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Emergent Spacetime and Gravitational Nieh-Yan Anomaly in Chiral $p+ip$ Weyl Superfluids and Superconductors

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Momentum transport is anomalous in chiral $p+ip$ superfluids and superconductors in the presence of textures and superflow. Using the gradient expansion of the semiclassical approximation, we show how gauge and Galilean symmetries induce an emergent curved spacetime with torsion and curvature for the quasirelativistic low-energy Majorana-Weyl quasiparticles. We explicitly show the emergence of the spin connection and curvature, in addition to torsion, using the superfluid hydrodynamics. The background constitutes an emergent quasirelativistic Riemann-Cartan spacetime for the Weyl quasiparticles which satisfy the conservation laws associated with local Lorentz symmetry restricted to the plane of uniaxial anisotropy of the superfluid (or superconductor). Moreover, we show that the anomalous Galilean momentum conservation is a consequence of the gravitational Nieh-Yan (NY) chiral anomaly the Weyl fermions experience on the background geometry. Notably, the NY anomaly coefficient features a nonuniversal ultraviolet cutoff scale Λ, with canonical dimensions of momentum. Comparison of the anomaly equation and the hydrodynamic equations suggests that the value of the cutoff parameter Λ is determined by the normal state Fermi liquid and nonrelativistic uniaxial symmetry of the p-wave superfluid or superconductor.

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Introduction.—Topological phases can be classified in terms of quantum anomalies that are robust to interactions and other perturbations [1–3]. Protected emergent quasirelativistic Fermi excitations coupled to gauge fields and geometry arise as dictated by topology and anomaly inflow [4–6]. In particular, gapless fermions with Weyl spectrum and chiral anomalies are a recent prominent example [7–17]. On the other hand, topological phases and their coupling to geometry (and gravity) is currently a rapidly advancing subject. The well-established results concern the Hall viscosity [18] and chiral central charge [19–21] related to gravitational anomalies [22–24] and thermal transport in quantum Hall systems, topological superfluids (SFs) and superconductors (SCs), as well as semimetals [25–55], the case of SFs and SCs being especially important due to the lack of conserved charge. Any purported topological response of geometrical origin is necessarily more subtle than that based on gauge fields with conserved charges due to the inherent dichotomy between topology and geometry.

Topologically protected Weyl quasiparticles also arise in three-dimensional chiral p-wave SFs and SCs from the gap nodes [5,56] and lead to nonzero normal density close to the Fermi surface zeros, even at zero temperature [57,58]. As any chiral fermion in three dimensions, they suffer from the chiral anomaly in the presence of nontrivial background fields, now as an anomaly where momentum is transferred from the order parameter fluctuations to the quasiparticles [56–62],

$$\partial_t \mathbf{P}_{\text{vac}} + \nabla \cdot \mathbf{\Pi}_{\text{vac}} = -\partial_t \mathbf{P}_{\text{qp}} - \nabla \cdot \mathbf{\Pi}_{\text{qp}} \neq 0, \qquad (1)$$

where $\mathbf{P}_{\text{vac,qp}}$ and $\mathbf{\Pi}_{\text{vac,qp}}$ refer, respectively, to the momentum and stress-tensor of the order parameter vacuum and quasiparticles (qps). Because of textures and superflow, the quasiparticles and -holes flow through the gap nodes transferring net momentum. The total momentum is conserved. The chiral anomaly (1) on vortices [63] has been measured in the early landmark experiment in ^{3}He-A [64]. See the Supplemental Material (SM) [65] for a detailed review of Eq. (1) in terms of SF hydrodynamics.

The anomaly (1) relates to the famous angular momentum "paradox" of the chiral superfluid or superconductor [57,59–61,66,67]: even though each Cooper pair carries net angular momentum $\hbar$, the pairs overlap substantially. In a fixed volume, the overlap is of the order $\sim a^2/\xi^2$ with mean pair separation a and coherence length ξ. This makes the local angular momentum contribution very small, of the order $\sim(\Delta_0/E_F)^2$, instead of (half) the total fermion density, where Δ_0 is the gap and E_F the Fermi energy. At equilibrium, the local variation of angular momentum is still well defined. Similarly, the total linear momentum density is well defined, but it is no longer conserved separately between the condensate and normal component, producing the anomaly [57,58]. This is in contrast to a system of nonoverlapping Bose Cooper pair "molecules" where the anomaly due to Weyl nodes also naturally vanishes [56,57].

