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Non-iterative Subspace-based Method for Estimating AR Model Parameters in the Presence of White Noise with Unknown Variance

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Abstract—We consider the problem of estimating the parameters of autoregressive (AR) processes in the presence of white observation noise with unknown variance, which appears in many signal processing applications. A new non-iterative subspace-based method named extended subspace (ESS) method is developed. The basic idea of the ESS is to estimate the variance of the observation noise via solving a generalized eigenvalue problem, and then estimate the AR parameters using the estimated variance. The major advantages of the ESS method include excellent reliability and robustness against high-level noise, and also estimating the AR parameters in a non-iterative manner. Simulation results help to evaluate the performance of the ESS method, and demonstrate its robustness.

Index Terms—Autoregressive signals, Noisy observations, Yule-Walker equations, Subspace-based method

I. INTRODUCTION

In many signal processing applications, the autoregressive (AR) modeling of random signals is used to describe the signal of interest (SoI) in a simple and effective way. The application areas cover array processing, spectral estimation, speech processing, noise cancellation, image processing, biomedical signal processing, and communication channel estimation [1]–[9]. The broad usage of AR modeling is due to its simplicity of computing unknown model parameters and its excellent resolution performance. In addition to the problem of one-dimensional AR estimation, the problems of multichannel AR estimation and nonlinear AR estimation have been subject of active research [10]–[13]. The conventional solution of the AR estimation problem is the standard least-squares (LS) derived by low-order Yule-Walker equations. However, in practical situations, the AR signal may be contaminated by noise. Because of white observation noise corruption of the measurements, the zero lag autocorrelation is biased, leading to a biased solution of Yule-Walker equations [14].

To estimate noisy AR parameters, three main types of techniques have been developed in the past decades. Techniques belonging to the first type aim to avoid zero lag autocorrelations by using high-order Yule-Walker equations. As the first step, the AR signal is modeled by the AR moving average (ARMA) model. Then, the AR parameters can be estimated by some methods such as the maximum likelihood

(ML) method [15], the recursive prediction error (RPE) [16], as well as using the modified Yule-Walker (MYW) equations [1]. Unfortunately, these methods suffer from the lack of data for computing high autocorrelation lags estimates, which causes the presence of error in those autocorrelation lags.

Methods of the second type use the bias compensation principle to estimate the noisy AR model. Removing bias from low-order Yule-Walker equations is the key then. In general, such methods can be divided into two subcategories. Methods in the first subcategory, which are known as subspace methods, model Yule-Walker equations as an eigenvalue problem and estimate both the observation noise variance and the AR model parameters [3], [17], [24]. Methods in the second subcategory attempt to find the best estimation of both observation noise variance and the AR model parameters by iterating between two sets of equations [18]–[22]. Such methods are called improved least-squares (ILS) based methods. In [19], an ILS method with a direct structure (ILSD) has been suggested, while in [20] it has been extended to achieve fast convergence. In [21], the inverse filtering based improved least-squares (IFILS) method, which uses inverse filtering equations in conjunction with Yule-Walker equations to find the desired solution, has been proposed. Recently, a novel iterative-based method which obtains a perfect solution by solving a nonlinear equation in order to achieve an unbiased estimate of AR parameters has been developed in [18]. It is claimed that this method is able to achieve efficient performance by picking the initial value of observation noise variance within a certain region [18]. However, our simulations showed that different initial values would lead to different estimates, which are not always the efficient ones.

The third type of methods exploit the concept named errors-in-variables (EIV) to estimate the noisy AR model [27]–[28]. For example in [27], the variance of observation noise is estimated by minimizing a cost function formed by high-order Yule-Walker equations, while the AR parameters are estimated via low-order Yule-Walker equations. Although, it is a non-iterative method, minimizing the proposed cost function leads to a one-dimensional search process, which turns this method to a computationally demanding one in comparison with the subspace-based methods.

In this paper, a novel subspace-based method for estimating the AR parameters contaminated by white noise is developed. This method tries to estimate the variance of the observation noise by solving a generalized eigenvalue problem, followed by using the estimated variance to estimate the AR parameters. This method is non-iterative, computationally efficient, and it demonstrates excellent robustness against high-level observation noise. In addition, unlike the iterative methods, the convergence problem is not existing here. Simulation results confirm our claims.

