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Published in:
IEEE Transactions on Smart Grids

DOI:
10.1109/TSG.2020.2991661

Published: 01/09/2020

Document Version
Peer-reviewed accepted author manuscript, also known as Final accepted manuscript or Post-print

Please cite the original version:
Jooshaki, M., Abbaspour, A., Fotuhi-Firuzabad, M., Muñoz-Delgado, G., Contreras, J., Lehtonen, M., & Arroyo, J. M. (2020). Linear Formulations for Topology-Variable-Based Distribution System Reliability Assessment Considering Switching Interruptions. *IEEE Transactions on Smart Grids*, *11*(5), 4032-4043. Article 9084092. https://doi.org/10.1109/TSG.2020.2991661

Linear Formulations for Topology-Variable-Based Distribution System Reliability Assessment Considering Switching Interruptions

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***Abstract*—Continuity of supply plays a significant role in modern distribution system planning and operational studies. Accordingly, various techniques have been developed for reliability assessment of distribution networks. However, owing to the complexities and restrictions of these methods, many researchers have resorted to several heuristic optimization algorithms for solving reliability-constrained optimization problems. Therefore, solution quality and convergence to global optimality cannot be guaranteed. Aiming to address this issue, two salient mathematical models are introduced in this paper for topology-variable-based reliability evaluation of both radial and radially-operated meshed distribution networks. Cast as a set of linear expressions, the first model is suitable for radial networks. The second model relies on mixed-integer linear programming and allows handling not only radial networks but also radially-operated meshed distribution grids. Therefore, the proposed formulations can be readily incorporated into various mathematical programming models for distribution system planning and operation. Numerical results from several case studies back the scalability of the developed models, which is promising for their further application in distribution system optimization studies. Moreover, the benefits of the proposed formulations in terms of solution quality are empirically evidenced.**

***Index Terms*—Electricity distribution network, linear formulations, topology-variable-based reliability assessment.**

NOMENCLATURE

Indices

i	Index for network components.
$l, l', \bar{l}$	Indices for feeder sections.
m	Index for paths.
n	Index for load nodes.

Sets

L	Set of all feeder sections.
SL	Subset of L containing feeder sections directly connected to substation nodes.
$\Psi_{l,\bar{l}}$	Set of all paths between branches l and $\bar{l}$.

Parameters

$\mathbf{A}$	$N_{lp} \times \lvert L \rvert$ matrix relating nodal power demands to branch flows.
D_l^r, D_l^{sw}	Repair and switching times.
M^F, M^H	Sufficiently large numbers.
N_c	Number of network components.
N_{lp}	Number of load nodes.
$\mathbf{NC}$	Vector of parameters NC_n.
NC_n	Number of customers connected to load node n.
$\mathbf{P}$	Vector of parameters P_n.
P_n	Power demand at load node n.
w_N, w_D, w_F	Weighting factors for $EENS$, $SAIDI$, and $SAIFI$.
$\chi_{l',m}$	Binary parameter, which is equal to 1 if feeder section l' is in path m, being 0 otherwise.
λ_i	Failure rate of component i.
$\xi_{\bar{l},l}$	Binary parameter, which is equal to 1 if feeder section l is in a feeder whose first branch is $\bar{l}$, being 0 otherwise.

Variables

$EENS$	Expected energy not supplied.
$\mathbf{f}$	Vector of variables f_l.
f_l	Power flow through feeder section l.
f_l^+, f_l^-	Non-negative variables used to model the absolute value of f_l.
$\mathbf{h}$	Vector of variables h_l.
h_l	Number of customers connected to the nodes downstream of feeder section l.
h_l^+, h_l^-	Non-negative variables used to model the absolute value of h_l.
$SAIDI$	System average interruption duration index.
$SAIFI$	System average interruption frequency index.
Uh_l	Number of customers connected to the nodes upstream of feeder section l if its switch is closed, being 0 otherwise.

The work of M. Jooshaki was supported by the Department of Electrical Engineering and Automation, Aalto University, Espoo, Finland. M. Fotuhi-Firuzabad would like to acknowledge the financial support from the Iran National Science Foundation (INSF). The work of G. Muñoz-Delgado, J. Contreras, and J. M. Arroyo was partly supported by the Ministry of Science, Innovation and Universities of Spain, under Projects RTI2018-098703-B-I00 and RTI2018-096108-A-I00 (MCIU/AEI/FEDER, UE).

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UP_l Total demand of the nodes upstream of feeder section l if its switch is closed, being 0 otherwise.

y_l Binary utilization variable of feeder section l, which is equal to 1 if feeder section l is in service, being 0 otherwise.

$z_{l,\bar{l}}$ Binary-valued continuous variable, which is equal to 1 if feeder section l is in a feeder whose first branch is $\bar{l}$ and the switch of feeder section l is closed, being 0 otherwise.

δ_n Average annual duration of customer outages at node n.

ν_n Average number of annual customer outages at node n.

I. Introduction

Due to the considerable share of electricity distribution system failures in customer interruptions, the reliability level of distribution networks has gained more attention in recent years. This is evidenced by the implementation of incentive regulations for reliability in many countries around the world. For instance, over 65 percent of the European countries investigated by the Council of European Energy Regulators (CEER) have implemented incentive schemes to motivate distribution companies to enhance their service reliability [1]. Such regulations, in general, provide a link between distribution companies' revenues and their service reliability [2]. This, alongside other economic factors and customer satisfaction, makes it crucial to consider reliability requirements in distribution system studies [3]–[5]. In this context, the calculation of quantitative reliability metrics is the first step. Classic concepts for reliability evaluation of distribution systems can be found in [3] and [5].