Here, we show that the momentum anomaly (1) is a manifestation of the so-called chiral gravitational Nieh-Yan (NY) anomaly [68–74] on emergent spacetime with torsion (and curvature) coupling to the low-energy Weyl fermions. The subtle role of this gravitational (geometric) anomaly term due to torsion has been debated in the literature since its discovery [75]. We show how it arises through the Galilean symmetries and hydrodynamics of the nonrelativistic system with an explicit ultraviolet completion. Therefore, taken at face value, the NY anomaly has been experimentally verified in the late 90's, albeit in the context of emergent condensed matter fermions and spacetime induced by the SF order parameter. Emergent spacetime in two-dimensional topological SCs was recently carefully discussed in Ref. [53]. Related recent work discusses emergent tetrads, gauge fields, and anomaly terms in Weyl SFs [67,76] and semimetals [54,77–81], without the framework of emergent conservation laws and geometry.

Model.—We consider an equal spin pairing p-wave SF at zero temperature on flat Euclidean space in the mean-field (MF) approximation [5,82] (see the SM [65] for more details). We comment below how to extend our results to SCs. The action for the spinless Grassmann fermion $\{\Psi(x), \Psi^\dagger(x')\} = 0$ is ($\hbar = 1$, summation over repeated indices)

$$S[\Psi, \Psi^\dagger, \Delta, \Delta^\dagger] = \int d^4x \Psi^\dagger i\partial_t \Psi - \mathcal{H}_{\mathrm{MF}},$$
$$\mathcal{H}_{\mathrm{MF}} = \epsilon(\Psi, \partial_i \Psi) + \frac{1}{2i}(\Delta^i \Psi^\dagger \partial_i \Psi^\dagger + \Delta^{*i}\Psi\partial_i\Psi). \quad (2)$$

The normal state energy is $\epsilon(\Psi, \partial_i\Psi) = (\partial_i\Psi^\dagger\partial_i\Psi/2m) - \mu_F\Psi^\dagger\Psi$, where m is the constituent mass, $\mu_F = p_F^2/2m$ is the normal state Fermi level. The spinless MF gap amplitude $\Delta^i(x) \propto (1/2i)\langle\Psi\partial_i\Psi\rangle$ is $\Delta^i = (\Delta_0/p_F)(\hat{\mathbf{m}} - i\hat{\mathbf{n}})^i \equiv c_\perp(\hat{e}_1 - i\hat{e}_2)^i$, the unit vector $\hat{e}_3 = \hat{\mathbf{m}} \times \hat{\mathbf{n}} = \hat{\mathbf{l}}$ being the axis of orbital angular momentum of the Cooper pairs. The dynamics of the SF free energy is ignored, and $\Delta(x)$, $\Delta^\dagger(x)$ represent given background fields. We ignore Fermi liquid corrections [82] for simplicity as we expect that these will not affect the arguments which are based on the symmetries, hydrodynamics, and anomalies of the system. For SCs, we perform minimal substitution in $\epsilon(\Psi, \partial_i\Psi)$.

Symmetries and Galilean transformations.—The unbroken continuous symmetries of the normal Fermi liquid state are $\mathrm{U}(1)_N \times \mathbb{R}^3 \times \mathrm{SO}(3)_L$, where the translations and rotations $\mathbb{R}^3 \times \mathrm{SO}(3)_L$ form a subgroup of the Galilean group. The gauge and rotational symmetries are broken to the combined gauge symmetry $\mathrm{U}(1)_{N/2+L_3}$ [83]. In addition, time-reversal symmetry is broken allowing for the emergence of Weyl quasiparticles.

In the SF, the global $\mathrm{U}(1)_N$ gauge symmetry leads to the conservation law $\partial_\mu J^\mu = -\Delta^i\Psi^\dagger\partial_i\Psi^\dagger + \Delta^{*i}\Psi\partial_i\Psi$, where $J^\mu = (\rho, J^i)$ is the normal state fermion current. A Galilean transformation from the SF comoving frame (cmf) is given as $\mathbf{x}' = \mathbf{x} + \mathbf{v}_s t$ and $t' = t$. In terms of SF velocity $\mathbf{v}_s$ and chemical potential μ_m, we transform [82,84]

$$\Psi(x) \to \Psi'(x') = e^{-i[m\mathbf{v}_s\cdot\mathbf{x} + \frac{1}{2}(m\mathbf{v}_s^2 - 2\mu_m)t]}\Psi(\mathbf{x} + \mathbf{v}_s t, t),$$
$$\Delta(x) \to e^{-i[2m\mathbf{v}_s\cdot\mathbf{x} + (m\mathbf{v}_s^2 - 2\mu_m)t]}\Delta(\mathbf{x} + \mathbf{v}_s t, t). \quad (3)$$

