characteristics of the electricity distribution networks, see Subsection III-B.

The following subsections present the details of the proposed approach. In A, a MILP model to obtain the initial solution of the algorithm is presented; in B, deriving a set of MICP models for each of the neighborhood structures, as well as a new constructive heuristic algorithm for the DG and ESD sizing-allocation are provided; and finally, C presents the procedure of the proposed VND-matheuristic approach.

A. Initial solution

The initial solution is generated through a MILP model, which is obtained via some relaxations and reductions from the original model (1)-(47). To this end, the power demands of the nodes are represented as their apparent power equivalent without considering the parameters of the lines nor the voltage drops between the nodes. Thus, the power balance equations (13)-(14) are replaced by equation (48), and the power flow equations (40) and (41) are removed from the formulation.

$$\sum_{j\in N(i)} S_{ij,c,t} = S^{SS}_{i,c,t} + S^{PV}_{i,c,t} + S^{W}_{i,c,t} + S^{GS}_{i,c,t} + S^{ESD}_{i,c,t} \quad (48)$$

$-S_{i,c,t}^{ESC}+S_{i,c,t}^{LS}-S_{i,c,t}^{D}$

In a similar way, the power injections of the DG and ESD, (15)-(24), current limits (42)-(43), and power limits in substations (30) are modeled as a function of apparent power. The objective function is reformulated to consider the operational costs as a function of the apparent power injections. The voltage limit constraints (25)-(29) and conic rotation (31)-(39) are also removed from the formulation.

B. Neighborhood structures

The optimal neighborhood structures of the DSEP problem are related to the optimized values of the binary decision variables that define the investment actions in substations, circuits, DG and ESD. These concepts are explained in detail as follows.

1) Network reconfiguration (N_1)

In the radial distribution system, the construction or reclosing of a branch ij in a period t can generate loops. In this case, the set $\boldsymbol{\Gamma}_t^L$ contains the candidate branches to be disconnected from the system in order to maintain the radiality. This set is initially empty ($\boldsymbol{\Gamma}_t^L \leftarrow \emptyset$) and is determined by a *backward sweep* on the set $\boldsymbol{\Gamma}_t^B$ that starts with the node i $\left(\boldsymbol{\Gamma}_t^B \leftarrow i\right)$ connected with the construction/reclosing of the branch ij. Thus, if $\left(\beta_{mn,t}=1 \mid m \in \boldsymbol{\Gamma}_t^B\right)$, then $\left(\boldsymbol{\Gamma}_t^B \leftarrow \boldsymbol{\Gamma}_t^B \cup \{n\}\right)$ and $\left(\boldsymbol{\Gamma}_t^L \leftarrow \boldsymbol{\Gamma}_t^L \cup \{mn\}\right)$. The *sweep* process continues until reaches any substation s, i.e. until $n=s$. This process is repeated taking into account the node j and using the set $\boldsymbol{\Gamma}_t^L$ previously determined. The construction/reclosing branch ij in period t is considered in (49), however, without specifying the conductor type a.

$$\sum_a z_{ij,a,t}^L = 1 \qquad (49)$$

By fixing all variables of the problem at their current value and optimizing the binary variables of radiality $\left(\beta_{ij,t}, \forall(ij \in \boldsymbol{\Gamma}_t^L, t)\right)$, branch investment in all the periods $\left(y_{ij,a,t^*}^L, \forall(ij \in \boldsymbol{\Gamma}_t^L, t^* \in \Omega_t)\right)$, and operating branch $\left(z_{ij,a,t}^L, \forall(ij \in \boldsymbol{\Gamma}_t^L, t)\right)$, the resulting optimization model is solved to determine the type of conductor to be installed in the new circuit ij, and the circuits to be disconnected from the current network.

This structure can be performed independently for each period because the topology may change for each one.

2) Network reconfiguration and conductor resizes (N_2)

This structure is carried out using the following steps in the current solution of the algorithm for each period t.

