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A Combined Waveform-Beamforming Design for Millimeter-Wave Joint Communication-Radar

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Abstract—Millimeter-wave (mmWave) joint communication-radar (JCR) simultaneously realizes a high data rate communication and a high-resolution radar sensing for applications such as autonomous driving. Prior JCR systems that are based on the state-of-the-art mmWave communications hardware, however, suffer from a limited angular field-of-view (FoV) and low detection rate for radars due to the employed directional beam. To address this limitation, we propose an adaptive and fast combined waveform-beamforming design for mmWave JCR with a phased-array architecture. We present a JCR beamformer design algorithm that permits a trade-off between communication rate and radar successful recovery rate in the angular domain. We show that distinct radar measurements can be obtained with circulant shifts of the designed JCR beamformer for compressed radar sensing. Numerical results demonstrate that our JCR design enables the angle-of-arrival/departure estimation of short-range radar targets with a high successful recovery rate and a wide FoV at the expense of a slight loss in the communication rate.

I. INTRODUCTION

Joint communication-radar (JCR) uses a common transmit (TX) signal for both communication and radar operations to enable hardware and spectrum reuse. A solution to realize JCR for next-generation applications, such as autonomous driving, is to exploit the large bandwidth at the millimeter-wave (mmWave) band [1]. In [2], [3], a practical mmWave WiFi-based JCR was proposed during the data transmission mode to achieve Gbps data rate simultaneously with a high range and velocity estimation accuracy for long-range radar applications. Unfortunately, the angular field of view (FoV) for short-range radar (SRR) applications was limited due to the employed directional beam.

Prior approaches to increase the radar FoV for mmWave JCR can be categorized into three types: (a) JCR during the communication beam training mode, (b) JCR with an adaptive beamforming design during the data transmission mode, and (c) multiple-input-multiple-output JCR with low resolution analog-to-digital converters. In the first approach [4], the IEEE 802.11ad control physical (PHY) layer frames along with beam scanning algorithm during the beam training mode was proposed for radar sensing with a wide FoV. In the second approach [5], the IEEE 802.11ad single-carrier (SC) PHY frames along with the adaptive random switching (RS) of TX antennas during the data transmission mode was proposed. In the RS-JCR, a coherent beam is formed towards the communication receiver, while simultaneously perturbing the

grating lobes of the resulting beam pattern for angle-of-arrival (AoA) estimation in SRR applications. In the last approach [6], a mmWave multiple-input-multiple-output JCR with 1-bit analog-to-digital converters per RF chain was proposed to achieve a high range and AoA estimation accuracy. The RS-JCR has a higher radar update rate and communication data rate than the first approach, and is based on a commercially available mmWave hardware unlike the third approach. The RS-JCR, however, employs TX antenna subsets instead of using all antennas, which decreases the net TX power for JCR operation under a per-antenna power constraint.

In this paper, we develop an adaptive combined waveform-beamforming design that exploits all the TX and receive (RX) antennas for mmWave JCR during the data transmission mode. Our mmWave JCR design enables a highly accurate AoA and angle-of-departure (AoD) estimation of SRR targets in a wide FoV without reducing communication rate much. For reduced hardware complexity, we assume a phased-array architecture at both the TX and RX. In particular, we present a TX beamformer design algorithm to generate a narrow coherent beam for communication and constant gain sidelobes in other directions for radar sensing. The TX beamformer design accounts for the trade-off between the sidelobe gain for radar and the mainlobe gain for communication. The radar receiver acquires distinct radar channel measurements using circulant shifts of the designed beamformer. To quantify the JCR trade-off for an adaptive combined waveform-beamforming design, we use the data rate metric for communication and a novel successful recovery rate metric for radar. Numerical results demonstrate that our JCR design performs significantly better than the RS-JCR extended for both AoA and AoD estimation, especially for a large number of antennas.

Notation: The operators $(\cdot)^*$ stands for conjugate transpose, $(\cdot)^T$ for transpose, and (\cdot) for conjugate of a matrix or a vector. $\mathcal{N}(\mu, \sigma^2)$ is used for a complex circularly symmetric Gaussian random variable with mean μ and variance σ^2 . $\mathbf{A} \star \mathbf{B}$ is defined as the 2D circular cross-correlation between matrices \mathbf{A} and \mathbf{B} [7], whereas $\langle \mathbf{A}, \mathbf{B} \rangle = \sum_{k,\ell} \mathbf{A}(k,\ell) \mathbf{B}(k,\ell)$ is defined as the inner product of \mathbf{A} and \mathbf{B} . $\mathbf{A} \odot \mathbf{B}$ is defined as the element-wise multiplication of \mathbf{A} and \mathbf{B} .

