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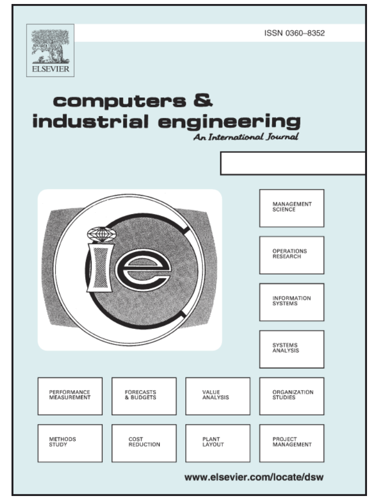

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A group learning curve model with motor, cognitive and waste elements

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Abstract

Nowadays, workers, individually or in groups, are continually learning new tasks. The speed at which they learn directly contributes to the success of their firms in competitive markets. Learning curve research has been either on the individual or organizational level. A few papers have developed learning curve models for a group of workers, even fewer that used empirical data for that purpose. However, none of the existing models comprises measurable elements from real industrial tasks. This paper aims to fill this gap in the literature by proposing a bivariate group learning curve model, an aggregation of three learning curves where the number of workers in a group and the number of repetitions are the independent variables. The dependent variable is the unit assembly time. The three learning curves represent motor, cognitive, and waste per unit assembled. The aggregated learning curve was fitted to experimental data consisting of different group sizes (1 to 4 students/workers), each performing four repetitions, and later compared to two log-linear learning curves, with and without plateauing. The results showed that the aggregated model represented the data the best and that segmenting waste into sub-elements (job familiarization, errors, and group coordination) improved the performance of the model. The parameter values affected by group sizes and repetitions for each task element provided insights that managers could use to improve the performance of their workforce.

Keywords: Learning curves; group size; motor/cognitive/waste elements; experimental data

1. Introduction

Manufacturing firms have increasingly been using workers in groups on the floor level (Moreland et al., 2002; Lantz et al., 2015). Working in groups is the practice, especially at the final assembly stage of large and complex products (Yazgan et al., 2011; Martignago et al., 2017). Lack of coordination and conflicts that might arise between group members impede their performance. Resolving those issues before initiating work, results in better worker utilization and, subsequently, group performance (Yilmaz & Yilmaz, 2016). Product customization is the trend for many manufacturing firms. Frequent changes in product design and production processes are the norms in such work environments, requiring workers to continually adapt to such changes and learning new tasks as they do through learning-by-doing (Uzumeri & Nemhard, 1998; Tilindis & Kleiza, 2017; Letmathe & Rößler, 2019). In such an environment, it is usually uncommon to have clear instructions on how to perform tasks effectively and efficiently. Other examples include rush orders (Engström et al., 1996) and rework (Badiru, 1995). Additionally, the number of workers assigned to a task may exceed the optimum, and group coordination becomes more complicated. All the above studies emphasize the importance of group learning, and further, its predictability by utilizing learning curve models.

The question about what affects learning has captured the attention of numerous researchers in various fields. The purpose has been to mathematically model learning as a function of known variables. Manufacturers produce products, typically, in lots. They have been using cumulative production, independent variable, as a proxy for measuring experience, which also has been the traditional approach for modelling the learning curve (Yelle, 1979; Jaber, 2011; Glock et al., 2019). There has been a debate in the literature, whether cumulative production, alone, appropriately represents learning. Some researchers have suggested learning to be time-dependent; others have stated that cumulative production overstates its persistence (Jaber & Sikström, 2004), while few have argued that having it alone underrepresents the learning data. However, none has proposed excluding it (Jaber & Sikström, 2004). The most commonly used univariate model is the Wright (1936) learning curve, henceforth WLC (Jaber, 2011). It has been popular among managers as it is easy to use; i.e., it could be transformed into a straight line once plotted on a log-log paper, and shown to fit many data sets well. However, it has a fundamental drawback as its results are not meaningful when learning ceases; i.e., it enters a plateau. Thus, this drawback has been an appropriate starting point for further developments of learning curve models (Jaber, 2011), ones that represent empirically gathered data better. The earliest along this road is the de Jong's model (1957), henceforth DJLC, who introduced a plateauing parameter that represents the minimum processing time, which is similar to the plateau model of Baloff (1971). Readers may refer to Glock et al. (2019) for a list of learning curves with plateauing. There has not been a consensus on what causes plateauing. Researchers have associated it with different causes (e.g., Yelle, 1979; Jaber & Guiffrida, 2004; Peltokorpi & Niemi, 2019a). Some researchers have modelled learning curves as bivariate or multivariate models (Badiru, 1992). For example, using a 4-year empirical data from an electronics manufacturing plant, Badiru (1995) presented a learning curve with cost per unit as the dependent variable and production level, the number of workers, downtime, and rework as the independent variables.

Thus, besides the numerical presentation of learning, industrial learning curves aim to show where improvement is needed. In this context, Yelle (1979) and Dutton & Thomas (1984) concluded that the factors underlying the learning curve are not well understood. Thomas & Yiakoumis (1987) continued with the same line of research and introduced a concept of the factor model for construction productivity. The model considers a learning curve for a crew of workers performing repetitive tasks. It states that many random factors disturb the work environment and, subsequently, crew performance. The study advocated that aggregating the factors that cause disturbance and representing them mathematically in one learning curve could result in an ideal model. Dar-El et al. (1995) and Jaber & Glock (2013), who combined motor and cognitive elements, and Jaber & Guiffrida (2004), who included the additional time to rework defective items, are examples of aggregated learning curves for individual performance.

Alongside individual learning, group learning has received growing attention. Argote et al. (2001, p.370) defined group learning as “the activities through which individuals acquire, share and combine knowledge through experience with one another.” Leavitt (1951) experimented on how knowledge sharing occurs among group members. The experiment consisted of a hundred students divided into groups of five, with each group member receiving a card having five symbols. The group’s task was to find which symbols appeared on all cards. The members were allowed to write messages and send them according to a communication pattern. Leavitt (1951) showed that not all communication patterns used were effective. Few researchers have expanded upon Leavitt’s experiment to explore the effects of group organization (Guetzkow & Simon, 1955) and planning (Shure et al. 1962) and to show that the WLC model describes well the performance of novice groups when learning tasks (Baloff & Becker, 1968). The above experimental studies on group learning did not consider varying group sizes. However, they improved our understanding of the dynamics of the transfer of knowledge among group members, implicitly linked to group size. In practice, the number of coordination links increases with increasing the group size, making a group inefficient (Steiner, 1972).

