Hinkkanen, Marko; Qu, Zengcai; Awan, Hafiz A.A.; Tuovinen, Toni; Briz, Fernando

Current control for IPMSM drives: direct discrete-time pole-placement design

Published in:
IEEE Workshop on Electrical Machines Design, Control and Diagnosis (WEMDCD)

DOI:
10.1109/WEMDCD.2015.7194524

Published: 26/03/2015

Please cite the original version:
Current Control for IPMSM Drives: Direct Discrete-Time Pole-Placement Design

Marko Hinkkanen†, Zengcai Qu∗, Hafiz Asad Ali Awan†, Toni Tuovinen‡, and Fernando Briz‡

†Aalto University, Espoo, Finland
‡ABB Drives, Helsinki, Finland
‡University of Oviedo, Gijón, Spain

Abstract—This paper deals with discrete-time models and current control methods for synchronous motors with a magnetically anisotropic rotor structure, such as interior permanent-magnet synchronous motors (IPMSMs) and synchronous reluctance motors (SyRMs). Dynamic performance of current controllers based on continuous-time models is limited, especially if the ratio of the sampling frequency to the fundamental frequency is low. An exact closed-form hold-equivalent discrete motor model is derived. The zero-order hold of the stator-voltage input is modeled in stationary coordinates, where it physically is. An analytical discrete-time pole-placement design method for a two-degree-of-freedom state-space current controller with an integral action is proposed. The proposed method is easy to apply: only the desired closed-loop bandwidth and the three motor parameters \((R_s, L_s, L_d)\) are required. The robustness of the proposed current control design against parameter errors is analyzed. The controller is experimentally verified using a 6.7-kW SyRM drive.

Index Terms—Current control, delay, discrete-time model, interior permanent-magnet synchronous motor (IPMSM), synchronous reluctance motor (SyRM), zero-order hold.

I. INTRODUCTION

Synchronous motors with a magnetically anisotropic rotor—such as interior permanent-magnet synchronous motors (IPMSMs), synchronous reluctance motors (SyRMs), and permanent-magnet-assisted SyRMs—are more and more applied in hybrid (or electric) vehicles, heavy-duty working machines, and industrial applications. In these applications, the maximum speeds and, consequently, the maximum operating frequencies can be very high (e.g., 12 000 r/min corresponding to the frequency of 1 000 Hz for a ten-pole machine). Since the switching frequency of the converter feeding the motor is limited due to the losses, the resulting ratio between the switching frequency and the maximum fundamental frequency can be even below ten. This will affect the sampling frequency, too, as it is typically equal to or twice the switching frequency.

Generally, the stator current of synchronous motor drives is controlled in rotor coordinates [1]–[11]. This coordinate system is a natural selection since the controllable quantities are DC in steady state, the inductance matrix and the PM-flux vector are (ideally) constant, and other parts of the control system typically operate in rotor coordinates. The most widely used current control approach is to use a synchronous-frame proportional-integral (PI) controller, often augmented with decoupling terms to compensate for the cross-coupling due to the rotating coordinate system. Typically, the controller is first designed in the continuous-time domain and then discretized for the digital implementation using, e.g., the Euler or Tustin approximation [1]–[5]. This approach is well understood and works well in most applications. However, the closed-loop control bandwidth and the maximum operating frequency are limited to below approximately 5% of the sampling frequency. Furthermore, heuristic compensations for the angular errors due to time delays are often needed. Higher dynamic performance at a given sampling frequency could be achieved by designing the controller directly in the discrete-time domain [6], [8]–[11].

A hold-equivalent discrete model—including the effects of the zero-order hold (ZOH) and a sampler—of the motor drive is needed for the direct discrete-time control design. An exact closed-form hold-equivalent model has been derived for induction motor drives in [12] and an approximate discrete model for the IPMSM drives has been proposed in [9], [10]. The exact closed-form hold-equivalent model for surface permanent-magnet synchronous motors (SPSM) drives can be found in [6], [8], but the closed-form expressions valid for IPMSM drives are not available in the literature.

In this paper, current control designs based on discrete IPMSM models are considered. First, the motor model and the control design in the continuous-time domain are reviewed in Section II. Then, the main results are presented as follows.

1) An exact closed-form hold-equivalent discrete model for IPMSM drives is derived in Section III. The ZOH of the stator-voltage input is modeled in stationary coordinates, where it physically is. The model provides the mapping between the continuous-time model (with physical parameters) and the discrete-time model. The derived closed-form expressions can be applied to the design, analysis, and implementation of controllers and observers (e.g., in connection with deadbeat [13] or predictive [14] direct torque control methods).

2) An analytical direct discrete-time pole-placement design method for a two-degree-of-freedom (2DOF) state-space current controller with an integral action is proposed in Section IV. The time delays are inherently taken into account in the design. The proposed method is easy to apply: only the desired closed-loop bandwidth and the three motor parameters are needed.

