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Published in:
IEEE Transactions on Industrial Informatics

DOI:
10.1109/TII.2019.2936280

Published: 01/06/2020

Document Version
Peer reviewed version

Please cite the original version:
Jooshaki, M., Farzin, H., Abbaspour, A., Fotuhi-Firuzabad, M., & Lehtonen, M. (2020). A Model for Stochastic Planning of Distribution Network and Autonomous DG Units. *IEEE Transactions on Industrial Informatics*, *16*(6), 3685-3696. [8807219]. https://doi.org/10.1109/TII.2019.2936280

A Model for Stochastic Planning of Distribution Network and Autonomous DG Units

Mohammad Jooshaki, Hossein Farzin, Ali Abbaspour, Mahmud Fotuhi-Firuzabad, *Fellow, IEEE*, and Matti Lehtonen

***Abstract*—This paper presents a mixed-integer linear stochastic model for optimal expansion planning of electricity distribution networks and distributed generation (DG) units. In the proposed framework, autonomous DG units are aggregated and modeled using the well-known energy hub concept. In this model, uncertainties of heat and electricity demand as well as renewable generation are represented using various scenarios. Although this is a standard technique to capture the uncertainties, it drastically increases the dimensions of this optimization problem and makes it practically intractable. In order to address this issue, a novel iterative method is developed in this paper to enhance the efficiency of the optimization model. The proposed framework is further utilized to assess the benefits of the collaborative distribution network and autonomous DG planning through various case studies performed on the 24-node distribution test grid. With 5.93% cost reduction, the obtained results indicate the importance of such collaborations in reaching an efficient network expansion solution. Moreover, total planning cost for the stochastic model is 1.23% lower than the deterministic case. Various sensitivity analyses are also carried out to investigate the impacts of parameters of the proposed model on the optimal planning solution. Scalability of the model is also assessed by its implementation on the 54-node distribution test network.**

***Index Terms*— Collaborative planning, distributed generation, electricity distribution system planning, energy hub, stochastic programming.**

NOMENCLATURE

Sets

CA^{co}	Candidate alternatives for investing in component *co*.
D_t	Representative days of year *t*.
DNC	Distribution network components.
EH	Energy hubs.
EHC	Energy hub components, including transformer, wind turbine, photovoltaic panel, CHP, and heat furnace.
LSc_d	Set of various scenarios for hourly electricity and heat load in representative day *d*.
SSc_d, WSc_d	Set of various scenarios for hourly solar radiation and wind pattern in representative day *d*, respectively.
TSc_d	Set of total scenarios which is Cartesian product of wind power, solar radiation and demand scenarios as $TSc_d= WSc_d\times SSc_d\times LSc_d$

Parameters

err	Maximum acceptable tolerance as the stopping criterion of loss factor.
$EDem$, $HDem$	Electricity and heat demands.
$EPr_{d,h}$, $Pr_{d,h}$	Electricity and natural gas prices.
i	Interest rate.
IC^{co}	Investment cost of component *co*.
IC_a^{co}	Investment cost of alternative *a* for component *co*.
$MCap_{eh}^{co}$	Maximum possible capacity of component *co* within energy hub *eh*.
N_d	Number of days of a year which are represented by day *d*.
$OMC_{(.)}^{(.)}$	Operation and maintenance cost.
p_{sc}	Probability of scenario *sc* as $p_{sc}=p_{ls}p_{ss}p_{ws}$. Where, p_{ls}, p_{ss}, p_{ws} are load, solar radiation, and wind pattern scenarios, respectively.
$TI_{eh,t}$	A binary parameter, which is 1 if energy hub *eh* is for the first time connected to the grid at year *t*, being 0 otherwise.
UL^{co}	Useful life of component *co*.
α	Sell-to-purchase electricity price ratio.
δ^{co}	Perpetuity factor.
$\gamma_{d,ws,h}^{WS}$, $\gamma_{d,ss,h}^{SR}$	Correction coefficients for calculation of available capacity of wind and solar generating units.
η_{GE}^{CHP}, η_{GH}^{CHP}	CHP Gas-to-electricity and Gas-to-heat conversion efficiencies.
η^{Fu}, η^{Tr}	Efficiency of heat furnace and transformer.
λ_t	Average loss cost at year *t*.
μ^{co}	A binary parameter, which is 1 if network component *co* is initially existent, being 0 otherwise.
ζ	Loss factor.

Variables

Cap_{eh}^{co}	Capacity of component *co* of energy hub *eh*.
$I_{b,\ldots}$	Branch currents.
Inv_t^{DN}, Op_t^{DN}	Investment and operating costs of distribution network.
Inv_t^{EH}, Op_t^{EH}	Investment and operating costs of EHs.
$Loss_t$	Total annual loss of year *t*.
$Loss_t^{act}$, $Loss_t^{est}$	Actual and estimated values of total network loss in year *t*, respectively.
$Loss_{t,md}^{CC}$, $Loss_{t,mg}^{CC}$	Network power loss associated with two critical load conditions.
OF^{EH}	Objective function of EH planner.
OF^{DN}	Objective function for electricity distribution network expansion planning.

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Op_T^{EH}, Op_T^{DN}	Operating costs of EHs and distribution network in the last stage of the planning horizon.
$P_{eh.t.md}^{CC}$, $P_{eh,t,mg}^{CC}$	Two critical conditions of electricity exchange between energy hub *eh* and grid, as maximum demand and maximum generation, respectively.
$P_{(.)}^{El,P}$, $P_{(.)}^{El,S}$	Electricity power purchased/sold from/to the grid.
$P_{(.)}^{El,PV}$, $P_{(.)}^{El,WT}$	Electricity power generated by photovoltaic and wind turbine, respectively.
$P_{(.)}^{G}$	Natural gas purchased from the grid.
$P_{(.)}^{G,CHP}$, $P_{(.)}^{G,Fu}$	Natural gas input of CHP and heat furnace.
$V_{n,\dots}$	Nodal voltages.
$\varphi_{0,t}^{co}$, $\varphi_{a,t}^{co}$	Binary utilization variables.
$\sigma_{a,t}^{co}$	Binary investment variables, which become 1 when investment alternative *a* should be performed on component *co* at year *t*, being 0 otherwise.
$\upsilon_{eh,\dots}^{md}$, $\upsilon_{eh,\dots}^{mg}$	Binary auxiliary variables.

I. Introduction

SHARE of distributed generation (DG) technologies in electricity production portfolio has increased in recent years. Integration of DG units in distribution grids can bring about various advantages such as transmission and distribution cost savings [1], congestion alleviation [2], deferral of investments in transmission and distribution grids [3], system reliability and resilience improvement [4], [5], as well as power quality enhancement [6]. Increased deployment of renewable energy sources (RESs) (mainly in the form of small-scale solar and wind generating systems) has been another major driving force for proliferation of DG units in distribution sector [7].

Considering the abovementioned benefits, it is anticipated that penetration of DG units in distribution grid would increase in the upcoming years. Since integration of these small-scale generating systems would considerably affect the performance of future distribution grids [8], they have to be incorporated in the planning studies of distribution companies (DISCOS). In this context, installation of DG units has been a common aspect of many publications in the field of electricity distribution planning.

