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Effect of positional errors on the accuracy of multi-probe roundness measurement methods $\stackrel{\text{\tiny{themax}}}{\longrightarrow}$

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ABSTRACT

Multi-probe roundness measurement methods can be used to measure cross section roundness profiles and dynamic behaviour of large flexible rotors such as paper machine rolls. Other roundness measurement methods are not suitable for such measurements, since the rotors are too large to be measured on precision spindles and the center point of the measured profile can move in an unpredictable and unrepeatable way during the measurement. Multi-probe roundness measurement methods can, to a limited extent, separate the center point movement (commonly also called error motion) and the roundness profile of a cross section of a rotating workpiece. This study compares the effect of positional errors and center point movement on the accuracy of three different multi-probe roundness measurement methods. The research included quantification of the effects of probe noise, positional errors and center point movement on the accuracy of the roundness profiles produced by the different methods. A novel method for generating continuous random center point movement is presented. Signals of a rotating workpiece with center point motion were simulated, and following GUM handbook supplement 1 guidelines, the Monte Carlo method was used to obtain distributions for the harmonic components of the methods. Distributions for different errors and their effects on roundness parameters are presented separately for each roundness measurement method.

The results of this research only show minor differences in lower order harmonic components between the methods. The results also show that diameter sampling is immune to horizontal positional errors although it is particularly sensitive to angular positional errors of the probes, which lead to more errors accumulating in the phases, not the amplitudes of the harmonic components. With higher order harmonic components, the obtained error distributions were observed to correspond well with theoretical error propagation rates for harmonic sensitivity.

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1. Introduction

Roundness is an important geometric property in many fields of engineering. Improving roundness of a machine part can have a direct effect on the quality of the end product in several fields of industry, such as paper making and the rolling of metals. Roundness and relevant parameters are defined in ISO standards 12181-1 and 12181-2 [1,2].

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Nomenc	lature
$\Delta r(\theta)$	Diameter variation profile
Ŧ	Fourier transform
μ	Mean
ϕ_n	Angle of the nth probe
σ	Standard deviation
θ	Workpiece angle of rotation
$m(\theta)$	Linear combination of probe signals
Ν	Total number of samples
п	Index number of sample
$N(\mu, \sigma^2)$	Normal distribution
r(heta)	Roundness profile
r_k	k ^{ui} complex Fourier coefficient of the roundness profile
$S_n(\theta)$	Reading of the <i>n</i> th probe
S_{nk}	k^{un} complex Fourier coefficient of the n^{th} probe signal
$\boldsymbol{x}(\theta)$	Profile center point <i>x</i> coordinate
x_k	k th complex Fourier coefficient of the <i>x</i> coordinate of the workpiece center point
$\boldsymbol{y}(\theta)$	Profile center point y coordinate
y_k	k th complex Fourier coefficient of the <i>y</i> coordinate of the workpiece center point
LS	Least squares method
LSCI	Least squares reference circle
PDF	Probability distribution function
RD	Redundant diameter method
RONT	Peak-to-valley roundness deviation
UPR	Undulations per revolution

Several methods exist for separating the rotating center point motion and the roundness profile of a rotating workpiece. An obvious solution is to use a high precision spindle with a negligible error. Such spindles are typically found in roundness measurement machines. If the spindle error motion is repeatable between revolutions, reversal/turn methods [3,4] can be used. However, precision spindle measurements, multi orientation methods, or reversal methods are not suitable for large flexible rotors too large to be placed onto precision roundness measurement machines.

When other factors such as random loads or bending stiffness variation contribute to the error motion or a large flexible rotor, the orientation of the rotor on its bearings will not determine the center point movement, which will be unrepeatable and unpredictable. In these situations when the workpiece center point motion is not repeatable between revolutions, multiprobe roundness measurement methods have to be used to determine the roundness profile accurately (application in Fig. 1). This leads to the main research question of the current study: how well do multi-probe roundness measurement methods separate the random center point movement and roundness profile from each other, and are there major differences between the methods?

In multi-probe roundness measurement, the expected probe signals from a rotating roundness profile can be constructed as a group of equations (one equation for each probe n) as follows:

$$S_n(\theta) = r(\theta + \phi_n) + x(\theta) \cdot \cos \phi_n + y(\theta) \cdot \sin \phi_n$$

This formulation of the equations is called the limaçon approximation, in which it is assumed that the center point of the part remains near the center point of the measurement system [6,7]. When the equations are formulated like this, there are three unknown variables: two coordinates for the position of the center point and one for the radius of the workpiece. Thus, signals from at least three probes are required to form a determined system of equations.