3) The robustness of the proposed current control design against parameter errors is analyzed in Section V. The operation of the controller is further investigated by means of simulations and experiments using a 6.7-kW SyRM drive.
Naturally, the model and the control design method are directly applicable to SPMSM drives as well.

II. CONTINUOUS-TIME MODEL AND CONTROL DESIGN

A. Continuous-Time Model

In order to model IPMSMs, real space vectors will be used throughout the paper. For example, the stator-current vector is \( \mathbf{i}_s = [i_d, i_q]^T \), where \( i_d \) and \( i_q \) are the components of the vector and the matrix transpose is marked with the superscript T. The identity matrix, the orthogonal rotation matrix, and the zero matrix are defined, respectively, as

\[
I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}
\]

(1)

Vectors are denoted using boldface lowercase letters and matrices using boldface uppercase letters. Space vectors in stator coordinates are marked with the superscript “s”, no superscript is used for space vectors in rotor coordinates.

The electrical angular speed and angle of the rotor are denoted by \( \omega_m \) and \( \vartheta_m(t) = \int \omega_m dt \), respectively. The time dependency of the variables is denoted by the argument \( t \). It is worth noticing that the rotor speed and the motor parameters dependency of the variables is denoted by the argument \( t \). When the continuous-time model is discretized.

In rotor coordinates, the state-space representation corresponding to the standard model of the IPMSM is

\[
\frac{d\psi_s(t)}{dt} = A\psi_s(t) + Bu_s(t) + B\psi_{pm} \tag{2a}
\]

\[
i_s(t) = C\psi_s(t) + d\psi_{pm} \tag{2b}
\]

where \( \psi_s \) is the stator flux vector, \( u_s \) is the stator voltage vector, and \( \psi_{pm} \) is the permanent-magnet (PM) flux. The system matrices are

\[
A = \begin{bmatrix} -R_s/L_d & \omega_m \\ -\omega_m/L_d & -R_s/L_q \end{bmatrix}, \quad B = I, \quad b = \begin{bmatrix} R_s/L_d \\ 0 \end{bmatrix}
\]

\[
C = \begin{bmatrix} 1/L_d & 0 \\ 0 & 1/L_q \end{bmatrix}, \quad d = \begin{bmatrix} -1/L_d \\ 0 \end{bmatrix}
\]

(3)

where \( R_s \) is the stator resistance, \( L_d \) is the direct-axis inductance, and \( L_q \) is the quadrature-axis inductance. The state-space representation in (2) has two inputs: the stator voltage \( u_s(t) \) and the PM flux \( \psi_{pm} \) (which is constant). If \( L_d = L_q \), the model represents the SPMSM. If \( \psi_{pm} = 0 \), the model of the SyRM is obtained.

The model (2) can be expressed in the Laplace domain as

\[
i_s(s) = Y_c(s) [u_s(s) - u_i(s)]
\]

(4)

where the transfer-function matrix is

\[
Y_c(s) = C(sI - A)^{-1} = \begin{bmatrix} R_s + sL_d & -\omega_mL_q \\ \omega_mL_d & R_s + sL_q \end{bmatrix}^{-1}
\]

(5)

1The notation is very similar to that obtained for complex space vectors: the rotation matrix \( J \) corresponds to the imaginary unit \( j \) and the coordinate transformation matrices can be expressed using matrix exponentials, i.e., \( e^{\theta J} = \cos \theta I + \sin \theta J \).

and \( u_i = [\omega_m\psi_{pm}]^T \) is the voltage induced by the PM flux. This induced voltage can be considered as a load disturbance from the current controller point of view.

For the purposes of Section III, the stator voltage input in (2a) can be expressed in stator coordinates, leading to

\[
\frac{d\psi_s(t)}{dt} = A\psi_s(t) + B'(t)u^s_s(t) + b\psi_{pm} \tag{6}
\]

where the time-varying input matrix is

\[
B'(t) = e^{-\vartheta_m(t)J} \tag{7}
\]

If constant \( \omega_m \) is assumed, \( \vartheta_m(t) = \vartheta_m(0) + \omega_m t \) holds.

B. Preliminaries: Continuous-Time Current Control Design

For starters, a continuous-time 2DOF PI-type current controller is reviewed. A state controller with an integral action and reference feedforward will be used as a design framework. In the Laplace domain, this control law, expressed in rotor coordinates, is

\[
u_{s,ref}(s) = K_{1c}i_{s,ref}(s) + \frac{K_{1c}}{s}[i_{s,ref}(s) - i_s(s)] - K_{1c}i_s(s)
\]

(8)

where \( u_{s,ref} \) is the resulting reference voltage, \( i_{s,ref} \) is the reference current, \( K_{1c} \) is the feedforward gain, \( K_{1c} \) is the integral gain, and \( K_{1c} \) is the state feedback gain. The gains are real 2×2 matrices. The voltage production of the inverter is assumed to be accurate and delayless, i.e., \( u_s = u_{s,ref} \) holds.