As per the group learning definition (Argote et al. 2001, p.370), many group learning curve models considered the transfer of knowledge as an additional measure to cumulative production or the number of repetitions (Ingram & Simons, 2002; Ryu et al. 2005; Wilson et al. 2007; Glock & Jaber, 2014; Méndez-Vázquez, 2019). Glock & Jaber (2014) proposed a group learning curve model that has two components, one describing individual learning and the other the transfer of knowledge among the group members. Furthermore, two factors in their model impact the success of knowledge transfer. The first is knowledge compatibility, and the second is the willingness of group members to share and absorb it among themselves. According to the prevailing theory, the model of Glock & Jaber (2014) assumes an increasing delay in the transfer of knowledge as a function of increasing group size. A group learning curve is formed by aggregating the learning curves of the individuals in a group (e.g., the sum of all WLCs), which happens when knowledge of the members is neither compatible nor transferable. Knowledge incompatibility and the unwillingness to share it impede its transfer. Their proposed model fitted experimental group learning data rather well. They also compared the fits to a model that was an aggregation of individual learning curves and their model outperformed it. However, the data sets fitted to models do not consider varying group sizes.

The model of Méndez-Vázquez (2019) is the only model that differentiates the effect of process loss from that of knowledge transfer in a group. Process loss occurs when the group's actual performance falls below potential because of coordination, motivation and relational processes between group members. Using the data from Peltokorpi & Niemi (2019a), the process loss parameter was estimated and assumes a fixed value, which increases with group size. Méndez-Vázquez (2019) tested various scenarios (degrees, percentages) of knowledge transfer and process loss and used the developed model for simulation and optimization purposes. For future research, she suggested the development of mathematical models with the effect of knowledge transfer derived from experimental data.

Camm & Womer (1987) developed a bivariate model that has the production rate as the dependent variable and crew size and the number of repetitions as the independent variables. They estimated the model using empirical production data. They did not fit the model to data as they have not mentioned so in their article. The model of Camm & Womer (1987), to the authors' knowledge, is the only one of its form in the literature.

The paper at hand considers a group task that is divisible into sub-tasks as per Steiner (1972), with group performance being the additive and interactive efforts of individuals in a group (Witte & Davis, 2013). Previous experimental studies for such assembly tasks (e.g., Ryall et al., 2004;

Sando et al., 2011; Staats et al., 2012; Peltokorpi & Niemi, 2019a) verify the diminishing returns in output with increasing group size, in line with the hypothesis of Steiner (1972, p.96). The group learning curve models in the literature associate diminishing returns in output to either delay in the transfer of knowledge (e.g., Glock & Jaber, 2014), process loss (Méndez-Vázquez, 2019) or overmanning (Camm & Womer, 1987), i.e., having more workers on a task than its optimal group size. However, the above models have not been fitted to empirical data containing varying group sizes. More importantly, there is a lack of group learning curve models comprising of measurable elements from real industrial tasks. By aggregating, for example, different waste elements into a learning curve would provide insights for managers on how to improve working and speed up learning.

This paper, therefore, addresses this research gap by proposing a bivariate group learning curve model, an aggregation of the motor, cognitive and waste elements. This study achieves this goal by conducting additional analysis of the data in Peltokorpi & Niemi (2019a,b). The experiment of Peltokorpi & Niemi (2019a) consisted of assembling a product whose components came from real industrial products. Students as surrogates for workers did the assembly, including the work assignment and management. The conductor of the experiment has not instructed them on how to. They measured the time it took to assemble the product by a worker and a group of workers of sizes 2, 3, and 4 for four consecutive repetitions. The results first showed that, for novice workers, assembly time decreases, and learning occurs rapidly through repetition. The learning data for each group size fitted de Jong's (1957) model almost perfectly, suggesting that learning plateaus. Second, productivity per worker decreased smoothly as a function of increasing group size, according to the hypothesis from Steiner (1972, p.96). Peltokorpi & Niemi (2019b) conducted a further analysis of the data to gain insights into the factors affecting group performance. More precisely, by using a video-based activity analysis the assembly time was broken down into the following elements: (1) value-added time (refers to part installation), (2) necessary movements,

(3) time used to read instructions, and (4) waste (comprising different types of inefficiencies at work). The results showed that much of the time for assembling the product for the first time contained reading instructions and waste due to inexperience. This further causes productivity losses with larger groups at first repetitions. Idleness that large groups experience negatively affects performance in later repetitions. This observation was due to a lack of meaningful tasks and working space for several workers at the end of processing.

Data from the aforementioned experiment is the first to account for the size of a group and the number of repetitions as independent variables. By utilizing this data, the present paper shows the following contributions:

1. The paper develops a bivariate group learning curve aggregated from three task elements: motor, cognitive, and waste. This model contributes to current literature that lacks group learning curve models comprising of measurable elements of real industrial tasks.
2. The developed aggregated model represents the data better than two non-aggregated models derived from literature. The parameter values for the effects of group size and repetition for each task element over the entire learning period provide insights that managers could use to improve the performance of their workforce.

The rest of the paper is structured as follows. Section 2 provides a background to the learning curves that are relevant to this study. Section 3 presents the group learning experiment. Section 4 develops five bivariate group learning curve models: three aggregated models and two non-aggregated log-linear models. The developed models are fitted to experimental data, and their results are compared. The parameter values of the models are analyzed to gain insights into the effects of group size and the number of repetitions on different elements of assembly work. Section 5 presents the conclusions and provides aspects for further research.

2. Background to learning curves

Learning is a natural phenomenon where human performance improves each time he/she repeats a task or activity (Jaber, 2011; Glock et al., 2019). Task repetition reduces the time to recall procedural information (Dar-El et al., 1995), improves familiarity with a product and process (Peltokorpi & Niemi, 2019b), and eliminates inefficiencies (i.e., errors and unnecessary and faulty activities and movements). Numerous learning curve models aim to represent empirically gathered learning data, and some of them represent, on average, better for a large number of data sets (Grosse et al., 2015). There are mainly three forms of learning curves, which are log-linear, exponential, and hyperbolic models (Glock et al., 2019). The log-linear learning curve form is the most popular for the reasons mentioned above, and relevant to this study. The WLC (Wright, 1936) is the first known industrial learning curve model. It is of the form:

$$Y_x = Y_1 x^{-b} \quad (1)$$

where Y_x is the time to produce xth unit, x is the repetition number or cumulative output, Y_1 is time to produce the first unit, and b the learning parameter, measuring the rate at which Y_x decreases as cumulative output doubles; i.e., $2^{-b} = {}^{Y_{2x}}/_{Y_x}$. Eq. (1), which is of a power-form, becomes log-linear as $\log{(Y_x)} = \log{(Y_1)} - b\log{(x)}$.

Eq. (1) has a drawback that $Y_x \to 0$ as $x \to \infty$. This result is not meaningful. Although mathematically correct, it is incorrect in reality since real learning data shows a plateau (e.g., Jaber, 2006, 2011; Glock et al., 2019). De Jong (1957) modified the WLC by forcing to plateau. For this purpose, he added an incompressibility factor ($0 \leq M \leq 1$) to determine the minimum processing or standard time. The DJLC model is of the form:

$$Y_x = Y_1\left(M + (1 - M)x^{-b}\right) \quad (2)$$

From Eq. (2), learning plateaus at $Y_1 M$ when x approaches a very large number.