II. DATA MODEL

The noisy p th order real AR model can be represented by

$$\begin{aligned} x(t) &= a_1 x(t-1) + a_2 x(t-2) + \dots + a_p x(t-p) + e(t) \\ &= \mathbf{a}^T \mathbf{x}_t + e(t) \end{aligned} \quad (1)$$

$$y(t) = x(t) + w(t) \quad (2)$$

where $e(t)$ is zero mean white stationary noise with variance σ_e^2 ; $\mathbf{a} = [a_1, a_2, \dots, a_p]^T$ is the vector of coefficients of the AR model (T denotes the transpose operation); $\mathbf{x}_t = [x(t-1), x(t-2), \dots, x(t-p)]^T$; and $w(t)$ is zero mean white stationary observation noise with variance σ_w^2 . Moreover, $w(t)$ in (2) is assumed to be uncorrelated with the driving noise $e(t)$, that is, $E\{w(t)e(n)\} = 0$ for all the N s and T s, where $E\{\cdot\}$ is the expectation operator.

The autocorrelation function of $y(t)$ can be presented by :

$$\begin{aligned} r_y[k] &= E\{y(t)y(t-k)\} \\ &= E\{(x(t) + w(t))(x(t-k) + w(t-k))\} \\ &= r_x[k] + \sigma_w^2 \delta[k] \end{aligned} \quad (3)$$

where $r_x[k]$ is the autocorrelation function of the noiseless AR process $x(t)$, and $\delta[k]$ is the delta function. The result obtained in (3) clearly indicates that the presence of the observation noise causes zero lag of the autocorrelation function of $y(t)$ to be biased. By taking into account the fact that $e(t)$ is white and also independent of $x(t-i)$, $i > 0$, (3) can be written for $k = 0$ as

$$\begin{aligned} r_y[0] &= E\{y(t)^2\} = E\{x(t)^2\} + \sigma_w^2 \\ &= E\{(x(t))(\mathbf{x}_t^T \mathbf{a} + e(t))\} + \sigma_w^2 = \mathbf{r}_x^T \mathbf{a} + \sigma_e^2 + \sigma_w^2 \end{aligned} \quad (4)$$

where $\mathbf{r}_x = [r_x[1], r_x[2], \dots, r_x[p]]^T$. In addition, according to [1], the well-known Yule-Walker equations can be written as

$$r_x[k] = \sum_{i=1}^p a_i r_x[k-i], \quad k \geq 1. \quad (5)$$

As a result, by evaluating (5) for $k = 1, \dots, p$, the following linear system of equations is obtained:

$$\mathbf{R}_x \mathbf{a} = \mathbf{r}_x \quad (6)$$

where

$$\mathbf{R}_x = \begin{bmatrix} r_x[0] & r_x[-1] & \dots & r_x[1-p] \\ r_x[1] & r_x[0] & \dots & r_x[2-p] \\ \vdots & \vdots & \ddots & \vdots \\ r_x[p-1] & r_x[p-2] & \dots & r_x[0] \end{bmatrix}. \quad (7)$$

Combining (3), (6) and (7), we obtain

$$\mathbf{R}_y \mathbf{a} = \mathbf{r}_y + \sigma_w^2 \mathbf{a} \quad (8)$$

where \mathbf{R}_y and \mathbf{r}_y are defined in a similar way as \mathbf{R}_x and \mathbf{r}_x , and also $\mathbf{r}_y = \mathbf{r}_x$. Note that p equations (8) are called the low-order Yule-Walker equations. By multiplying both sides of (8) by \mathbf{R}_y^{-1} from the left, we obtain

$$\mathbf{a} = \mathbf{R}_y^{-1} \mathbf{r}_y + \sigma_w^2 \mathbf{R}_y^{-1} \mathbf{a} \quad (9)$$