A mixed integer non-linear programming (MINLP) model is developed in [9] for distribution network expansion planning (DNEP). In this paper, installation of DG units is considered as an alternative alongside traditional measures such as substation and feeder reinforcements. The results confirmed that optimal deployment of DG units can minimize total expansion cost of DISCO, improve the voltage profile, reduce network losses, and increase feeders' lifetime by reducing their loading levels. This model is further extended in [10] as a dynamic planning problem, and it is solved using genetic algorithm (GA). DNEP considering DG units is also investigated in [11] using a nonlinear model. It should be noted that construction of new feeders and substations are not modeled in the abovementioned works.

A MINLP model for DNEP considering gas-fired DG units is introduced in [12], where construction as well as reinforcement of network assets, and installation of DG units are considered as network expansion alternatives.

In contrast to the above-mentioned papers that only consider conventional DG units, renewable DGs are also modeled in [13]. Reactive capability limits of different RESs including wind turbine (WT), solar photovoltaic (PV), and biomass generating units are incorporated in the planning model of this paper, and uncertainties of load demand, wind speed, and solar radiation are captured using probabilistic models.

DG integration as well as conventional alternatives such as rewiring, network reconfiguration, and installation of new protection devices are considered in the expansion planning model of [14]. Uncertainties associated with DG power generation and load growth are taken into account, and multiple objectives such as reliability, energy losses, and power import cost are modeled in the optimization problem.

However, all the above mentioned models are nonlinear, and thus there are no guaranteed global optimal solutions for them. On this basis, some papers have focused on linearization of planning models. For example, a linearization technique to perform multistage DNEP including DGs is presented in [15]. The proposed mixed integer linear programming (MILP) model can be efficiently solved using mathematical methods such as branch-and-bound algorithm [15]. It is worth mentioning that locations and installation times of DG units are considered as input parameters of the planning model. The MILP model of [15] is further extended in [16] to incorporate DG integration plans. Both conventional and wind generating units are considered in this paper, although uncertainty of wind speed is neglected. This model is further developed in [17]–[19] where uncertainty of RESs and demand as well as network reliability costs, demand response (DR), and energy storage systems (ESSs) are taken into account.

A MILP formulation for multi-stage and multi-load scenario expansion planning of active DN is presented in [20]. This paper models the integration of new DG units and construction of new feeders at planning level, and addresses topology reconfigurations at the operation level.

Careful review of all the above-mentioned works reveals that they have assumed DG units are installed and operated by DISCOS. However, this is not the case in practice, as DISCOS are not legally allowed to own generating units in many countries [21]. Therefore, developing an efficient mathematical model is crucial for studying the interaction between autonomous DG units and DISCOS. Such a model should meet the following requirements:

- Guaranteed convergence to the optimal solution.
- Providing a tool for assessing the collaboration plans between DG owners and DISCOS.
- Systematically modeling the effects of various DG technologies on DN expansion.
- Capturing the inherent uncertainties.

On these bases, this paper aims at presenting such a mathematical model. The proposed model comprises efficient models for planning and operation of DG units and DN, which are in MILP form. Hence, they can be readily solved with global optimal solutions by the standard mathematical programming algorithms. In this model, energy hub (EH) concept is used for systematic integration of various DG technologies, and uncertainties of demand and renewable generation are represented using various scenarios. On this basis, the main contributions of this paper can be listed as follows:

- Deriving a linear model for joint expansion planning of electricity DN and DG units. This model can be used as a benchmark for assessment of collaboration plans between DG owners and DISCOS.
- Accounting for uncertainties of RESs output power as well as electricity and heat demands.
- Using EH concept to systematically model the effects of various DG technologies and customer heat and electricity demands on distribution grid expansion requirements.
- Proposing an efficient iterative method for incorporating power flow equations in the planning studies and accurate estimation of network energy loss costs.

The remainder of this paper is organized as follows. The problem under study is described in Section II. Detailed formulation of the proposed model is provided in Section III. Several case studies are presented in Section IV, followed by conclusions in Section V.

II. Problem Description

Traditional passive DNs were responsible for delivering electricity from transmission networks to medium and low voltage consumers. In this respect, for many years such networks were planned in anticipation of load growth in order to efficiently (from cost and reliability viewpoints) serve their customers. However, like most of the industries, DISCOS have also been affected by new technological advances. Integration of various DG technologies has been one of the major changes in this field. In fact, recent technical developments alongside regulatory supports have resulted in increased economic efficiency of small-scale generating units. Therefore, as schematically depicted in Fig. 1, customers are not forced to purchase all their electricity needs from distribution grid anymore. Moreover, as illustrated, they can sell their excess power to the grid. Consequently, electricity consumption pattern at various load points are changed owing to the local electricity generation. This, in turn, can have positive or negative influences on DN planning and operation [22]. In other words, integration of DG units can positively or negatively affect DN investment cost, energy losses, and voltage profile, depending on various factors such as their penetration, location, size, and operating conditions [22]. In order to fully exploit potential DG benefits for network or at least avoid their drawbacks, collaboration between DISCOS and DG owners would be crucial.

Regarding DG integration, DISCOS face a new situation which is depicted in Fig. 2. Accordingly, there are some new generating units as well as upcoming loads to be connected to the grid for which network must be expanded to accommodate them. To handle this new situation, beside prevalent alternatives including construction and reinforcement of feeders, substations and connections to the transmission network, distribution system planners can also collaborate with DG owners to achieve more effective solutions. In this respect, an efficient mathematical model is essential to assess all profits of such collaborations. To this end, efficient formulations for optimal planning and operation problem of DG owners and DISCOS are derived in this paper. Subsequently, benefits of collaborative planning are quantified employing the proposed models.

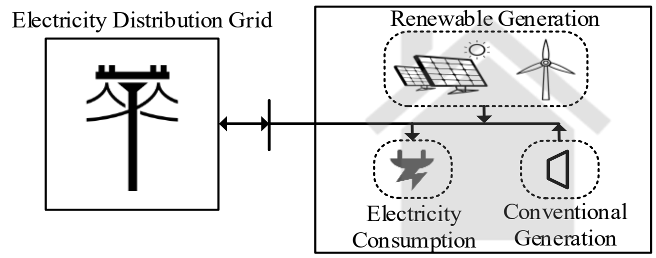

Fig. 1. Schematic structure of customers in an active distribution network.

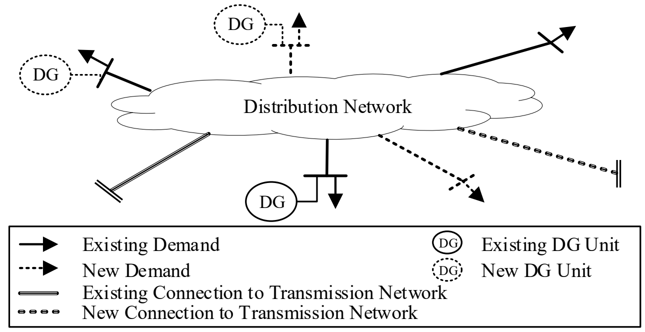

Fig. 2. Distribution companies' responsibilities in the new situation: they need to provide enough capacity for upcoming local generation and demands.