Using (4) and (8), the closed-loop current response becomes

\[
i_s(s) = H_c(s)i_{s,ref}(s) - Y_{ic}(s)u_i(s)
\]

(9)

The closed-loop transfer-function matrices for the reference following and disturbance rejection are

\[
H_c(s) = (s^2I + sA_{1c} + A_{0c})^{-1}(sB_{1c} + B_{0c})
\]

Y_{ic}(s) = (s^2I + sA_{1c} + A_{0c})^{-1}(sC)

(10)

(11)

where

\[
A_{0c} = CK_{1c}, \quad A_{1c} = C(K_{1c} - AC^{-1})
\]

B_{0c} = CK_{1c}, \quad B_{1c} = CK_{1c}

(12)

General control objectives for current controllers are: 1) no cross-coupling between the d- and q-axis and 2) the same closed-loop dynamics for both the axes. Hence, the non-diagonal elements of \( H_c(s) \) should be zero due to the first objective and the diagonal elements should be equal due to the second objective. The resulting desired closed-loop transfer-function matrix is of the form

\[
H_{cs}(s) = \frac{b_{1c}s + b_{0c}}{s^2 + a_{1c}s + a_{0c}}I
\]

(13)

Based on (12) and (13), the closed-loop poles and zeros can be placed using the relations

\[
K_{1c} = b_{1c}C^{-1}, \quad K_{1c} = a_{0c}C^{-1}
\]

(14)
where the hat indicates parameter estimates, e.g.:
\[
\hat{C}^{-1} = \begin{bmatrix}
\hat{L}_d & 0 \\
0 & \hat{L}_q
\end{bmatrix}
\] (15)

Choosing the design parameters\(^2\)
\[
a_{0c} = \alpha^2, \quad a_{1c} = 2\alpha, \quad b_{1c} = \alpha
\] (16)

the transfer-function matrix (13) reduces to the first-order unity-gain low-pass filter
\[
H_{c*}(s) = \frac{\alpha}{s + \alpha} I
\] (17)

where \(\alpha\) is the desired closed-loop control bandwidth. If accurate parameter estimates in (14) are assumed, \(H_c(s) = H_{c*}(s)\) holds. It can be easily shown that the controller corresponding to (8), (14), and (16) is equivalent to the 2DOF PI-type current controller considered in [2], [15], [16].

Advantages of this pole-placement design are its simplicity and easy tuning: only the desired bandwidth \(\omega_c\) and three parameter estimates \((\hat{R}_s, \hat{L}_d, \text{and } \hat{L}_q)\) are needed. For digital implementation, the algorithm has to be discretized using, e.g., the Euler or Tustin methods. Unfortunately, unless the sampling frequency is much higher than the closed-loop bandwidth and the maximum operating frequency, the actual closed-loop system deviates significantly from (17) due to discretization errors, leading to the cross-coupling between the d- and q-axis, oscillations, or even instability.

Assuming accurate parameter estimates in (14), the disturbance-rejection transfer-function matrix in (11) reduces to \(Y_{ic*}(s) = s/(s + \alpha)^2C\). Alternatively, the poles and zeros could be placed using the complex vector design [3], which leads to the same desirable reference-following transfer-function matrix (17), but the disturbance-rejection transfer-function matrix becomes different. According to [3], the complex vector design reduces the sensitivity to parameter mismatch.

III. DISCRETE-TIME MODELS

A. Exact Model

For the development of an exact discrete-time model, sampling of the stator currents is assumed to be synchronized with the pulse-width modulation (PWM). The switching-cycle averaged quantities are considered. Under these assumptions, the stator voltage in stator coordinates is piecewise constant during the sampling period as \(\text{ZOH}\) delay properly into account.

Choosing the design parameters\(^3\)
\[
\alpha = \frac{\omega_s}{2}, \quad \alpha_1 = 2\alpha, \quad \beta_1 = \alpha
\] (16)

the transfer-function matrix (13) reduces to the first-order unity-gain low-pass filter
\[
H_{c*}(s) = \frac{\alpha}{s + \alpha} I
\] (17)

where \(\alpha\) is the desired closed-loop control bandwidth. If accurate parameter estimates in (14) are assumed, \(H_c(s) = H_{c*}(s)\) holds. It can be easily shown that the controller corresponding to (8), (14), and (16) is equivalent to the 2DOF PI-type current controller considered in [2], [15], [16].

Advantages of this pole-placement design are its simplicity and easy tuning: only the desired bandwidth \(\omega_c\) and three parameter estimates \((\hat{R}_s, \hat{L}_d, \text{and } \hat{L}_q)\) are needed. For digital implementation, the algorithm has to be discretized using, e.g., the Euler or Tustin methods. Unfortunately, unless the sampling frequency is much higher than the closed-loop bandwidth and the maximum operating frequency, the actual closed-loop system deviates significantly from (17) due to discretization errors, leading to the cross-coupling between the d- and q-axis, oscillations, or even instability.

Assuming accurate parameter estimates in (14), the disturbance-rejection transfer-function matrix in (11) reduces to \(Y_{ic*}(s) = s/(s + \alpha)^2C\). Alternatively, the poles and zeros could be placed using the complex vector design [3], which leads to the same desirable reference-following transfer-function matrix (17), but the disturbance-rejection transfer-function matrix becomes different. According to [3], the complex vector design reduces the sensitivity to parameter mismatch.