II. SYSTEM MODEL

We consider the use case where a source vehicle sends a mmWave JCR waveform to communicate with a recipient

III. BEAMFORMER DESIGN FOR JCR

In this section, we explain our approach to construct a collection of beamformers that are well suited to the JCR application. Our method first constructs one sequence for each of the TX and RX. These sequences are designed according to the JCR specification. Then, our method constructs the collection of beamformers by circularly shifting the sequences constructed at the TX and the RX. As circulant shifts of a vector preserve the magnitude of its discrete Fourier transform (DFT), the proposed method ensures that the beams constructed according to our procedure achieve the desired JCR specifications. The collection of circularly shifted beamformers help to acquire distinct radar channel measurements.

Now, we explain the key idea underlying the proposed transmit beamformer design technique. For tractability, we design the beamformer by considering a DFT grid with N discrete angles. For ease of exposition, we assume the communication direction is 0° . The transmit beamformer design problem in JCR is to design a sequence $\mathbf{f}_0 \in \mathbb{Q}_q^N$ whose beampattern has an energy of δ along 0° . The remaining energy in the beamformer must be distributed to enable radar channel reconstruction with fewer channel measurements. Prior work has shown that beamformers with close to uniform gain along the desired sensing directions enable fast channel reconstruction through compressed sensing [7]. To this end, the proposed construction distributes the energy of $1 - \delta$ “uniformly” across the remaining DFT grid locations.

We show here how the Gerchberg-Saxton (GS) algorithm [9] can be used to construct the desired TX beamformer. We define $\delta_{\text{rad}} = (1 - \delta)/(N - 1)$. The DFT magnitude vector associated with the desired beamformer is then $\mathbf{b}_{\text{mag}} = [\sqrt{\delta}, \sqrt{\delta_{\text{rad}}}, \sqrt{\delta_{\text{rad}}}, \dots, \sqrt{\delta_{\text{rad}}}]$. The inverse DFT of \mathbf{b}_{mag} , however, may not be an element in \mathbb{Q}_q^N . The GS algorithm is an alternating projection method that finds a sequence in \mathbb{Q}_q^N such that the magnitude of its DFT is close to \mathbf{b}_{mag} . We use $\text{phase}_q(x)$ to denote the q -bit phase quantized version of x . The GS algorithm is summarized in Algorithm 1.

Algorithm 1 GS algorithm to find \mathbf{f}_0

- 1: **Inputs:** δ , N , q , and T_{GS} .
 - 2: **Initialize:** Set $t_{\text{iter}} = 1$ and \mathbf{f}_0 to a Zadoff-Chu sequence.
 - 3: **while** $t_{\text{iter}} < T_{\text{GS}}$ **do**
 - 4: $\mathbf{b}_{\text{phase}} \leftarrow \text{phase}(\text{DFT}(\mathbf{f}_0))$
 - 5: Constraint on the discrete beam pattern:
 $\mathbf{b} \leftarrow \mathbf{b}_{\text{mag}} \odot \exp(j\mathbf{b}_{\text{phase}})$
 - 6: Constraint on the antenna weights:
 $\mathbf{f}_{\text{phase}} \leftarrow \text{phase}_q(\text{IDFT}(\mathbf{b}))$
 - 7: $\mathbf{f}_0 \leftarrow \exp(j\mathbf{f}_{\text{phase}})/\sqrt{N}$
 - 8: **end while**
 - 9: **return** \mathbf{f}_0 .
-

The proposed GS-based beamformer design procedure can be generalized for any communication direction $\theta \neq 0^\circ$. The transmit beamformer in such a case is defined as $\mathbf{f}_0 \odot \mathbf{a}(\theta)$.

For the radar receiver, a good compressed sensing (CS)-based beamformer is one that has equal energy at all DFT-grid locations [7]. We propose to use a Zadoff-Chu sequence in \mathbb{Q}_q^N to be the RX combiner vector \mathbf{w}_0 because the DFT of a Zadoff-Chu sequence has a constant amplitude.