The roundness profile of a profile can be presented as a Fourier series, with the first component representing a small eccentric movement. The profile can be presented in polar coordinates (Fig. 4), the radius is suppressed in such presentations and displayed is the roundness deviation as a function of the rotational angle.

Multiple sources have discussed solving the roundness profile from probe signal equations as formulated in Eq. 1. Ozono [8] presented a solution by summing the signals with a weighted linear combination based on a geometrical connection that results in the first harmonic component not showing in the combination. The equations have also been solved without using the Fourier transform by Chen [9]. Replacing displacement probes with angle probes has also been demonstrated in [10,11].

When more than three probes are used, the system of equations will be overdetermined. To solve the overdetermined system, one option is to make a least squares estimate to determine the coefficients using all probe signals [12]. The equations have been formulated in matrix form by Jansen [11], also demonstrating the possibility of using angle probes and a least squares estimate to combine more than three probe signal equations.

(1)



Fig. 1. A multi-probe measurement device setup with four laser displacement sensors on a paper machine roll. Laser sensors are used for dynamic measurements. For more accurate roundness measurement in manufacturing, tactile sensors are used.

An alternative approach to solving an overdetermined system are redundant methods, which combine multiple probe signals by calculating the harmonic components separately from different combinations of the probes, the combination selected for each harmonic component based on an estimate of smallest sensitivity to errors [13,14]. In the hybrid redundant diameter four-point method, the four probe signals are merged by calculating odd component coefficients from three probes, and even component coefficients are from diameter sampling.

A major limitation of multi probe methods is harmonic suppression, which has been analyzed widely. Harmonic suppression occurs when a wavelength in the roundness profile is close to a multiple of the probe angles, which is also the explanation to why diameter sampling cannot be used to detect odd harmonic components of a profile. Whitehouse [15] presented equations for three probes and analyzed harmonic sensitivity of different angle combinations. Kato [5] optimized probe angles based on a performance index. Attempts at optimizing the probe angles have also been made by Cappa [16] and Hale [17].

In addition to harmonic suppression, discrete sampling introduces limitations to multi-probe roundness measurement. Chen [9] notes that when the probe angles are not exact multiples of the sampling interval, a rounding error will be introduced, but it can be considered negligible if the number of samples is sufficiently large. The maximum number of harmonics is also limited by the sampling frequency. A suitable sampling can be determined based on sampling theory (Nyquist frequency). The number of samples needs to be more than twice than the wanted largest harmonic number [7,18].

A similar approach has been taken in previous studies [19,20] which have assessed error distributions in the phase and amplitude of harmonic components of the redundant diameter four-point method. However, these studies have failed to consider that positional errors combined with the movement of the center point have a dynamic effect on the measurement setup, which will have an effect on the roundness profile components; the results obtained in this research differ significantly from the previous studies. Additionally, a comprehensive analysis comparing the error sensitivity of the different roundness measurement methods has not been published before.

The aim of this research is twofold. Firstly, the aim is to investigate the effect of error positions and center point movement on the accuracy of three different multi-probe roundness measurement methods. Secondly, this research aims to establish a consistent method for comparing multi-probe roundness measurement methods.

2. Methods

The research was conducted by obtaining estimates of uncertainty for different parameters in the measurement setup, following the principles presented in the GUM handbook [21] and GUM handbook supplement 1 [22]. The GUM handbook

describes a framework for expressing uncertainty in measurement. The GUM supplement 1 describes an approach for obtaining uncertainty values with Monte Carlo simulations by selecting typical realistic distributions for errors based on available prior information. The GUM Monte Carlo simulation method was selected for the research, since it is suitable for quantifying the accuracy of non-linear models, such as the ones studied in this case.

The probe signals were generated by simulating a rotating reference profile and displacement probes. The reference profile and its harmonic content is presented in Fig. 4. Previous results by Viitala et al. [19] suggest that the error distributions for the methods will be equal regardless of the harmonic content of the profile.