1. Set all the integer variables of the problem to their current value.
2. Define the set of branches $\boldsymbol{\Gamma}_t^L$ which are not operating or have not been installed yet in period t.
3. For period t, let the radiality and operational branch variables in $\boldsymbol{\Gamma}_t^L$ be optimized. Also, the branch investment variables for all the periods. *i.e.*, $\beta_{mn,t}$ $\forall(mn \in \boldsymbol{\Gamma}_t^L)$; $z_{mn,a,t}^L$ $\forall(mn \in \boldsymbol{\Gamma}_t^L, a)$; y_{mn,a,t^*}^L $\forall(mn \in \boldsymbol{\Gamma}_t^L, a, t^* \in \Omega_t)$;
4. Define an existing branch ij to be reconducted and optimize the operation variable $z_{ij,a,t}^L$ $\forall(a)$;
5. Solve the optimization problem (1)-(47) by fixing the binary variables at their current values while optimizing the investment, radiality, operational, and lines binary variables.

This algorithm is repeated for each period t and all installed branches ij in the system. This criterion determines the optimal conductor size for each branch and reconfiguration system whether the branch ij is disconnected.

3) A constructive heuristic algorithm (CHA) for DG and ESD sizing-allocation (N_3)

A neighborhood structure that determines the proper allocation and sizing of a diverse set of available RES, DG, and ESD is a highly complex problem. This is due to the large differences in the operation of these devices. To this end, a CHA is proposed to design this structure. Starting in the current solution of the DSEP problem, the CHA uses the optimization model (1)-(47) by fixing the binary variables of radiality, construction, and operation of the branches, and construction of substations ($\beta_{ij,t}$, $y_{ij,a,t}^L$, $z_{ij,a,t}^L$, $y_{s,k,t}^{SS}$) to their current values while relaxing the integrality of the variables that define the installation of RES, dispatchable DG and ESD ($y_{p,k,t}^{PV}$, $y_{w,k,t}^{W}$, $y_{g,k,t}^{GS}$, $y_{e,k,t}^{ES}$). The proposed CHA determines the installation of one device at a time, until the stopping criterion is reached. Defining $M=\{PV, W, GS, ES\}$, the installation of generation or storage technology m at node i is modeled by (50).

$$\sum_t \sum_{k \in \Omega_k^M} y_{i,k,t}^m = 1 \qquad (m \in M)\,(50)$$

The CHA consists of the following steps:

1. Set the current solution of the DSEP problem x as a global incumbent solution;
2. Set x' as the CHA's incumbent solution, considering that there is no RES, DG or ESD installed;
3. Relax the integrality of all the $y_{i,k,t}^M$ and select a technology m to be analyzed;
4. Set all the $y_{i,k,t}^M$ that have not been analyzed yet to 0;
5. Solve the resulting conic programming model;
6. Use the relaxed model solution to determine the most appropriate node i and period t based on the maximum value of $y_{i,k,t}^m$ $\forall(t)$;
7. For the technology m, restore the integrality of these variables and add the constraint (50) to the formulation;
8. Solve the resulted MICP and define this solution as a neighboring solution x_m';
9. If all the technologies have already been analyzed, go to step 10, otherwise, choose another technology m and return to step 4;

10. If the best solution $x_M^{'}$ does not improve the CHA's incumbent solution $(x^{'})$ go to step 11, otherwise, set $x^{'} = x_M^{'}$ and return to step 4;
11. If $f(x^{'}) < f(x)$, then set $x = x^{'}$;
12. Report the results.

The final solution provided by the CHA does not depend on the order in which the available generation and storage technologies are analyzed; this is because the best solutions are always chosen after analyzing all possible technologies.

4) Substations reinforcement (N_4)

In this structure, the variables that determine the substations investment $y_{s,k,t}^{SS}\ \forall(s, k \in \Omega_k^{TR}, t)$ become the control variables while all other integer variables of the problem are fixed at their current value. The optimization problem (1)-(47) is solved to provide the optimal substation capacity for the current system topology.