It is worth noting that, although discussion of practical tools (such as various kinds of contracts) for realizing such collaborations is out of the scope of this paper, the proposed model can also be utilized to evaluate the effectiveness of those tools. In this context, results of the collaborative plans obtained here can be employed as a benchmark for assessing the effectiveness of any kind of contracts between DISCOS and DG owners.

III. Problem Formulation

A. Planning and Operation of DG Units Modeled as EH

In order to systematically model various DG technologies, the well-known EH concept is utilized in this paper. This concept has been developed in response to the enormous demand for improving energy efficiency through considering the interaction between various energy carriers [23], [24]. Using this model, it is possible to systematically explore the effects of combined heat and power (CHP) units on net energy demand of customers. Structure of the intended EH is depicted in Fig. 3. As shown, various energy sources including electricity and natural gas from distribution grids as well as solar and wind power are considered in the model. These energy sources are then processed by EH components to serve electricity and heat demands at the output layer [25]. In order to achieve efficient performance of EHs, various components should be optimally planned and utilized. In this respect, present value of total investment and operating costs of supplying EH demands, as expressed in (1a), is considered as the objective function of the EH planner. As a typical method for evaluating the net present value of a cash flow, an infinite perpetuity is considered [16].

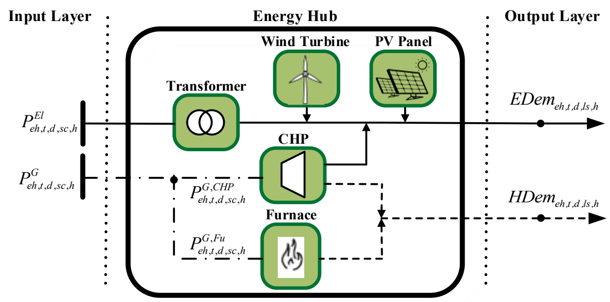

Fig. 3. Energy hub structure.

According to this technique, operating cost of the last year of planning horizon T is repeated in the following years as expressed by the second term on the right-hand side of (1a) [16]. Moreover, by considering perpetuity factor δ^{co}, an infinite stream is considered for investment costs, i.e. it is assumed that each equipment is replaced by the same one after reaching its useful lifetime (UL^{co}).

$$OF^{EH} = \sum_{t=1}^{T}\left(\frac{Inv_t^{EH} + Op_t^{EH}}{(1+i)^{t-1}}\right) + \frac{Op_T^{EH}}{i(1+i)^{T-1}} \tag{1a}$$

$$Inv_t^{EH} = \sum_{eh\in EH}\sum_{co\in EHC} TI_{eh,t}\delta^{co} IC^{co} Cap_{eh}^{co} \tag{1b}$$

$$Op_t^{EH} = \sum_{eh\in EH}\sum_{co\in EHC}\left(OMC^{co}Cap^{co}\sum_{\tau=1}^{t} TI_{eh,t}\right) + \sum_{eh\in EH}\sum_{d\in D_t}\sum_{sc\in TSc_d}\left(N_d p_{sc}\sum_{h=1}^{24}\left(P^{El,P}_{eh,t,d,sc,h} - \alpha P^{El,S}_{eh,t,d,sc,h}\right)EPr_{d,h} + P^{G}_{eh,t,d,sc,h}GPr_{d,h}\right) \tag{1c}$$

$$\delta^{co} = (1+i)^{UL^{co}} \Big/ \left((1+i)^{UL^{co}} - 1\right) \tag{1d}$$

As can be inferred from (1b), it is also assumed that the total investment costs of a given EH are spent at the time step when the EH operator is going to connect the EH to the distribution grids in order to start operation. Operating cost is also formulated as (1c) which comprise operation and maintenance costs of various equipment as well as the costs associated with electricity transactions with power DN and purchasing natural gas from the grid. It is worth mentioning that parameter α is employed to model the difference between purchasing/selling prices from/to DN owing to the network use-of-system charges. It should be noted that in each hour, only one of the two non-negative variables $P^{El,S}_{eh,t,d,sc,h}$ and $P^{El,P}_{eh,t,d,sc,h}$ can take a non-zero value. However, there is no need to explicitly impose this constraint, since it will be automatically enforced during the optimization process. This is because the electricity power selling price is lower than the purchasing price, so it is not optimal to inject power to the grid while the EH needs to purchase electricity.

As can be seen from (1c), in order to reduce the computational burden, some representative days of a year are considered. Subsequently, various scenarios are taken into account for hourly wind power, solar radiation and load patterns in the representative days to model the corresponding uncertainties. Therefore, the cost calculated by (1c) is expected value of annual operating costs of the EHs for all scenarios.

The above objective function is subject to various investment and operating constraints, as follows:

$$Cap_{eh}^{co} \le MCap_{eh}^{co} \tag{2}$$

$$\eta^{Tr}P^{El}_{eh,t,d,sc,h} + \eta^{CHP}_{GE}P^{G,CHP}_{eh,t,d,sc,h} + P^{El,PV}_{eh,t,d,sc,h} + P^{El,WT}_{eh,t,d,sc,h} = EDem_{eh,t,d,ls,h} \tag{3a}$$

$$P^{El}_{eh,t,d,sc,h} = P^{El,P}_{eh,t,d,sc,h} - P^{El,S}_{eh,t,d,sc,h} \tag{3b}$$

$$\eta^{CHP}_{GH}P^{G,CHP}_{eh,t,d,sc,h} + \eta^{Fu}P^{G,Fu}_{eh,t,d,sc,h} = HDem_{eh,t,d,ls,h} \tag{4}$$

$$P^{G,CHP}_{eh,t,d,sc,h} + P^{G,Fu}_{eh,t,d,sc,h} = P^{G}_{eh,t,d,sc,h} \tag{5}$$

$$P^{El,P}_{eh,t,d,sc,h} + P^{El,S}_{eh,t,d,sc,h} \le Cap^{Tr}_{eh} \tag{6a}$$

$$\eta^{CHP}_{GE}P^{G,CHP}_{eh,t,d,sc,h} \le Cap^{CHP}_{eh}\ ,\ \ P^{Fu}_{eh,t,d,sc,h} \le Cap^{Fu}_{eh} \tag{6b}$$

$$P^{El,WT}_{eh,t,d,sc,h} \le \gamma^{WS}_{d,ws,h}Cap^{WT}_{eh}\ ,\ \ P^{El,PV}_{eh,t,d,sc,h} \le \gamma^{SR}_{d,ss,h}Cap^{PV}_{eh} \tag{6c}$$

Equation (2) indicates the maximum possible capacity (due to practical limitations such as space availability) of each equipment which can be installed in a given EH. Electricity, heat and natural gas balance in the EHs are formulated in (3)−(5), respectively. Capacity limitations of various EH equipment are also modeled by (6a)−(6c). Note that for renewable units, equation (6c) ensures that power production do not exceed the assoicated maximum available power. In this context, coefficient γ ($0\le \gamma \le 1$), which is the ratio of the renewable unit's available power to its capacity at each scenario, is used for determination of the maximum power available from these units. The γ values are input data of the problem, and are determined based on solar radiation and wind speed data [26].

