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Robust optimization of multi-probe roundness measurement probe angles

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ABSTRACT

Positional errors of a multi-probe roundness measurement frame will result in the probe angles deviating from the nominal angles. The actual probe angles can be accurately determined after placing the measurement frame using cross correlation or other methods. However, since multi-probe methods suffer from harmonic suppression, determining the angles is not enough for accurate reconstruction of the roundness profile, if the actual angles are situated in an area with poor harmonic characteristics. To find suitable areas for the probe angles to allow for deviations from the nominal angles, this paper presents a robust optimization for probe angles to avoid harmonic suppression regardless of errors in the probe angles. Suggestions for optimal probe angles are presented for the three-point and the four-point redundant diameter method.

1. Introduction

Roundness is an important concept in many fields of engineering. Relevant roundness parameters are defined in ISO 12181-1 [1] and standard filters in ISO 12181-2 [2]. When determining roundness profiles of cross sections of large flexible rotors such as paper machine rolls, there are several conditions which restrict the available roundness measurement methods. Rotors can be too large to be placed onto precision spindles, and under rotation the cross section center point movement can be unpredictable and unrepeatable. Under these conditions, multi-probe methods can be used to accurately determine the roundness profile of a cross section. From at least three probe signals obtained from a rotating workpiece (Fig. 1), the methods aim to separate the center point movement (also called *spindle error motion* [3]) and the roundness profile, which both contribute to the probe signals. Several multi-probe roundness measurement methods have been published [4–9].

Multi-probe methods are based on forming a system of equations from the probe signals as a function of the rotational angle, center point movement and the roundness error and then solving the roundness profile. Several roundness measurement methods rely on the Fourier transform in solving the related equations [3,4], but the equations have also been solved in time domain [9]. A roundness profile can be presented as a Fourier series, with the first component representing small eccentric center point movement, the second component two undulations per revolution (UPR) etc.

If more than three probes are used, a least squares estimate can be made to determine the coefficients using all probe signals [3]. An alternative to a least squares estimate are redundant methods, which combine multiple probe signals by calculating the harmonic components from different combinations of the probes, the combination selected for each harmonic component based on smallest error propagation [10]. Furthermore, if two probes are placed on the same line, even harmonic components can be calculated from the diameter variation profile [7,11] and then merged in frequency domain with odd components calculated from the three point measurement. Using angle probes instead of displacement probes is also possible and has been demonstrated [3,12]. Angle probes will have an effect on the transfer function and harmonic sensitivity of the probing arrangement.

When a fixed measurement frame is used and there is any error in the positioning of the frame, the actual probe angles will deviate from the nominal probe angles (Fig. 3). Except for the angular errors, the slight misalignment errors of the probes caused by the incorrect positioning are assumed not to affect the probe readings except for a negligible cosine error.

First, this paper shows measurements to accurately determine the probe angles in a multi-probe roundness measurement frame using cross correlation, after positioning the probes. The effects of correcting the probe angles on the produced roundness profile are presented using a simulated and a measured workpiece data.

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Nomenclature

$\mathbf{S}_k$	Matrix of with Fourier coefficients of probe signals
$\mathbf{r}_k$	Matrix of k^{th} Fourier coefficient of center point movement and roudness profile
$\mathbf{H}_k$	Matrix for k^{th} harmonic component describing probe arrangement
k	Harmonic component k of the roundness profile
κ_k	Condition number of the matrix for the k^{th} harmonic component
θ	The angle of rotation of the workpiece
n	Displacement probe n
ϕ_n	The angle of the n^{th} displacement
T	Function for the weighted sum of condition numbers
h_k	The weighing factor for the k^{th} harmonic component

Determining the probe angle with cross correlation has been presented before by Shi [13] for a multi-step method, where one probe was used to obtain multiple signals by changing the probe position between measurements. Determining the probe angles with other techniques is also possible.

The determination of the probe angles leads to the second part of the research problem of this paper. Multi-probe methods suffer from a phenomenon called harmonic suppression, where some probe angle combinations are insensitive to certain harmonic components of the roundness profile [6]. Complete harmonic suppression occurs when the harmonic component wavelength is equal to the spacing of the probes.