The gauge and Galilean transformations are not independent for coordinate dependent transformation parameters. For infinitesimal constant velocity, the action changes in the cmf with $\Psi'(x')$ as $\delta S_{\mathrm{cmf}}[\mathbf{v}_s, \mu_m] = \int d^4x\, m\mathbf{v}_s \cdot J^i - \mu_m\rho + O(\mathbf{v}_s^2)$. Equivalently, $\partial_\mu\Psi \to (\partial_\mu - imv_\mu)\Psi$, where $v_\mu = (-\mu_m/m, \mathbf{v}_s)$. Rotations along $\hat{e}_3$ act as $\Delta^i \to e^{i\varphi}\Delta^i$ and lead to the combined gauge symmetry [83]. The SF velocity is determined as

$$2mv_s^i = -\hat{e}_1 \cdot \partial_i\hat{e}_2 = \partial_i\varphi, \quad 2\mu_m = \hat{e}_1 \cdot \partial_t\hat{e}_2 = -\partial_t\varphi, \quad (4)$$

where $-\varphi$ is the rotation angle. In addition, the SF velocity satisfies the Mermin-Ho relations [56,85]

$$\nabla \times \mathbf{v}_s = -\frac{\kappa}{4\pi}\epsilon^{ijk}\hat{\mathbf{l}}_i\nabla\hat{\mathbf{l}}_j \times \nabla\hat{\mathbf{l}}_k,$$
$$\partial_t\mathbf{v}_s + \nabla\mu_m = -\frac{\kappa}{2\pi}\epsilon^{ijk}\hat{\mathbf{l}}_i\partial_t\hat{\mathbf{l}}_j\nabla\hat{\mathbf{l}}_k. \quad (5)$$

where $\kappa = h/2m$ is the circulation quantum. The translation symmetry corresponds to the energy-momentum tensor conservation law, $S = \int d^4x\mathcal{L}$,

$$\partial_\mu\Pi^\mu_\nu = \partial_\nu\mathcal{L}|_{\mathrm{explicit}}, \quad (6)$$

where $\Pi^\mu_\nu = -(\partial\mathcal{L}/\partial(\partial_\mu\Psi))\partial_\nu\Psi + \mathrm{H.c.} + \mathcal{L}\delta^\mu_\nu$. In particular, this conservation law is broken by the anomaly (1) for $\nu = i$ along $\hat{\mathbf{l}}$. Moreover, Π^i_j is not symmetric in the plane determined by Δ^i, as is well known in ^{3}He-A [53,56,82]. See the SM [65] for more details on the symmetries of the model.

Linearized comoving quasiparticle action.—We define the Bogoliubov transformation $\Psi(x) = \sum_s u_s(x)a_s + v_s^*(x)a_s^\dagger$, where s is a generalized index in quasiparticle (particle-hole) space with $\{a_s^\dagger, a_s'\} = \delta_{ss'}$. The Bogoliubov-de Gennes (BDG) quasiparticles form a spinless two-component Grassman Nambu spinor $\Phi(x) \sim \sum_s (\,u_s(x)v_s(x)\,)^T a_s$, with the Lagrangian

$$\mathcal{L}_{\mathrm{BDG}} = \Phi^\dagger(x)\begin{pmatrix} i\partial_t - \epsilon(-i\partial_i) & \frac{1}{2}\{\Delta^i, i\partial_i\} \\ \frac{1}{2}\{\Delta^{*i}, i\partial_i\} & i\partial_t + \epsilon(i\partial_i) \end{pmatrix}\Phi(x). \quad (7)$$

If $(u_s v_s)$ is a solution to the equation of motion with energy ε_s, $(v_{-s}^* u_{-s}^*)$ satisfies the particle-hole conjugate solution $\varepsilon_{-s} = -\varepsilon_s$. This implies the Majorana relation $\tau^1\Phi^\dagger \sim \Phi$. For a homogenous state, the dispersion vanishes on the

Fermi surface at $\mathbf{k} = \pm p_F \hat{\mathbf{l}}$. The same BDG action applies for SCs with the replacements $\partial_t \to \partial_t - iA_0\tau^3$, $\tau_3\epsilon(\mp i\partial_i) \to \tau^3\epsilon[\mp (i\partial_i + A_i)]$, where the signs are according to eigenstates of τ^3 and $v_\mu \to v_\mu - A_\mu$, where A_μ is the electromagnetic gauge potential.