2.1. The roundness measurement methods

This section presents the different roundness measurement methods assessed in this research. The equations for multiprobe roundness measurement were presented in the introduction and the calculation of the roundness profile in each method is briefly covered.

2.1.1. Three-point method

The three-point method solution first presented by Ozono [8] can be used to separate the roundness profile and the motion of the center points in a rotating workpiece. The method is based on a trigonometric relationship and weighted linear combination of the three probe signals, which can be used to eliminate the profile center point motion from the linear combination.

Given three probe equations:

$$S_{1}(\theta) = r(\theta + \phi_{1}) + x(\theta) \cdot \cos \phi_{1} + y(\theta) \cdot \sin \phi_{1}$$

$$S_{2}(\theta) = r(\theta + \phi_{2}) + x(\theta) \cdot \cos \phi_{2} + y(\theta) \cdot \sin \phi_{2}$$

$$S_{3}(\theta) = r(\theta + \phi_{3}) + x(\theta) \cdot \cos \phi_{3} + y(\theta) \cdot \sin \phi_{3}$$
(2)

Which are linearly combined with weighting factors *a* and *b*:

$$m(\theta) = S_1(\theta) + a \cdot S_2(\theta) + b \cdot S_3(\theta) \tag{3}$$

When *a* and *b* are selected to satisfy the constraints:

$$\sin \phi_1 + a \sin \phi_2 + b \sin \phi_3 = 0$$

$$\cos \phi_1 + a \cos \phi_2 + b \cos \phi_3 = 0$$
(4)

It has been shown [5,8] that with the right weighting factors, the first component (eccentric movement) of the Fourier transform of the linear combination is eliminated and *m* will contain only components from the second term onwards.

2.1.2. Least squares four-point method

When more than three probes are used, there will be more than three equations. Combining more than three equations is possible by making a least squares estimate of the solution [11]. The advanced spindle runout-roundness separation method presented by Jansen [11] demonstrated a matrix formulation of multi-probe roundness measurement which allows the use of an arbitrary number of displacement and angle probes to calculate the roundness profile Fourier components, using a least squares minimization when more than three probes are used.

Jansen [11] starts from the same equations and after a Fourier transform the equations can be presented in matrix form. An equation is formulated for each of the probe signal Fourier components (S_{nk}), which can be directly calculated from the probe signal. The calculation of coefficients in the least squares method is as follows:

Let **S** be a matrix containing the complex Fourier coefficient S_{nk} of the probe signal, **H** a matrix describing the probe configuration and **r** a matrix containing complex Fourier coefficients of the center point movement coordinates (x_k, y_k) and the roundness profile Fourier coefficient r_k . As a whole, the equations can be written in the form $\mathbf{S} = \mathbf{H} \cdot \mathbf{r}$

For three probes, the results yielded from the solution $\mathbf{r} = \mathbf{H}^{-1} \cdot \mathbf{S}$ are equal to the three point method. If there are four or more probes, a least squares estimate for \mathbf{r} can be made: $\hat{\mathbf{r}} = (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \cdot \mathbf{S}$.

2.1.3. Redundant diameter four-point method

The redundant diameter four-point method (also called the hybrid four-point method [19,20]) combines diameter measurement (which cannot detect odd components and is suitable for calculation of even Fourier components [7]) and the three-point roundness measurement method (which is used for odd components) [20]. The diameter variation profile of the workpiece can be calculated from two probes placed opposite to each other:

$$\Delta r(\theta) = \frac{1}{2} (S_1(\theta) + S_4(\theta)) \tag{6}$$

The even harmonic components can then directly be calculated by taking the Fourier transform of diameter variation profile. As stated in literature [7, p. 229], the diameter variation profile can only be used to transfer even components and it cannot detect odd harmonic components. It can be shown, that a curve consisting of only odd harmonic components will have a constant width [23].

Odd components are calculated as in the three point method and the roundness profile Fourier components are then merged in frequency domain from the three-point method coefficients calculated from S_1 , S_2 , and S_3 and diameter measurement coefficients calculated from S_1 and S_4 as shown in Fig. 3.

A redundant three point method could be used to obtain the roundness profile by combining the components in frequency domain from components calculated from different three probe combinations from a total set of four or more probes, by selecting for each harmonic the combination with the best harmonic transfer properties [13]. In the configuration used in this research, when there are four probes and S_1 and S_4 are placed opposite to each other (Fig. 2), all available three probe combinations will have the same harmonic sensitivity. In other words, multiple valid combinations for a redundant threepoint method are not available with these probe angles.