Robust optimization is a subset of optimization problems, which tries to account for uncertainties in the variables of an optimization. In robust optimization, the variables are assumed to reside inside a known uncertainty set [14]. The goal is to obtain a robust (allowing for deviation), but conservative estimate for the variables to be optimized. In this article, the principles of robust optimization were applied by selecting a window of uncertainty for the probe angles and finding the optimal angles to minimize the maximum value inside that window.

Several suggestions of optimal probe angles based on optimizing a performance index have been published [3,15–17]. Cappa [17] optimized the angles considering the first 150 harmonics and stated the transfer function shows a self-similar pattern. As is shown in this paper, the pattern is not self-similar, but each harmonic component adds a layer of suppressed areas. Hale [16] considered a tolerance in the angles in the optimization, but also considered an overly wide range of harmonics for practical roundness measurement in most engineering purposes.

This research addresses two connected sources of error in multi-probe roundness measurement. When probes are positioned around a workpiece, in reality there will be an uncertainty associated with their actual positions. It is shown with measurements how probe angles can be determined with cross correlation to avoid using the incorrect nominal angles in the calculation of the roundness profile. Because multi-probe methods suffer from harmonic suppression, it may happen that the determined angles are situated in an area which results in harmonic suppression. To avoid suppression, an optimization is performed to find suitable probe angles tolerant to deviations in the probe angles. Probe angles are optimized for the three-point method and for a redundant method where even harmonic components are calculated from the diameter variation profile.

2. Methods

A paper machine roll with a diameter of 320 mm was used for the measurements of the probe angles, with the middle cross section used for the measurements. The measurement setup consisted of four triangulation reflective laser sensors (Matsushita NAIS LM 300) placed around the rotor (Fig. 2). The nominal probe angles in the measurement frame are 0°, 38°, 67° and 180° and are based on an optimization by Kato [15].

2.1. Determining probe angles with cross correlation

To determine the probe angles, a marker was attached to the roll. The roll was rotated at a slow speed to eliminate any dynamic behaviour and measurement signals were obtained from the laser probes. Each probe measurement was triggered by rotary pulse encoder with 1024 steps in a round. 100 rounds of displacement data were acquired with the laser sensors.

After acquiring data, cross correlation was used to calculate the phase differences between the signals to obtain the actual probe angles. By definition, "cross correlation is a measure of similarity between two signals" [18]. It can be used to determine the phase difference between two similar signals signals. For a discrete signal, the phase difference can be determined by offsetting one of the signals with a lag value and calculating the cross correlation. The lag which produces the highest value of cross correlation will correspond, in this case, to the correct probe angle.

Several similar measurements were also performed with a rotor shaft of an electrical drive. The angular errors observed in these measurements were larger than with the paper machine roll, which can be explained with the lower diameter of the electrical machine rotor shaft. These measured error values were used to select an error tolerance window for optimization of the probe angles.

2.2. Roundness profile calculation

The obtained angles and the probe data were then used to compare the roundness profile produced by the assumed (erroneous) angles to the roundness profile produced by the determined (correct) angles. The effect of the angle error is demonstrated both on the real data and simulated workpiece data. The simulated workpiece contained harmonic components 2…100 with 10 μm amplitude, the phases selected so to minimize the total roundness error. In this paper, the roundness profiles were calculated as follows.

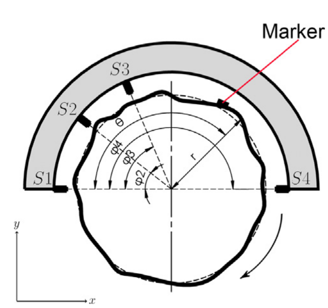

Fig. 1. Schematic picture of the measurement setup.

For a system with displacement probes (Fig. 1), when assuming that there is only small center point movement, when the work-pieces is rotated by the angle θ, the expected probe signal $S_n(\theta)$ for the n:th probe can be formulated (refer to Fig. 1):

$$S_n(\theta) = r(\theta + \phi_n) + x(\theta) \cdot \cos\phi_n + y(\theta) \cdot \sin\phi_n \quad (1)$$