The learning curves in Eqs. (1) and (2) are univariate models where Y_x is the dependent variable and x is the independent one. Multivariate models have several independent variables. They have received very little attention in the literature, which is perhaps due to the complexity of implementing them as practical productivity assessment tools and multicollinearity problems. Bivariate models have been used instead for their simple mathematics and ease of analysis (Badiru, 1992; p.180). A bivariate model is of the form $Y_x = \beta_0 x_1^{\beta_1} x_2^{\beta_2}$, where Y_x has been defined above, x_1 and x_2 are independent variables and β_0, β_1, and β_2 are model parameters. The idea of having more independent variables is to improve the quality of fits of the learning curve to data and the understanding of which factors other than cumulative production explain the behavior of Y_x. The bivariate learning curve of Camm & Womer (1987) is relevant to this paper. Their model has the production rate as the dependent variable and the crew size (number of workers in a group) and the number of repetitions as independent variables. The model is of the form:

$$P(x)_{xj} = B n_{xj}^{1/r} x^{\delta} \qquad (3)$$

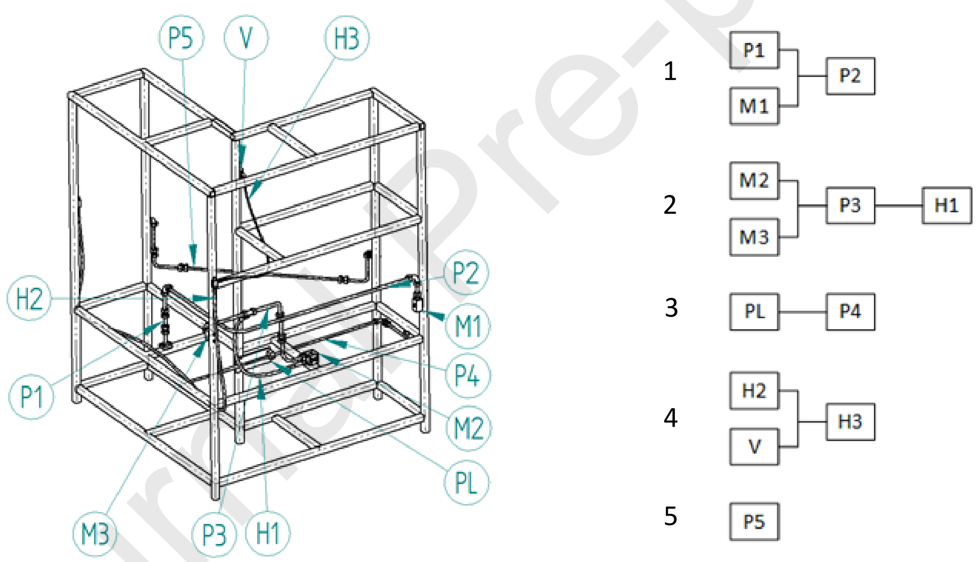
where $P(x)_{xj}$ is production rate for repetition x by group j, and n_{xj} size of group j assigned to repetition x, *r* is a model parameter describing the returns to group size, δ is the learning exponent and B is a scale constant. The model assumes that knowledge about how to perform a task assigned to group j increases through repetition, and the group produces at a faster rate, i.e. $\delta >$ 0. The model also assumes that the attempt to increase the output rate by increasing the group size is subject to diminishing returns, due to overmanning, i.e. *r* > 1. In a later section, a group learning curve is developed by aggregating three bivariate learning curves representing motor, cognitive and inefficiencies (waste). The three learning curves have group size and the number of repetitions as independent variables.

3. Experiment on group learning

This section briefly describes the experiment conducted by Peltokorpi & Niemi (2019a). It starts with the product structure and assembly steps, followed by an overview of the participants, the assembly procedure, and the experimental data, respectively.

3.1 Case assembly product

Fig. 1(a) is a pictorial of the product used in the assembly experiment. It consists of 13 components in total, assembled into a frame. They comprise pipe sub-assemblies (P1-P5), modules (M1-M3), hoses (H1-H3), plate (PL) and valve (V). The product's size and structure made it possible for several workers to work simultaneously. Fig. 1(b) is a pictorial of the five subsystems (1-5), assembled in parallel, and their precedence constraints of parts forming the product.



(a) (b)

Fig. 1. (a) The case assembly product; (b) Precedence constraints of parts

3.2 Participants

The number of students, male undergraduates, who participated in the laboratory assembly experiment, was 68. They had no prior knowledge of how to assemble the product and which members would be in their assigned group. The students were provided with a printed instructional assembly drawing. The information it contained was sufficient for them to assemble the product.

3.3 Procedure

The 68 participants formed 31 groups. The assignment of participants to groups was random. The groups varied in sizes, from 1 to 4 members. Sample sizes, N, for each group size and repetition are given in Table 1. Each group repeated the assembly up to four times, with breaks in between. While resting, the laboratory staff disassembled the product and placed the parts/components in their designated positions. The participants did not show fatigue, and the breaks were of enough length to alleviate any. Therefore, this study ignores fatigue effects.

The experimenter briefly introduced the participants to the assembly task, just before performing their first repetition. The assembly drawing included a list and descriptions of all parts and the tools to be used. The experimenter told the participants that they were free to manage their work without him intervening. There was one quality criterion, which is to assemble a complete product as described in the instructions. A final quality check included the tightening of screws and bolts. The experimenter checked the assembled product and notified each group of any detected faults. Groups were video recorded using one camera. The time to perform one repetition is the time difference between starting and finishing the assembly. The experimenter controlled the recording of sessions, meaning he identified their start and end times. The activity analysis was later done manually and solely by the experimenter to maintain consistency. A slow-motion (0.25x) mode of the analysis software (AviX) was used to distinguish, with reasonable accuracy, different activities (i.e., task elements). The status of each worker was based on the physical activity (and inactivity) of that worker over time. The accuracy of activity analysis is subject to variation due to human conducting the analysis. Readers may refer to Peltokorpi & Niemi (2019a,b) for detailed descriptions of the experiment and activity analysis.