B. *Multistage Distribution System Expansion Planning*

This problem generally aims at finding optimal investment plan to provide adequate network capacity to serve upcoming demands and DGs, which are assumed to be balanced [15], [16], [19]. Such investment solution includes optimal sets of feeders and substations, which should be reinforced or constructed in each year. Accordingly, objective function of DNEP problem can be expressed as (7). Analogous to (1a), expression (7) is derived based on an infinite perpetuity. Thus, the second term on the right-hand side of (7) represents the present value of the infinite stream of Op_T^{DN} following the last stage of the planning horizon.

$$OF^{DN} = \sum_{t=1}^{T}\left(\frac{Inv_t^{DN} + Op_t^{DN}}{(1+i)^{t-1}}\right) + \frac{Op_T^{DN}}{i(1+i)^{T-1}} \tag{7}$$

Assuming various investment options for each network component, and considering a binary variable corresponding to each investment alternative, *a,* in a given year, *t*, the investment cost is formulated as (8a). It should be noted that subscript *t* of the binary investment variables $\sigma^{co}_{a,t}$ implies that the problem is a multistage planning study. In other words, solving this problem determines not only the optimal investment alternatives, but also the year at which each of these plans should be performed. Moreover, as a common assumption in the literature of DNEP, it is assumed that only one investment action is allowed to be performed on each network component during the planning horizon, as expressed by (8b) [15], [16], [19].

$$Inv_t^{DN} = \sum_{co\in DNC}\sum_{a\in CA^{co}} \delta^{co}\sigma^{co}_{a,t}IC^{co}_{a} \tag{8a}$$

$$\sum_{t=1}^{T}\sum_{a\in CA^{co}} \sigma^{co}_{a,t} \le 1 \tag{8b}$$

Since some network assets might not be utilized during the operation (for example some feeder sections are switched off to ensure the network radiality), some binary utilization variables should also be considered for each component. In this respect, in addition to considering binary variables regarding the utilization of various investment alternatives $\varphi_{a,t}^{co}$, a binary variable, $\varphi_{0,t}^{co}$ is also assigned to indicate the utilization of initial state of each component. Using these variables, operating cost of DN is formulated as (9a). However, these binary variables are also subject to some logical constraints. Equation (9b) restricts the utilization of component alternative a prior to performing the corresponding investment. Since after doing any of the candidate investment alternatives on a component, its initial state is not available anymore, (9c) must be imposed. For instance, candidate alternatives for an existing feeder might be various options for enhancing its capacity. From the year when one of these alternatives is selected, only the state associated with the chosen alternative can be utilized. Accordingly, as an investment alternative is chosen, the associated binary variable $\sigma_{a,t}^{co}$ becomes one, and makes the right-hand side of (9c) zero. It should be noted that according to (8b) only one of the binary variables $\sigma_{a,t}^{co}$ can be one. Moreover, (9d) forces $\varphi_{0,t}^{co}$ of all components which are not initially existent (e.g. a new feeder section which is candidate to be constructed) to zero.

$$Op_t^{DN} = \sum_{co\in DNC}\left(\varphi_{0,t}^{co}OMC_0^{co} + \sum_{a\in CA^{co}}\varphi_{a,t}^{co}OMC_a^{co}\right) + \lambda_t Loss_t \tag{9a}$$

$$\varphi_{a,t}^{co} \le \sum\nolimits_{\tau=1}^{t}\sigma_{a,\tau}^{co} \tag{9b}$$

$$\varphi_{0,t}^{co} \le 1 - \sum\nolimits_{a\in CA^{co}}\sum\nolimits_{\tau=1}^{t}\sigma_{a,\tau}^{co} \tag{9c}$$

$$\varphi_{0,t}^{co} \le \mu^{co} \tag{9d}$$

The DNEP problem is also subject to various technical constraints associated with DN operation such as capacity limits of feeders and substations, and nodal voltage limits. In order to obtain the nodal voltages and line currents for formulating these constraints and also calculating network losses, power flow equations should also be incorporated into the model. Although power flow equations are nonlinear in nature, various consistent linearized approximate models can be found in the literature. In this paper, we have employed the linearization technique proposed in [15]. Using this method, power flow equations of a DN can be expressed in terms of mixed-integer linear equality and inequality constraints, as generally stated in (10a) and (10b).

$$\varepsilon(V_{n,t,d,sc,h}, I_{b,t,d,sc,h}, P_{eh,t,d,sc,h}^{El}, \varphi_{0,t}^{co}, \varphi_{a,t}^{co}) = 0 \tag{10a}$$

$$u(V_{n,t,d,sc,h}, I_{b,t,d,sc,h}, P_{eh,t,d,sc,h}^{El}, \varphi_{0,t}^{co}, \varphi_{a,t}^{co}) \ge 0 \tag{10b}$$

Moreover, by solving these power flow equations and calculating the loss associated with each state using a piecewise linear approximation [16], the annual network energy loss can be formulated as follows:

$$Loss_t = \sum\nolimits_{d\in D_t}\sum\nolimits_{sc\in TSc_d}\sum\nolimits_{h=1}^{24} N_d p_{sc} Loss_{t,d,sc,h} \tag{11}$$

Nonetheless, solving the resulting optimization problem is not practically possible owing to the enormous number of decision variables and constraints associated with power flow equations. This is because power flow must be performed for all scenarios and time stages.

The main reasons of incorporating power flow equations into the optimization model can be listed as follows:

1) Obtaining nodal voltages and current flow of various feeder sections to impose operational constraints.
2) Calculating network energy loss.

In order to reach the first goal in an efficient way, instead of calculating the load flow equations for all states, we only consider two critical cases: maximum load and maximum generation conditions. The first case represents the situation in which the nodal electricity consumption at each load point has its maximum value, i.e. the annual peak load. For a passive network, this is the only critical condition to be considered. Hence, passive DNEP models are traditionally performed based on this load condition [27]. However, for an active DN, there is another critical condition: a situation in which the net power generation of each node (power injection to the grid) is at the maximum value. Accordingly, based on Fig. 3, it can be said that these two critical conditions are equivalent to the maximum and minimum values of $P_{eh,t,d,sc,h}^{El}$, as below:

$$P_{eh,t,md}^{CC} = max\{P_{eh,t,d,sc,h}^{El}\}_{\forall d\in D_t, sc\in TSc_d, h\in\{1,\dots,24\}} \tag{12a}$$

$$P_{eh,t,mg}^{CC} = min\{P_{eh,t,d,sc,h}^{El}\}_{\forall d\in D_t, sc\in TSc_d, h\in\{1,\dots,24\}} \tag{12b}$$