Now, we consider the low-energy fermions in the presence of slowly varying order parameter texture and Galilean transformation parameters $\mathbf{v}_s(x), \mu_m(x)$ in the semiclassical approximation, see, e.g., [86]. We assume that the BDG fermions with gap $\gtrsim mc_\perp^2$ have been integrated out and restrict ourselves to the linear expansion close to the Weyl nodes $\pm p_F \hat{\mathbf{l}}$. The dispersion is

$$\Phi^\dagger(x)\epsilon(-i\partial)\Phi(x) \approx \sum_\pm \tilde{\chi}_\pm^\dagger(x)\left\{\frac{v_F}{2}\hat{\mathbf{l}}^i(x), -i\partial_i\right\}\tilde{\chi}_\pm(x), \quad (8)$$

and we neglect terms of order $O(\partial^2\tilde{\chi})$, where $\Phi(x) = \sum_\pm e^{\pm i p_F \int^{\mathbf{x}} \hat{\mathbf{l}}(\mathbf{x}')\cdot d\mathbf{x}'}\tilde{\chi}_\pm(x)$ and $\tilde{\chi}_\pm(x)$ is slowly varying compared to p_F. The order parameter vectors form the spatial part of an inverse tetrad $e_a^{\ i} = \{(c_\perp/c_\parallel)\hat{\mathbf{m}}, (c_\perp/c_\parallel)\hat{\mathbf{n}}, \hat{\mathbf{l}}\}$ with uniaxial symmetry. Note that, after linearization of $\epsilon(-i\partial)$, we perform in (8) the transformation $\Phi \to \tilde{\chi}$, i.e., $\{e_3^i, (-i\partial_i - \mathbf{e}^3 p_F)_i\} \to \{e_3^i, \partial_i\}$ corresponding to a chiral rotation in momentum space due to the node. This is precisely the anomalous chiral symmetry in the system and produces the quasirelativistic anomaly (18) below, proportional to $\Lambda \propto p_F$ in the path-integral representation [72,73,87].

Further, in the presence of SF velocity $v_\mu = \partial_\mu \varphi = 2m(\mu_m/m, -\mathbf{v}_s)$, the fermions transform in the comoving frame as $\tilde{\chi} \to e^{-i\varphi\tau^3/2}\tilde{\chi}$, $e_1^i - ie_2^i \to e^{-i\varphi}(e_1 - ie_2)^i$ where $\varphi(x)$ is slowly varying compared to p_F. The derivative operator transforms to $\tau^a i e_a^\mu \partial_\mu \to \tau^a i e_a^\mu(\partial_\mu - \frac{i}{2}\partial_\mu\varphi\tau^3)$. These coincide with local spin-1/2 Lorentz transformations in the 12 plane, and we can attempt to associate the linearized action to a nontrivial spacetime. Denoting the Pauli matrices $\tau^a, \bar{\tau}^a = (1, \pm\tau^i)$ in Nambu space, velocities $c_\parallel = v_F = p_F/m$ and $c_\perp = (\Delta_0/p_F)$, and $\partial_\mu = (\partial_t, c_\parallel \partial_i)$, the linearized action close to the node $+p_F\hat{\mathbf{l}}$, written in explicitly Hermitian form, becomes

$$S[\tilde{\chi}^\dagger, \tilde{\chi}, \Delta, \mu_m, \mathbf{v}_s] = \int d^4x \frac{1}{2}(\tilde{\chi}^\dagger i\partial_t\tilde{\chi} - i\partial_t\tilde{\chi}^\dagger\tilde{\chi}) + (\mu_F - \mu_m)\tilde{\chi}^\dagger\tau^3\tilde{\chi} - \frac{p_F\hat{\mathbf{l}}^i}{2}\tilde{\chi}^\dagger v_F\hat{\mathbf{l}}^i\tau^3\tilde{\chi}$$
$$+ \frac{v_F\hat{\mathbf{l}}^i}{2}(\tilde{\chi}^\dagger\tau^3 i\partial_i\tilde{\chi} - i\partial_i\tilde{\chi}^\dagger\tau^3\tilde{\chi}) - \frac{1}{2}[\tilde{\chi}^\dagger(e_1^i\tau^1 i\partial_i + e_2^i\tau^2 i\partial_i)\tilde{\chi} + \text{H.c.}] + p_F\hat{\mathbf{l}}\cdot\mathbf{v}_s\tilde{\chi}^\dagger\tilde{\chi} - \frac{v_s^i}{2}(\tilde{\chi}^\dagger i\partial_i\tilde{\chi} - i\partial_i\tilde{\chi}^\dagger\tilde{\chi}). \quad (9)$$