Additionally, q high-order Yule-Walker equations can be obtained by evaluating (5), for $p+1 \leq k \leq p+q$, and using (3) as follows:

$$\mathbf{R}_q \mathbf{a} = \mathbf{r}_q \quad (10)$$

where $\mathbf{r}_q = [r_y[p+1], r_y[p+2], \dots, r_y[p+q]]^T$, and

$$\mathbf{R}_q = \begin{bmatrix} r_y[p] & r_y[p-1] & \dots & r_y[1] \\ r_y[p+1] & r_y[p] & \dots & r_y[2] \\ \vdots & \vdots & \ddots & \vdots \\ r_y[p+q-1] & r_x[p+q-2] & \dots & r_y[q] \end{bmatrix} \quad (11)$$

III. EXTENDED SUBSPACE METHOD

In this section, we propose a novel non-iterative subspace-based method for the problem above. This method is primarily based on combining the low- and high-order Yule-Walker equations, which are respectively given in (8) and (10), and estimating σ_w^2 via converting the resultant linear system of equations to a generalized eigendecomposition problem. After obtaining σ_w^2 , \mathbf{a} can be computed as the LS solution of the aforementioned system of equations.

To begin with, by combining (8) and (10), we obtain

$$\mathbf{A} \mathbf{a} = \mathbf{b} + \sigma_w^2 \mathbf{c} \quad (12)$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{R}_y \\ \mathbf{R}_q \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} \mathbf{r}_y \\ \mathbf{r}_q \end{bmatrix}, \quad \mathbf{c} = \begin{bmatrix} \mathbf{a} \\ \mathbf{0}_q \end{bmatrix} \quad (13)$$

where $\mathbf{0}_q$ denotes a $q \times 1$ vector whose all entries are zero. Since \mathbf{b} is a $(p+q) \times 1$ vector, there exist $p+q-1$ vectors \mathbf{d}_i ($i = 1, 2, \dots, p+q-1$) which satisfy the following conditions:

$$\mathbf{d}_i^T \mathbf{b} = 0 \quad i = 1, \dots, p+q-1 \quad (14)$$

$$\mathbf{d}_i^T \mathbf{d}_j = 0 \quad i \neq j. \quad (15)$$

Consequently, we can construct a $(p+q-1) \times (p+q)$ matrix \mathbf{D} whose rows are \mathbf{d}_i^T . By premultiplying (12) by \mathbf{D} , and taking advantage of (14), we obtain

$$\mathbf{D} \mathbf{A} \mathbf{a} = \sigma_w^2 \mathbf{D} \mathbf{c}. \quad (16)$$

In addition, by taking the definition of \mathbf{c} in (13) into account, (16) can be rewritten as

$$(\mathbf{D}\mathbf{A} - \sigma_w^2 \mathbf{E})\mathbf{a} = \mathbf{0}_{p+q-1} \quad (17)$$

where \mathbf{E} denotes the matrix constructed by the first p columns of \mathbf{D} . As it can be observed, (17) appears to have the form of the generalized eigendecomposition problem.

Multiplying both sides of (17) by $(\mathbf{D}\mathbf{A} - \sigma_w^2 \mathbf{E})^T$, we obtain a quadratic eigenvalue problem as follows:

$$(\mathbf{G}_0 + \sigma_w^2 \mathbf{G}_1 + (\sigma_w^2)^2 \mathbf{G}_2)\mathbf{a} = \mathbf{0}_p \quad (18)$$

where

$$\begin{aligned} \mathbf{G}_0 &= \mathbf{A}^T \mathbf{D}^T \mathbf{D} \mathbf{A}, \quad \mathbf{G}_1 = -(\mathbf{A}^T \mathbf{D}^T \mathbf{E} + \mathbf{E}^T \mathbf{D} \mathbf{A}), \\ \mathbf{G}_2 &= \mathbf{E}^T \mathbf{E}. \end{aligned} \quad (19)$$