Equation (12a) can be written in a linear form as follows:

$$P_{eh,t,md}^{CC} \ge P_{eh,t,d,sc,h}^{El} \tag{13a}$$

$$P_{eh,t,md}^{CC} \le P_{eh,t,d,sc,h}^{El} + M(1-\upsilon_{eh,t,d,sc,h}^{md}) \tag{13b}$$

$$\sum\nolimits_{d\in D_t}\sum\nolimits_{sc\in TSc_d}\sum\nolimits_{h=1}^{24}\upsilon_{eh,t,d,sc,h}^{md} = 1 \tag{13c}$$

Constraint (13a) indicates that for a given load point connected to hub eh, peak demand of year t, $P_{eh,t,md}^{CC}$, is greater than or equal to the demand of all time steps within year t. This forces the $P_{eh,t,md}^{CC}$ to take a value greater than or equal to the annual peak load. In addition, expressions (13b) and (13c) state that $P_{eh,t,md}^{CC}$ must be lower than or equal to exactly one of the hourly loads of year t. This is because in case binary variable $\upsilon_{eh,t,d,sc,h}^{md}$ is zero, constraint (13b) is relaxed due to the presence of big number, M, in the right-hand side of the equation. On the other hand, when this binary variable equals to one, the big number, M, is eliminated and (13b) enforces $P_{eh,t,md}^{CC}$ to be lower than or equal to $P_{eh,t,d,sc,h}^{El}$. In addition, according to (13c), only one of the binary variables, $\upsilon_{eh,t,d,sc,h}^{md}$, can be equal to one. Hence, the set of equations (13) states that $P_{eh,t,md}^{CC}$ is greater or equal to all $P_{eh,t,d,sc,h}^{El}$ and at the same time, is lower than or equal to one of them. Therefore, these equations set the $P_{eh,t,md}^{CC}$ to the maximum value of $P_{eh,t,d,sc,h}^{El}$ in year t. Similarly, equation (12b) can be expressed by (14a)–(14c).

$$P_{eh,t,mg}^{CC} \le P_{eh,t,d,sc,h}^{El} \tag{14a}$$

$$P_{eh,t,mg}^{CC} \ge P_{eh,t,d,sc,h}^{El} - M(1-\upsilon_{eh,t,d,sc,h}^{mg}) \tag{14b}$$

$$\sum\nolimits_{d\in D_t}\sum\nolimits_{sc\in TSc_d}\sum\nolimits_{h=1}^{24}\upsilon_{eh,t,d,sc,h}^{mg} = 1 \tag{14c}$$

Now, the power flow equations need to be calculated merely for these two critical cases, as generally expressed by (15a) and (15b).

$$\varepsilon(V_{n,t,k}, I_{b,t,k}, P_{eh,t,k}^{CC}, \varphi_{0,t}^{co}, \varphi_{a,t}^{co}) = 0, \quad k\in\{md, mg\} \tag{15a}$$

$$u(V_{n,t,k}, I_{b,t,k}, P^{CC}_{eh,t,k}, \varphi^{co}_{0,t}, \varphi^{co}_{a,t}) \geq 0, \quad k \in \{md, mg\} \tag{15b}$$

In order to achieve the second goal of incorporating the power flow equations into the model, i.e. annual network energy loss calculation, the following equation is utilized:

$$Loss^{est}_{t} = 8760\left(\frac{Loss^{CC}_{t,md} + Loss^{CC}_{t,mg}}{2}\right)\zeta \tag{16}$$

However, the accurate value of loss factor, ζ, depends on the network topology and operating condition, which both are the outcomes of the optimization model. Hence, in order to obtain the precise value of this parameter, an iterative method is proposed in this paper. Detailed discussion of this technique is presented in the following subsection.

C. Proposed Iterative Framework

Structure of the proposed procedure for solving the presented optimization formulation is illustrated in Fig. 4. As shown, the algorithm starts with setting an initial value for loss factor. Then, this value is sent to General Algebraic Modeling System (GAMS) environment, where the mixed-integer linear models proposed for EH and DNEP problems are solved using the CPLEX solver. It should be noted that the arrow between electricity distribution and EH planning models represents the interdependence of these cost functions, which is determined by the type of collaboration. In this respect, two extreme scenarios will be investigated in this paper, as follows:

- ***Independent Planning:*** This is equivalent to the traditional state of DN and DG planning. In this framework, EH planners autonomously perform the expansion planning of DGs. Their required connection capacity is then announced to the grid planners and the network is expanded to serve these demands. Accordingly, in our model, the EH planning model is solved at first. Then, the obtained results for $P^{El}_{eh,t,d,sc,h}$ are utilized as the input data for solving the DNEP problem.
- ***Ideal Collaboration:*** In this framework, the objective is to minimize the total expansion cost of serving customers' demand. Hence, both planning models should be simultaneously solved, considering the total objective function as the sum of OF^{EH} and OF^{DN}.

It is worth mentioning that these two cases are evaluated as the benchmarks for quantifying the benefits of collaborative planning. However, the proposed model is not limited to these frameworks and can be utilized to assess other types of collaborative plans, as well.

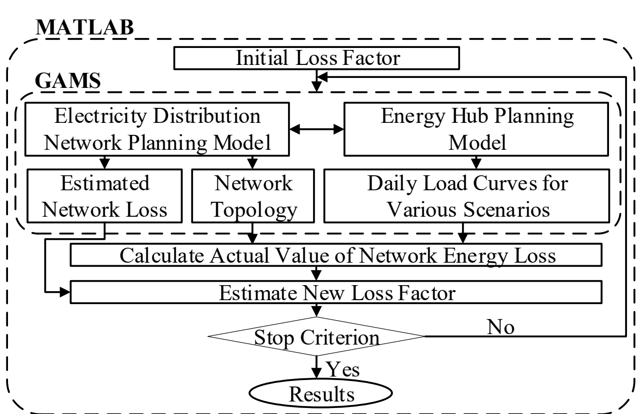

Fig. 4. Flowchart of the proposed optimization technique.

Once the solutions are achieved, the results are returned to MATLAB and by solving the power flow equations for all scenarios and time steps, the actual value of annual energy losses are obtained using (11). Subsequently, the new loss factor is calculated by (17).

$$\zeta^{new} = \left(\sum\nolimits_{t=1}^{T} Loss^{act}_{t} \Big/ \sum\nolimits_{t=1}^{T} Loss^{est}_{t}\right)\zeta^{old} \tag{17}$$

Finally, the stopping criterion is checked by comparing this value of loss factor with the previous one, as expressed by (18). Accordingly, as the rate of loss factor variation becomes less than a specific value, *err,* the algorithm would stop.

$$\frac{\left|\zeta^{new} - \zeta^{old}\right|}{\zeta^{old}} \leq err \tag{18}$$

IV. Numerical Results

This section is devoted to the application of the proposed model to two distribution test networks with 18 and 54 nodes. As previously mentioned, the optimization models are implemented in MATLAB and GAMS (Version 24.7.4). CPLEX 12.6 is used for solving the GAMS model, and the associated optimal gap tolerances were respectively set to 0.1% and 1% for the 24-node and 54-node test grids. Stopping criterion (*err* in (18)) is set to 0.01, and the optimization models are implemented on a Fujitsu Celsius W530 POWER PC with a Quad 3.30 GHz Intel Xeon E3-1230 processor and 32 GB of RAM.