3.4 Experimental data

Table 1 shows descriptive statistics for specific group size and repetition (Rep), for which N defines the sample size. The statistics for total assembly times (in minutes) and subcomponent times of the elements, i.e., motor, cognitive, and waste per unit assembled, in terms of the mean value (mean), variation (CV), and the minimum (min) and maximum (max) value, are presented. Table 2 presents the mean subcomponent times of different waste elements for each group size and repetition (Rep). The statistical method used here is similar to that in Peltokorpi & Niemi (2019b, Section 3). To illustrate the different task element times, Fig. 2 shows the mean time (per worker) to complete each task from the entire learning period (four repetitions) as a timeline for specific group sizes. The timelines represent a rough sequence of task elements, as observed in the experiments (see a sample timeline in Peltokorpi & Niemi, 2019b; Fig. 6). Different task elements are defined as follows:

- Motor: installing (actual installation movement) parts correctly [value-added time], picking (reaching, grasping) correct parts and tools, travelling to the assembly location, aligning parts, returning tools (travelling back, leaving tools) [required movements];
- Cognitive: looking at (reading) assembly drawing;
- Waste: the following inefficiencies in assembly work:
    - *Familiarizing*: unnecessary handling of (looking at, turning, etc.) parts, observing assembly locations and other's working, unnecessary travelling;
    - *Faulty installing*: Installing part incorrectly (or installing wrong part) so that part has to be disassembled;
    - *Wrong tools*: picking, travelling with and returning a tool that is inappropriate for installing the present part;
    - *Dropping*: dropping and picking up tools or parts;

- Co-worker: waiting for co-worker's help (e.g., handing out tools, travelling to assembly location), or completion of the preceding task;
- *Idleness*: no meaningful tasks left or working space for the worker at the end of the entire process.

<Insert Table 1 about here (provided on p.28) >

<Insert Table 2 about here (provided on p.29) >

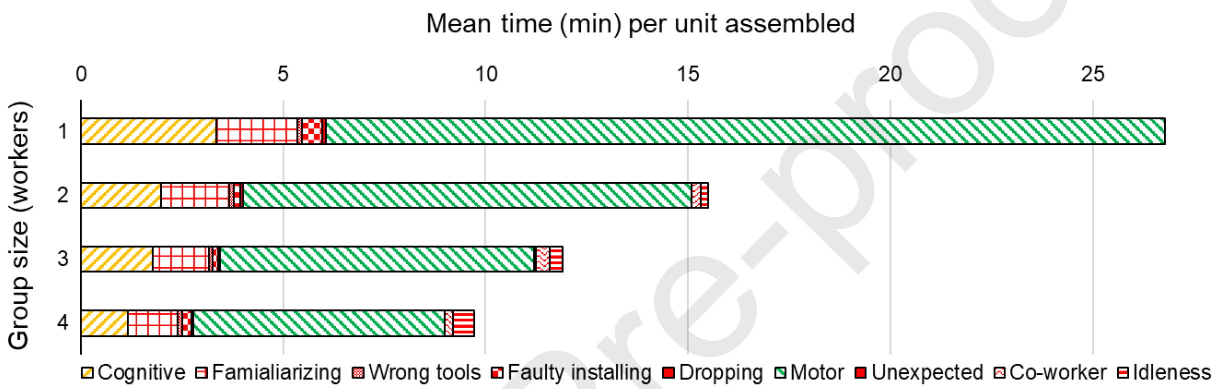


Fig. 2. Mean task element times (motor in green color, cognitive in yellow, waste in red) from the entire learning period (four repetitions) for specific group sizes.

4. A group learning curve model

In this section, five group learning curve models are developed and fitted, in the next subsection, to the experimental data of Table 1 and 2. This is followed by two subsections that provide additional analysis of the values of the learning curve parameters and a comparison of the models, respectively.

4.1 Models and fits

The bivariate group learning curve models studied and compared in this paper are presented in Table 3. For each of the models, the dependent variable is the time to assemble one unit, and the independent variables are the number of workers in a group (group size) and the number of repetitions. The first model is the single learning curve, SLC, model, which is the reference model similar in form to that of Camm & Womer (1987), in Eq. (3), except for, in their model, the dependent variable is the production rate instead of the unit time, and n and x are the independent variables, each raised to a non-negative (positive) parameter. The second model is a modification of the SLC, where it has a plateau factor, henceforth referred to as SLC-P. Adding a plateau to the SLC is in conformance with Peltokorpi & Niemi (2019a), who found that de Jong's model in Eq. (2) fitted the experimental data for each group size almost perfectly.

The last three models aggregate and represent different types of elements of assembly work in one learning curve, each of which is a bivariate learning curve contributing a fraction to the total unit assembly time. The first aggregate model, ALC, has three learning curves representing motor, cognitive and waste per unit assembled (as in Table 1). The second model further divides waste into seven sub-elements (as in Table 2), and is henceforth ALC-7W, which follows the concept of Thomas & Yiakoumis (1987) who advocated aggregating inefficiency causing factors (or waste elements) in one learning curve, as mentioned in Section 1. The third aggregate model combines similar waste elements and reduces them into three categories, and is henceforth ALC-3W, which are workers getting familiar with the job (referred to as familiarization and is described in Section 3.4), errors, and group coordination. Errors include faulty installation, using wrong tools, and dropping the equipment. Coordination comprises of co-worker related waste, idleness, and infrequent and unexpected disruptions, with this approach making the analysis straightforward and easy. Table 3 lists the five learning curves, their mathematical expressions, and their parameters along with definitions.

<Insert Table 3 about here (provided on p.30) >

For each model, the following constraints hold: γ or $\gamma_i > 0$, $-10 \leq \alpha$ or $\alpha_i \leq 10$, and, $-10 \leq \beta$ or $\beta_i \leq 10$. In addition, for SLC-P, $0 \leq M \leq 1$. The fits of the models were compared and the sumsquare of errors (SSE), SSE = $\sum_{i=1}^{x}(O_i - P_i)^2$ where O_i and E_i are the observed and estimated values for repetition $i = 1,\cdots,x$, for SLC, SLC-P, ALC, ALC-7W and ALC-3W were found to be SSE_{SLC} = 24.685, $SSE_{SLC\text{-}P}$ = 11.360, SSE_{ALC} = 4.449, $SSE_{ALC\text{-}7W}$ = 4.259, and $SSE_{ALC\text{-}3W}$ = 4.363, respectively. The accuracy of the SLC model improves when the plateauing effect (SLC-P) is considered, $SSE_{SLC\text{-}P}$ = 11.360 < SSE_{SLC} = 24.685. The results also show that breaking the learning curve data into more elements and then aggregating them into one learning curve significantly improves the learning curve accuracy. The more aggregated elements are the better the accuracy of the learning curve becomes; i.e., $SSE_{ALC\text{-}7W}$ = 4.259 < $SSE_{ALC\text{-}3W}$ = 4.363 < SSE_{ALC} = 4.449. The values of the parameters and coefficients for those learning curve models are given in Table 4.

<Insert Table 4 about here (provided on p.31) >

4.2 Analysis of the parameters of group size and learning

Fig. 3 is a pictorial of the effects of parameter values of group size (α) and learning (β) from Table 4 on each task element.