5) Construction of new substations (N_5)

The construction of a new substation s in period t must be carried out with the construction of new circuits and the shutdown of some existing circuits to maintain the radiality of the system for the later periods t^*. The set $\mathbf{\Gamma}_{t^*}^{L}$ that determines the candidate lines to be constructed to connect the loads in substation s is initially empty ($\mathbf{\Gamma}_{t^*}^{L} \leftarrow \emptyset$) and is determined by checking the circuits that can be connected directly to s via $\left(\mathbf{\Gamma}_{t^*}^{L} \leftarrow \mathbf{\Gamma}_{t^*}^{L} \cup \{ij\} \middle| \forall(ij, t^* | i = s \ \vee \ j = s, t^* \geq t)\right)$, subsequently, the auxiliary set $\mathbf{\Gamma}_{t^*}^{B}$ of load nodes, which can be directly connected to this substation, is determined. The set of lines $\mathbf{\Gamma}_{t^*}^{L^*}$ that should be disconnected from the system to maintain the radiality of the network is initially empty ($\mathbf{\Gamma}_{t^*}^{L^*} \leftarrow \emptyset$) and is determined by a *backward sweep* starting at each of the nodes in $\mathbf{\Gamma}_{t^*}^{B}$ checking the indices of the radiality variables. Thus, if $\left(\beta_{mn,t} = 1 \,|m \in \mathbf{\Gamma}_{t^*}^{B}\right)$, then $\left(\mathbf{\Gamma}_{t^*}^{B} \leftarrow \mathbf{\Gamma}_{t^*}^{B} \cup \{n\}\right)$ and $\left(\mathbf{\Gamma}_{t^*}^{L^*} \leftarrow \mathbf{\Gamma}_{t^*}^{L^*} \cup \{mn\}\right)$. For each one of the initial nodes of $\mathbf{\Gamma}_{t^*}^{B}$, the *sweep* process continues until reaches any substation s.

The construction of substation s in period t and the lines for connecting the loads in this substation are considered through the constraints (51) and (52), respectively.

$$\sum_{k \in \Omega_{TR}} y_{s,k,t}^{SS} = 1 \tag{51}$$

$$\sum_{a} z_{ij,a,t}^{L} = 1 \qquad \forall(ij \in \boldsymbol{\Gamma}_{t^*}^{L}) \tag{52}$$

Finally, the MICP model (1)-(47), (51)-(52) is solved by fixing the binary variables in their current values, while the radiality variables $\left(\beta_{ij,t^*}, \forall(ij \in \mathbf{\Gamma}_{t^*}^{L^*}, ij \in \mathbf{\Gamma}_{t^*}^{L}, t^* |\, t^* \geq t)\right)$, the circuit variables $\left(y_{ij,a,t^*}^{L}, z_{ij,a,t^*}^{L} \forall(ij \in \mathbf{\Gamma}_{t^*}^{L^*}, ij \in \mathbf{\Gamma}_{t^*}^{L}, a, t^* \,|t^* \geq t)\right)$, and substation variables $\left(y_{s,k,t^*}^{SS}, \forall(s,\ k \in \Omega_{TR},\ t^* \geq t)\right)$ are optimized.

6) Elimination of substations (N_6)

In the DSEP problem, this structure is valid if the substation s does not exist at the beginning of the planning horizon. In period t, the constraint (53) eliminates the substation s by fixing the substation investment variable to 0 for the previous periods t^*.

$$y_{s,k,t^*}^{SS} = 0 \qquad \forall\left(k \in \Omega_k^{TR}, t^* \middle| t^* \leq t\right) \tag{53}$$

The circuits that are directly connected to s should be removed via (54) and (55), which set the operation and radiality variables of these circuits to zero.

$$y_{ij,a,t^*}^{L} = 0 \qquad \left(ij, a, t^* | (i = s \ \vee \ j = s) \wedge (t^* \leq t)\right) \tag{54}$$

$$\beta_{ij,t^*} = 0 \qquad \forall\left(ij, t^* | (i = s \ \vee \ j = s) \wedge (t^* \leq t)\right) \tag{55}$$

To establish the connection of the nodes that are disconnected from the system with the elimination of substation s, the set $\mathbf{\Gamma}_{t^*}^{L}$ of candidate lines to be constructed or reconnected is initially empty ($\mathbf{\Gamma}_{t^*}^{L} \leftarrow \emptyset$) and is defined by analyzing the radiality variables. Thus, if $\beta_{ij,t^*} = 0$ and $\beta_{ji,t^*} = 0\ \forall(ij, t^* | t^* \leq t)$, then $\left(\mathbf{\Gamma}_{t^*}^{L} \leftarrow \mathbf{\Gamma}_{t^*}^{L} \cup \{ij\}\right)$. The resulting MICP model optimizes the radiality, investment, and operation of circuits variables that belong to the set $\mathbf{\Gamma}_{t^*}^{L}$ while all the other integer variables are fixed at their current value.