2.2. Simulation research

This section describes the simulation research and in detail the application of the selected positional errors. 100 000 cases of probe signals measured from a rotating reference profile were simulated for each error separately and all errors together. Each case was distorted with random parameters from realistic distributions selected based on the GUM handbook supplement 1 guidelines. The resulting distributions in roundness parameters were obtained by calculating the roundness profile with each of the roundness measurement methods. Distributions for roundness parameter RONt and harmonic component amplitudes and phases were calculated based on the results.

The reference profile was simulated as a circular polygon constructed from 1024 points with the roundness profile shown in Fig. 4 imposed on it. The probe signal generation was simulated as follows for each sample:

1. Probe angular errors were applied by determining new random probe positions by rotating the probes around the center point of the measurement frame origin

2. Positional errors and center point movement were applied by calculating a combined translation matrix with fixed positional errors and center point movement. Center point movement was applied separately for each sample with the number of center point movement points corresponding to the number of samples.

3. The profile was rotated by the angle corresponding to the sampling interval $(\frac{2\pi}{N} \operatorname{rad})$

4. Four probe signals were obtained by calculating the distance between the probe and the intersection of the profile polygon boundary, along lines drawn from the probes to the center of the measurement frame (Fig. 6



Fig. 2. The roundness profile $\mathbf{r}(\theta)$ and center point movement coordinates $\mathbf{x}(\theta)$, $\mathbf{y}(\theta)$ and can be constructed from at least three displacement probe signals \mathbf{S}_n and their respective probe angles ϕ_n . The probe angles used in this research are the same as in an existing laboratory roundness measurement frame (Fig. 1) and are based on a numerical optimization done by Kato in 1991 [5].



Combine roundness profile in frequency domain

Fig. 3. Schematic diagram of the redundant diameter four-point method. Three-point coefficients are used for odd Fourier components and diameter measurement sampling for even Fourier components. The coefficients are merged in frequency domain.



Fig. 4. The roundness profile of the reference profile. The reference profile has a diameter of 500 mm distorted with a roundness profile consisting of harmonic components from 2 to 100 with an amplitude of 10 μm. The phases of the reference profile were selected by numerical optimization to minimize the total roundness of the profile, which resulted in a total roundness of 235.074 μm.

A sampling rate N = 1024 samples per round was used in this research. In practical roundness measurements of large flexible rotors, rotary encoders are used to trigger readings from the probe signals. In practice, the rotary encoder is attached to the workpiece center point and moves with the rotor, therefore the rotation of the profile was performed before translation in the simulation.

Previous research [19] has identified several error sources common in multi-probe roundness measurement. For the relevant positional errors, similar distributions were used in this research. In total, three different positional errors, center point movement and probe scale error were introduced to the system in the simulations. The errors and their selected distributions are presented in Table 1. The positional errors were introduced to the simulation by translating and rotating the profile or the probes. The probe scale error was introduced by adding a randomly sampled valued in each probe sample.

The probe angles used in the research were 0° , 38° , 67° and 180° . The angles were based on an optimization by Kato [5] and are the same ones as used in an existing measurement frame (Fig. 2).

2.2.1. Center point movement

Linearly decreasing amplitudes were selected for the research to resemble the behaviour of an actual flexible rotor. A decreasing trend of center point harmonic component amplitudes has been measured by Viitala [24] in a large flexible rotor, although in practice the components can largely depend on excitations present in the system and can be arbitrary.

A method for generating center point movement resembling flexible rotor center point movement is presented. The amplitudes of the harmonic components of the center point movement in both directions were set to decrease linearly from

Table 1

Positional errors and their distributions used in the research.

Error source	PDF	Parameters	Parameters	
		μ	σ	
1. Probe scale error	$N(\mu, \sigma^2)$	0 µm	0.3 μm	
2. Center point movement (c)	$N(\mu, \sigma^2)$	100 µm	30 µm	
3. Angular position error of the probes	$N(\mu, \sigma^2)$	0°	0.25°	
4. Vertical (y) position error of the frame	$N(\mu, \sigma^2)$	0 mm	0.35 mm	
5. Horizontal (x) position error of the frame	$N(\mu, \sigma^2)$	0 mm	0.25 mm	

c to 0 between components from 2 to 6 (Fig. 5 a). The phases were randomized and an inverse Fourier transform was used to obtain x and y coordinates of the center point in each sampling point (Fig. 5 b). In the simulation, the center point movement was applied by translating each point of the profile based on these coordinates. A new random center point path was generated for each case.

