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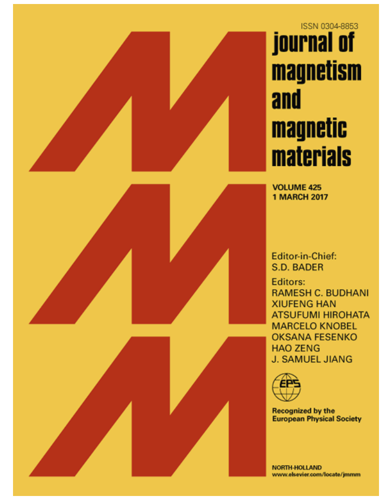

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Determination of stress dependent magnetostriction from a macroscopic magneto-mechanical model and experimental magnetization curves

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In this paper, we propose a method to identify the magnetostrictive behavior of electrical steel sheet submitted to a mechanical loading. The technique relies on the use of a magneto-mechanical model including the magnetostrictive phenomenon, namely the anhysteretic Jiles-Atherton-Sablik (JAS) model, and experimental macroscopic stress dependent magnetization curves. The method is illustrated with measured magnetization curves of a non-oriented (NO) electrical steel sheet under different stresses. Furthermore, the influence of a bi-axial mechanical loading on the magnetostrictive behavior is analyzed with the help of an equivalent stress.

Index Terms **— Magnetostriction model identification, Magneto-mechanical modeling, Jiles-Atherton-Sablik model**

I. INTRODUCTION

The magnetic properties of electrical steel sheets depend strongly on the mechanical stress. Different research works have been carried out to address this magneto-mechanical coupling, where the effects of both elastic and plastic stresses have been investigated. In electrical machines, the magnetic losses and permeability are the most affected parameters [1]; moreover, the magnetostriction, which is an intrinsic characteristic of the material has also an elastic stress dependency. Thus, when the stress is relatively high, it is considered as the main source of acoustic noise in rotating electrical machines and transformers [2-4].

The magnetostriction in ferromagnetic materials corresponds to the fractional change in the length $\Delta l/l$ of a sample when it is submitted to a magnetic field. In general, the magnetostriction is a tensor describing the material deformation under the action of an arbitrary magnetization. Moreover, an applied mechanical stress can alter the magnetostrictive behavior and leads to a change in the magnetic properties of the material [5].

To describe this double dependency with respect to the magnetization and the mechanical stress, various research works revealed that the magnetostriction is an even function of the magnetization, with a nonlinear dependency on the compressive and tensile stresses [6]. According to these results, models of varying complexity have been developed. However, they often require complicated measurements for identification and validation [7].

In this work, we propose a method to identify the parameters of a magnetostrictive model. The technique does not require any special measurements; it is based on the correlation of the magnetostrictive phenomenon with the magnetic behavior using the Jiles-Atherton-Sablik (JAS) model. Measured magnetization curves of a non-oriented (NO) electrical steel sheet submitted to different elastic stresses have been used for this purpose. Furthermore, the influence of bi-axial mechanical loading on the magnetostrictive behavior is analyzed.

This paper is organized as follows: In Section II JAS model and the used magnetostriction model are presented. In Section III, the identification procedure of the magnetostriction model is presented. In Section IV the study of the magnetostrictive behavior under biaxial stress is presented. Finally, in Section V the conclusion is drawn on the proposed identification technique that can be extended to predict the behavior of all components of the magnetostriction tensor.

II. MAGNETO-MECHANICAL MODELING

A. Uniaxial JAS model

The JAS model was developed for hysteretic phenomena; it is based on the thermodynamic equilibrium [8]. For an isotropic polycrystalline material with no pinning sites, the anhysteretic magnetization M_{an} is expressed as:

$$M_{an} = M_s\left(\coth\left(\frac{H_e}{a}\right) - \frac{a}{H_e}\right) \quad (1)$$

Where M_s, H_e and a are the saturation magnetization, the effective field and the anhysteretic scaling factor respectively. The effective field derives from thermodynamics as the derivative of the free energy with respect to magnetization.

$$H_e = H + \alpha M_{an} + H_\sigma \quad (2)$$

where H is the applied magnetic field, α is the coupling parameter of the magnetic domains, and H_σ refers to the component of the effective field due to the mechanical stress and is given by

$$H_\sigma = \frac{3\,\sigma}{2\mu_0}\left(\frac{\partial\lambda}{\partial M_{an}}\right)_T \tag{3}$$

where μ_0 and σ are the permeability of the vacuum and the applied mechanical stress. $\lambda = \lambda_\parallel$ is the magnetostriction in the direction of the applied magnetic field.