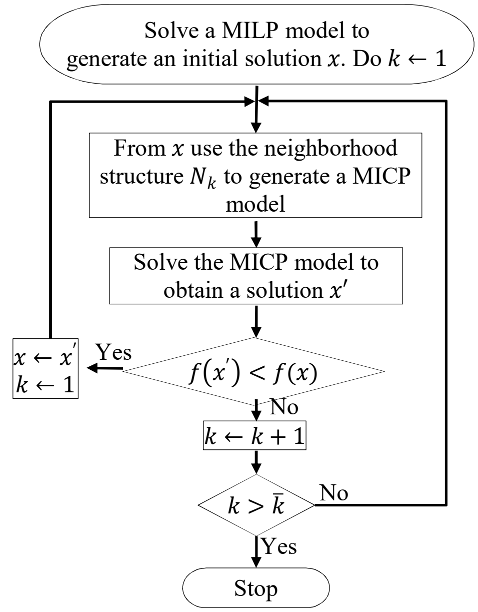

Fig.1. Proposed VND-based matheuristic algorithm

C. Procedure of the VND-based matheuristic technique

To solve the proposed DSEP model, first, the initial solution is obtained by the proposed MILP model in subsection III-A, then the VND algorithm is utilized to the set of neighborhood structures presented in subsection III-B.

In order to explore the search space, the neighborhood structures with the greatest impact on the incumbent solution are used as the last criteria to be analyzed. Thus, defining $\bar{k}$ as the number of neighborhood structures, the order $N_k = \{1 \dots \bar{k}\}$ is established to analyze the neighborhood structures in the VND algorithm as follows: Network reconfiguration; Conductor resizes; DG and ESD sizing-allocation (CHA); Substations reinforcement; Construction of new substations; and Elimination of substations.

TABLE II
CONDUCTOR TYPE OF THE 24-NODE SYSTEM AT DIFFERENT STAGES— CASES A AND B

				Case A						Case B					
				CPLEX			VND-Matheuristic			CPLEX			VND-Matheuristic		
Branch	Node i	Node j	a_0	Stage 1	Stage 2	Stage 3	Stage 1	Stage 2	Stage 3	Stage 1	Stage 2	Stage 3	Stage 1	Stage 2	Stage 3
1	1	5	-	-	-	-	-	2	0	-	2	2	-	-	-
2	1	9	-	-	-	-	-	-	1	1	1	1	-	-	-
3	1	14	-	-	-	2	-	-	-	-	1	1	1	1	0
4	1	21	1	2	2	2	1	0	2	2	2	2	2	2	2
5	2	3	-	-	-	-	-	1	1	-	-	-	1	1	1
6	2	12	-	-	1	1	-	1	1	-	1	1	-	1	1
7	2	21	1	1	1	1	1	2	2	1	1	1	2	2	2
8	3	10	-	-	-	-	-	-	-	1	1	1	-	-	-
9	3	16	-	-	-	-	-	-	-	-	-	-	1	1	1
10	3	23	-	2	2	2	1	0	0	1	1	1	-	-	-
11	4	7	-	-	-	-	-	-	-	-	-	-	-	-	-
12	4	9	-	1	1	1	1	1	0	-	-	-	1	1	1
13	4	15	-	-	-	-	1	1	1	1	0	1	1	1	1
14	4	16	-	1	1	1	-	-	-	1	1	1	-	-	-
15	5	6	1	1	1	1	1	0	1	1	0	0	1	1	1
16	5	24	-	-	-	-	-	2	0	-	-	-	-	-	-
17	6	13	-	1	1	1	-	1	1	-	2	2	-	1	1
18	6	17	-	-	-	-	-	-	-	-	-	-	-	-	-
19	6	22	1	1	1	1	1	1	2	1	1	1	1	1	1
20	7	8	1	-	-	1	-	-	1	-	1	1	1	2	2
21	7	11	-	-	-	-	-	-	-	-	-	-	-	1	1
22	7	19	-	-	-	-	-	-	-	-	-	1	-	-	1
23	7	23	-	2	2	0	1	1	0	2	0	0	-	-	-
24	8	22	1	1	1	1	1	1	1	1	2	2	2	2	2
25	10	16	-	1	1	1	-	-	1	1	1	0	1	1	1
26	10	23	-	2	2	2	1	1	1	-	-	-	-	-	-
27	11	23	-	1	1	1	-	1	1	-	1	1	-	-	-
28	13	20	-	-	-	-	-	-	1	-	1	1	-	1	0
29	14	18	-	-	2	0	-	1	1	-	1	0	1	1	1
30	15	17	-	-	1	1	1	1	1	2	2	2	1	1	1
31	15	19	-	-	-	1	-	-	1	-	-	-	-	-	-
32	17	22	-	-	2	2	2	2	2	-	1	1	1	1	1
33	18	24	-	-	2	2	-	2	2	-	-	1	-	-	2
34	20	24	-	-	-	2	-	-	-	2	0	0	-	-	1