B. Approximation Based on Series Expansion

The exact model \(\Phi\) in (18) can be expressed using the series expansion [18]
\[
\Phi = I + T_s A \Psi
\] (23)

where
\[
\Psi = I + T_s A + \frac{T_s^2 A^2}{2!} + \frac{T_s^3 A^2}{3!} + \ldots
\] (24)

The exact voltage input matrix \(\Gamma\) cannot be easily expressed as a series expansion due to the time-varying matrix \(B'(t)\) in (7). If the \(\text{ZOH}\) of the stator voltage were in rotor coordinates, the matrix \(\Gamma\) would equal \(T_s \Psi B\). However, the voltage is kept constant in stator coordinates during the sampling period as

\footnotetext[3]{Unlike the IPMSM model, the SPMSM model could be expressed using complex space vectors by replacing the matrix \(J\) with the imaginary unit.}
discussed before. In [17], an approximate compensation for this effect was derived. Applying this compensation, the input matrix for the voltage can be approximated as

$$\Gamma \approx T_s \Psi B \frac{\omega_m T_s / 2}{\sinh(\omega_m T_s / 2)} e^{-(\omega_m T_s / 2)}$$

(25)

Since the PM flux is constant in rotor coordinates, the input matrix for the PM flux is

$$\gamma = T_s \Psi b$$

(26)

Typically, only the first two terms of (24) are needed, i.e., $\Psi = I + (T_s / 2) A$. Choosing $\Psi = I$ yields the Euler approximation, which suffices only at high sampling frequencies.

C. Current as a State Variable

The current control design becomes more straightforward, if the stator current is chosen as a state variable. With this selection, the state equation can be expressed as

$$i_s(k + 1) = \mathbf{F} i_s(k) + \mathbf{G} u_{s,ref}(k) + g \psi_{pm}$$

(27)

where the new system matrices are

$$\mathbf{F} = C \Phi \Phi^{-1}, \quad \mathbf{G} = C \Gamma, \quad \mathbf{g} = (I - \mathbf{F}) d + C \gamma$$

(28)

D. Inclusion of the Control Delay

Fig. 1 shows the plant model from the control system point of view. As shown in the figure, the digital control system and PWM update have (at least) one-sampling-period time delay due to the finite computation time, i.e., $u^s_{s,ref}(k) = u^s_{s,ref}(k - 1)$ in stator coordinates, or, when transformed into rotor coordinates, $u_{s,ref}(k) = e^{-\omega_m T_s J} u_{s,ref}(k - 1)$. To simplify the notation, $u_{s,ref} = e^{-\omega_m T_s J} u_{s,ref}$ is defined, giving $u_{s}(k) = u_{s,ref}(k - 1)$. The effect of the time delay on the voltage angle can be easily compensated for in the coordinate transformation of the reference voltage (cf. Fig. 1).

For control design, the time delay can be included in the plant model as [18]

$$\begin{bmatrix} i_s(k + 1) \\ u_s(k + 1) \end{bmatrix} = \begin{bmatrix} \mathbf{F} & \mathbf{G} \\ \mathbf{O} & \mathbf{O} \end{bmatrix} \begin{bmatrix} i_s(k) \\ u_s(k) \end{bmatrix} + \begin{bmatrix} \mathbf{O} \\ \mathbf{I} \end{bmatrix} u_{s,ref}(k) + \begin{bmatrix} g \\ \mathbf{0} \end{bmatrix} \psi_{pm}$$

(29)

It is worth noticing that both the states are readily available as feedback signals in the state feedback control: $i_s$ is the measured feedback and $u_s$ is obtained from the previous value of the reference voltage $u_{s,ref}$.

In the following section, the current controller will be designed based on the reference-following dynamics (similarly as in Section II-B). The effect of the disturbance voltage $u_d$ on the stator current will be omitted for simplicity. If needed, its effect can be analyzed separately based on the superposition principle. Hence, from (29), the stator current in the $z$-domain can be expressed as

$$i_s(z) = Y(z) u_{s,ref}(z), \quad \text{where}$$

$$Y(z) = z^{-1} (z I - F)^{-1} G$$

(30)

IV. DISCRETE-TIME CURRENT CONTROL DESIGN

A. Framework

A 2DOF state-space controller with an integral action, shown in Fig. 1, is considered. The control law is

$$x_i(k + 1) = x_i(k) + i_{s,ref}(k) - i_s(k)$$

(31a)

$$u_{s,ref}(k) = K_i i_{s,ref}(k) + K_i x_i(k)$$

(31b)

$$K_1 i_s(z) - K_2 u_{s,ref}(z)$$

where $x_i$ is the integral state, $K_i$ is the integral gain, $K_1$ is the feedforward gain, $K_1$ and $K_2$ are the state-feedback gains, and $u_{s,ref}(k + 1) = u_{s,ref}(k)$. Since all the states are directly available, the closed-loop poles can be placed arbitrarily. The control law (31) can be expressed in the $z$-domain as

$$u_{s,ref}(z) = K_i i_{s,ref}(z) + \frac{K_i}{z - 1} [i_{s,ref}(z) - i_s(z)] - K_1 i_s(z) - \frac{K_2}{z} u_{s,ref}(z)$$