When a mechanical stress $\sigma = \sigma_\parallel$ is applied parallel to the magnetization, the magnetostrictive behavior is a function of both magnetization and mechanical stress.

B. Magnetostriction model

The magneto-mechanical coupling in the Jiles-Atherton-Sablik (JAS) model is described by the term H_σ, which depends on the magnetostriction $\lambda_\parallel$. The magnetostriction as mentioned in the introduction depends on magnetization and has a non-uniform dependency with the compressive and tensile applied stresses.

Different magnetostriction models are proposed in the literature, such as the multiscale model [7] which is a physical approach based on the modeling of the magneto-elastic coupling by considering the free energy of the material at the domain scale, or the invariant model presented in [10]. In this study, the proposed model, given by expression (4), is inspired from [9]; it consists in the product of two distinct functions; the first one is a polynomial of degree $2p$ depending on the anhysteretic magnetization M_{an} and the second function scales the magnetostriction depending on the stress, using the hyperbolic tangent which is controlled by the parameters C, τ, σ_0.

$$\lambda_\parallel = \left(\sum_{j=1}^{p} a_j M_{an}^{\;2j}\right)\left(C + \tanh\left(\frac{\sigma + \sigma_0}{\tau}\right)\right) \tag{4}$$

This mathematical model allows to reproduce the macroscopic magnetostrictive behavior observed experimentally. It also presents a good compatibility with the proposed identification technique, where macroscopic magnetization curves under different stress levels are considered.

C. Equivalent stress

To take into account the effect of a multiaxial stress on the magnetostriction behaviour, the uniaxial stress σ in (3) and (4) is replaced by the equivalent stress σ_{eq} defined in (5). This equivalent stress correspends to a fictive uniaxial stress that would change the magnetic behavior in a similar manner as the real multiaxial stress [11].

$$\sigma_{eq} = \frac{3}{2}\boldsymbol{h}^{\boldsymbol{t}}\boldsymbol{\sigma}\,\boldsymbol{h} - \frac{1}{2}\mathrm{tr}(\boldsymbol{\sigma}) \tag{5}$$

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} \tag{6}$$

where $\boldsymbol{h}$ denotes the direction of the applied field and $\boldsymbol{h}^{\boldsymbol{t}}$ the transpose of $\boldsymbol{h}$ .The term $\mathrm{tr}(\boldsymbol{\sigma})$ represents the trace of the stress tensor $\boldsymbol{\sigma}$.

III. MAGNETOSTRICTION MODEL IDENTIFICATION

To identify the magnetostriction model parameters, measured anhysteretic magnetization curves under different mechanical stresses ∓ 30 MPa are used. The measurements were performed on a M400-50A non-oriented electrical steel sheet by using a custom made single sheet tester device. The detail of the measurement setup and experimental procedure can be found in [12].

A. Identification steps

There are two distinct steps to follow during the identification procedure. The first step, corresponding to the unloaded state, concerns the material's parameters identification, these are common to the biaxial and uniaxial stress cases. The second step deals with magnetostriction model's parameters, it is carried out separately for each type of mechanical loading. To carry out the curve fitting in this study, the least square error was used. The identification illustrated below considers the uniaxial stress. The same procedure will be followed in next section to consider the biaxial stress.

First step: without considering the stress dependence in (2) ($H_\sigma = 0$), the initial parameters of the JAS model: M_s, a and α were identified by fitting the model with the zero stress magnetization curve, their values are given in Table I

TABLE I
JAS MODEL INITIAL PARAMETERS

Parameter	Value
M_s	1.12×10^6 A/m
a	112.40
$\boldsymbol{\alpha}$	2.51×10^{-4}

Second step: the parameters previously determined are kept fixed and the magnetostriction model parameters are identified. The expressions (2), (3) and (4) are combined in (1) to fit the global expression of magnetization with measured magnetization curves for different elastic uniaxial stress levels. Fig. 1 gives the fitted and measured anhysteretic magnetization curves for different uniaxial stresses used to identify the magnetostriction model.