Emergent Riemann-Cartan spacetime.—The action (9) is (after a rotation of τ^a in the 12 plane) equivalent to a relativistic chiral (right-handed) Majorana-Weyl fermion on Riemann-Cartan spacetime [69,88,89], see the SM [65] for a review of Rieman-Cartan spacetimes and our conventions on relativistic fermions,

$$S_{\text{Weyl}}[\chi, \chi^\dagger, e, \omega] = \frac{1}{2}\int e d^4x \chi^\dagger \tau^a e_a^\mu i\vec{D}_\mu \chi + \text{H.c.}, \quad (10)$$

with the identifications $\chi = e^{-1/2}\tilde{\chi}$, $e = \det e_\mu^a = (c_\parallel^2/c_\perp^2)$,

$$e_0^\mu = (1, -\mathbf{v}_s), \qquad e_1^\mu = \frac{c_\perp}{c_\parallel}(0, \hat{\mathbf{m}}),$$
$$e_2^\mu = \frac{c_\perp}{c_\parallel}(0, \hat{\mathbf{n}}), \qquad e_3^\mu = (0, \hat{\mathbf{l}}), \quad (11)$$

and $\omega_\mu^{12} = 2mv_{s\mu} = 2m(-\mu_m/m, \mathbf{v}_s^i) = \partial_\mu\varphi$. The covariant derivate is $\vec{D}_\mu = \partial_\mu - (i/4)\omega_\mu^{ab}\sigma_{ab}$, $\sigma^{ab} = (i/2)[\bar{\tau}^a, \tau^b]$. Note that the Galilean invariance leads to both the shift $e_0^i = -v_s^i$, although it is $O(\partial^2)$ in the action, and the spin connection term $e_3^i\omega_i^{12}\tilde{\chi}^\dagger\tilde{\chi} \propto p_F\hat{\mathbf{l}}\cdot\mathbf{v}_s$ in Eq. (9). We also emphasize that we explicitly retained the momentum density $p_F^2\hat{\mathbf{l}}^2/2m\tilde{\chi}\tau^3\tilde{\chi}$ in the action, which cancels with the Fermi level $\mu_F\tilde{\chi}\tau^3\tilde{\chi}$, since this term represents nonzero physical Galilean momentum present at the Weyl node, see Eq. (16) below.

Equations (10), (11), and the spin-connection ω_μ^{12} define a Weyl fermion on an emergent Riemann-Cartan spacetime [69,88], with the uniaxial metric $g^{\mu\nu} = e_a^\mu e_b^\nu \eta^{ab}$, $e_\mu^a e_a^\nu = \delta_\nu^\mu$, and $\nabla_\mu e_\nu^a \equiv \partial_\mu e_\nu^a + \omega_{\mu b}^a e_\nu^b - \Gamma_{\mu\nu}^\lambda e_\lambda^a = 0$. The nonzero torsion and curvature tensors are given by $T_{\mu\nu}^a = \partial_\mu e_\nu^a - \partial_\nu e_\mu^a + \omega_{\mu b}^a e_\nu^b - \omega_{\nu b}^a e_\mu^b$ and $R_{\mu\nu}^{12} = \partial_\mu\omega_\nu^{12} - \partial_\nu\omega_\mu^{12} = 2m(\partial_\mu v_{s\nu} - \partial_\nu v_{s\mu})$. Although seemingly pure gauge, the latter is, in general, nonzero by the Mermin-Ho relations Eq. (5). This correspondence is the first major result of the Letter. See the SM [65] for a glossary of Riemann-Cartan spacetimes [88,89].

Conservation laws on curved spacetime.—Now, we formulate the SF conservation laws in terms of the Weyl fermions coupled to emergent spacetime [38,53,69]. The current (ρ, J^i) in terms of the Weyl quasiparticles is $(\tilde{\chi}^\dagger\tau^3\tilde{\chi}, p_F\hat{\mathbf{l}}^i\tilde{\chi}^\dagger\tilde{\chi} - \frac{1}{2}[\tilde{\chi}^\dagger i\partial_i\tilde{\chi} - i\partial_i\tilde{\chi}^\dagger\tilde{\chi}])$. To first order in gradients, $\partial_\mu J^\mu$ is equal to $\frac{1}{2}(\tilde{\chi}^\dagger[e_1^i\tau^1 i\partial_i + e_2^i\tau^2 i\partial_i]\tilde{\chi} + \text{H.c.})$. On the other hand, this becomes (assuming only $\omega_\mu^{12} \neq 0$ and $e = \text{const}$)

$$e\partial_\mu S^\mu_{12} = -eT_{12} + eT_{21}, \tag{12}$$

in terms of the Weyl fermions [53]. Here, the currents

$$T^a{}_\mu = \frac{1}{e}\frac{\delta S_W}{\delta e^\mu_a} = \frac{1}{2}\chi^\dagger\tau^a i\vec{D}_\mu\chi - \frac{1}{2}i\chi^\dagger\overleftarrow{D}_\mu\bar{\tau}^a\chi$$
$$= \frac{1}{2}(\chi^\dagger\tau^a i\partial_\mu\chi - i\partial_\mu\chi^\dagger\tau^a\chi) + \frac{1}{8}\omega^{bc}_\mu\chi^\dagger\{\tau^a,\sigma_{bc}\}\chi, \tag{13}$$