(-) Indicates that the branch has not been installed. (0) Indicates that the branch has been opened.

The proposed VND-based matheuristic approach to solve the MICP model (1)-(47) is illustrated via flowchart, see Fig. 1. In this flowchart, the current solution of the algorithm, x, represents the value of all the integer and continuous variables of the problem; and the $f(x)$ stands for the objective function value. The VND algorithm ends when after analyzing all the proposed neighborhood structures, there is no possibility to improve the incumbent solution of the problem.

IV. TESTS AND RESULTS

The proposed model and solution approach are tested and thoroughly analyzed on the adapted 24-node distribution system [29]. This system operates at 20.0 kV with six existing branches and 28 candidate branches. The system has 20 load nodes attended by 2 existing substations, and there are two candidate nodes for the construction of substations. The planning horizon is 15 years, which is divided into three 5-years stages. Moreover, the scalability of the proposed model and solution technique is tested on a 182-node system. At the first stage of the planning horizon, this system operates one substation at 10kV, with 44 branches to attend 135 load nodes. The projected expansion problem considers the construction of two new substations and 163 branches to attend 180 load nodes at the end of the planning horizon. The planning horizon is 9 years that divided into three 3-year stages. For the 24-Node system, Table II presents the comparison of the conductor gauges installed in the system along the planning horizon for each similar case study, Table III presents the comparison in the cost components of the objective function, and Table IV presents the CO_2 emissions of each case study and solution technique. Detailed results of the cases, as well as the full data for the 24-node and 182-node systems, are provided in [30].

A. Solution via commercial optimization solver

The proposed MICP optimization model (1)-(47) and the VND algorithm are implemented in AMPL [31] and solved with the commercial solver CPLEX 12.8.0 [32]. The numerical experiments were processed on a computer with XEON E5 2650 @2.20GHz processor and 64 GB of RAM. For all the cases, the CPLEX stopping criterion is reaching to 1.0% relative optimality gap or a time limit of 72 hours of processing.

1) 24-Node; case A: without RES

In this case, the solution has been reported after reaching the time limit, 72 hours. The obtained feasible integer solution presents an OF value of M$ 88.500 with an optimality gap of about 1.17%. At the first stage, the expansion plan proposes the installation of a new substation of 12 MVA at node 23, four dispatchable generators of 0.4 MVA at nodes 4, 9, 11 and 13, the construction of four circuits with conductor type 2, and five circuits with conductor type 1. At the second stage, the construction of a new substation of 8 MVA at node 24, three circuits with conductor type 2, and two circuits with conductor type 1 is proposed. Finally, at stage 3, the reinforcement of the two existing substations at nodes 21 and 22 by adding 8 MVA in each of them, the construction of two circuits with conductor type 2, and one circuit with conductor type 1 are proposed.