A. The 18-node Test Grid

In order to demonstrate the application of the proposed model, several case studies are defined for the 18-node test distribution system (Fig. 5). Feeder sections of this network are categorized into three groups: *Fixed branches* which exist at the beginning of planning horizon, and there are no plans for their reinforcement. *Candidate branches for reinforcement* are initially existent likewise, but the network planner has some candidate plans to reinforce them. Finally, in order to serve new load points, there are some candidate plans for network expansion through construction of new feeder sections. These branches are called *Candidate branches for construction.*

Six load points are considered as the candidate places for establishment of EHs, which are depicted as shaded circles in Fig. 5. A planning horizon of three years is considered. Two representative days in summer and winter are used for planning, where each day consists of 24 hourly load steps. In order to capture different system uncertainties, Monte Carlo simulation method is used to generate a suitable set of scenarios, and they are reduced to 12 final scenarios using the backward reduction technique [28]. In this context, three scenarios for wind speed,

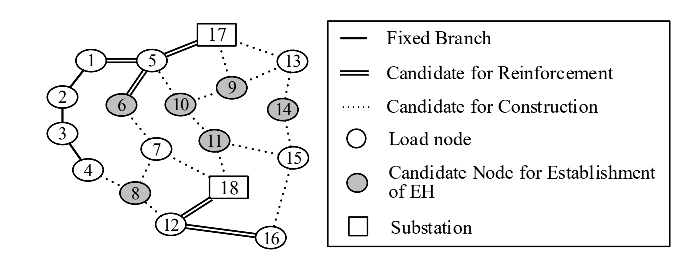

Fig. 5. 18-node test distribution system.

two scenarios for solar radiation, and two scenarios for electricity as well as heat demand are considered as representative scenarios. It is worth mentioning that the Cartesian product of sets of wind speed, solar radiation, as well as demand scenarios yields the set of the final scenarios. Hence, the number of the final scenarios is equal to the product of the number of each set of scenarios. Moreover, it is assumed that prediction errors of electricity as well as heat demands, wind, and solar power follow the normal distributions with standard deviations equal to 3%, 3%, 10%, and 5% of the forecasted values, respectively.

Based on the electricity price in Finland, a time of use (ToU) tariff including 4 levels with an average price of 120 $/MWh is assumed. Average natural gas price is also set based on Finland's prices, and its value is 23 $/MWh. Additionally, feed-in tariff of DG units is assumed to be 50% of the retail price. The other case study parameters can be found in [29].

In order to better demonstrate the benefits of collaborative planning of distribution system and EHs, three cases are analyzed as follows. The former two cases are based on independent planning and the third case is according to the ideal collaboration plan introduced earlier. It should be noted that due to space limitation, detailed topology of optimal expansion plans in different cases have been uploaded at [29].

1) Case I: Passive Distribution System Planning

In this case, it is assumed that EHs do not install DG units. In other words, they purchase natural gas and electricity to serve their demands. On this basis, investment cost of EHs is comprised of transformer and heat furnace installation costs. The results of multistage planning of distribution grid and EHs are reported in Table I (Note: Trans. denotes EH transformer). As can be observed, total cost of EHs' objective function is considerably higher than sum of the associated investment and operation costs. This is due to consideration of an infinite perpetuity, as mentioned in (1a). Moreover, in contrast to distribution grid, a significant share of EH objective function is related to the perpetuity of operating cost. In other words, high costs are required for fulfilling the demand of passive EHs.

2) Case II: Separate Planning

In this case, it is assumed that the DISCO has no control over the installation as well as operation of DGs. In other words, EHs are planned in the first step, without considering DN expansion. The obtained results are then used as the input data for DNEP.

The results are summarized in Table II. As can be seen, objective function of EHs has significantly decreased due to the low cost electricity production of RESs as well as high overall efficiency of combined heat and power (CHP) units. Interestingly, operating cost of EHs in the first and second stages are negative, which indicates that EHs have gained profit from selling electricity to the grid. On the other hand, objective function of distribution system is also decreased as a result of DG deployment. This observation is a result of overall loss reduction

TABLE II. OUTCOMES FOR CASE II (ALL COSTS ARE IN M$)

		t=1	t=2	t=3	Total
OF^{EH}	Inv_t^{EH}	1.606	1.112	0	25.9560
	Op_t^{EH}	-0.659	-0.226	0.944	
OF^{DN}	Inv_t^{DN}	14.731	5.263	0	19.7803
	Op_t^{DN}	0.0135	0.0201	0.0257	
Total Cost		**45.7363**			
Component	Trans.	Furnace	CHP	PV Panel	Wind Turbine
Total Capacity (MW)	5.01	1.59	1.91	3.70	2.70

in the grid (annual loss costs (M$) for Case II are respectively 0.0131, 0.0195 and 0.0251, while the associated values for Case I are 0.0129, 0.02, and 0.0326). Total capacity of transformers has increased with respect to Case I, as EHs need more transformer capacity to export their excess electricity to the grid. On the contrary, total capacity of heat furnaces is decreased owing to the heat production of CHP units. The results also reveal that EHs would install maximum allowable capacity of renewables in Case II (total allowable capacities of PV and WT at all buses were set to 3.7MW and 2.7MW, respectively).

3) Case III: Collaborative Planning

Collaborative planning of EHs and electricity DN is investigated in Case III. In other words, the objective function is to minimize the sum of EHs' and DN's objective functions. This case can be imagined as ideal collaboration between EH and distribution grid planners. Hence, the obtained results in Case III can be considered as a benchmark to compare the effectiveness of different contracts between these entities.

As per Table III, although EHs' cost is slightly increased compared to Case II, DN's cost is significantly decreased and as a result, total costs have fallen. Similar to Case II, all available capacity of RESs are utilized. However, total capacity of installed transformer is lower than Case II. This is because the peak injected power of EHs' to the grid is decreased, thus, a lower transformer capacity is required. Accordingly, a lower network capacity is needed and as a result, network investment cost is decreased compared to Case II.

On the other hand, installed capacity of CHP is slightly lower than Case II, while furnace capacity is significantly increased. This is a result of reduced EHs' injected power for network expansion cost saving. Therefore, a lower capacity of CHP units are installed and furnaces are used to fulfil the heat load instead.

4) Discussion of the Proposed Optimization Model

In this subsection, salient features of the proposed optimization model are discussed based on the conducted case studies. The associated information are summarized in Table IV.