As shown by Jansen [3], after a Fourier transform of the probe signals, the complex Fourier coefficients of the measured probe signals S_{nk} and the complex coefficients of the roundness profile r_k and center point movement x_k and y_k can be presented as a matrix in the form:

$$\mathbf{S}_k = \mathbf{H}_k \cdot \mathbf{r}_k \quad (2)$$

$$\begin{bmatrix} S_{1k} \\ S_{2k} \\ S_{3k} \end{bmatrix} = \begin{bmatrix} \cos\phi_1 & \sin\phi_1 & e^{-i(k\phi_1)} \\ \cos\phi_2 & \sin\phi_2 & e^{-i(k\phi_2)} \\ \cos\phi_3 & \sin\phi_3 & e^{-i(k\phi_3)} \end{bmatrix} \cdot \begin{bmatrix} x_k \\ y_k \\ r_k \end{bmatrix} \quad (3)$$

The solution is calculated separately for each harmonic: $\mathbf{r}_k = \mathbf{H}_k^{-1} \cdot \mathbf{S}_k$. The inverse Fourier transform of these calculated complex Fourier coefficients can then be used to obtain the roundness profile $r(\theta)$ and center point movement coordinates $x(\theta)$ and $y(\theta)$.

2.3. Performance index for probe angle optimization

A metric for the harmonic suppression of single harmonic components is defined via a matrix condition number. A higher condition number indicates higher sensitivity to errors. Complete harmonic suppression for a component occurs when the condition number rises to infinity. The condition numbers.

The characteristics of this function are analyzed and harmonic suppression as a function of the probe angles is also investigated also on the level of single harmonic components. The results of measurements of the probe angle errors are used as basis for the optimization criteria of the probe angles to obtain probe angles which are in a harmonically sensitive region regardless of small positioning errors of the probes.

The condition number of the matrix $\mathbf{H}_k$ can be used as a metric for the harmonic sensitivity [3]. When the matrix is close to singular, κ_k will get a higher value and small errors in the probe signals will lead to large errors in the end result.

$$\kappa_k = \|\mathbf{H}_k\| \cdot \|\mathbf{H}_k^{-1}\| \quad (4)$$

A performance index to be used in the optimization can be defined:

$$T(\phi_2, \phi_3) = \sum_{k=2}^{\infty} h_k \cdot \kappa_k \quad (5)$$

h_k is a transfer function that can be used to give weights for selected harmonics. Kato [15] used a low pass filter with a cutoff frequency at the 15th harmonic as the transfer function, but in this research a linearly decreasing function for harmonics from 2 to 20 is used ($h_k = 1$ for $k = 1$ and $h_k = 0$ for $k = 21$).

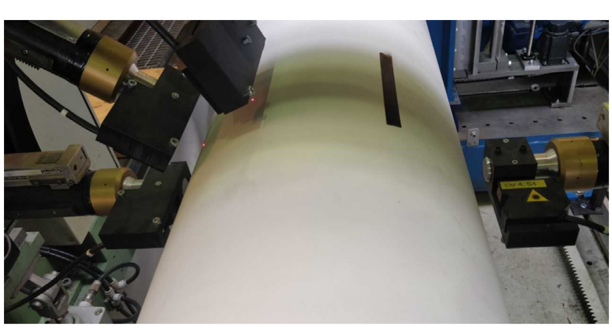

Fig. 2. Measurement setup with rotor, marker and four laser sensors.

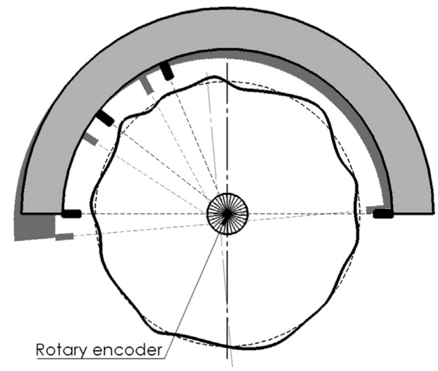

Fig. 3. Schematic picture of positioning error showing how a placement error of a fixed measurement frame results in altered probe angles.