Several approaches have been presented in the literature in order to solve (18) and find σ_w^2 [26]. We can rewrite (18) as a generalized eigenvalue problem in the following way [26]:

$$(\mathbf{P} - \sigma_w^2 \mathbf{Q})\bar{\mathbf{a}} = \mathbf{0}_{2p} \quad (20)$$

where

$$\mathbf{P} = \begin{bmatrix} \mathbf{G}_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_p \end{bmatrix}, \quad \mathbf{Q} = \begin{bmatrix} -\mathbf{G}_1 & -\mathbf{G}_2 \\ \mathbf{I}_p & \mathbf{0} \end{bmatrix}, \quad \bar{\mathbf{a}} = \begin{bmatrix} \mathbf{a} \\ \sigma_w^2 \mathbf{a} \end{bmatrix} \quad (21)$$

where \mathbf{I}_p is the $p \times p$ identity matrix. After solving (20), since the resultant $2p$ eigenvalues are real or complex conjugate [26], the real eigenvalue with the smallest modulus should be chosen as σ_w^2 . However, in practical scenarios, due to the finite number of samples utilized to estimate the autocorrelation matrix, all of the eigenvalues obtained by solving (20) may be complex. Therefore, it is reasonable to choose the modulus of the eigenvalue whose absolute value of the imaginary part is minimum, as the estimated σ_w^2 .

The only issue remaining is to define a method for estimating \mathbf{a} . Making use of the estimated σ_w^2 , (12) can be rearranged as

$$\mathbf{H}\mathbf{a} = \mathbf{b} \quad (22)$$

where

$$\mathbf{H} = \begin{bmatrix} \mathbf{R}_y - \sigma_w^2 \mathbf{I}_p \\ \mathbf{R}_q \end{bmatrix}. \quad (23)$$

Thus, the LS solution of (23) with respect to \mathbf{a} is given by

$$\mathbf{a} = (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \mathbf{b}. \quad (24)$$

After calculating σ_w^2 and \mathbf{a} , σ_e^2 can be obtained via (4).

IV. SIMULATION RESULTS

In this section, the performances of the proposed ESS algorithm is compared with that of the subspace method (SS) [24], the IFILS method [21], and Xia and Zheng's method [18] by means of simulations. In order to evaluate the accuracy of the aforementioned methods in estimating AR parameters, two numerical examples are considered. The number of trials is set to $M = 1000$. Moreover, the parameter estimation methods are evaluated in terms of relative error (RE), and normalized root

mean squared error (NRMSE), which are defined through the upcoming procedure:

$$\begin{aligned} RE &= \frac{\|m(\hat{\mathbf{a}}) - \mathbf{a}\|}{\|\mathbf{a}\|} \\ m(\hat{\mathbf{a}}) &= \frac{1}{M} \sum_{m=1}^M \hat{\mathbf{a}}_m \\ NRMSE &= \frac{\sqrt{\left(\sum_{m=1}^M \|\hat{\mathbf{a}}_m - \mathbf{a}\|^2 \right) / M}}{\|\mathbf{a}\|} \end{aligned}$$

where $\hat{\mathbf{a}}_m$ denotes the estimate of \mathbf{a} in the m th trial.

It should be mentioned that $\sigma_w^{2(0)}$ is chosen to be $\eta \lambda_{\min}(\mathbf{R}_y)$ in the Xia-Zheng method; η is picked within the interval $[0.55, 0.99]$ (see [18]). As mentioned before, the main drawback of the Xia-Zheng method is that there is an uncertainty about choosing the value of η . Therefore, in each scenario, we choose the appropriate values for parameters η , δ_1 and δ_2 , which lead to the best performance of the Xia-Zheng method.

The initial parameters of each simulated method are as follows. In the IFILS method, the parameter q is set to 2, and the value of parameter δ , which determines when to terminate the iteration process, is set to 0.001 in all the examples. The parameter q is set to 8 for the SS method. In addition, q is set to 3 for the proposed ESS method.

Example 1. Consider a fourth-order noisy AR process with $\mathbf{a} = [1.6771, -1.6875, 0.9433, -0.3164]^T$ and $\sigma_e^2 = 1$. It should be noted that $e(t)$ is assumed to be a zero mean, white Gaussian process in all the examples. Two scenarios are considered. In the first one, $w(t)$ is assumed to be a zero mean, white Gaussian process with $\sigma_w^2 = 0.056$, making signal-to-noise ratio (SNR) as follows:

$$SNR = 10 \log_{10} \frac{m(x(t)^2)}{\sigma_w^2} \approx 20 \text{ dB}$$

The sample size is set to $N = 100$ in this situation. Table I displays the means and standard deviations of the estimates of the AR parameters obtained from 1000 independent trials together with RE, and NRMSE for this case. As it can be noticed, the Xia-Zheng method and the ESS show better performance in comparison with two other methods. Note that η is set to 0.96, and δ_1 and δ_2 are set to 0.001 and 0.01, respectively, for the Xia-Zheng method.