***Running Time*:** Among the three cases, simulation time of Case I is the lowest, since the network is passive and there are no decision variables associated with planning and operation of DG units. In contrast, Case III where EH and DN planning are simultaneously conducted as a single optimization problem, has

TABLE I. OUTCOMES FOR CASE I (ALL COSTS ARE IN M$)

		t=1	t=2	t=3	Total
OF^{EH}	Inv_t^{EH}	0.473	0.368	0	84.8445
	Op_t^{EH}	0.798	1.596	3.192	
OF^{DN}	Inv_t^{DN}	15.422	4.741	0	20.0657
	Op_t^{DN}	0.0133	0.0205	0.0331	
Total Cost		**104.9102**			
Component	Trans.	Furnace	CHP	PV Panel	Wind Turbine
Total Capacity (MW)	3.920	3.916	-	-	-

TABLE III. OUTCOMES FOR CASE III (ALL COSTS ARE IN M$)

		t=1	t=2	t=3	Total
OF^{EH}	Inv_t^{EH}	1.603	1.121	0	26.0167
	Op_t^{EH}	-0.652	-0.214	0.946	
OF^{DN}	Inv_t^{DN}	13.280	3.811	0	17.0063
	Op_t^{DN}	0.0154	0.0202	0.0251	
Total Cost		**43.0230**			
Component	Trans.	Furnace	CHP	PV Panel	Wind Turbine
Total Capacity (MW)	4.74	1.72	1.89	3.70	2.70

TABLE IV. COMPARISON OF DIFFERENT CASES (ALL COSTS ARE IN M$)

	Case I	Case II	Case III
Simulation Time of Each Iteration (min)	4.70	15.29	612.59
Loss Factor	0.1310	0.1690	0.1543
Final Mismatch	2.11E-16	0	1.28E-3
Number of Iterations	2	4	4

the longest run time. This is due to the increased number of variables and constraints in the associated problem.

***Loss Factor*:** As expected, the value of loss factor varies for different planning and operating conditions. Hence, instead of using a predetermined value for loss factor in the net planning studies (such as [27]), it is more accurate to employ the proposed iterative method. The obtained results confirmed the efficiency of the introduced method, where all cases converged in a few iterations.

***Iterations*:** The number of iterations for Case I is lower than the other cases. This is because in Cases II–III, electricity demand of EH nodes ($P^{El}_{eh,t,d,sc,h}$) are decision variables of the optimization problem. These variable demands, in turn, can change the network losses. Hence, there are more variables influencing the network loss in Cases II–III, which increases the number of iterations needed to obtain the accurate value of loss factor.

***Convergence of Power Flow Equations*:** In the proposed model, it is assumed that if network constraints are satisfied for the two critical situations (maximum demand and generation), they would be satisfied for all other time steps as well. During calculation of the actual value of energy loss in MATLAB environment, we solved the power flow equations for all time steps and checked the results. In all occasions, power flow equations were successfully converged, which validates our assumption.

5) Sensitivity Analysis

In this part, a sensitivity analysis is provided to explore the impacts of electricity and natural gas prices on the obtained results. In this regard, four situations are compared with the base case (S0) as follows:

- **S0:** Base case, in which average electricity and natural gas prices are based on the prices in Finland (120$/MWh and 23$/MWh, respectively).
- **S1:** Average electricity price is half of that of **S0**.
- **S2:** Average electricity price is 1.5 times of that of **S0**.
- **S3:** Average gas price is half of that of **S0**.
- **S4:** Average gas price is 1.5 times of that of **S0**.

The results are given in Table V and Figs 6 and 7. As can be seen in the figures, Case I is more sensitive to the electricity price, while the other two cases are more sensitive to the gas price. The reason for this is that in Case I customers are forced to purchase all their electricity demand from the grid and thus any increase in the electricity price will highly affect their costs. It is worth noting that in the simulations, the elasticity of electricity demand was neglected. In other words, it was assumed that customers would not change their consumption behavior when electricity price is increased. In contrast, customers in Cases II–III can produce part of their required electricity and even sell their excess generation to the grid. Hence, they are more sensitive to the natural gas price in these cases, since they produce part of their electricity demand using gas-fired CHP units.

TABLE V. SENSITIVITY ANALYSIS RESULTS (ALL COSTS ARE IN M$)

		S1	S2	S3	S4	S0
Case I	OF^{EH}	47.15	122.54	80.52	89.17	84.84
	OF^{DN}	19.90	20.23	20.07	20.07	20.07
	Total Cost	67.05	142.77	100.59	109.23	104.91
Case II	OF^{EH}	21.67	30.14	18.48	33.42	25.96
	OF^{DN}	19.65	19.91	19.78	19.78	19.78
	Total Cost	41.32	50.05	38.26	53.20	45.74
Case III	OF^{EH}	21.89	30.22	18.56	33.45	26.02
	OF^{DN}	16.61	17.13	17.00	17.00	17.01
	Total Cost	38.50	47.35	35.56	50.45	43.02

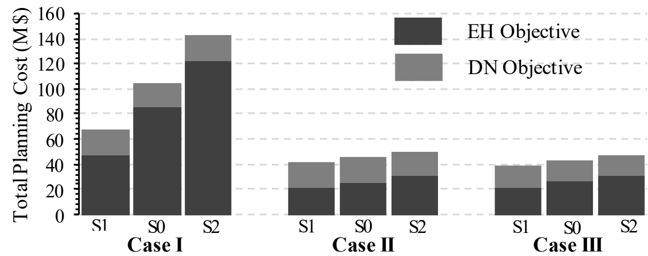

Fig. 6. Sensitivity analysis on electricity price.

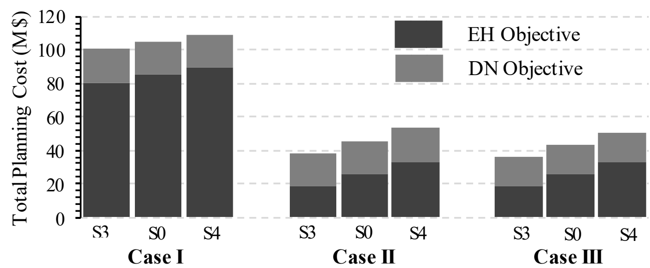

Fig. 7. Sensitivity analysis on gas price.

According to Table V, it can be claimed that at higher DG penetrations, an increased value of electricity price would result in higher expansion costs, as DG owners tend to inject more power to the DN (compare OF^{DN} in S1 and S2 with that of S0 for Cases II and III). Moreover, it can be concluded that DN expansion cost is not sensitive to the natural gas price. On the other hand, installation of DG units decrease DN expansion costs, whether DGs are controlled by DISCO or not (in all states, OF^{DN} in Case I is higher than that of Cases II and III). Finally, it can be concluded that the value of DN expansion cost reduction caused by DG integration would be higher, if DISCO determines its expansion plan with collaboration of DG owners (compare OF^{DN} of Case III with that of Case II).

6) Sensitivity Analysis on the Number of Scenarios

In order to assess the effect of the uncertainty modeling on the obtained solution, two cases, namely a deterministic case and a case with more scenarios are considered in this part. In the deterministic case, the uncertain input parameters including wind and solar patterns as well as daily heat and electricity demand curves for each representative day are set to their expected values. Running the collaborative planning model for this deterministic case yields the results provided in Table VI.

The total number of scenarios is increased to 24 for the latter case, and, hence, is doubled with respect to the base case with 12 scenarios. The outcomes of the collaborative planning model for this case are also represented in Table VII.