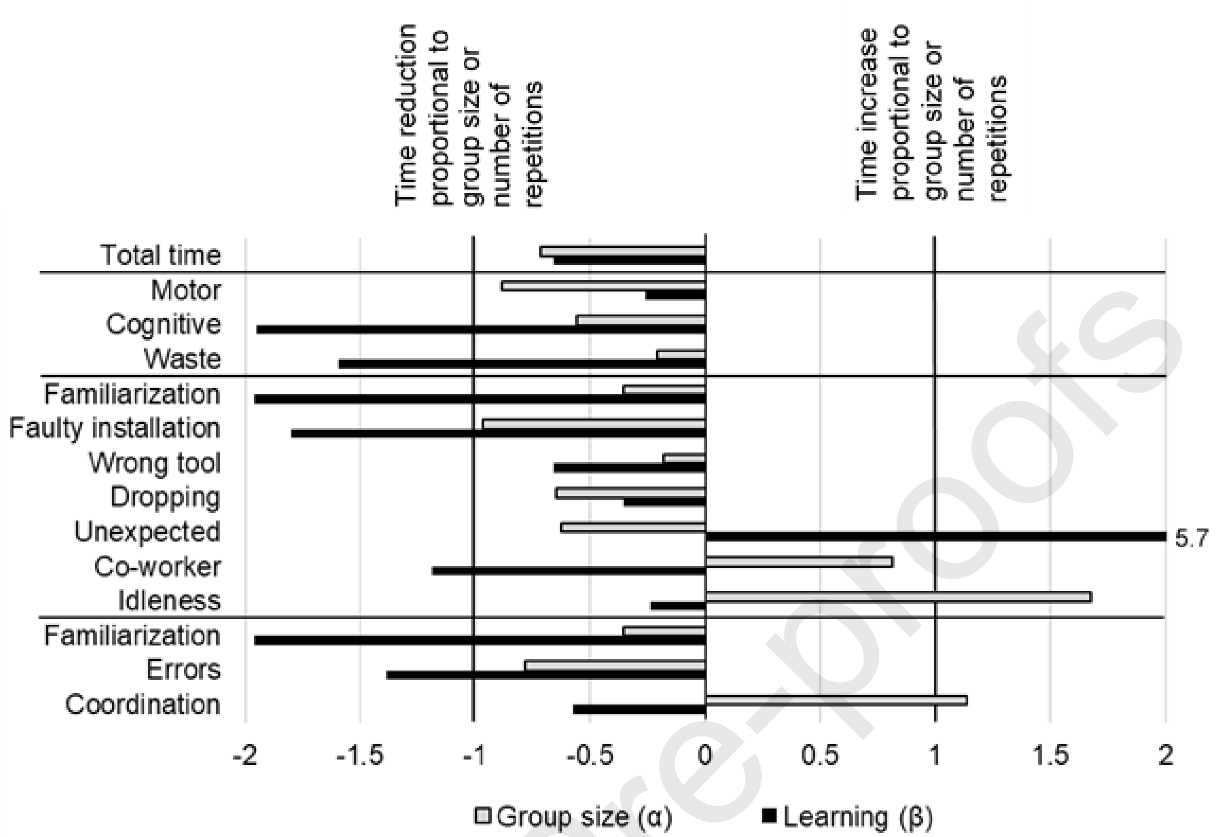


Fig. 3. Effects of parameter values of group size and learning on each task element

The negative and positive values of α (group size, grey bar) and β (learning, black bar) in Fig. 3, represented by bars, mean that additional workers and repetitions either decrease or increase the portion of the unit assembly time each element contributes. The length of a bar represents the magnitude of the effect a parameter (α or β) has on a task element. To illustrate and as an example, β (learning from repetition) affects (reduces) the cognitive unit assembly time element much more than α (group size) does. Noteworthy is that an $\alpha > -1$ means, on average, groups do not perform better than individuals do for each task element.

The analysis begins by examining the first three task elements. Motor learning is the slowest of the three, and the parameter value corresponds to motor learning rate $LR_M = 2^{-0.8846} = 83.6\%$. Learning for cognitive ($LR_C = 25.9\%$) and waste ($LR_W = 33.2\%$) elements are much faster. When fitted separately to the data from each group size, LR_C = 30.9% for a single worker, 24.0% for a group of two, 19.9% for a group of three, and 12.6% for a group of four. LR_C is the speed at which workers read the drawings reduces. It becomes faster with each repetition because of a faster recall of information; i.e., workers spend less time examining the drawings. For a single worker, the rate is similar to what Watson et al. (2010) observed. Noteworthy is that they used text instructions, and their experimental setting was different. Learning for the cognitive element being faster for larger groups has to do with the fact that there is less information to recall per worker in their dedicated tasks.

The effect of group size is much more than that of learning for the motor element. The reduction in motor element-time is proportional to group size. Group performance of a motor task, on average, does not exceed the combined performance of individuals, suggesting that one worker is sufficient to install the parts. Large group sizes experience more losses due to cognitive and waste elements than small ones. Cognitive elements comprise of the group members going over the details in the product assembly drawing, as the experimenter did not provide workers with dedicated instructions. Waste elements increase the inefficiency of the learning process and impede performance, and therefore should be eliminated, which is investigated in much detail next.

Waste is first analyzed for seven sub-elements. The sub-elements “familiarization” and “faulty installation” have the fastest learning rates, LR_F = 25.7% and LR_{FI} = 28.7%. In this context, learning is largely associated with “successful perception of instructional information about a motor task in order to carry out that action quickly and accurately” (Watson et al. 2010). Fig. 4 shows a strong correlation between cognitive elements and the familiarization and faulty installation, with coefficients 0.977 and 0.885, respectively. This finding suggests that, by reading instructions, workers learned to identify the assembly parts and their locations, and how to install them correctly. See Fig. 4 for other correlations coefficients.

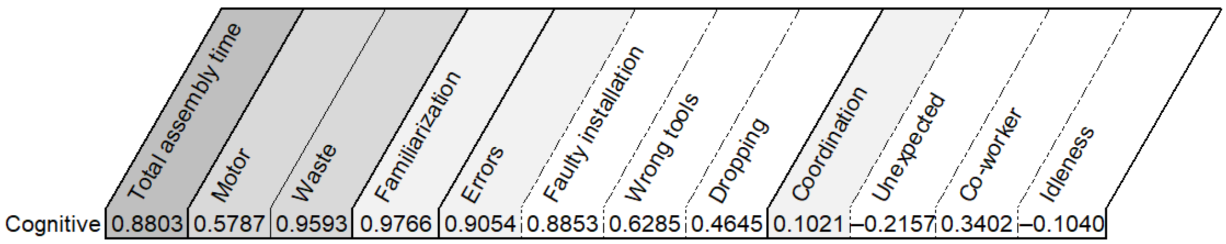


Fig. 4. Correlation coefficients describing the strength of the relationships between cognitive and the other elements. A coefficient value of one (1) shows perfect positive correlation, i.e., element times change in the same direction and proportion through repetition. A zero (0) value shows no relationship between the elements, and minus one (-1) perfect negative correlation.