In the second case, in order to demonstrate the reliability and robustness of the ESS, we increase σ_w^2 to 4.6, which leads SNR to be 1 dB. In this case, N is set to be 4000. Table II shows the means and standard deviations of the estimates of the AR parameters obtained from 1000 independent trials together with RE, and NRMSE for this case. As it can be observed, the performance of the ESS is much better than that of the other three methods. The IFILS has very bad performance in this scenario. In this case, both δ_1 and δ_2 are considered to be 0.01 for the Xia-Zheng method. Moreover, η is set to 0.96.

TABLE I
COMPUTED RESULTS OF ESTIMATED PARAMETERS FOR $SNR = 20$ dB
WITH $N=100$ IN EXAMPLE 1

True value	SS	IFILS	Xia-Zheng method	ESS
$a_1 = 1.6771$	2.0546 ± 0.2567	1.5771 ± 0.2289	1.6538 ± 0.1962	1.7045 ± 0.1873
$a_2 = -1.6875$	-2.3323 ± 0.4017	-1.5266 ± 0.4073	-1.6668 ± 0.3521	-1.7219 ± 0.3522
$a_3 = 0.9433$	1.4755 ± 0.3053	0.8154 ± 0.3733	0.9429 ± 0.3232	0.9650 ± 0.3388
$a_4 = -0.3164$	-0.4996 ± 0.1011	-0.2723 ± 0.1677	-0.3259 ± 0.1453	-0.3183 ± 0.1592
$\sigma_w^2 = 0.0560$	0.2218 ± 0.0652	0.0563 ± 0.1093	0.0863 ± 0.0598	0.1103 ± 0.0520
$\sigma_e^2 = 1$	0.2273 ± 0.4176	1.1292 ± 0.4088	1.0023 ± 0.3043	0.8542 ± 0.2798
$RE(\%)$	36.2746	9.0271	1.2636	1.9031
$NRMSE(\%)$	42.5794	25.7180	20.8500	21.2980

TABLE II
COMPUTED RESULTS OF ESTIMATED PARAMETERS FOR $SNR = 1$ dB
WITH $N=4000$ IN EXAMPLE 1

True value	SS	IFILS	Xia-Zheng method	ESS
$a_1 = 1.6771$	1.9133 ± 0.3722	0.3981 ± 0.0829	1.5039 ± 0.1547	1.6088 ± 0.1445
$a_2 = -1.6875$	-2.0425 ± 0.6860	-0.0481 ± 0.0321	-1.3542 ± 0.3379	-1.5638 ± 0.2456
$a_3 = 0.9433$	1.2087 ± 0.6175	-0.1416 ± 0.0288	0.6325 ± 0.3721	0.8369 ± 0.2035
$a_4 = -0.3164$	-0.3966 ± 0.2508	-0.0425 ± 0.0189	-0.1841 ± 0.2103	-0.2772 ± 0.0745
$\sigma_w^2 = 4.6$	4.6255 ± 0.1901	-0.6246 ± 2.4832	4.5279 ± 0.2342	4.5783 ± 0.1892
$\sigma_e^2 = 1$	0.5190 ± 0.6857	9.1475 ± 2.7775	1.3014 ± 0.3862	1.1369 ± 0.2546
$RE(\%)$	19.7227	91.5637	19.5889	7.0255
$NRMSE(\%)$	44.4177	91.6373	29.4308	15.5602