Comparing these results with those reported in Table III reveals that the investment cost of the distribution grid Inv^{DN}_{t} is identical for all cases. Thus, uncertainties of the input

TABLE VI. OUTCOMES FOR THE DETERMINISTIC CASE (ALL COSTS ARE IN M$)

		t=1	t=2	t=3	Total
OF^{EH}	Inv_t^{EH}	1.583	1.103	0	26.0199
	Op_t^{EH}	-0.652	-0.221	0.948	
OF^{DN}	Inv_t^{DN}	13.280	3.811	0	17.0432
	Op_t^{DN}	0.0172	0.0221	0.0288	
Total Cost		**43.0631**			

Component	Trans.	Furnace	CHP	PV Panel	Wind Turbine
Total Capacity (MW)	4.57	1.90	1.90	3.70	2.70

TABLE VII. OUTCOMES FOR THE CASE WITH INCREASED NUMBER OF SCENARIOS (ALL COSTS ARE IN M$)

		t=1	t=2	t=3	Total
OF^{EH}	Inv_t^{EH}	1.605	1.121	0	25.5108
	Op_t^{EH}	-0.658	-0.229	0.927	
OF^{DN}	Inv_t^{DN}	13.280	3.811	0	17.0235
	Op_t^{DN}	0.016	0.021	0.027	
Total Cost		**42.5343**			

Component	Trans.	Furnace	CHP	PV Panel	Wind Turbine
Total Capacity (MW)	4.84	1.69	1.89	2.7	3.7

parameters do not affect the network expansion plans. However, the investment as well as operating costs of the EHs and the operating cost of the distribution grid are influenced by these uncertainties. According to the results provided in Tables III, VI, and VII, increasing the number of scenarios, which, in turn, enhances the precision of the model, results in lower total costs. In other words, the deterministic case (Table VI) has the highest total cost, whereas the case with 24 scenarios (Table VII) features the lowest total planning cost.

With regard to the convergence of the proposed model, the deterministic case converges after 6 iterations and reaches to the final loss factor of 0.0919, whereas the case with increased number of scenarios converges to the final loss factor of 0.0819 after 4 iterations. The simulation time for each iteration of the deterministic case and the case with increased number of scenarios are 0.24 and 10.76 hours, respectively.

7) Sensitivity Analysis on the Number of Representative Days

In this part, the number of representative days is doubled to investigate its effect on the planning solution. In other words, four representative days, i.e., one day per season, are considered. It should be noted that a representative day is not a specific day in the associated season. In fact, we have selected four months in different seasons, namely February (winter), May (spring), August (summer), and November (fall). Subsequently, mean values of daily wind and solar patterns as well as daily heat and electricity demand curves for each representative day are set to the associated average values in that month. For example, average values of variable parameters (wind speed, solar radiation, and load) in February are used for the day that represents winter, and average values of variable parameters in May are used for the day representing spring. Nonetheless, this is not a general rule, and the representative days can be selected in a different fashion based on the depth of study and the desired accuracy.

Solving the collaborative planning model for this case results in the outcomes presented in Table VIII. As per this table, the total objective of the EHs is considerably increased compared to Case III (Table III). Consequently, the total planning cost is

TABLE VIII. OUTCOMES FOR THE CASE WITH INCREASED NUMBER OF REPRESENTATIVE DAYS (ALL COSTS ARE IN M$)

		t=1	t=2	t=3	Total
OF^{EH}	Inv_t^{EH}	1.580	1.102	0	30.3769
	Op_t^{EH}	-0.344	-0.034	1.100	
OF^{DN}	Inv_t^{DN}	11.828	5.263	0	17.1218
	Op_t^{DN}	0.0270	0.0374	0.0494	
Total Cost		**47.4987**			

Component	Trans.	Furnace	CHP	PV Panel	Wind Turbine
Total Capacity (MW)	4.62	1.55	1.86	3.70	2.70

markedly higher for the case with four representative days, which is more accurate than the previous case with two representative days. This reveals the significance of selecting appropriate representative days for performing the planning studies.

It is worth mentioning that for the case presented in this part, the model converges after two iterations reaching the final loss factor of 0.1531; and the simulation time for each iteration is 9.14 hours.

B. *The 54-node Test System*

In order to investigate the scalability of the proposed model, it is applied to a test distribution network with 54 nodes in this part. This system comprises 50 load nodes, 4 substation nodes as well as 63 branches, of which 9 are fixed branches, 8 are candidates for reinforcement, and the remainder are candidates for construction. The whole data for this system can be downloaded from [29].

Running the proposed model for collaborative planning of EHs and electricity DN for this test network yields the results presented in Table IX. Negative values for the operating cost as well as the total cost of the EHs indicate that the EHs make profit from selling their surplus electricity to the grid. This is because in this case, the total installed capacity of the generating units within the EHs is higher than the overall demand of the corresponding nodes. Again, the total installed capacities of the wind and solar units are equal to the considered upper limits, revealing that it is optimal to deploy the entire available capacity of such units. Moreover, the system net loads for the maximum demand case are 5.0540, 9.6956, and 13.3053 MW in years 1–3, respectively, whereas for the maximum generation case the system net loads are respectively 5.0540, 5.3643, and 6.0031 MW. It is worth mentioning that in the first year, installed DG capacity is zero (as can be inferred from the zero investment cost of the EHs in the first year, as per Table IX), and, hence, the system net load for both cases are identical, i.e., 5.0540 MW.

It is worth mentioning that for this simulation, the initial loss factor is set to 0.150. The proposed model converges after 6 iterations, reaching the final loss factor of 0.138, with the final mismatch error equal to 0.0016. In addition, the simulation time

TABLE IX. OUTCOMES FOR THE 54-NODE TEST NETWORK (ALL COSTS ARE IN M$)

		t=1	t=2	t=3	Total
OF^{EH}	Inv_t^{EH}	0	1.962	1.243	–2.370
	Op_t^{EH}	0	–0.114	–0.198	
OF^{DN}	Inv_t^{DN}	0.068	0.380	0.196	0.845
	Op_t^{DN}	0.092	0.020	0.027	
Total Cost		**–1.525**			

Component	Trans.	Furnace	CHP	PV Panel	Wind Turbine
Total Capacity (MW)	6.65	0.26	1.26	6.50	3.30

for each iteration is 44.62 minutes. The detailed set of outcomes is also available from [29].

V. CONCLUSION

In this paper, a stochastic MILP model for joint expansion planning of DN and autonomous DG units was developed. Autonomous DGs were aggregated as EHs, and the impacts of collaboration programs on expansion cost savings were investigated using several case studies. Moreover, an efficient iterative method was proposed for incorporating power flow equations in the planning model and accurate estimation of network energy loss costs. It is worth mentioning that although the ideal collaboration between DISCO and EHs was considered in this paper, the proposed model can also be used to assess other types of collaboration programs such as tariff setting, capacity provision contracts, and generation control contracts. This is the topic of authors' future work, where this model will be deployed for investigating the effectiveness of such collaboration plans.

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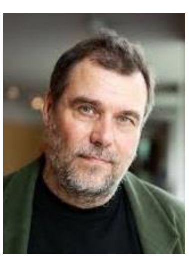

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