2.2.2. Probe scale error

A realistic probe scale error achievable with accurate tactile probes was selected (Table 1). The scale error of the probe was applied separately for every sample of each sensor.

2.2.3. Angular position error

A schematic picture of the angular position error of the probes is shown in Fig. 6 b. Each probe was estimated to have an angular error distribution with zero mean and 0.25°standard deviation. The angular position error was applied to the sensor positions by rotating them around the sensor frame center point by the angle error.

Vertical and horizontal position error of the frame The vertical and horizontal position errors were applied by translating all of the profile points with the error value in each sampling angle. This corresponds to having the whole measurement frame misaligned by a fixed distance.

3. Results

The simulation was performed and six datasets of 100 000 samples were obtained. A set was generated separately for each error source (Table 1). Finally, a set of further 100 000 cases were simulated with all of the errors enabled. The RONt value was used as a metric for the convergence of the Monte Carlo simulation (Fig. 8 and 100 000 cases were estimated to be sufficient for the simulation results to converge. If only components under 30 or 16 were considered, convergence was much faster and 10 000 rounds would have been deemed sufficient. The slower convergence when considering the whole range of harmonics is due to larger errors in the higher order components, especially in the least squares four-point method, which produced very large errors for some harmonic components in some of the cases.



(a) Harmonic components of center (b) One example of generated random point movement center point movement with randomly selected phases and c

Fig. 5. Ratios of harmonic component amplitudes for center point x and y direction movement (a) and generated center point coordinates with randomized phases (b).



(a) Profile center point movement



(b) Probe angle error



(c) Horizontal position error of frame (d) Vertical position error of frame

Fig. 6. Positional errors used in the research. A smaller horizontal error (c) than vertical error (d) was selected since in practice S_1 and S_4 can be used to accurately position the frame horizontally.



(a) Histogram of second harmonic (b) Histogram of RONt from dataset component from dataset with all errors with all errors included, bin spacing 0.1 µm µm

Fig. 7. Distributions of the outputs of the roundness measurement methods. The amplitudes of the harmonic components were assumed to be normally distributed (Figs. 10–15).

The distributions of the outputs were examined from the dataset simulated with all errors. For the harmonic components, the outputs of the roundness measurement me were assumed to be normally distributed (Fig. 7a).

The obtained error distributions for amplitude and phase are presented in Figs. 10–14 for each error separately and in Fig. 15 for a dataset simulated with all of the error sources together. The figures show the deviation of the calculated result



Fig. 8. Convergence of the Monte Carlo simulation with mean RONt of N randomly selected samples from dataset simulated with all errors. Based on the convergence of the simulation, 100 000 simulated cases for each error type were considered to be enough.



Fig. 9. Theoretical error propagation rates for the probe angles used by Kato et al. [5]. In three probe measurement, harmonic suppression will occur when the harmonic wavelength is a multiple of the probe angles.



Fig. 10. Simulated results of 100 000 cases with only probe scale error ($d \sim N(\mu, \sigma^2)\mu = 0\mu m\sigma = 0.3\mu m$) showing the difference between the actual value and calculated value (the amplitudes of all harmonic components of the reference profile are 10 μm).



Fig. 11. Simulated results of 100 000 cases with only center point movement error ($c \sim N(\mu, \sigma^2)\mu = 100\mu m\sigma = 30\mu m$) showing the difference between the actual value and calculated value (the amplitudes of all harmonic components of the reference profile are 10 μ m).



Fig. 12. Simulated results of 100 000 cases with only probe angle error ($\alpha \sim N(\mu, \sigma^2)\mu = 0\sigma = 0.3$) showing the difference between the actual value and calculated value (the amplitudes of all harmonic components of the reference profile are 10 µm).



Fig. 13. Simulated results of 100 000 cases with only vertical error position ($y \sim N(\mu, \sigma^2)\mu = 0\mu m\sigma = 0.35mm$) showing the difference between the actual value and calculated value (the amplitudes of all harmonic components of the reference profile are 10 μ m).

from the actual 10 µm component amplitudes and phases of the reference profile components. The error bars correspond to 95% confidence intervals of the median for the harmonic components.