$$S^\mu_{ab} = 2\frac{1}{e}\frac{\delta S_W}{\delta\omega^{ab}_\mu} = \frac{1}{4}e^\mu_c\chi^\dagger\{\tau^c,\sigma^{ab}\}\chi, \tag{14}$$

are derived from the relativistic Weyl action Eq. (10). In particular $S^\mu_{12} = \frac{1}{4}\chi^\dagger e^\mu_a\{\tau^a,\tau^3\}\chi$ and $T^a{}_b = e^\mu_b T^a_\mu$. The relativistic conservation law Eq. (12) follows from the local Lorentz symmetries

$$\delta\chi = -\frac{i}{4}\Lambda_{ab}\sigma^{ab}\chi, \qquad \delta e^a_\mu = \Lambda^a_b e^b_\mu,$$
$$\delta\omega^{ab}_\mu = \Lambda^a{}_c\omega^{cb}_\mu + \omega^{ac}_\mu\Lambda^b{}_c - \partial_\mu\Lambda^{ab}, \tag{15}$$

which, when restricted to the 12 plane, coincide with the Galilean transformations. Indeed, the energy momentum tensor Π^μ_ν was not symmetric either in this plane, the linearization of which is equal to, see the SM [65],

$$\Pi^\nu_\mu = (\Pi^{(1)} + \Pi^{(2)})^\nu_\mu = p_F\hat{\mathbf{l}}^i ej^\nu\delta_{i\mu} - ee^\nu_a T^a_\mu + e\omega^{12}_\mu S^\nu_{12}. \tag{16}$$

The Galilean term $\Pi^{(1)}$ proportional to $ej^\nu = \tilde{\chi}^\dagger e^\nu_a\tau^a\tilde{\chi}$ arises due to the finite momentum density $+p_F\hat{\mathbf{l}}$ at the node and, therefore, contributes to energy momentum. The corresponding relativistic conservation law related to Π^μ_ν of the linearized Weyl action follows from spacetime diffeomorphisms and leads to

$$\partial_\mu\left(-e^\mu_a T^a_\nu + \frac{1}{2}\omega^{ab}_\nu S^\mu_{ab}\right) = \partial_\nu e^\mu_a T^a_\mu + \frac{1}{2}\partial_\nu\omega^{ab}_\mu S^\mu_{ab}, \tag{17}$$

i.e., $\partial_\mu\Pi^{(2)\mu}_\nu = \partial_\nu\mathcal{L}$. The field theory conservation equation for the energy-momentum Eq. (6) is, then, equivalent to Eq. (17) and the conservation of the quasiparticle current density $ej^\mu = (\tilde{\chi}^\dagger\tilde{\chi}, \tilde{\chi}^\dagger[-\mathbf{v}^i_s + v_F\hat{\mathbf{l}}^i\tau^3]\tilde{\chi})$ at the node [up to subleading terms $O(\partial/p_F)$, see, e.g., [58,60]]. Although the Weyl action (10) implies the classical conservation law $\partial_\mu j^\mu = 0$, this suffers from the chiral anomaly at the quantum level due to the emergent spacetime (11).

Nieh-Yan anomaly.—Adding both chiralities $\pm p_F\hat{\mathbf{l}}$, the conservation law for momentum is broken since, in spite of Eq. (17), $\partial_\mu j^\mu_5 = \partial_\mu(j^\mu_+ - j^\mu_-) \neq 0$ at the quantum level, i.e., the conservation law suffers from the axial anomaly (however, the Weyl qp number is conserved $\partial_\mu\sum_\pm j^\mu_\pm = 0$) and leads to the observed momentum nonconservation Eq. (1) in the system. The gravitational NY anomaly is [34,68,70,72], for a chiral pair of Weyl fermions, with $e^a = e^a_\mu dx^\mu$,

$$\partial_\mu(ej^\mu_5 d^4x) = \frac{\Lambda^2}{4\pi^2}(T^a\wedge T_a - e^a\wedge e^b\wedge R_{ab}), \tag{18}$$

where the higher order term $O(R^2)$ is neglected. For the $p+ip$ SF, the anomalous chiral Weyl action is (10) with spacetime defined by Eq. (11). Note that, although the quasiparticles $\tilde{\chi}$ are Majorana-Weyl contributing one-half of Eq. (18) per node, a factor of 2 comes from accounting for spin degeneracy. The temporal torsion $T^0 = 0$, and we compute the spatial contribution,