(32)

From (30) and (32), the closed-loop dynamics become

$$i_s(z) = H(z) i_{s,ref}(z)$$

(33)

where

$$H(z) = (z^2 I + z A_2 + z A_1 + A_0)^{-1} (z B_1 + B_0)$$

(34)
The matrices are
\[
A_0 = G(K_2 G^{-1} F + K_1 - K_1)
\]
\[
A_1 = F + G[K_1 - K_2 G^{-1} (I + F)]
\]
\[
A_2 = G K_2 G^{-1} - I - F
\]
\[
B_0 = G(K_1 - K_1), \quad B_1 = G K_1
\]  
(35)

A closed-loop state-space representation can be formed from (29) and (31) as
\[
\begin{bmatrix}
  i_s(k+1) \\
  u_s(k+1) \\
  x_1(k+1)
\end{bmatrix} =
\begin{bmatrix}
  F & G & O \\
  -K_1 & -K_2 & K_1 \\
  -I & O & I
\end{bmatrix}
\begin{bmatrix}
  i_s(k) \\
  u_s(k) \\
  x_1(k)
\end{bmatrix}
\]  
(36)

where the inputs are omitted for simplicity. The poles of (34) equal the eigenvalues of the system matrix in (36). The system is stable if the eigenvalues are inside the unit circle. When analyzing the robustness of the system, it is worth noticing that \( F \) and \( G \) should be the exact system matrices calculated using the actual motor parameters, while the gains can be based on approximations and erroneous parameter estimates (depending on the controller under analysis). If the control design is based on the exact model and the motor parameters are perfectly known, the eigenvalues of (36) equal the desired closed-loop poles.

### B. Approximation of the Continuous-Time Design

The gains of the discrete-time controller (31) can be determined by approximating the continuous-time controller (cf. Section II-B) with the Euler method. In the framework of Fig. 1, the angular error of \( \omega_m T_s \) due to the computational delay is compensated for in the coordinate transformation. When approximating continuous-time designs, the angular error of \( \omega_m T_s / 2 \) caused by the ZOH delay should also be taken into account [17]. Embedding this compensation into the gains yields
\[
K_1 = e^{(\omega_m T_s / 2) J} \left( 2 \alpha C^{-1} - \hat{R}_e I - \omega_m J C^{-1} \right), \quad K_2 = O
\]  
\[
K_1 = e^{(\omega_m T_s / 2) J} \alpha C^{-1}, \quad K_i = e^{(\omega_m T_s / 2) J T_s \alpha^2 C^{-1}}
\]  
(37)

The performance of this design is acceptable if the sampling frequency is about twenty times higher than the closed-loop bandwidth and the operating frequency. At lower sampling frequencies, direct discrete-time design methods are preferred.

### C. Proposed Direct Discrete-Time Design

General control objectives for current controllers in the case of the discrete-time design are the same as in the continuous-time case, cf. Section II-B. Hence, the non-diagonal elements of \( H(z) \) in (34) should be zero in order to avoid cross-coupling of the axes and the diagonal elements should be equal in order to achieve the same dynamics for both the axes. The desirable closed-loop transfer-function matrix is of the form
\[
H_s(z) = \frac{b_1 z + b_0}{z^3 + a_2 z^2 + a_1 z + a_0} I
\]  
(38)

Due to the time delay, \( a_0 = 0 \) is selected. The gain matrices can be solved using (35) and (38):
\[
K_1 = K_1 + (1 + a_2) \hat{C}^{-1} \hat{F} + \hat{C}^{-1} \hat{F}^2
\]
\[
K_2 = (1 + a_2) I + \hat{G}^{-1} \hat{F} \hat{G}
\]
\[
K_t = b_1 \hat{C}^{-1}, \quad K_i = (1 + a_1 + a_2) \hat{C}^{-1}
\]  
(39)

Using these expressions, the poles and zero in (38) can be arbitrarily placed. The gains depend on the rotor speed via the matrices \( \hat{F} \) and \( \hat{G} \).

Choosing the coefficients \( a_1 = \beta^2, \ a_2 = -2 \beta, \) and \( b_1 = 1 - \beta \) leads to
\[
H_s(z) = \frac{1 - \beta}{z^2} I
\]  
(40)

where \( \beta = e^{-\alpha T_s} \) is the exact mapping in the discrete domain of the intended real pole of the system. The diagonal matrix consists of the delay and the first-order unity-gain low-pass filter; the pole in (38) is cancelled by the zero in a similar manner as in the continuous-time case in (17), cf. Section II-B.