Fig. 3 shows that co-worker related waste increases as a function of group size, as expected, but reduces even faster through repetition (LR_{Cw} = 44%). This finding relates to workers becoming more experienced and working better in less congested work areas. The data show that the frequency of workers dropping tools and parts became less with each repetition. However, the improvement is rather slow (LR_D = 78.2%). Increasing group size results in more idle time due to the significant workload imbalance. Idleness in large groups does not disappear with repetition (experience). Fig. 4 shows that idleness is, negatively, correlated with the cognitive element (coefficient value of –0.104), confirming that the instructions do not provide information on how to manage the temporal and spatial coordination of workers. A well-planned division of work among group members reduces idleness. This approach includes sequencing and scheduling the tasks based on their precedence constraints and work contents.

According to Fig. 3, larger groups waste relatively more resources in familiarization with a job in comparison to single-worker groups do (α_F = –0.3518). Group members not been given instructions on how to manage work among themselves may have attributed to this result. It may also have to do with supervision conflict, meaning who should instruct whom in the group to do what. This issue is nonexistent in a group of one person as the line of command is clear. In the group of most members (i.e., four), workers spent relatively less time reading the instructions, especially at the last three repetitions (cognitive element in Table 1). A possible reason for this is less space per worker in the assembly drawing. For such group sizes, workers more frequently learned from errors (e.g., picking wrong tools). Fig. 3 shows that the reduction in the time for faulty installations is proportional to the group size, which suggests that, in general, this element is independent of the group size and more dependent on the individual performing it.

Table 4 shows that the fits of the model to the data of unexpected disruptions are poor (R^2 = 0.2428, very low). Exogenous reasons caused workers disruptions, which were seldom. Therefore, an analysis of the unexpected disruptions parameters is not appropriate, if not invaluable.

Lastly, Fig. 3 shows the analysis of waste for familiarization, errors and coordination. This approach allows for a straightforward and summative analysis of the parameter values for them. Familiarization has the fastest learning rate (LR_F = 25.7%) followed by errors (LR_E = 38.3%) and coordination (LR_{CO} = 67.2%). Larger groups are subject to great loss with familiarization (α_F = –0.3518) and less with errors (α_E = –0.7856). The coordination time-element increases proportionally to group size (α_{CO} = 1.1358, roughly one), and the group size effect is twice as much as the learning effect (β_{CO} = –0.5739). This result indicates that learning through repetition is not sufficient to offset the coordination problems with increasing group size.

4.3 Comparison of models

Fig. 5 presents the deviations (%) of models from the observed data in each experimental setup. Fig. 6 illustrates the behavior of the learning curves generated from the observed data and the predicted data from SLC, SLC-P and ALC-3W. ALC and ALC-7W behave similar to ALC-3W, and, therefore, are omitted from Fig. 6.

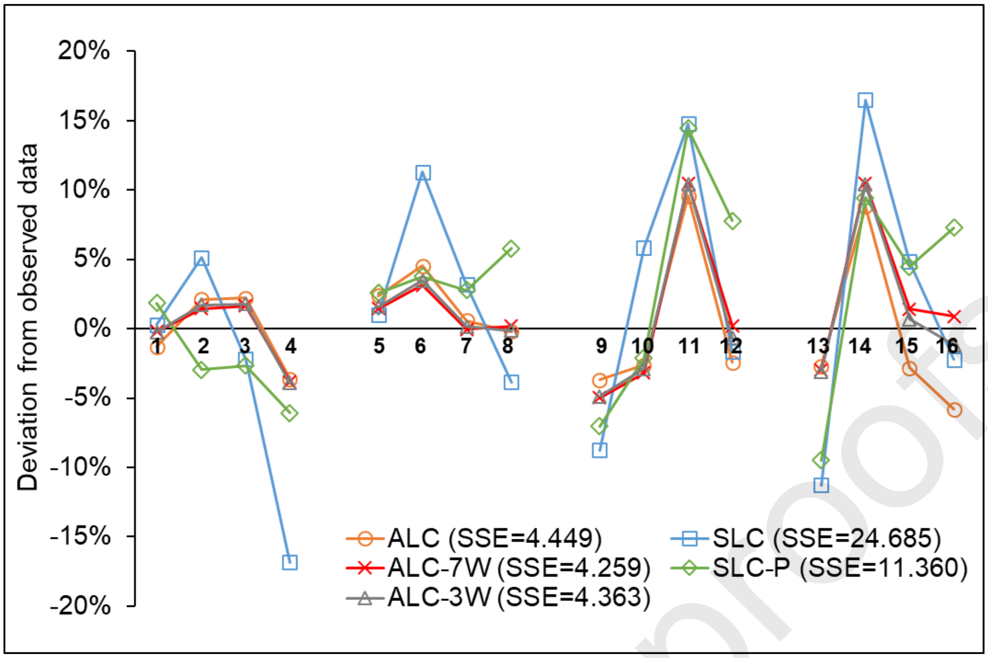


Fig. 5. Deviations (%) of models from the observed data in each experimental setup (1-4 = Rep. 1-4 for 1 worker, 5-8 = Rep. 1-4 for 2 workers, 9-12 = Rep. 1-4 for 3 workers, 13-16 = Rep. 1-4 for 4 workers).

As Fig. 5 shows, the predicted performances of aggregate models (ALC, ALC-7W and ALC-3W) are much more accurate than those of a single learning curve model, SLC, and its plateau version, SLC-P. ALC significantly improved performance by 82% and 60.8% over SLC and SLC-P, respectively. Further, dividing waste elements into three (ALC-3W) or seven (ALC-7W) sub-elements improve the performance by 1.9% or 4.3%, respectively. The accuracy of a learning curve model improves when its data comprises of aggregated elements. However, not all sub-elements would be appropriate for analysis as some of them have insignificant effects of performance.

SLC becomes less accurate for the second repetitions and, especially, the fourth repetition for groups of one worker. In general, learning is fastest for the second repetition and slows as repetitions increase (observed data in Fig. 6). In this regard, the SLC poorly reflects those characteristics that are typical of assembly processes. SLC-P improves the accuracy in general, and especially for one- and two-worker groups. However, it predicts, relatively, poorly the first repetition of each group size. It appears that all the models predict a lower learning effect than the observed between the first and second repetition for three- and four-worker groups. The SLC predicts the worst performance of single-worker groups, while the other models that of the three-worker groups (Fig. 5 and 6). The observed performance deviates from the predicted values, especially for the three-worker groups at the third repetition and for the four-worker groups at the second repetition. For the first deviation, Peltokorpi & Niemi (2019a) did not find a practical explanation. However, they were able to explain, partially, the second deviation and has to do with the small sample size for four-worker groups (N = 3 in each repetition, Table 1).