$$T^1\wedge T^1 + T^2\wedge T^2$$
$$= 2\left(\frac{c_\parallel}{c_\perp}\right)^2\epsilon^{0ijk}[(\hat{\mathbf{l}}\cdot\mathbf{v}_s)\partial_i\hat{\mathbf{l}}_a - \partial_i\mathbf{v}_{sa}]\hat{\mathbf{l}}_j\partial_k\hat{\mathbf{l}}_a d^4x$$
$$= 2\left(\frac{c_\parallel}{c_\perp}\right)^2\epsilon^{0ijk}\hat{\mathbf{l}}_i\partial_j\mathbf{v}_s\cdot\partial_k\hat{\mathbf{l}}d^4x \approx 0 + O(\partial^3),$$

where $\epsilon^{0xyz} = -1$ and

$$T^3\wedge T^3 = \{2\epsilon^{0ijk}[\partial_t\hat{\mathbf{l}}_i - \partial_i(\mathbf{v}_s\cdot\hat{\mathbf{l}})]\partial_j\hat{\mathbf{l}}_k\}d^4x$$
$$= 2\epsilon^{0ijk}\{[\partial_t\hat{\mathbf{l}}_i - (\mathbf{v}_s\cdot\nabla)\hat{\mathbf{l}}_i]\partial_j\hat{\mathbf{l}}_k - \hat{\mathbf{l}}_a\partial_i\mathbf{v}_{sa}\partial_j\hat{\mathbf{l}}_k\}d^4x$$
$$\approx 2\epsilon^{0ijk}\{[\partial_t\hat{\mathbf{l}}_i - (\mathbf{v}_s\cdot\nabla)\hat{\mathbf{l}}_i]\partial_j\hat{\mathbf{l}}_k\}d^4x + O(\partial^3). \tag{19}$$

The curvature term $-e_a\wedge e_b\wedge R^{ab}$ is

$$-\frac{4\pi}{\kappa}\left(\frac{c_\parallel}{c_\perp}\right)^2\epsilon^{0ijk}\{\hat{\mathbf{m}}_i\hat{\mathbf{n}}_j(\partial_0 v_{sk} - \partial_k v_{s0})$$
$$+ [(\hat{\mathbf{m}}\cdot\mathbf{v}_s)\hat{\mathbf{n}}_i - (\hat{\mathbf{n}}\cdot\mathbf{v}_s)\hat{\mathbf{m}}_i](\partial_j\mathbf{v}_{sk})\}d^4x$$
$$= \left(\frac{c_\parallel}{c_\perp}\right)^2\epsilon^{0ijk}[2\hat{\mathbf{m}}_i\hat{\mathbf{n}}_j(\hat{\mathbf{l}}\cdot\partial_t\hat{\mathbf{l}}\times\partial_k\hat{\mathbf{l}})$$
$$+ (\hat{\mathbf{l}}\times\mathbf{v}_s)_i(\hat{\mathbf{l}}\cdot\partial_j\hat{\mathbf{l}}\times\partial_k\hat{\mathbf{l}})]d^4x. \tag{20}$$

To lowest order in gradients, we arrive to

$$e\partial_\mu j^\mu_5 = \frac{\Lambda^2}{2\pi^2}e\left(1 - \frac{c_\perp^2}{c_\parallel^2}\right)\epsilon^{0ijk}[\partial_t\hat{\mathbf{l}}_i - (\mathbf{v}_s\cdot\nabla)\hat{\mathbf{l}}_i]\partial_j\hat{\mathbf{l}}_k, \tag{21}$$

where $e = (c_\parallel/c_\perp)^2$. Matching the expression with the hydrodynamic anomaly [56,57,62] in the SM [65], the anisotropic cutoff is $\Lambda = (c_\perp/c_\parallel)p_F$ and applies in a Weyl SF with the nodes at $\pm p_F\hat{\mathbf{l}}$, such as ^{3}He-A, or in a Weyl SC after minimal substitution [53,62]. The expression is a Galilean invariant and the coefficient is proportional to the weak-coupling normal state density [without the logarithm $\ln(E_F/\Delta_0)$ due to the neglected gapped fermions [57]]. This the central result of the Letter. The NY anomaly equation can also be derived with simple arguments using

Landau levels and spectral flow in the case of a torsional magnetic field $T^3_{\mu\nu}$ [9,11,34,58]. In general, the dimensional coefficient Λ is seen simply to follow from the fact that torsion couples to momentum and that the density of states of the anomalous chiral lowest Landau level branches is momentum dependent. Lorentz invariance would require that the Weyl nodes are symmetrically at $p^\mu = 0$ which leads to $\Lambda = 0$ at the node. For chiral Weyl nodes with a nonzero separation $2p^\mu$ in momentum space, the coefficient of the torsion anomaly is $\Lambda \propto |p|$ according to the spectral flow calculation.