Additionally, Tables 2–4 present the effect of the different errors on the total peak-to-valley roundness deviation (RONt, LSCI) parameter as defined in ISO 12181–1:200 for each of the roundness measurement methods. RONt is defined as the difference between the maximum and minimum values of the roundness profile. As can be seen from Fig. 7 b, the distribution of RONt values is positively skewed. The authors suggest that this is a result of how RONt is defined as the difference between maximum and minimum values of the profile. Thus, negative RONt is not possible and any spike in the profile, be it negative or positive, will result in a higher RONt.

The results show mainly symmetric error distributions for the harmonic components, with almost equal mean values for all components. This was assumed to be due to the arrangement of the measurement setup and selected symmetric normal



Fig. 14. Simulated results of 100 000 cases with only horizontal error position ($x \sim N(\mu, \sigma^2)\mu = 0\mu m\sigma = 0.25$ mm) showing the difference between the actual value and measured value (the amplitudes of all harmonic components of the reference profile are 10 µm).



Fig. 15. Simulated results of 100 000 cases with all errors combined showing the difference between the actual value and measured value (the amplitudes of all harmonic components of the reference profile are 10 μm).

distributions for the error sources. The center point movement was observed to only have minor effect on the lower order components.

From Fig. 10 and Tables 2–4 it can be seen that probe scale error caused only minor errors with all levels of filtering, although the effects of probe noise can be seen in the components which have a high error propagation rate (for example 37 and 39).

Diameter sampling provided very good estimates for even harmonic components under all conditions except for angular errors, which were harmful to the diameter sampling results. Diameter sampling results in especially small errors under horizontal positioning error, which can be seen from the even component amplitudes and phases in Fig. 14.

Especially in higher order components, the error amplitudes caused by positional errors become relatively large. For single components, the least squares minimization yielded the largest standard deviations. This can be explained by the probe angles having been optimized for the three-point method based on a smallest total error propagation rate for the lower order components. Increasing phase deviations can be explained by higher order components having a shorter wavelength. Absolute phase deviations are expressed in the component's coordinate system become larger as the component's wavelength decreases.

In practical roundness measurement of form, the higher order components are usually not so much of interest. They were included in the results to show that the positional errors and center point movement combined correspond to theoretical error propagation rates (Fig. 9), for example higher errors in harmonics 37, 39 and 75 in Fig. 15.

To summarize, the main results of the research are:

1. When considering lower order harmonic components, the differences between the methods are negligible and the methods provide uncertainties of fractions of a micrometer regardless of minor frame misalignment or random center point movement.

2. Angular misalignment errors of the probes are especially harmful for diameter sampling, and lead to a large phase errors. In the other methods, these errors were spread between the phase and the amplitude of the component.

3. As seen in Fig. 14, the diameter sampling used for even components in the redundant diameter method is immune to horizontal positioning error.

Table 2

RONt values for different error sources and different roundness measurement methods filtered from 2 to 16 UPR. The actual RONt value at this filter level is 105.96 µm.

	RONt (µm)					
Method	Three-point		Four-point LS		Four-point RD	
Error source	μ	σ	μ	σ	μ	σ
Probe scale error	-0.02	0.09	-0.02	0.08	-0.02	0.08
Center point movement	-0.02	0.16	-0.02	0.10	-0.02	0.11
Angular position error of the probes	-0.02	0.00	-0.02	0.00	-0.04	0.23
Vertical position error of the frame	-0.02	0.10	-0.03	0.20	-0.03	0.23
Horizontal position error of the frame	-0.02	0.09	-0.02	0.01	-0.02	0.07
All errors combined	-0.02	0.23	-0.03	0.28	-0.04	0.42

Table 3

RONt values for different error sources and different roundness measurement methods filtered from 2 to 30 UPR. The actual RONt value at this filter level is 163.51 µm.