In digital control, the computational time delay \( z^{-1} \) cannot be avoided in practice. It is worth noticing that the same input parameters for the design are needed as in the continuous-time case (\( R_e, \hat{L}_d, \hat{L}_q, \) and \( \alpha \)). The proposed design can be seen as a discrete-time counterpart to the control law presented in Section II-B.

### V. Results

A transverse-laminated 6.7-kW four-pole SyRM is considered. The rated values of the motor are: speed 3175 r/min; frequency 105.8 Hz; line-to-line rms voltage 370 V; and rms current 15.5 A. Four different current control designs have been evaluated:

- **Design 1**: approximation of the continuous-time design;
- **Design 2**: proposed design based on the approximate model with \( \Psi = I \);
- **Design 3**: proposed design based on the approximate model with \( \Psi = I + (T_s / 2) A \);
- **Design 4**: proposed design based on the exact model.

For brevity, only selected results are presented in the following.

#### A. Robustness Analysis

The robustness of the four current control designs against parameter errors is analyzed by calculating the eigenvalues of (36). The controller gains have been calculated using the parameter estimates \( \hat{L}_q = 2.0 \) p.u., \( \hat{L}_q = 0.3 \) p.u., and \( \hat{R}_e = 0.04 \) p.u. The desired bandwidth \( \alpha \) is varied in a range from 0 to 2π-500 rad/s.

Fig. 2 shows the stability maps as a function of the desired bandwidth \( \alpha \) and the ratio \( L_q / \hat{L}_q \). The actual inductance \( L_q \) is varied in a range from 0 to 2.5\( L_q \), while other actual parameters perfectly match with their estimates. Figs. 2(a) and 2(b) show the stability maps at zero speed when the sampling frequency is 2 kHz and 1 kHz, respectively. It can be seen that Design 1 has clearly the smallest stable regions: the
Fig. 2. Stability maps for the four different current control designs as a function of the desired bandwidth $\alpha$ and the ratio $L_q/\hat{L}_q$: (a) electrical angular speed $\omega_m = 0$ of the rotor, the sampling frequency $f_s = 2$ kHz; (b) $\omega_m = 0$, $f_s = 1$ kHz; (c) $\omega_m = 2\pi \cdot 200$ rad/s, $f_s = 2$ kHz; and (d) $\omega_m = 2\pi \cdot 200$ rad/s, $f_s = 1$ kHz.

Fig. 3. Stability maps for the four different current control designs as a function of the desired bandwidth $\alpha$ and the ratio $R_s/\hat{R}_s$. The speed is $\omega_m = 2\pi \cdot 200$ rad/s and the sampling frequency is $f_s = 1$ kHz.

The robustness against erroneous $\hat{L}_d$ and $\hat{R}_s$ has also been analyzed. In the case of $\hat{L}_d$, the results are very similar to those in Fig. 2 and are not shown here. All the current control designs are almost insensitive to errors in $\hat{R}_s$ in the whole speed range. As an example, Fig. 3 shows the stability maps as a function of the desired bandwidth $\alpha$ and the ratio $R_s/\hat{R}_s$. The actual resistance $R_s$ is varied in a range from 0 to $2.5\hat{R}_s$, while other actual parameters perfectly match with their estimates. The speed is $\omega_m = 2\pi \cdot 200$ rad/s and the sampling frequency is 1 kHz. It can be seen that the stable region of Design 2 is almost independent of the stator resistance error. Further, the stable regions of Designs 3 and 4 effectively cover the whole area. Design 1 is unstable due to the low sampling frequency (independently of the stator resistance error).

It is worth noticing that the actual parameters were assumed to be constant (but erroneous) in this robustness analysis. In practice, the actual inductances may vary significantly (due to the magnetic saturation) even during one sampling period, desired bandwidth $\alpha$ is limited to about $2\pi \cdot 150$ rad/s when the sampling frequency is 2 kHz and to about $2\pi \cdot 75$ rad/s when the sampling frequency is 1 kHz. The stable regions of Designs 3 and 4 basically overlap with those of Design 2, i.e., there are no significant differences between Designs 2, 3, and 4. A comparison of Figs. 2(a) and 2(b) shows that decreasing the sampling frequency from 2 kHz to 1 kHz makes the stable regions smaller in all the designs and the difference between Designs 3 and 4 increases.

Further, the stable region of Design 2 is very small: $\alpha$ is limited to about $2\pi \cdot 50$ rad/s, if there are no parameter errors. The stable regions of Designs 3 and 4 are comparatively large. A comparison of Figs. 2(c) and 2(d) show that decreasing the sampling frequency from 2 kHz to 1 kHz makes the stable regions smaller in all the designs and the difference between Designs 3 and 4 increases.
which causes additional bandwidth limitations.

B. Simulation Results

Figs. 4 and 5 show time-domain simulation results of the current waveforms. The electrical angular speed of the rotor is \( \omega_m = 2\pi \cdot 200 \text{ rad/s} \). The desired bandwidth is \( \alpha = 2\pi \cdot 100 \text{ rad/s} \) and the sampling frequency is 1 kHz. The current references \( i_{d,\text{ref}} \) and \( i_{q,\text{ref}} \) are changed stepwise. The sampled values of the current components \( i_d \) and \( i_q \) are shown (but the ripple between the sampling instants is fairly large at this low sampling frequencies, cf. [7]).