In condensed matter systems, however, the Weyl description of the quasiparticles and the chiral anomaly breaks down at some cutoff scale. This is in contrast to fundamental Weyl fermions, where the conventional chiral anomalies satisfy IR-UV independence: the anomaly is the same at each energy scale since it can be computed by comparing to a theory with no anomaly simply by adding a high-energy chiral fermion that cancels the anomaly of the original theory [23]. On the other hand, for ^{3}He-A the UV completion is fully known in terms of the Fermi-liquid theory and the anomalous SF hydrodynamics of ^{3}He-A [5]. In the idealized p-wave BCS pairing model (2), the cutoff energy scale $E_{\rm IR} = \Delta$ for the SF is determined from the MF gap equation $c_\perp \sim (E_{\rm UV}/p_F)e^{-m_3 n/g} \sim (\Delta/p_F)$, where g is the δ-function interaction coupling constant and $E_{\rm UV} \sim v_F p_F = c_\parallel p_F$ the normal state Fermi energy. The linear quasirelativistic Weyl regime emerges when $E \ll E_{\rm W} = mc_\perp^2 = (c_\perp/c_\parallel)\Delta$. Therefore, the uniaxial anisotropy is simply the relative scale $(c_\perp/c_\parallel) = (E_{\rm IR}/E_{\rm UV})$, while the linear Weyl regime is suppressed by an additional factor of $c_\perp/c_\parallel$ compared to $E_{\rm IR}$ leading to the value of $\Lambda = (c_\perp/c_\parallel)p_F$. In ^{3}He-A, $c_\perp/c_\parallel$ is of the order 10^{-3} [82]. Remarkably, the hydrodynamic anomaly (1) is the same as in Eq. (18) when all states beyond the linear quasirelativistic Weyl approximation are taken into account.

Outlook.—We have revisited the anomalous momentum transport in chiral p-wave SFs and SCs in terms of a consistent hydrodynamic and low-energy effective theory description. Using the gauge and Galilean symmetries of the system, we have shown how the quasirelativistic Weyl approximation, emergent spacetime, and symmetries appear in the semiclassical derivative expansion. The anomalous transport is a consequence of the axial gravitational NY anomaly due to the chiral Weyl fermions on an emergent Riemann-Cartan spacetime with torsion.

Here, we have shown that the emergent spacetime formulation satisfies all the symmetries and conservation laws of the effective field theory required for the gravitational NY anomaly and saturates the nonzero value from SF hydrodynamics [64]. The early papers [9–12] treat the anomaly in terms of a momentum space axial gauge field; this follows from our formalism via the substitution of the tetrad $e^i_3 = \hat{\mathbf{z}} + \delta e^i_3$, formally equivalent to gauge field $\sim p_F \delta\hat{\mathbf{l}}_\mu$ in the Hamiltonian [34,54,90]. In contrast, Refs. [67,76] inconsistently consider both contributions independently. Moreover, the emergent gauge field does not correspond to physical symmetry in the system and p_F is an explicit UV scale. This is distinct from the emergent spacetime (11), which, in addition, is valid for arbitrary (semiclassical) textures and superflow, or considerations of other Weyl systems, where the emergent tetrads or gauge fields (e.g., elastic deformations, Fermi velocities, node separation) are independent [54,77,78]. On the other hand, Ref. [54] sets the NY cutoff to the lattice scale and neglects the breakdown of the linear Weyl spectrum.

Interestingly, for the emergent spacetime in Eq. (11), the anomaly coefficient seems to vanish in the relativistic case $c_\perp = c_\parallel$, but this is probably an artefact of the breakdown of the (weak-coupling) BCS model. Our findings corroborate the subtle interplay of broken Lorentz invariance, anisotropic dispersion, renormalization, and the NY-anomaly coefficient Λ and should be verified by detailed field theory computations [34,53,55,72,91,92]. Similarly, the relation of the emergent quasirelativistic (or uniaxial) spacetime to Newton-Cartan geometries should be clarified [84,93–96]. We did not consider the dynamics of the SF order parameter or Goldstone modes, orbital nonanalycity [5,56,97], nor derive the Wess-Zumino consistency equation and action for the chiral NY anomaly. This will be a gravitational Chern-Simons term for the tetrad and spin connection [16,98–100]. Likewise, we did not consider singular vortices, which will lead to additional curvature and zero modes, as well as the Iordanskii force and gravitational Aharonov-Bohm phase [5] for the quasiparticles. The connection of emergent spacetime and thermal transport should be explored [40,76]. In particular, it is possible that the UV scale Λ is supplemented by the IR temperature scale in the anomaly, which can be universal [101]. These, and other considerations extending previous results in the literature, will be left for the future.

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