2) 24-Node; Case B: system with RES and dispatchable DG

In this case, the CPLEX reaches the time limit while finding a feasible integer solution with an OF value of M$ 78.885 and an optimality gap of about 3.04%. At the first stage, the expansion plan proposes the installation of a new substation of 8 MVA at node 23, three wind turbines of 2.0 MVA at nodes 5, 6, and 16, two PV of 1.6 MVA at nodes 7 and 17 and four ESD of 0.5 MVA at nodes 5, 10, 15, and 18; the construction of three circuits with conductor type 2 and six circuits with conductor type 1. At the second stage, the reinforcement of the substation at node 21 by adding 12 MVA, the installation of one dispatchable DG of 0.4 MVA at node 9, one wind turbine of 2.0 MVA at node 14 and two PV of 1.6 MVA at nodes 12 and 20; the construction of three circuits with conductor type 2, and six circuits with conductor type 1 is proposed. At stage 3 the expansion plan proposes the reinforcing of existing substation at node 22 with 12 MVA, constructing a new substation of 8 MVA at node 24, installing three dispatchable DG of 0.4 MVA at nodes 4, 11, 13, and constructing of two circuits with conductor type 1.

3) 182-Node system

For this system, the commercial solver CPLEX could not reach a solution within the time limit of 72 hours. This condition highlights the difficulties that the solver faces in solving the proposed MICP model for large-scale systems.

B. Solution via VND-based matheuristic

In this subsection, the proposed VND-based matheuristic optimization algorithm is implemented under the same conditions as the commercial solver-based approach. Similar case studies are investigated, and results are reported.

1) 24-Node; Case a: without RES

The output of the MILP model provides an optimal solution with the OF of M$ 89.741 within only 157 seconds. Starting at this point, the VND-based matheuristic algorithm provides a solution of M$ 89.072 with a computational time of about 1.87 hours. At the first stage, the expansion plan proposes the installation of a new substation of 8 MVA at node 23, three dispatchable generators of 0.4 MVA at nodes 4, 9, 11, construction of a circuit with conductor type 2 and six circuits with conductor type 1. At the second stage, construction of a new substation of 8 MVA at node 24, four circuits with conductor type 2, and five circuits with conductor type 1 is proposed. At stage 3, the reinforcement of two existing substations at nodes 21 and 22 by adding 8 MVA in each of them, construction of two circuits with conductor type 2, and four circuits with conductor type 1 are considered.

2) 24-Node; Case B: with RES and dispatchable DG

Similar to case *a*, by solving the MILP model within 133 seconds, an initial solution for the optimization process with the OF value of M$ 83.347 is provided. Starting at this point, the VND-based matheuristic algorithm obtains a solution of M$ 78.309 within 10.95 hours. At the first stage, the expansion plan proposes the installation of two dispatchable DG of 0.4 MVA at nodes 4 and 9, four wind turbines of 2.0 MVA at nodes 5, 6, 14 and 16, two PV of 1.6 MVA at nodes 7 and 17, four ESD of 0.5 MVA at nodes 5, 10, 15 and 18, and construction of three circuits with conductor type 2, and eight circuits with conductor type 1. At the second stage, the substation at node 21 is reinforced by adding 8.0 MVA while taking into account the installation of two dispatchable DG of

TABLE III
TOTAL COSTS OF 24-NODE SYSTEM ($$10^6$)

Costs	Technique	Case A	Case B
OF	CPLEX	88.500	78.885
OF	VND	89.072	78.309
Investment	CPLEX	3.457	6.039
Investment	VND	3.385	5.342
Production	CPLEX	74.003	62.678
Production	VND	74.578	62.723
Maintenance	CPLEX	0.061	0.754
Maintenance	VND	0.053	0.855
CO_2	CPLEX	10.979	9.415
CO_2	VND	11.055	9.389

Table IV
TOTAL CO_2 EMISSIONS OF THE 24-NODE SYSTEM (KTON)

CO_2 emissions	Technique	Case A	Case B
DG	CPLEX	41.630	16.000
DG	VND	37.388	21.237
Main grid	CPLEX	1868.704	1636.852
Main grid	VND	1884.198	1630.756
Total	CPLEX	1910.333	1652.852
Total	VND	1921.587	1651.992

0.4 MVA at nodes 11 and 13, and two PV of 1.6 MVA at nodes 12 and 20, construction of one circuit with conductor type 2 and four circuits with conductor type 1. At stage 3, the expansion plan proposes the reinforcement of the existing substation at node 22 with 12 MVA, construction of a substation of 8 MVA at node 24, construction of one circuit with conductor type 2, and two new circuits with conductor type 1.