Figs. 4(a) and 4(b) show the results for Designs 3 and 4, respectively. The actual parameters perfectly match with their estimates. Some cross-coupling and overshoot appears in Fig. 4(a), while the results in Fig. 4(b) completely agree with the desired performance. If the sampling frequency were increased to 2 kHz, the results of Design 3 would become very close to those of Design 4. In accordance with the stability maps in Fig. 2(d), the time-domain simulations for Designs 1 and 2 were unstable under these operating conditions.

Fig. 5 demonstrates the effects of parameter mismatches on the step responses in the case of Design 4. The actual inductance is \( L_q \approx 0.7L_{q}\) in Fig. 5(a), where significant oscillations appear. These oscillations could also be anticipated based on Fig. 2(d), where the given operating condition is close to the stability boundary. In Fig. 5(b), the actual inductance is \( L_q \approx 1.5L_{q}\). The step response is now well damped, but the actual bandwidth is much less than the desired bandwidth.

C. Experimental Results

The discrete-time current controllers were experimentally investigated using the 6.7-kW SyRM drive. A servo induction machine was used as a loading machine in the speed-control mode. The four current control designs (described in the beginning of the section) were implemented in a dSPACE DS1104 PPC/DSP board. The sampling was synchronized with the PWM. The sampling and switching frequencies were 2 kHz.

The actual inductances \( L_d(i_d, i_q) \) and \( L_q(i_d, i_q) \) of the SyRM depend significantly on the current components due to the magnetic saturation. The saturation effects were modeled in the estimates \( \hat{L}_d \) and \( \hat{L}_q \) by means of the power functions [19]. In order to properly model the effect of the differential inductances, a one-step prediction of the inductance estimates would be needed, since the transformations in (28) become \( \dot{\Phi}(k) = C(k + 1)[\Phi(k)\dot{C}^{-1}(k)] \) and \( \dot{G}(k) = C(k + 1)[\Phi(k)] \). Here, for simplicity, the effect of the differential inductances was omitted, i.e., the matrices were calculated as \( \dot{F}(k) = \hat{C}(k)\hat{\Phi}(k)\hat{C}^{-1}(k) \) and \( \dot{G}(k) = C(k)\hat{\Phi}(k) \).
Fig. 6. Experimental results at the rotor speed $\omega_{m} = 0$ for Design 4. The desired bandwidth is $\alpha = 2\pi \cdot 100 \text{ rad/s}$ and the sampling frequency is $f_s = 2 \text{ kHz}$. Sampled values of $i_d$ (blue), $i_q$ (red), and their references (black) are shown.

Fig. 7 shows examples of experimental results at the rotor speed $\omega_{m} = 2\pi \cdot 200 \text{ rad/s}$: (a) Design 2; (b) Design 4. The desired bandwidth is $\alpha = 2\pi \cdot 100 \text{ rad/s}$ and the sampling frequency is $f_s = 2 \text{ kHz}$. Sampled values of $i_d$ (blue), $i_q$ (red), and their references (black) are shown.

model or a series expansion (where one more term than in the Euler method already gives good results). According to the results of eigenvalue analysis, simulations, and experiments, the proposed design improves the dynamic performance and robustness especially at high speeds as compared to the benchmark methods. The design method is directly applicable to SPMSM drives as well.

APPENDIX

EXACT DISCRETE-TIME MODEL

The closed-form solutions for the elements of $\boldsymbol{\Phi}$ in (19) are

$$
\begin{align*}
\phi_{11} &= e^{-\sigma T_s} \left[ \cosh(\lambda T_s) - \delta \frac{\sinh(\lambda T_s)}{\lambda} \right] \\
\phi_{22} &= e^{-\sigma T_s} \left[ \cosh(\lambda T_s) + \delta \frac{\sinh(\lambda T_s)}{\lambda} \right] \\
\phi_{21} &= -\phi_{12} = -\omega_m e^{-\sigma T_s} \frac{\sinh(\lambda T_s)}{\lambda}
\end{align*}
$$

(41)

where $\lambda = \sqrt{\delta^2 - \omega_m^2}$ and\footnote{If $\omega_m^2 > \delta^2$, then $\lambda = j \lambda_m = j \sqrt{\omega_m^2 - \delta^2}$ is imaginary. All the matrix elements remain real since $\cosh(j \lambda_m T_s) = \cos(\lambda_m T_s)$ and $\sinh(j \lambda_m T_s)/j \lambda_m = \sin(\lambda_m T_s)/\lambda_m$ hold due to the properties of hyperbolic functions. Furthermore, for $\lambda = 0$, these functions reduce to $\cosh(\lambda T_s) = \sinh(\lambda T_s)/\lambda = 1$.}

$$
\sigma = \frac{R_s}{2} \left( \frac{1}{L_d} + \frac{1}{L_q} \right), \quad \delta = \frac{R_s}{2} \left( \frac{1}{L_d} - \frac{1}{L_q} \right)
$$