The final goal of the optimization is to minimize the worst case cost, in other words the maximum value inside a region $(\phi_2 + \phi_{tol2}, \phi_3 + \phi_{tol3})$ representing selected angle deviation tolerance. The optimization problem can be formulated as follows:

$$\begin{aligned} \min_{\phi_2,\phi_3} \max\, & T(\phi_2 + \phi_{tol2}, \phi_3 + \phi_{tol3}) \\ \text{subject to } & -2 < \phi_{tol2} < 2 \\ & -2 < \phi_{tol3} < 2 \end{aligned} \quad (6)$$

To find the optimal probe angles, with $\phi_1 = 0$, this performance index is numerically evaluated over the 2D parameter space of ϕ_2 and ϕ_3 (evaluated for $0^\circ < \phi_n < 180^\circ$ and $\phi_2 > \phi_3$). For practical reasons, the optimization was restricted to angles under 180°; for example it is advantageous to design a fixed tactile probing head for angles not exceeding 180° in total. Based on the cross correlation measurements of the probe angle errors, the selected angle deviation tolerance in this research was ±2°.

3. Results

3.1. Probe angle determination and effect on roundness profile

One round of the measured data (1024 samples) is shown in Fig. 5 and the cross correlation of the signals with each other with different lag values in Fig. 6. Signal means were removed from the signals and cross correlation was used to determine the lag value, that is the maximum argument of the cross correlation, with the highest correlation with respect to S1. In Fig. 6 the black dotted line corresponds to zero lag, vertical red lines are the maximum values of the cross correlation. The determined lag values for this round of data were 105 samples for S2, 189 samples for S3 and 510 samples for S4 which correspond to angles {0°, 36.91°, 66.45° and 179.30°}. Lag values were calculated in the same way for 100 rounds of data. The measured mean angles, their standard deviations and the mean deviations from the nominal angles are shown in Table 1. The results in this table served as input for the later research: the mean deviations dive an indication of the minimum tolerance zone for the probe placement. The standard deviations in the determined angles describe the uncertainty in the probe angles.

The effect of using incorrect angles in the calculation of roundness profiles is presented in Fig. 7 for a simulated profile and Fig. 8 for a measured roundness profile of the roll after removing the marker. Both of the plots display the roundness profile are filtered to contain only the first 100 harmonic components. The deter-

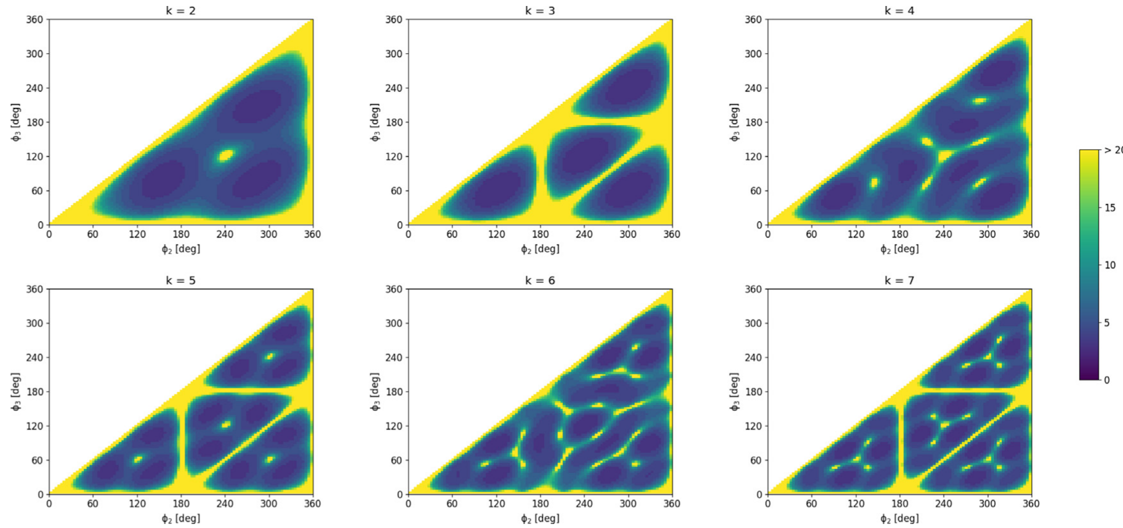

Fig. 4. κ_k calculated separately for single harmonics $k = 2, \ldots, 7$, showing how each added harmonic results in added areas of suppression with the distance between the suppressed areas decreasing as the harmonic number increases. It can also be seen that odd harmonics are suppressed when at least one of the angles is equal to 180°. The final performance index T is a weighted sum of the single harmonics' indexes (Eq. (5).