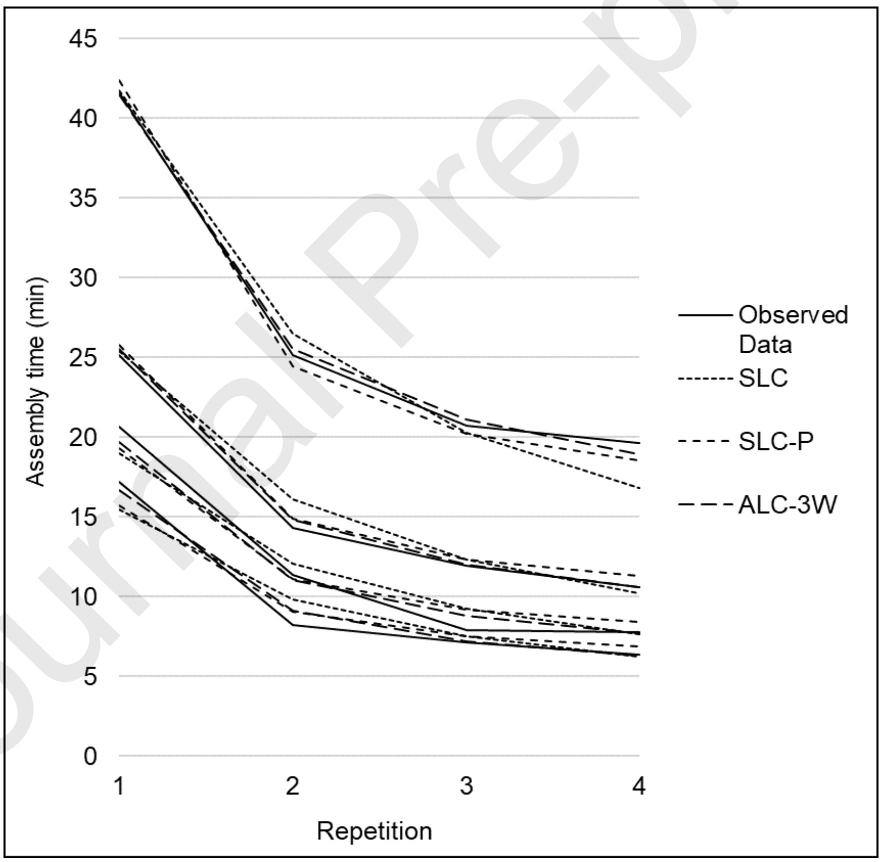


Fig. 6. Learning curves generated from the observed data and SLC, SLC-P and ALC-3W. The topmost curves are for single workers, the second topmost curves for two-worker groups, etc.

5. Conclusions

This paper developed an aggregated bivariate group learning curve model of motor, cognitive and waste elements. The number of workers in a group and the number of repetitions are the independent variables for each bivariate model. The dependent variable is the time per unit to assemble a unit. Each of the bivariate models contributed a fraction to the time per unit. The fits of the developed models were tested using the empirical data of Peltokorpi & Niemi (2019a,b). The results showed that the developed aggregated learning curve fits the data better than a non-aggregated group learning curve and its plateau version. The performance of the aggregated curve improved by dividing the waste element data into additional ones. The three sub-elements of waste were (1) workers' familiarization with the job, (2) errors from faulty installation, picking the wrong tools, and dropping the equipment, and, (3) coordination of activities among the group members.

The parameter values for the effects of group size and the number of repetitions for each task element provided insights into how industrial managers could improve the performance of worker groups at the shared task. What is especially valuable are the findings regarding the practical effects of group size. Noteworthy is that for each task element of the industrial-like product assembly, the performance of the groups did not exceed the combined effect of individuals. The unit assembly time of the motor element reduced proportionally to the number of workers in a group. Larger groups suffered from more performance losses. This deterioration had to do with the time the members spent reviewing the drawings (cognitive) and getting familiar with the job requirements (waste). Groups of smaller sizes did better in this regard. These observations highlight the importance of group work management and dedicated task instructions. Errors were found to have little to do with the group size and more with the individuals. The results showed that when groups are self-managed, and working is not standardized, the number of repetitions is not sufficient to offset coordination problems with increasing group size. Managing temporal and spatial coordination in groups requires a well-planned division of work among group members.

The study presented in this paper has limitations. One is that its results could not be generalized, as doing so requires considering a variety of different factors that affect the group learning process. Among others, those factors include characteristics of worker groups and individuals as well as tasks and environments at which each group operates. Therefore, it is recommended to consider the effects of group size and learning on task elements on a case-by-case basis. The precision with which the developed aggregate model represents the group learning process depends on the available data. When data on appropriate task elements are not available, one may use a single learning curve model to assess the effects of group size and learning of the entire task cycle. This study developed a group learning curve model based on measurable task elements representing the activities of workers. The modelling and analysis approaches used in this paper are different from those studies on knowledge transfers in groups (Ingram and Simons, 2002; Ryu et al., 2005; Wilson et al., 2007; Glock & Jaber, 2014; Méndez-Vázquez, 2019). The group learning data used in the present paper gives little evidence on knowledge sharing. For example, verbal interaction between the workers, which strongly associates to knowledge sharing and learning, was omitted, as the physical activities of workers have been the base of the analysis. Despite the limitations, the modelling approach used herein is suitable for predicting manufacturing performance for groups where their sizes and the numbers of repetitions affect it. The parameter values for the sub-elements in the aggregated model also showed where improvement is needed, which is one of the purposes of learning curves. This study, in a way, responds to the call of Thomas & Yiakoumis (1987), who advocated that aggregating and mathematically representing inefficiency (waste) causing factors in one learning curve could result in an ideal model.

The work presented in this paper seeds for further research. Depending on the research environment and interest, one could consider dividing the task elements further (e.g., single subtasks). How learning occurs in industrial worker groups is parsimonious, and more research is needed. The question returns to knowledge sharing among group members; however, its occurrence and practical effects on group learning are not well understood. One interesting aspect would be learning-by-observation in groups. A promising research direction is to investigate how workers turnover affects group performance and what makes novices learn effectively. Most importantly, further developments of group learning curves need more empirical data with a large number of repetitions and a variety of group sizes, which is a lack of current literature.

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Table 1. Descriptive statistics for motor, cognitive and waste elements for each group size and repetition.