IV. COMPRESSED SENSING USING THE DESIGNED BEAMFORMERS

Now, we explain how radar channel measurements are acquired using the beamformers \mathbf{f}_0 and \mathbf{w}_0 . The radar channel $\mathbf{H} \in \mathbb{C}^{N \times N}$ encodes the AoA and AoD information of the targets, and it is sparse at mmWave. The sparse channel \mathbf{H} can be reconstructed from fewer measurements according to the model in (3). The measurements in the proposed CS technique are acquired by applying random circulant shifts of $\mathbf{f}_i = \mathbf{J}_i \mathbf{f}_0$ and $\mathbf{w}_j = \mathbf{J}_j \mathbf{w}_0$ at the TX and the RX, where \mathbf{J}_i is the i^{th} circulant-delay matrix. Such a CS technique is known as convolutional CS [10], [11]. The base matrix in convolutional CS is defined as $\mathbf{B}_{i,j} = \mathbf{w}_j^* \mathbf{f}_i$. Therefore, the noiseless measurement corresponding to \mathbf{f}_i and \mathbf{w}_j is given as

$$z[i, j] = \mathbf{w}_j^* \mathbf{H} \mathbf{f}_i = \langle \mathbf{H}, \mathbf{w}_j \mathbf{f}_i^* \rangle = \langle \mathbf{H}, \mathbf{B}_{i,j} \rangle. \quad (4)$$

The received measurements when the TX and the RX apply all combinations of circulant shifts can be represented as $\mathbf{Z} = \mathbf{H} \star \mathbf{B}_{0,0}$. We assume that 2D-DFT of \mathbf{H} , defined as \mathbf{X} , is k -sparse. Using the properties of the 2D-DFT, the 2D-DFT of \mathbf{Z} can be expressed as $\mathbf{S} = \mathbf{X} \odot \mathbf{D}$, where \mathbf{D} is the 2D-DFT of $\mathbf{B}_{0,0}$. As \mathbf{X} is k -sparse, it can be shown that \mathbf{S} is at most k -sparse. Such a property can be exploited to recover \mathbf{S} from fewer projections than N^2 .

In 2D-convolutional CS, the TX and RX apply $M \ll N^2$ distinct pairs of circulant shifts of \mathbf{f}_0 and \mathbf{w}_0 . We define Ω as a set containing the pair of circulant shifts used at the RX and the TX. For example, $M = 3$ and $\Omega = \{(0, 0), (1, 2), (2, 1)\}$. The operator $\mathcal{P}_\Omega(\mathbf{A}) \rightarrow \mathbb{C}^M$ returns the entries of a matrix \mathbf{A} at the locations in Ω . The noisy radar channel measurements with convolutional CS can be expressed as

$$\mathbf{y} = \mathcal{P}_\Omega(\mathbf{Z}) + \mathbf{e}, \quad (5)$$

where the additive noise $e[m]$ is defined in (3).

Next, we describe a CS algorithm and corresponding recovery guarantees to estimate \mathbf{H} from the channel measurements in (5). A reasonable approach for radar channel estimation is to minimize the ℓ_1 norm of \mathbf{S} subject to the constraint corresponding to (5). Such a method encourages channels that are faithful to the measurements in (5) and whose 2D-DFT is sparse [10]. Let $\hat{\mathbf{H}}$ be the solution to the ℓ_1 minimization program and D_{min} denote the minimum magnitude of \mathbf{D} , i.e., the 2D-DFT of $\mathbf{B}_{0,0}$. For the construction in Section III, it can be shown that $D_{\text{min}} = \sqrt{\delta_{\text{rad}}}$. We use [7, Theorem 1] to obtain a guarantee on the radar channel reconstruction performance.

Theorem 1. *For a fixed constant $\epsilon \in (0, 1)$, the solution $\hat{\mathbf{H}}$ obtained with ℓ_1 norm minimization over \mathbf{S} satisfies*

$$\|\mathbf{H} - \hat{\mathbf{H}}\|_F \leq C_1 \frac{N\sigma}{\sqrt{\delta_{\text{rad}}}}, \quad (6)$$

with a probability of at least $1 - \epsilon$ if $M \geq Ck \max \{2\log^3(2k) \log(N), \log(\epsilon^{-1})\}$. The constants C and C_1 are independent of all the other parameters.

Proof. The result follows by using the assumption that \mathbf{X} is k -sparse and the fact that $D_{\min} = \sqrt{\delta_{\text{rad}}}$ in [7, Theorem 1]. \square

The bound in (6) indicates that sparse radar channels can be recovered from sub-linear channel measurements that are acquired using the proposed beamformers. The result in Theorem 1 also indicates that the MSE of the channel estimate scales inversely with δ_{rad} , i.e., the fraction of energy used for sensing in JCR.