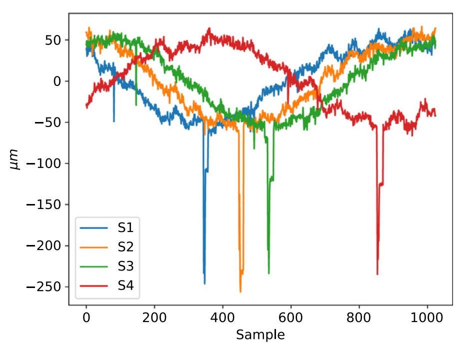

Fig. 5. One round (1024 samples) of measured displacement signals. The notches in the signals are caused by the marker.

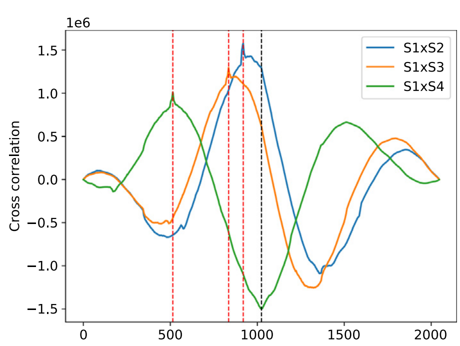

Fig. 6. One round (1024 samples) of measured signals cross correlated with each other. The red line shows the maximum, which is the probe position.

Table 1
Measured mean angles for the probes, their standard deviations and the mean deviations from the nominal angles. The values have been calculated from 100 rounds of data.

Sensor (nominal angle)	Mean measured angle	σ	Mean deviation
S1 (0°)	0.00°	0.00°	0.00°
S2 (38°)	36.91°	0.45°	−1.09°
S3 (67°)	66.45°	1.26°	−0.55°
S4 (180°)	179.30°	0.75°	−0.70°

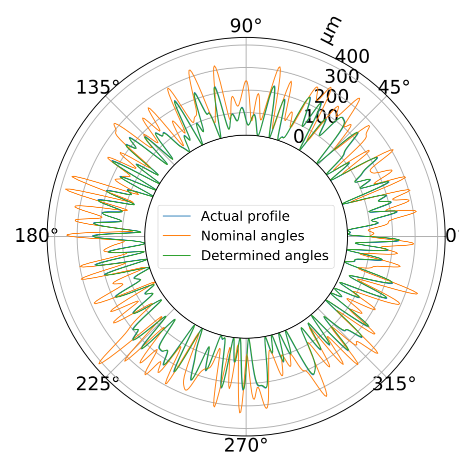

Fig. 7. Roundness profile calculated from simulated signals with incorrect angles and determined corrected angles. The nominal and incorrect angles used in the calculation are shown in Table 1.

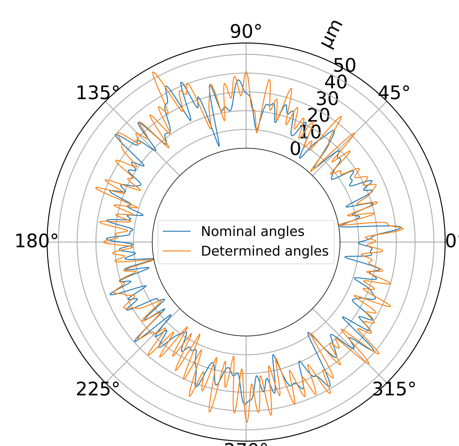

Fig. 8. Effect of correcting the angles on roundness profile calculated from measured data from the roll. The nominal and incorrect angles used in the calculation are shown in Table 1.

mined (correct) angles in the figures are the mean measured angles; the angles with errors are the nominal probe angles.

The results show that using the incorrect angles in the calculation of the roundness profile leads an error in the produced profile, which is demonstrated in Fig. 7 for a simulated case and in Fig. 8 the measurement of the actual profile. The actual profile of the real workpiece is not known, so for the measured data, only the difference between the two cases can be shown. In the simulation, the actual profile is known, and the figure shows how using the correct angles produces a better estimate of the roundness profile.

3.2. Robust optimization of the probe angles

An optimization of the probe angles was performed for two roundness measurement methods: the three-point method and the redundant diameter four-point method. In the probe angle optimization for the redundant diameter four-point method, it was assumed that the fourth probe is located exactly opposite to the first probe. In practice, this can be achieved for example by placing the frame to a location where the difference between the diameter probe reading is the greatest, or by using vertical line contact probes allowing for vertical positioning error. Thus, the optimization of the redundant diameter four-point method becomes a problem of optimizing the three-point method considering only odd harmonic components.