3) 182-Node system

In this case study, the MILP model provides an initial solution with an OF value of M$ 39.500. Starting at this point, the matheuristic optimization algorithm finds an expansion plan with a total cost of M$ 37.062 within about 27.20 hours. The OF is composed of the investment cost of MUS$ 1.838, the production and maintenance costs of M$ 30.357 and MUS$ 0.212, respectively, and CO_2 emission costs of MUS$ 4.654. The total pollutant emission is 773.148 kTon of CO_2. At the first planning period, the investment plan determines the construction of a new substation of 15 MVA at node 182, installation of two wind turbines of 0.91 MVA at nodes 123 and 133, installation of a dispatchable generator of 0.8 MVA at node 120, two photovoltaic plants of 1.0 MVA at nodes 119 and 135, and two ESD of 0.5 MW at nodes 45 and 56, construction of 5 circuits with conductor type 2 and 112 circuits with conductor type 1. At stage 2, the algorithm proposes the installation of a dispatchable generator of 0.8 MVA at node 44, and construction of 14 branches with conductor type 1. Finally, at stage 3, the construction of a new substation with the capacity of 17 MVA at node 164, and the construction of 6 with branches with conductor type 1 are proposed. The final topology disregards the connection between the transfer nodes 105 and 107. The complete investment plan for this system is shown in Fig. 2, in which