In numerical simulations, we show that the sparse matrix \mathbf{S} can be efficiently recovered from a subsampled version of \mathbf{Z} using a low-complexity OMP algorithm that exploits the partial 2D-DFT nature of our radar CS problem [10].

V. PERFORMANCE METRICS

We consider achievable data rate, $R_c(\delta)$, as the performance metric for the communication system in (2) with δ as the fraction of TX power for communication. For $\delta = 1$, we achieve maximum effective SNR at the communication receiver, ζ_c , and maximum data rate using the ideal beampattern for communication. Assuming $s_c[n, p]$ is distributed as $\mathcal{N}_{\mathbb{C}}(0, 1)$, the maximum achievable data rate is given by

$$R_c(\delta) = \frac{1}{T_s} \log_2(1 + \delta\zeta_c). \quad (7)$$

For radar performance evaluation of the proposed JCR system with a probability of successful recovery, $P_r(\delta, M)$, in a CPI of T seconds, we define a novel normalized radar successful recovery rate metric as

$$R_r(\delta, M) = \frac{P_r(\delta, M)}{\eta_r M T_{\text{int}}}, \quad (8)$$

where η_r is the normalization factor. The metric $R_r(\delta, M)$ is analogous to the detection rate metric [12], which is used to study the trade-off between integration time and scanning rate. We choose η_r such that the maximum value of $R_r(\delta, M)$ is 1 for better interpretation of the radar detection results.

The trade-off curve for our JCR combined waveform-beamforming design can be obtained using the data-rate constrained problem with a minimum required data rate Υ_c , which is given as

$$\begin{aligned} & \underset{\delta}{\text{maximize}} && R_r(\delta, M) \\ & \text{subject to} && R_c(\delta) \geq \Upsilon_c \\ & && \mathbf{W} \in \mathcal{W}(\delta) \\ & && \mathbf{F} \in \mathcal{F}(\delta) \\ & && M \geq Ck \log^3(2k) \log(N), \end{aligned}$$

where \mathbf{F} is the matrix of M TX precoder vectors and \mathbf{W} is the matrix of M RX precoder vectors at the source vehicle during a CPI. The TX codebook $\mathcal{F}(\delta)$ and the receive codebook $\mathcal{W}(\delta)$ are based on our JCR beamformer design described in Sections III and IV.

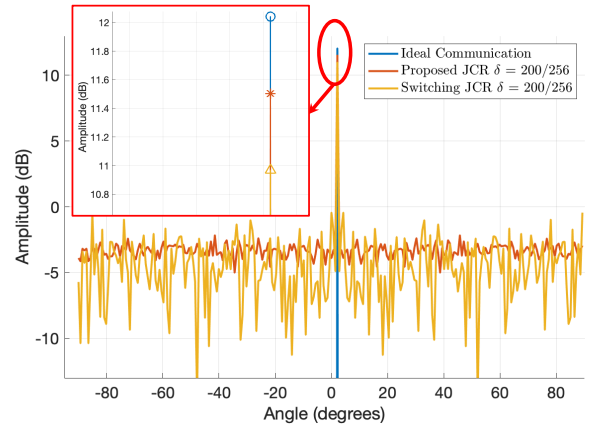


Figure 2: Comparison between the TX beampattern achieved by our JCR design with RS-JCR and the ideal communication. Our JCR achieves higher mainlobe gain than RS-JCR for data communication under a per-antenna power constraint.

VI. NUMERICAL RESULTS

In this section, the numerical results of the proposed adaptive combined waveform-beamforming design for mmWave JCR are presented. The TX/RX antenna arrays are considered to be uniform linear arrays with 256 elements. We assume the grid size of 256 with 180° FoV, the recipient vehicle distance $d_c = 100$ m, and the radar cross-section as 10 dBsm [13]. The interval for one independent TX-RX beam pair is considered as $T_{\text{int}} = 0.2$ ms. We employed the OMP algorithm [7] for its simplicity to estimate the support of virtual radar channel in the angular domain, \mathbf{X} .