(42)
The closed-form solutions for the elements of $\mathbf{G}$ in (20) are

$$
\gamma_{11} = G [g_{11} \cos(\omega_m T_s) - g_{12} \sin(\omega_m T_s) - g_{11} \phi_{11}]
+ (\sigma + \delta) \omega_m^2 (\phi_{11} - \phi_{22})
$$

$$
\gamma_{12} = G [g_{12} \cos(\omega_m T_s) + g_{11} \sin(\omega_m T_s) - g_{12} \phi_{11} + g_{22} \phi_{21}]
$$

$$
\gamma_{21} = G [g_{21} \cos(\omega_m T_s) - g_{22} \sin(\omega_m T_s) - g_{21} \phi_{22} - g_{11} \phi_{11}]
$$

$$
\gamma_{22} = G [g_{22} \cos(\omega_m T_s) + g_{21} \sin(\omega_m T_s) - g_{22} \phi_{22}]
+ (\sigma - \delta) \omega_m^2 (\phi_{22} - \phi_{11})
$$

where $G = 1/[(\sigma^2 - \delta^2) \omega_m^2 + 4 \sigma^2 \omega_m^2]$ and

$$
g_{11} = (\sigma - \delta)^2 (\sigma + \delta) + 4 \sigma^2 \omega_m^2,
\quad g_{12} = 2(\sigma - \delta) \omega_m,
\quad g_{21} = 2(\sigma + \delta) \omega_m,
\quad g_{22} = (\sigma + \delta)^2 (\sigma - \delta) + 4 \sigma^2 \omega_m^2
$$

In the previous derivations, it is important to notice that $e^{\sigma+\delta} = e^\sigma e^\delta$ does not hold for matrix exponentials in general. The elements of $\gamma$ in (21) are given by

$$
\gamma_1 = H [(\sigma - \delta)(1 - \phi_{11}) - \omega_m \phi_{21}]
$$

$$
\gamma_2 = H [-\sigma \phi_{21} + \omega_m \left( \frac{\phi_{11} + \phi_{22}}{2} - 1 \right)]
$$

where $H = (\sigma + \delta)/[(\sigma + \delta)(\sigma - \delta) + \omega_m^2]$.

ACKNOWLEDGMENT

The authors would like to thank Mr. Daniel Koslopp for preliminary analyses of discrete models. The work was supported in part by ABB Oy and in part by the Academy of Finland.

REFERENCES


Marko Hinkkanen (M’06–SM’13) received the M.Sc.(Eng.) and D.Sc.(Tech.) degrees from Helsinki University of Technology, Espoo, Finland, in 2000 and 2004, respectively. Since 2000, he has been with Helsinki University of Technology (part of Aalto University, Espoo, since 2010). He is currently an Assistant Professor (tenure track) with the Aalto University School of Electrical Engineering. His research interests include control systems, electric drives, and power converters.

Zengcai Qu received the B.Sc. degree in electrical engineering and automation from Shanghai Jiao Tong University, Shanghai, China, in 2007 and the M.Sc. degree in space science and technology jointly from Lulea University of Technology, Kiruna, Sweden, and Helsinki University of Technology, Espoo, Finland, in 2009. Since 2009, he has been working towards the D.Sc.(Tech.) degree at Aalto University, Espoo. His research interests are efficiency control and sensorless control of electric drives.

Hafiz Asad Ali Awan received the B.Sc. degree in electrical engineering from University of Engineering & Technology, Lahore, Pakistan in 2012. Since 2013, he has been working towards his Masters degree in electrical engineering at Aalto University, Espoo, Finland. His main research interest is the control of electric drives.

Toni Tuovinen received the M.Sc. degree from the University of Helsinki, Helsinki, Finland, in 2005 and the M.Sc.(Eng.) degree from Helsinki University of Technology, Espoo, Finland, in 2009. He received the D.Sc.(Tech.) degree from Aalto University, Espoo, in 2014. He is currently a Senior Design Engineer with ABB Oy, Helsinki. His main research interests include the control of electric drives.

Fernando Briz (A’96–M’99–SM’06) received the M.S. and Ph.D. degrees from the University of Oviedo, Gijón, Spain, in 1990 and 1996, respectively. From June 1996 to March 1997, he was a Visiting Researcher with the University of Wisconsin, Madison. He is currently a Full Professor with the Department of Electrical, Computer and Systems Engineering, University of Oviedo. His topics of interest include control systems, power converters and ac drives, machine diagnostics and digital signal processing. Dr. Briz received the 2005 IEEE TRANSACTIONS ON INDUSTRY APPLICATIONS Third Place Prize Paper Award and was the recipient of five IEEE Industry Applications Society Conference and IEEE Energy Conversion Congress and Exposition prize award papers. He is currently Program Chair and Associate Editor of the Industrial Drives Committee of the IAS-IPCSD.