	RONt (µm)					
Method	Three-point		Four-point LS		Four-point RD	
Error source	μ	σ	μ	σ	μ	σ
Probe scale error	-0.15	0.13	-0.17	0.10	-0.16	0.10
Center point movement	-0.15	0.24	-0.15	0.23	-0.15	0.25
Angular position error of the probes	-0.15	0.01	-0.15	0.01	-0.16	1.05
Vertical position error of the frame	-0.13	0.36	-0.17	0.13	-0.13	0.82
Horizontal position error of the frame	-0.14	0.51	-0.15	0.16	-0.16	0.27
All errors combined	-0.10	0.63	-0.16	0.41	-0.13	1.35

Table 4

RONt values for different error sources and different roundness measurement methods filtered from 2 to 100 UPR. The actual RONt value at this filter level is 235.07 µm.

	RONt (µm)					
Method	Three-point		Four-point LS		Four-point RD	
Error source	μ	σ	μ	σ	μ	σ
Probe scale error	-6.35	0.22	-6.50	0.17	-6.49	0.18
Center point movement	-0.06	3.07	-1.03	2.56	-0.96	2.62
Angular position error of the probes	-6.78	0.10	-6.78	0.07	10.51	14.08
Vertical position error of the frame	-1.65	4.05	4.20	8.43	-0.21	5.40
Horizontal position error of the frame	-2.53	3.54	-0.92	5.01	-1.32	4.57
All errors combined	10.23	19.31	25.57	71.6	17.89	19.64

4. When considering lower order components, center point movement caused smaller errors in the four-point methods than in the three-point method. The four-probe methods are better at eliminating random center point movement.
5. In this research, the three-point method yielded the smallest total error distributions for the harmonic components and the most accurate and least uncertain estimate for total roundness. This can partly be explained by the used probe angles, but more probes to lead to more sources of error in the system.

6. The errors in harmonic components caused by the selected errors in the research corresponded well to theoretical error propagation rates (Fig. 9). This can be seen especially in increased errors in harmonic components 37–39, 75–78 and 83–86.

Finally, it must be noted that discrete sampling causes a base error in all of the examined cases and methods. Discrete sampling both causes angle mismatch and limits the number of available harmonics. Using lower sampling rates than used in the simulations was observed to lead to even larger errors. Since calculating solutions for multi-probe roundness measurement equations is not computationally very intensive with modern hardware, a higher rather than a lower sampling rate should be selected when possible.

4. Discussion

The results show that when measuring the roundness profiles of workpieces with random center point motion, the combined effect of center point movement, discrete sampling and positional errors can cause relatively large errors, which will have a significant effect on the roundness parameters and will prevent measuring the higher order harmonic components accurately. Even when positional errors are carefully minimized, discrete sampling will cause an error in the result. Therefore, a sufficiently high sampling rate should be used.

In this research, one particularly large source of uncertainty was the angular misalignment error of the probes, which caused major errors especially in the methods using four probes. Their effects can be well seen in the figures and tables with all of the errors combined. These large errors are caused by the relatively large 0.5°standard deviation distribution that was selected for the angular misalignment in this research. In practice, even larger angular errors have been measured, although it is possible to determine the angles accurately *after* positioning the measurement device using cross correlation, as demonstrated by Shi for the multi-step method [25]. The same principle can be applied to determine the probe angles and aid in the positioning of the measurement frame in a multi-probe system. Before performing the roundness measurement, sensor angles should be determined as accurately as possible. Determining the angles will eliminate any positional errors in the setup and a good estimate for roundness parameters can be obtained if the determined angles will not cause harmonic suppression.

Filtering is an important topic in form and surface metrology, and separating form features from surface features is not unambiguous. In this research, a Fourier window filter was applied (in Tables 2 and 3) by setting higher harmonic components to 0. This can be problematic, since certain shapes which contain high order components, such as sharp corners, are obviously a part of the form, not surface. Fourier-based roundness measurement is limited to nearly circular workpieces.

When all errors were applied (Fig. 15), although there were observable differences between the methods in the standard deviations of the amplitudes and phases of the harmonic components, the means re approximately as accurate. To get a more accurate mean value, time synchronous averaging (TSA) can be used to filter out phenomena which do not occur similarly each round [26]. However, it must be noted that if the error is synchronous, i.e. repeats in the same manner each round, averaging will have no effect on the error.

In roundness measurement, the center point movement is sometimes called *eccentric movement*. This term is perhaps used because on a precision spindle with no play, misalignment of the workpiece will cause the workpiece center point to rotate in a circle around the spindle center point. However, the authors argue that in multi-probe roundness measurement, the center point motion should not be called eccentric movement because when measuring for example large flexible rotors, the center point movement can be unpredictable and unrepeatable.