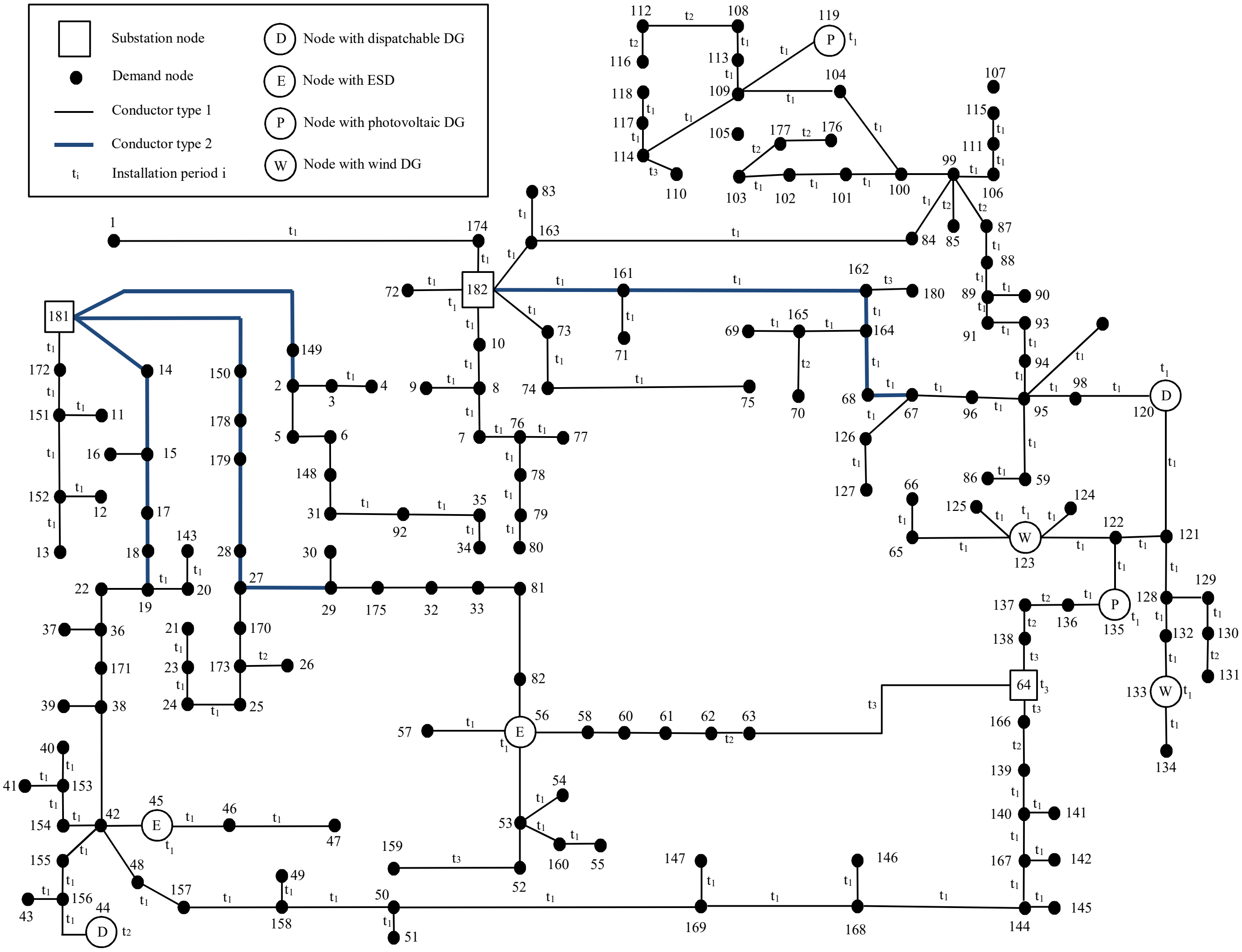

Fig.2. Proposed topology for the 182-node system via VND-based Matheuristic approach

the devices without demonstration of investment period (t_i) belong to the initial network (before the optimization). In all stages, the system has a radial operation, however, since the operational state of the branches has not been depicted in Fig. 2, radiality is not observable in this figure.

V. Discussion

For the *24-Node* system, both the commercial solver and the proposed VND-based matheuristic technique provide a feasible solution for the system; however, the computational efficiency of the solver-based approach is much lower than the proposed VND-based matheuristic algorithm. For the *24-Node* system, in case *a*, about 0.65% improvement in the objective function, via commercial solver, resulted in a CPU time increase for about 70.13 hours. On the other hand, in case *b*, the VND-based matheuristic algorithm provides an investment plan that is MUS$ 0.576 lower than the solution found by CPLEX using the MICP model. On the other hand, as expected, the proper consideration of the RES in the distribution system effectively mitigates the CO_2 emission, although a higher investment cost is incurred.

The optimality gap provided by CPLEX to solve MICP indicates that the obtained solution is still far from the optimal solution of the problem, thus, it is inferred that by providing a stronger PC and within much higher computational time, the CPLEX might find the same or better solution than the proposed matheuristic methodology, while the convergence is yet questionable.

All in all, it can be seen that while for the small-scale system the commercial solver-based model obtained a feasible integer solution, although within a high CPU time, for the large-scale system the commercial solver failed in finding a feasible solution within the predefined acceptable computational resources. However, the proposed matheuristic technique obtains a feasible solution for both test systems and all the case studies, even in case *b* of the 24-node system, the proposed matheuristic algorithm conquered the solver-based approach by finding a better solution with higher computational efficiency. Although the proposed methodology can solve the DSEP problem for several instances, the methodology is bound to the optimization solver efficiency and the available computational resources. Proper modeling and implementation techniques to enhance the adaptability with the commercial solvers can be considered as effective remedies to this issue.

VI. Conclusions

A mixed-integer conic programming (MICP) model has been presented for the problem of multi-period expansion planning of distribution systems with renewable/dispatchable distributed generation and energy storage devices. The MICP model is a convex programming-based model that guarantees finding the global solution via the existing commercial solvers. However, this model shows some issues such as intractability or reaching to a solution with a very low computational efficiency. In this regard, a VND-based matheuristic algorithm has been proposed to propose higher-quality solutions than the solutions obtained by the commercial solvers with a significantly higher computational efficiency. In this paper, the limitations of the solver have been addressed without abandoning the main goal of solving the mathematical model. The proposed approach has been tested on the 24- and 182-node systems considering renewable sources. Results reveal the potential of the proposed approach in solving large-scale stochastic distribution planning problems. This solution technique is extensible for other DSEP models with different objectives and constraints, including electric vehicles, multi-energy components, etc., or even to problems related to distribution systems such as reconfiguration, device allocation/reallocation, etc.

VII. References

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