A performance index as described in Eq. (5) was calculated for the sensor angles over a 2-dimensional parameter space with ϕ_2 and ϕ_3 between 0° and 180°. A robust optimization was then performed with the goal to minimize the worst case conditions inside an angle tolerance of ±2° (Eq. (6)).

The condition numbers were also investigated on the level of single harmonic components. Fig. 4 shows κ_k calculated for harmonic components from 2 to 7. It can be seen that each added harmonic results in added zones of harmonic suppression. Mathematical formulation for harmonics with complete suppression occurring at periodic intervals has been presented in literature [13]. Also, it can be seen from Fig. 4 that for the odd harmonic components there is a suppressed area when the probes are located at 180 degrees.

The probe angles suggested by Kato [15] {0°, 37°, 68°}, and Hale [16] {0, 90°, 133.8°} are presented in Figs. 10 and 9 for reference. Marsh [19] {0°, 99.84°, and 202.5°} and Cappa [17] have also mentioned suitable angles, which were not included in the figures due to the maximum difference between the angles exceeding 180°.

The optimization was performed for all harmonics for the three-probe method (9) and for odd harmonics only (Fig. 10) for the redundant diameter sampling, where even harmonics are calculated from the diameter variation profile.

When considering all harmonics in the range 2,...,20, the global minimum 52.83 was encountered at {0°, 60.5°, 105.5°} and the robust minimum 55.47 at {0°, 49°, 115°}. When considering only odd harmonics in the range 3,...,20, the global minimum 24.23 was encountered at {0°, 45°, 105°} and the robust minimum 25.56 at {0°, 59.5°, 135°}.

4. Discussion

The problem addressed in this paper was twofold. One source of error can be using the nominal angles in calculating roundness profiles, when actually during the measurement, the angles are not the nominal angles. These errors can be caused by engineering imperfections of the measurement setup. Cross correlation or other techniques can be used to determine the actual probe angles, but if the probe angles have not been correctly selected, the actual angles can reside in an area with poor harmonic transmission characteristics.

If the probe angles are measured accurately after positioning the probes and these determined angles are used when calculating the roundness profile, tolerances of the probe alignment do not need to be as tight during manufacturing and positioning of the measuring equipment.

Possible sources of error in determining the probe angles are discrete sampling and center point movement of the workpiece. These were assumed not to contribute significantly to the angle determination. It must also be noted that cross correlation is not the only possibility to determine phase differences between the signals. Any method which can be used to calculate the phase difference between similar signals could have been used, and the probe angles could be calibrated with an electrical contact marker, which could be used to detect at which encoder position the probes make contact with the marker.

The results of such optimization as performed in this research will largely depend on the selected optimization criteria, which include the range of harmonics, selected weighting function h_k and selected angle deviation tolerances ϕ_{tol2} and ϕ_{tol3}. Performing a robust optimization over a large angular tolerance or selecting a wide range of harmonics will obviously worsen the total performance index.

Kato [15] used a low pass filter on the performance index to give emphasis on the lower order harmonic components, but in this case a linearly decreasing function was used. An alternative could have been to use the low pass filters defined for roundness