TABLE III
COMPUTED RESULTS OF ESTIMATED PARAMETERS FOR $SNR = 10$ dB
WITH $N=100$ IN EXAMPLE 2

True value	SS	IFILS	Xia-Zheng method	ESS
$a_1 = 1.352$	2.0411 ± 1.1262	0.7839 ± 0.3288	1.2366 ± 0.3637	1.3933 ± 0.3017
$a_2 = -1.338$	-2.4366 ± 1.8283	-0.5626 ± 0.4276	-1.1973 ± 0.5554	-1.4045 ± 0.3836
$a_3 = 0.662$	1.6026 ± 1.5322	0.0645 ± 0.3514	0.5722 ± 0.4811	0.7010 ± 0.3884
$a_4 = -0.24$	-0.5992 ± 0.4939	-0.0428 ± 0.1616	-0.2224 ± 0.2127	-0.2553 ± 0.1787
$\sigma_w^2 = 0.38$	0.5524 ± 0.1246	-0.7137 ± 2.3966	0.2020 ± 0.8515	0.4554 ± 0.3231
$\sigma_e^2 = 1$	-0.0158 ± 1.2487	2.9012 ± 2.8150	1.3353 ± 1.1903	0.8411 ± 0.2947
$RE(\%)$	80.9451	56.6415	10.0421	4.3774
$NRMSE(\%)$	155.1096	65.4127	42.9537	32.2851

TABLE IV
COMPUTED RESULTS OF ESTIMATED PARAMETERS FOR $SNR = 2$ dB
WITH $N=4000$ IN EXAMPLE 2

True value	SS	IFILS	Xia-Zheng method	ESS
$a_1 = 1.352$	1.3481 ± 0.4356	0.4662 ± 0.1229	1.4521 ± 0.0928	1.3497 ± 0.1194
$a_2 = -1.338$	-1.3364 ± 0.6445	-0.2083 ± 0.1071	-1.5117 ± 0.2185	-1.3325 ± 0.1802
$a_3 = 0.662$	0.6559 ± 0.5383	-0.1644 ± 0.0511	0.8263 ± 0.2466	0.6562 ± 0.1503
$a_4 = -0.24$	-0.2413 ± 0.2016	0.0046 ± 0.0295	-0.3173 ± 0.1557	-0.2365 ± 0.0596
$\sigma_w^2 = 2.4$	2.3480 ± 0.2361	0.3662 ± 0.8487	2.4215 ± 0.0870	2.3942 ± 0.1348
$\sigma_e^2 = 1$	1.0540 ± 0.7885	4.4457 ± 1.0818	0.8827 ± 0.0466	1.0161 ± 0.2278
$RE(\%)$	0.3711	82.5522	13.3357	0.4449
$NRMSE(\%)$	47.6875	82.9949	22.8363	13.3178

Example 2. Consider a fourth-order noisy AR process with $\mathbf{a} = [1.352, -1.338, 0.662, -0.24]^T$ and $\sigma_e^2 = 1$. Two scenarios are studied here. In the first one, σ_w^2 is set to 0.38, giving rise to $SNR=10$ dB. Moreover, N is set to 100. Table III shows the means and standard deviations of the estimates of the AR parameters. Similar to the previous examples, the ESS has much better performance in this case. For the Xia-Zheng method, η , δ_1 , and δ_2 are respectively set to 0.7, 0.001, and 0.01.

In the second scenario, σ_w^2 is assumed to be 2.4, yielding $SNR=2$ dB. In addition, N is set to 4000. Table IV shows the means and standard deviations of the estimates of the AR parameters. Analogous to the previous scenarios, the performance of the ESS is much better than the other three methods. Note that η is set to 0.96, and both δ_1 and δ_2 are set to 0.05 here for the Xia-Zheng method.

Example 3. In this example, the performance of the aforementioned methods are compared for a more general

simulation setup to provide more insights into accuracy of parameter estimation by these methods. The parameters N and σ_e^2 are set to 4000 and 1, respectively. Additionally, η is set to 0.94, and both δ_1 and δ_2 are set to 0.05 here for the Xia-Zheng method. A hundred sets of poles, each set containing four poles selected randomly inside the unit circle, are considered as the poles of a hundred different AR processes. Then, for each AR process, the AR parameters are estimated via the aforementioned methods for 1000 trials, and NRMSE is calculated. The mean of resultant NRMSEs of a hundred different AR signals is then taken as the total NRMSE in this simulation example. We repeat this scenario for different values of SNR that varies from 0 dB to 10 dB and plot the results in Fig. 1.