	Group size = 1				Group size = 2				Group size = 3				Group size = 4			
Rep.	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
N	9	9	9	2	10	10	10	7	9	9	9	7	3	3	3	3
	Total				Total				Total				Total			
Mean	41.61	25.14	20.75	19.65	25.14	14.30	11.96	10.63	20.65	11.35	7.88	7.78	17.19	8.19	7.15	6.38
CV	0.16	0.11	0.12	0.11	0.33	0.32	0.27	0.23	0.21	0.10	0.11	0.18	0.02	0.09	0.12	0.16
Min	33.44	21.96	16.76	17.42	15.24	8.45	7.96	7.49	13.36	9.56	6.91	6.58	16.73	7.39	6.03	4.96
Max	54.18	31.79	27.11	21.88	47.37	24.40	18.42	15.12	27.20	12.79	9.55	10.37	17.71	9.15	8.20	7.23
	Motor				Motor				Motor				Motor			
Mean	25.68	20.17	18.21	18.91	13.38	10.95	10.50	9.55	9.56	8.05	6.75	6.66	8.12	5.94	5.92	4.89
CV	0.11	0.07	0.12	0.12	0.35	0.26	0.24	0.17	0.16	0.10	0.11	0.14	0.07	0.09	0.16	0.08
Min	21.53	17.39	14.76	16.67	9.91	7.77	7.42	7.33	7.40	6.33	5.68	5.75	7.44	5.16	4.58	4.34
Max	30.14	22.30	23.33	21.15	26.35	17.50	14.99	12.65	11.44	9.07	8.11	8.36	8.82	6.45	6.78	5.24
	Cognitive				Cognitive				Cognitive				Cognitive			
Mean	8.66	2.90	1.44	0.37	5.64	1.37	0.54	0.35	5.48	1.24	0.24	0.12	3.91	0.48	0.20	0.04
CV	0.28	0.38	0.40	0.48	0.25	0.63	1.13	1.23	0.27	0.37	0.43	0.62	0.10	0.48	0.26	0.60
Min	5.54	1.72	0.88	0.19	3.41	0.31	0.06	0.02	3.07	0.26	0.05	0.03	3.55	0.22	0.15	0.01
Max	13.62	5.22	2.79	0.54	7.80	3.41	2.23	1.33	7.41	1.73	0.46	0.24	4.45	0.78	0.27	0.07
	Waste				Waste				Waste				Waste			
Mean	7.28	2.06	1.09	0.37	6.12	1.99	0.93	0.73	5.61	2.07	0.88	1.00	5.17	1.78	1.04	1.43
CV	0.54	0.77	0.68	0.49	0.48	0.56	0.32	0.59	0.40	0.22	0.34	0.61	0.14	0.25	0.24	0.41
Min	2.70	0.58	0.41	0.19	1.93	0.36	0.40	0.14	2.31	1.26	0.34	0.47	4.44	1.15	0.69	0.61
Max	15.53	5.13	2.45	0.55	13.22	3.57	1.45	1.44	9.00	2.69	1.47	2.38	6.17	2.17	1.25	1.92

Table 2. Mean times (min) of different waste elements per unit assembled for each group size and repetition.

	Group size = 1				Group size = 2				Group size = 3				Group size = 4			
Rep.	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Familiarizing	5.59	1.20	0.90	0.33	4.74	1.11	0.49	0.41	3.81	1.09	0.38	0.30	3.30	0.90	0.47	0.22
Faulty installing	1.37	0.60	0.05	0.00	0.37	0.15	0.06	0.01	0.44	0.16	0.00	0.00	0.71	0.01	0.01	0.26
Wrong tools	0.21	0.11	0.11	0.00	0.24	0.12	0.06	0.07	0.11	0.09	0.07	0.05	0.08	0.20	0.11	0.07
Dropping	0.10	0.14	0.04	0.04	0.09	0.06	0.07	0.06	0.05	0.02	0.03	0.04	0.02	0.02	0.02	0.07
Unexpected	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.20	0.00	0.00	0.00	0.00
Co-worker	0.00	0.00	0.00	0.00	0.36	0.28	0.10	0.10	0.80	0.33	0.11	0.14	0.41	0.18	0.04	0.19
Idleness	0.00	0.00	0.00	0.00	0.33	0.22	0.12	0.08	0.39	0.37	0.28	0.27	0.63	0.48	0.38	0.62

Table 3. The studied group learning curve models

Model	Formula	Definitions (when terms appear first time)
SLC	$y_x = \gamma n^{\alpha} x^{\beta}$	y_x= time per unit to assemble the product for repetition x n = group size x = repetition number (cumulative output) γ = parameter α = group size parameter β = learning parameter
SLC-P	$y_x = \gamma[M + (1 - M)n^{\alpha} x^{\beta}]$	M = plateau factor
ALC	$y_x = y_x^M + y_x^C + y_x^W = \sum_i \gamma_i n^{\alpha_i} x^{\beta_i}$	y_x^M= time per unit for the motor elements; y_x^C= time per unit for the cognitive elements; y_x^W= time per unit for the waste elements of repetition x by group *j*.
ALC-7W	$y_x = y_x^M + y_x^C + y_x^F + y_x^{FI} + y_x^{WT} + y_x^D + y_x^U + y_x^{CW} + y_x^I = \sum_i \gamma_i n^{\alpha_i} x^{\beta_i}$	y_x^F= time per unit for the familiarization elements; y_x^{FI}= time per unit for the faulty installation elements; y_x^{WT}= time per unit for the wrong tool elements; y_x^D= time per unit for the dropping equipment elements; y_x^U= time per unit for the unexpected elements; y_x^{CW}= time per unit for the co-worker related elements; y_x^I= time per unit for the idleness elements of repetition x by group *j*.
ALC-3W	$y_x = y_x^M + y_x^C + y_x^F + y_x^E + y_x^{CO} = \sum_i \gamma_i n^{\alpha_i} x^{\beta_i}$	y_x^E= time per unit for the errors elements; y_x^{CO}= time per unit for the coordination elements of repetition x by group *j*.

Table 4. Values of the parameters and coefficients for the models.

SLC		γ	α	β		R^2
		41.7301	–0.7165	–0.6558		0.9813
SLC-P		γ	α	β	M	R^2
		42.3891	–0.7162	–1.6093	0.3692	0.9914
ALC	i	γ_i	α_i	β_i		R^2
Motor	M	25.1787	–0.8846	–0.2588		0.9919
Cognitive	C	8.7949	–0.5595	–1.9489		0.9805
Waste	W	7.0962	–0.2093	–1.5924		0.9827
ALC-7W (Waste)	i	γ_i	α_i	β_i		R^2
Familiarization	F	5.6659	–0.3518	–1.9618		0.9936
Faulty installation	FI	1.3203	–0.9663	–1.8010		0.8167
Wrong tool	WT	0.1936	–0.1842	–0.6590		0.4479
Dropping	D	0.1123	–0.6461	–0.3548		0.4377
Unexpected	U	1.1E-05	–0.6272	5.7027		0.2428
Co-worker	CW	0.1965	0.8148	–1.1850		0.6213
Idleness	I	0.0622	1.6759	–0.2373		0.8855
ALC-3W (Waste)	i	γ_i	α_i	β_i		R^2
Familiarization	F	5.6659	–0.3518	–1.9618		0.9936
Errors	E	1.6201	–0.7856	–1.3846		0.8629
Coordination	CO	0.2524	1.1358	–0.5739		0.7670

Highlights

- An aggregate bivariate group learning curve model was developed.
- The model describes motor, cognitive and waste elements from real assembly work.
- For each element, unit time is dependent on the number of workers and repetitions.
- The aggregate model outperformed a non-aggregate model and its plateau version.
- Segmenting waste into sub-elements further improved the performance of the model.

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CRediT author statement

Jaakko Peltokorpi: Conceptualization, Methodology, Validation, Formal analysis, Investigation, Resources, Data Curation, Writing - Original Draft, Writing - Review & Editing, Visualization, Project administration, Funding acquisition

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