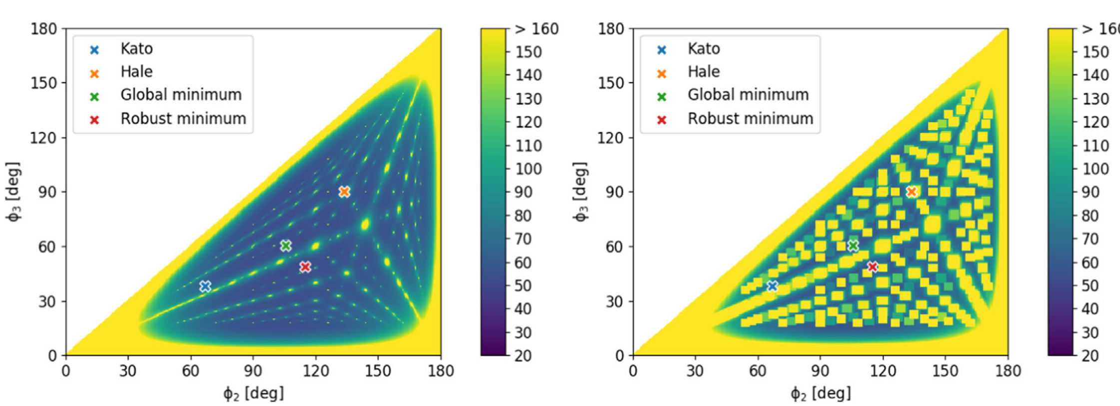

Fig. 9. On the left, performance index T (Eq. (5)) as a function of the probe angles with harmonics 2,...,20 with linearly decreasing weighting h_k. The robust optimization (Eq. (6)) is shown on the right.

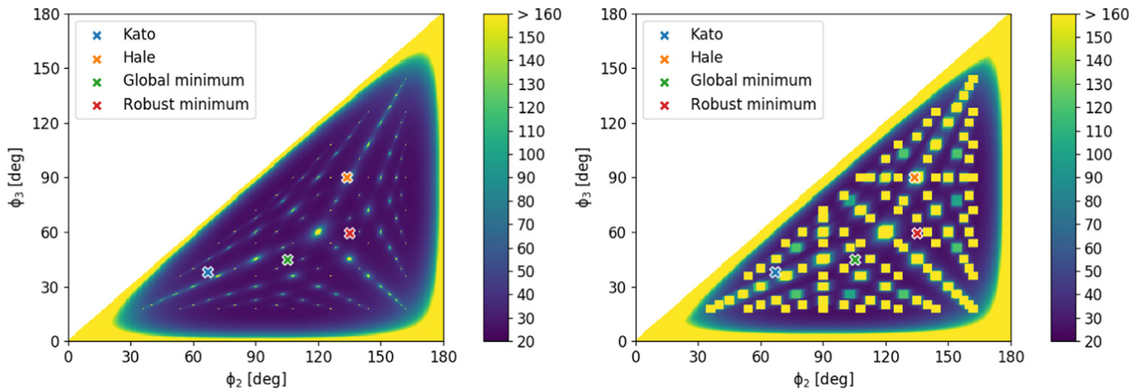

Fig. 10. On the left, performance index T (Eq. (5)) as a function of the probe angles with only odd harmonics in the range 2,. . .,20 with linearly decreasing weighting h_k. The robust optimization (Eq. (6)) is shown on the right. The results of this optimization are intended for a redundant roundness measurement method, where even harmonics are calculated from a diameter variation profile.

profiles in ISO 12181-2 [1], but the authors consider these filters not to have been designed for weighing performance indexes. Nevertheless, the optimization of the probe angles largely depends on which harmonics are of interest. Selecting a large range of harmonics will lead to worse results when considering the whole range.

Parameters selected for the optimization largely depend on the application. A similar absolute positioning error will lead to different angular errors with workpieces of different diameters. The same applies both for the mean angle deviation and the standard deviation. Whatever the selected parameters for the optimization, assessing the quality of the recommended probe angles is difficult, and will be limited by the application and also characteristics of the workpiece being measured. Previous work by the author shows that there is a connection between the error propagation rate and the error that different kinds of positional errors and center point movement cause in a harmonic component [20].

5. Conclusion

Deviations from the nominal probe angles can result in an incorrect roundness profile in two ways: using incorrect angles in the calculation of the roundness profile, or using the correct angles with poor harmonic characteristics. The results of this research address both of these problems. It was first shown how probe angles can be determined accurately after the probes have been positioned for measurement. To avoid errors, these determined actual angles can be used in the calculation of the roundness profile. However, because multi-probe methods suffer from harmonic suppression, it may be that the determined angles are in an area where a harmonic is suppressed. A robust optimization was performed to find areas for the angles where deviations will not result in suppression.

To conclude, determining and using the correct angles for roundness profile calculation will lead to reduced error in calculated roundness profiles. However, the prerequisite for the correction is that the actual angles are not in an area with harmonic suppression. When choosing the nominal probe angles, a robust optimization can be performed with criteria to allow deviations in the nominal angles without suppression.

CRediT authorship contribution statement

Tuomas Tiainen: Methodology, Software, Formal analysis, Visualization, Writing - original draft. **Raine Viitala:** Conceptualization, Methodology, Writing - review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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