It can be observed from Fig. 1 that the proposed method shows more robust overall performance in dealing with various kind of AR processes in comparison with the IFILS and Xia and Zheng's methods for all SNRs. Note that since the performance of the SS method is very bad in this example, we omitted it. Moreover, since the selected value of η is not

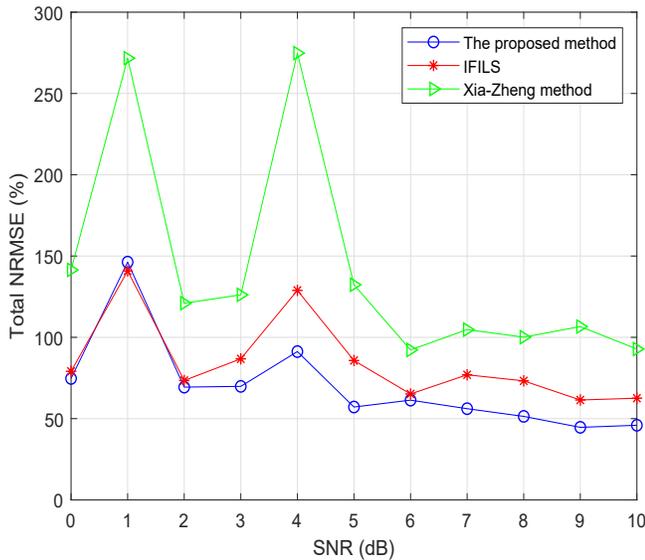


Fig. 1. Total RMSE versus SNR for Example 3.

appropriate for all of AR signals generated in this example, the Xia and Zheng’s method does not have a good performance. Thus, this simulation example confirms the dependency of Xia and Zheng method’s performance to the choice of initial value.

V. CONCLUSION

In this paper, a novel non-iterative subspace-based method was proposed for estimating the parameters of AR processes in the presence of white observation noise with unknown variance. The main idea of this method is to estimate the variance of the observation noise by solving a generalized eigenvalue problem as the first step, and then estimate the AR vector of parameters by finding the LS solution of a linear system of equations. The performance of the proposed method has been evaluated and compared with that of three other methods presented in the literature by means of simulations. The simulation results have demonstrated the superiority of the proposed method in terms of having smaller NRMSE, and also shown better robustness against high level of the observation noise. Moreover, the proposed method is non-iterative, and thus there are no convergence issues, unlike it is with the iterative-base methods.

REFERENCES

- [1] S. M. Kay, *Modern Spectral Estimation*. Prentice-Hall, Englewood Cliffs, NJ, 1988.
- [2] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. Englewood Cliffs, NJ: Prentice-Hall, 1993.
- [3] M. D. Srinath, P. K. Rajasekaran, and R. Viswanathan, *Introduction to Statistical Signal Processing with Applications*. Englewood Cliffs, NJ: Prentice-Hall, 1996.
- [4] R. Sameni, “Online filtering using piecewise smoothness priors: Application to normal and abnormal electrocardiogram denoising”, *Signal Processing*, vol. 133, pp. 52–63, 2017.
- [5] L. Legrand and E. Grivel, “Jeffrey’s divergence between autoregressive processes distributed by additive white noise”, *Signal Processing*, vol. 149, pp. 162–178, 2018.