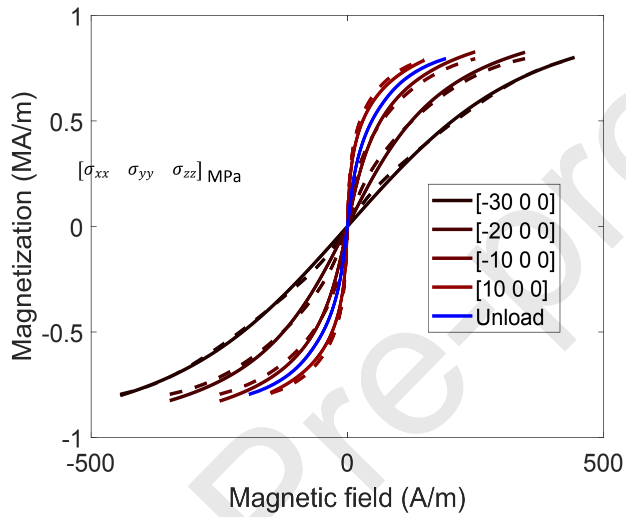

Fig. 1. Modeled (solid line) and Measured (dashed line) anhysteretic magnetization curves for different uniaxial stress applied parallel to the direction of magnetization.

Due to the implicit nature of the Langevin function in (1), the Newton Raphson method is used for the computation. To ensure the convergence of the iterative process, the degree of polynomial function in (4) is set p = 1. Table II gives the fitted parameters of $\lambda_{\parallel}$.

TABLE II
FITTED MAGNETOSTRICTION PARAMETERS

Parameter	Value
a_1	$-1,28 \times 10^{-16}$
C	-0.99
$\boldsymbol{\tau}$ (MPa)	105,45
$\boldsymbol{\sigma_0}$ (MPa)	233,41

The modeled magnetostriction function for different stress levels is given in Fig.2.

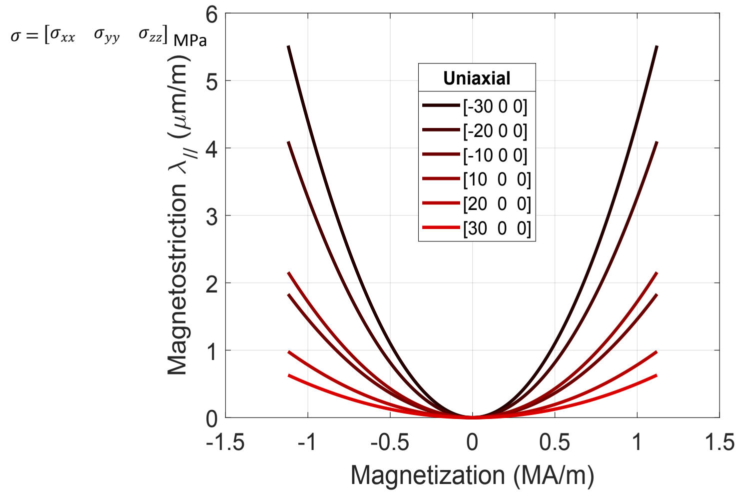

Fig. 2. Stress dependency of the magnetostriction $\lambda_{\parallel}$ identified using the proposed method

To validate the identification approach, the modeling result is compared to the measured magnetostriction given in [9]. The magnetization curves used in the proposed identification technique and the magnetostrictive measurement are both performed on NO electrical steel sheet with grade M400-50A.