Fig. 2 compares the TX beampattern for ideal communication, our proposed JCR with $\delta = 200/256$, and the RS-JCR proposed in [5] with the number of TX antennas switched on as 200. We see that communication TX power achieved by our proposed JCR using GS-algorithm improves by 1 dB as compared to the RS-JCR. The mainlobe for both the JCR beampatterns, however, has reduced gain compared to the ideal communication beampattern. Therefore, the best communication rate would be achieved by the ideal communication, followed by our JCR, and lastly the RS-JCR. The sidelobe gains achieved by the ideal communication beampattern, however, is the lowest and results in the worst radar performance. We observe random sidelobes with several grating lobes and nulls for the RS-JCR, whereas we observe constant sidelobe gain for our proposed JCR. Additionally, over an ensemble of beamformers constructed according to our JCR design, the net TX power along SRR directions is smaller for the RS-JCR than that for our proposed JCR.

Fig. 3 shows the variation of probability of successful recovery with the number of measurements and antennas for both the proposed JCR and the extended RS-JCR for AoA/AoD estimation. The extended RS-JCR is realized by incorporating phase randomization in the RX combiner vector in addition to the RS at the TX proposed in [5]. We see that our proposed JCR needs much lesser number of measurements than the RS-JCR to achieve a high probability of successful

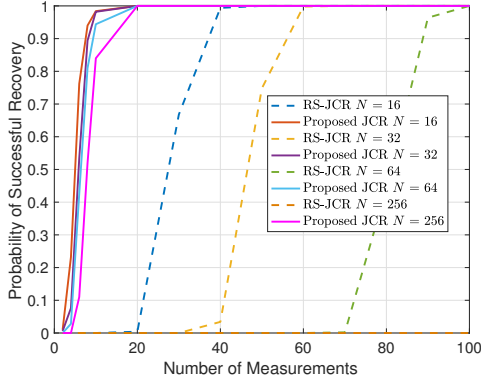


Figure 3: Comparison of RS-JCR with our proposed JCR for varying number of measurements and antennas at $d = 12$ m and $\delta = 200/256$. Our JCR performs better than the RS-JCR.

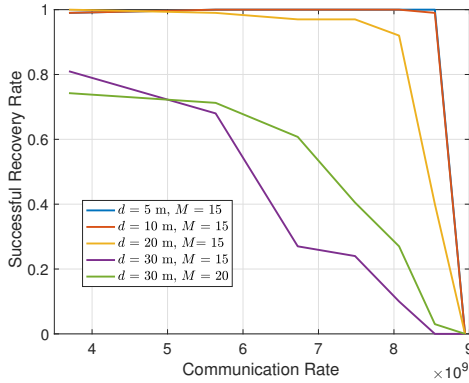


Figure 4: The trade-off curve achieved by our JCR with $N = 256$ with $\eta_r = 1/(15 T_{\text{int}})$. The radar successful recovery rate improves with decreasing δ , while lowering $R_c(\delta)$.

recovery. The difference between the radar successful recovery rate achieved by these approaches increases with the number of antennas. Our proposed JCR achieves the probability of successful recovery close to one within a CPI of less than 10 ms, which is desirable in automotive radars [13].

Fig. 4 shows the trade-off curve between communication rate, $R_c(\delta)$, and radar successful recovery rate, $R_r(\delta, M)$, with different values of $0.1 \leq \delta \leq 1$ for $N = 256$. The radar successful recovery rate improves with decreasing δ due to increase in the radar SNR, while lowering communication data rate due to decrease in the communication SNR. From Fig. 4, we observe that the radar successful recovery rate for a given communication rate reduces with distance. For a distance of 30 m, although $P_r(\delta, M) = 0.99$ for $M = 20$ and $P_r(\delta, M) = 0.82$ for $M = 15$ at $\delta = 0.1$ with $R_c(\delta) = 4$ Gbps, $R_r(\delta, M)$ is higher for $M = 15$ than $M = 20$. Additionally, we can see from Fig. 4 that large M is preferred at low δ , whereas small M is preferred at high δ for our JCR design.

VII. CONCLUSION

In this paper, we proposed a TX-RX JCR beamformer design for the mmWave JCR system with a phased-array architecture to estimate radar AoA/AoD with a wide FoV at

the cost of a small reduction in communication data rate. Our AoA and AoD estimation technique for SRR targets exploits mmWave channel sparsity and has a low complexity due to the partial DFT CS. Our proposed TX precoder and RX combiner design for mmWave JCR performs better than the random switching-based one, especially for a large number of antennas. The results in this paper can be used to develop low-power, small size, spectrum-efficient, and high-performance mmWave devices that will enable next-generation automotive sensing and communication needs. Future work includes an extension of our work for simultaneous range, velocity, and direction estimation, as well as experimental demonstration.

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