- [6] S. Dutta, M. Singh, and A. Kumar, “Automated classification of non-motor mental task in electroencephalogram based brain-computer interface using multivariate autoregressive model in the intrinsic mode function domain”, *Biomedical Signal Processing & Control*, vol. 43, pp. 174–182, 2018.
- [7] S. Ganapathy, “Multirate autoregressive spectrogram modeling for noisy speech recognition”, *IEEE Signal Processing Letters*, vol. 24, no. 9, pp. 1373–1377, 2017.
- [8] K. Upadhyaya and S. A. Vorobyov, “An array processing approach to pilot decontamination for massive MIMO”, in *Proc. 6th IEEE Int. Workshop Computational Advances in Multi-Sensor Adaptive Processing, CAM-SAP’15*, Cancun, Mexico, Dec. 2015, pp. 465–468.
- [9] A. Alkhateeb, O. El Ayach, G. Leus, and R. W. Heath, “Channel estimation and hybrid precoding for millimeter wave cellular systems”, *IEEE Journal of Selected Topics in Signal Processing*, vol. 8, no. 7, pp. 831–846, 2014.
- [10] A. Mahmoudi and M. Karimi, “Estimation of the parameters of multichannel autoregressive signals from noisy observations”, *Signal Processing*, vol. 88, no. 11, pp. 2777–2783, 2008.
- [11] S. Maanan, B. Dumitrescu, and C. D. Giurcaneanu, “Conditional independence graphs for multivariate autoregressive models by convex optimization: Efficient algorithms”, *Signal Processing*, vol. 133, pp. 122–134, 2017.
- [12] A. Amanat, A. Mahmoudi, and M. Hatam, “Two-dimensional noisy autoregressive estimation with application to joint frequency and direction of arrival estimation”, *Multidim. Syst. Sign. Process.*, vol. 29, no. 2, pp. 671–685, 2018.
- [13] M. Kallas, P. Honeine, C. Francis, and H. Amond, “Kernel autoregressive models using Yule-Walker equations”, *Multidim. Signal Processing*, vol. 93, no. 11, pp. 3053–3061, 2013.
- [14] S. M. Kay, “Noise compensation for autoregressive spectral estimates”, *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 28, no. 3, pp. 292–303, 1980.
- [15] H. Tong, “Autoregressive model fitting with noisy data by Akaike information criterion”, *IEEE Trans. Inf. Theory*, vol. 21, no. 4, pp. 476–480, 1975.
- [16] A. Nehorai and P. Stoica, “Adaptive algorithms for constrained ARMA signals in the presence of noise”, *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 36, no. 8, pp. 1282–1291, 1988.
- [17] W. X. Zheng, “On estimation of autoregressive signals in the presence of noise”, *IEEE Trans. Circuits Syst. II*, vol. 53, no. 12, pp. 1471–1475, 2006.
- [18] Y. Xia and W. X. Zheng, “Novel parameter estimation of autoregressive signals in the presence of noise”, *Automatica*, vol. 62, pp. 98–105, 2015.
- [19] W. X. Zheng, “Autoregressive parameter estimation from noisy data”, *IEEE Trans. Circuits Syst. II: Analog Digital Signal Process.*, vol. 47, no. 1, pp. 71–75, 2000.
- [20] W. X. Zheng, “Fast identification of autoregressive signals from noisy observations”, *IEEE Trans. Circuits Syst. II*, vol. 52, no. 1, pp. 43–48, 2005.
- [21] A. Mahmoudi and M. Karimi, “Inverse filtering based method for estimation of noisy autoregressive signals”, *Signal Processing*, vol. 91, no. 7, pp. 1659–1664, 2011.
- [22] A. Mahmoudi and M. Karimi, “Parameter estimation of autoregressive signals from observations corrupted with colored noise”, *Signal Processing*, vol. 90, pp. 157–164, 2010.
- [23] M. H. A. Davis and R. B. Vinter, *Stochastic Modeling and Control*. London, U.K.: Chapman and Hall, 1985.
- [24] C. E. Davila, “A subspace approach to estimation of autoregressive parameters from noisy measurements”, *IEEE Trans. Signal Process.*, vol. 46, no. 2, pp. 531–534, 1988.
- [25] C. Z. Jin, L. J. Jia, Z. J. Yang, and K. Wada, “On convergence of a BCLS algorithms for noisy autoregressive process estimation”, in *Proc. 41st IEEE Conf. Decision and Control*, Las Vegas, NV, USA, Dec. 2002, pp. 4252–4257.
- [26] F. Tisseur and K. Meerbergen, “The quadratic eigenvalue problem”, *SIAM Review*, vol. 43, no. 2, pp. 235–286, 2001.
- [27] R. Diversi, R. Guidorzi, and U. Soverini, “Identification of autoregressive models in the presence of additive noise”, *Int. Journal of Adaptive Control and Signal Processing*, vol. 22, pp. 465–481, 2008.
- [28] R. Diversi, R. Guidorzi, and U. Soverini, “A noise-compensated estimation scheme for AR processes”, in *Proc. 44th IEEE Conf. Decision and Control and 8-th European Control Conf.*, Melia Sevilla, Seville, Spain, Dec. 2005, pp. 4146–4151.