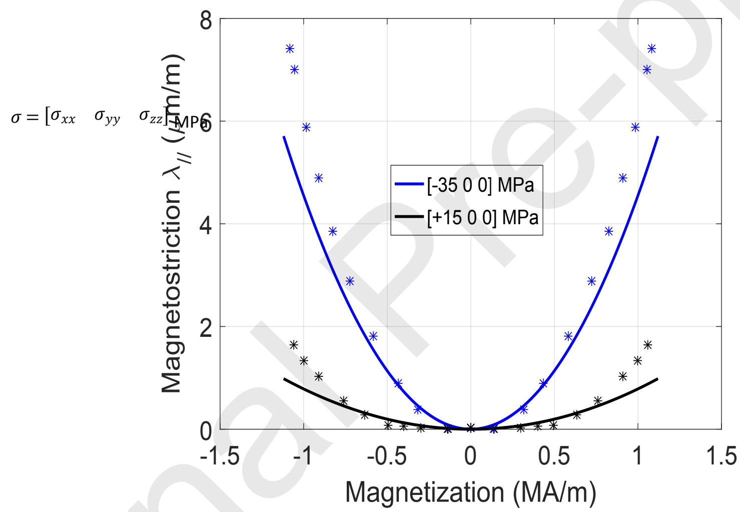

Fig. 3. Measured (dashed line) and modeled (solid line) magnetostriction $\lambda_{\parallel}$ curves under compressive and tensile stress.

The comparison carried out in Fig.3 shows a reasonable agreement of the qualitative magnetostriction behavior with the measurements. Indeed, the nonlinear dependency on the compressive and the tensile stress is well reproduced. However, the quantitative comparison shows a difference in the magnetostriction magnitude. This difference is explained by the fact that experimental and modeled data are from two different studies. Despite that the used magnetic material presents the same grade (M400-50A) in both studies, the magnetic properties can be different from one steel sheet to another. Moreover, the experimental error must be taken in to account during the comparison.

IV. Results and Discussion

In this section, the effect of the multiaxial stress on the behavior of the magnetostriction $\lambda_{\parallel}$ is analyzed. The same identification steps as described above are considered here, the first step being the identification of the initial parameters at zero stress, it remains unchanged. In the second step, first the model describing $\lambda_{\parallel}$ is writing in function of σ_{eq} instead of σ, then the measured magnetization curves under biaxial stress are used to fit the biaxial magnetostrictive model. The effect of two types of biaxial stresses are analyzed.

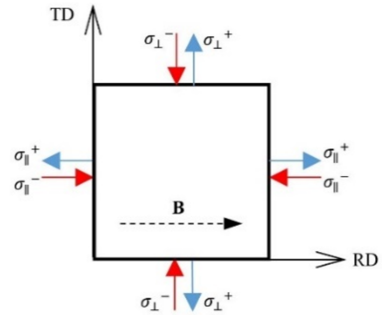

Fig. 4. Schematic of applied biaxial stress

Figure 4 represents the schematic of the applied biaxial stresses, where the measured magnetization and the magnetostriction are parallel to the rolling direction. The signs + and − refer to tensile and compression stress respectively, while ∥ and ⊥ refer to the direction of the stress parallel or transvers with respect to the magnetization.

A. Biaxial stress

In Fig. 5 the modeled magnetostriction $\lambda_\parallel$ is represented when the material is submitted to uniaxial and equi-biaxial stress. In Fig. 6, the modeled magnetostriction $\lambda_\parallel$ is given for uniaxial stress and shear-biaxial stresses. For more convenience, and because $\lambda_\parallel$ it is an even function of magnetization, the modeled magnetostriction curves under different biaxial stress levels are represented in the same magnetization axis. One varies with respect to the positive values of magnetization and the other one varies with respect to the negative values.

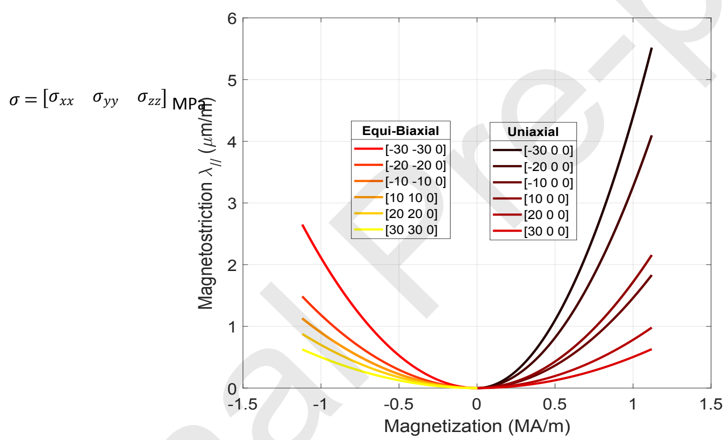

Fig. 5. Comparison between the modeled magnetostriction $\lambda_\parallel$ when the material is submitted to uniaxial and equi-biaxial stress.