Since a probe signal will include both the roundness profile and the center point movement of a rotating workpiece, the center point movement in a probe's direction can be calculated by subtracting the roundness profile from the probe signal. For determining the dynamic behaviour of a large flexible rotor, it is therefore important to be able to measure the roundness profile accurately. When performing measurements of dynamic behaviour, the aim is not to minimize the center point movement; it is the measurand. Therefore, the distortion of the roundness profile due to the center point movement needs to be considered when measuring the dynamic behaviour of large flexible rotors, especially in resonant conditions where the center point movement can grow relatively large.

Another way to mitigate the effects of positional errors is to use line contact displacement probes in the measurement. When vertically positioned cylindrical probe heads are used, diameter sampling will not be as sensitive to horizontal error positions of the frame. A setup with vertically positioned probes will be insensitive to the horizontal error position. In the redundant diameter method, this will lead to very small errors for the even harmonic components. The cost of using probes with line contacts is that they act as mechanical filters and prevent the measurement of higher order harmonic components.

The results can also be used to assess several claims about multi-probe roundness measurement in literature. Whitehouse [15] has suggested that adding more probes will only incur limited benefits, since additional probes will also give rise to additional tilts and shifts to the system. The results support this claim. Zhang [12] claims that generally speaking the measurement errors are caused by angular misalignment of the probes are. The results obtained here conflict with this statement, given that the selected distribution was selected to resemble the error of tactile probes. Furthermore, time-synchronous averaging over multiple rounds can be used to reduce probe noise the and eliminate any non-synchronous phenomena in the measurement.

Additionally, the results of the research indicate that errors in the harmonic component amplitudes correspond well with previously defined theoretical error propagation rates when the errors are sufficiently small. The positional errors lead to high uncertainties in the ranges where the error propagation rates predict poor harmonic reconstruction. The obtained error distributions correspond well with theoretical metrics for harmonic sensitivity.

Previous optimizations of the probe angles have been based on only analyzing theoretical error propagation rates and transfer functions of the methods. Therefore, further research of optimal probe positioning could be directed at combining theoretical error propagation rates with simulations of error positions to find the most suitable angles. Angle combinations with especially good error propagation rates might or might not be sensitive to common errors in roundness measurement, such as misalignment of the measuring frame. All positional errors which are not in the probe direction will actualize as angular errors. An optimization could be performed with the goal to find probe angles which are insensitive to error positions.

5. Conclusion

Determining roundness profiles accurately is important for manufacturing purposes such as roll grinding as well as accurate measurements of dynamic behaviour of large flexible rotors. When the roundness profile is known, calculating the center point movement in the probe direction from a probe signal is possible by subtracting the roundness profile from the probe signal. Multi-probe methods can eliminate the need for deterministic center point movement or a precision spindle, which other roundness measurements results depend on.

Via means of uncertainty quantification, this research compared differences in the results produced by three different methods for calculating the roundness profile from simulated displacement probe signals. Several informative observations have been drawn from the results which can aid in practical multi-probe roundness measurement. Findings about which errors cause significant errors in the end results and in which ways are important to obtain an accurate estimate of a workpiece profile in practice.

Uncertainty in roundness measurement can arise from several sources. Accurate roundness measurement requires knowledge of the most common sources of error and their effects on the results. In this research, simulation experiments were used to obtain estimates of uncertainty for roundness profile parameters. Minimizing positional errors in the measurement setup is extremely important to obtain an accurate result, especially when accurate estimates of higher order harmonic components are needed.

In real roundness measurement, there will be many other sources of error, such as vibration, electrical noise, thermal expansion and the frame not being perpendicular to the workpiece. When measuring flexible rotors, dynamic changes in the roundness profile may also cause errors. In paper machine rolls the roundness profile of a roll cross section has also been observed to change as a function of the running speed [27,28]. Nevertheless, and results obtained in this research still provide useful information on the accuracy and characteristics of multi-probe roundness measurement methods.

CRediT authorship contribution statement

T. Tiainen: Methodology, Software, Formal analysis, Visualization, Writing - original draft. **R. Viitala:** Conceptualization, Methodology, Writing - review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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