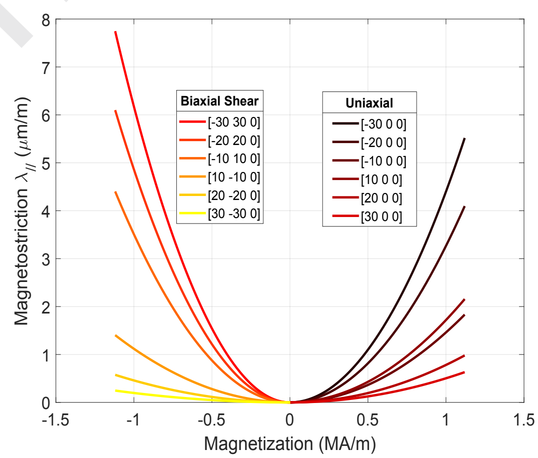

Fig. 6. Comparison between the modeled magnetostriction $\lambda_\parallel$ when the material is submitted to uniaxial and shear-biaxial stress.

As represented in Fig.2 under uniaxial stress, $\lambda_\parallel$ presents a maximum value at –30MPa which corresponds to the most degraded magnetization curve (Fig. 1) then decrease till the minimum value at +30 MPa which corresponds to the improved magnetization curve (Fig. 1). This magnetostrictive behavior with respect to the magnetization and stress is classically observed for electrical steel sheets [9].

The magnetostriction $\lambda_{\parallel}$ under a biaxial stress which is modeled using the equivalent stress shows the same behavior as in the uniaxial stress case. Indeed, the dependency of magnetostriction with the magnetization and the non-uniform dependency with the compressive and tensile applied stresses are respected in both types of biaxial stresses. However as shown in Fig. 5 and 6, the $\lambda_{\parallel}$ curves compared to the ones obtained under uniaxial stress, they shift down in the case of equi-biaxial stress and they shift up in the case of shear-biaxial stress respectively.

From there, the most interesting result of this study is the compatibility of σ_{eq} with the proposed identification technique. Contrary to other definitions of the equivalent stress such as *Von Mise* stress, σ_{eq} leads to a coherent representation of the magnetostriction where the nonlinear dependency on the compressive and tensile stress is respected. This result can be more investigated and extended to predict the behavior of the whole magnetostriction tensor. It will require further magnetic measurements under pure shear stresses.

VI. Conclusion

An identification method for the magnetostriction $\lambda_{\parallel}$ was presented. The technique requires only measured magnetization curves for different uniaxial stress levels and gives results in good agreement with direct measured magnetostriction.

The use of equivalent stress to take into account the effect of multiaxial stress gives coherent magnetostrictive behavior. This technique can be extended to identify the whole components of the magnetostrictive strain tensor.

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Highlights

- The magnetostrictive behavior of electrical steel sheet submitted to mechanical loading is modeled.
- The magnetostriction model is identified using a technique based on a macroscopic magneto-mechanical model and measured magnetization curves.
- The identification technique is extended to the biaxial stress case using an equivalent stress.
- The modeling result is coherent with the magnetostriction measurements,

- The nonlinear dependency of the magnetostriction on the compressive and the tensile stress is well represented.

AUTHOR STATEMENT

Nabil M'ZALI: Conceptualization, Methodology, Software, Writing - Original Draft
Floran MARTIN: Software, Validation
Ugur AYDIN: Resources, Investigation
Anouar BELAHCEN: Validation, Writing- Reviewing and Editing,
Abdelkader BENABOU: Validation, Writing- Reviewing and Editing,
Thomas HENNERON: Writing- Reviewing and Editing,