Based on these concepts, a wide range of approaches have been developed for the calculation of reliability indices for both radial and meshed networks such as analytical methods [5]–[13] and Monte Carlo simulation [14]–[16]. However, an essential prerequisite, which restricts the application of these methods, is that the network topology must be specified, typically in the form of an ordered set of nodes [5]–[8], [13]–[16] or constant matrices [9]–[12]. Nevertheless, in most of the key studies, such as optimal distribution system expansion planning and operational problems, the network topology is an outcome of the study. In order to address this issue, many researchers have resorted to employing heuristic optimization techniques. Accordingly, the network topology is known along the optimization process, and it is possible to calculate reliability indices using regular topology-parameterized approaches [5]–[16]. Relevant examples can be found in [9], [17]–[21] for expansion planning and in [22]–[25] for network reconfiguration.

In [9], [20], and [21], mixed-integer nonlinear models solved by a genetic algorithm have been proposed for reliability-constrained distribution expansion planning considering distribution automation. In [18], a multi-objective tabu search algorithm is employed to solve the multistage expansion planning problem considering reliability. A similar concept can be found in [25], where artificial immune systems are employed to solve the optimal reconfiguration of radially-operated meshed networks to minimize network losses and enhance the service reliability. Reliability-constrained network reconfiguration has also been addressed by simulated annealing [22], [23] and particle swarm optimization [24]. However, such heuristic optimization methods are unable to acknowledge the attainment of global optimality.

A novel approach to consider reliability-related costs in the expansion planning problem has been presented in [17] and further employed in [19]. In this method, a pool of solutions for the planning problem is obtained using standard mathematical programming in the first step. Then, reliability indices and interruption cost are calculated for each solution in the next step. Subsequently, the best solution is determined based on the trade-off between expansion and reliability-related costs. Nonetheless, this technique does not necessarily provide the global optimal solution, since the reliability model is not integrated into the planning model.

Motivated by the above shortcomings featured by topology-parameterized approaches [5]–[16], researchers have begun to develop alternative mathematical models for reliability assessment wherein, rather than parameters, variables are used to explicitly represent the network topology. Relevant examples of this recent avenue of research are [26]–[30]. As a major advantage over topology-parameterized reliability assessment models [5]–[16], topology-variable-based expressions can be readily incorporated into the mathematical formulations of reliability-constrained optimization models for distribution system operation and planning. As a consequence, the resulting mathematical programs are suitable for sound techniques with well-known properties in terms of solution quality and convergence and for which off-the-shelf software is readily available.

The first topology-variable-based formulation for analytical reliability assessment is presented in [26], which addresses the network reconfiguration problem of radially-operated meshed networks. However, the reliability assessment in [26] is limited to considering the effects of failures occurring in the shortest upstream path between each load node and the corresponding substation. In other words, López *et al.*'s approach neglects switching interruptions, i.e., those with out-of-service duration equal to the switching time associated with the isolation of the faulty portion of the network. In [27], a pioneering model is proposed to derive a linear formulation for the calculation of reliability indices of radial systems while considering switching interruptions. This linear model can be incorporated into various distribution system optimization problems. In [28], the model described in [27] is applied to distribution system expansion planning. However, although this method is capable of calculating all load-node and system-level reliability indices, it can considerably increase the dimension (number of decision variables and constraints) of the resulting optimization model. This, in turn, can negatively affect the simulation time, especially in the case of large-scale distribution networks. In order to overcome this shortcoming, an enhanced algebraic approach is proposed in [29] for radial grids. Similar to [27], the technique proposed in [29] relies on the calculation of load-node indices to eventually compute system-oriented metrics. This can cause various challenges for modeling the impact of distributed generating units on reliability indices. As an alternative, a mixed-integer linear programming (MILP) formulation is presented in [30] for the straight calculation of system-oriented analytical reliability indices. However, as done in [26], switching interruptions are disregarded in [30]. The adoption of such a far drastic simplification yields too optimistic reliability metrics, thereby giving rise to an inaccurate estimation of the planning and operational costs. This is particularly relevant for non-automated and semi-automated distribution systems [20].

Within the context of topology-variable-based approaches for distribution reliability assessment [26]–[30], this paper presents alternative innovative expressions for the efficient, systematic, as well as straightforward calculation of distribution system reliability metrics while precisely modeling switching interruptions. More specifically, two novel formulations are proposed to calculate widely-used system-level reliability indices for radial and radially-operated mesh-designed distribution networks. The first model comprises a set of linear expressions and is suitable for radial networks. The second model relies on mixed-integer linear programming and allows handling both radial grids and radially-operated meshed networks. It is worth emphasizing that existing approaches [26]–[30] are outperformed in terms of both computational efficiency and solution quality. To that end, the formulation for reliability assessment devised in [30] is extended in a non-trivial fashion. Major modeling differences are twofold. First, an extended set of decision variables comprising not only binary but also continuous decision variables is considered. Secondly, additional constraints are incorporated to characterize the behavior of the healthy portion of the system upstream of the fault. Moreover, it should be noted that, unlike the formulation described in [27] and subsequently applied in [28], the proposed model does not require the computationally expensive consideration of system operational constraints under every contingency, which is beneficial for practical implementation purposes.

The main contribution of this paper is to develop novel formulations for topology-variable-based analytical reliability assessment of radial and radially-operated mesh-designed distribution systems. The main advantages of the proposed linear reliability assessment are:

1) In contrast to widely-used topology-parameterized reliability assessment methods for both radial and radially-operated mesh-designed distribution systems [5]–[16], the incorporation of the proposed formulations in reliability-constrained distribution operational and planning models gives rise to optimization problems that can be tackled by sound mathematical-programming-based techniques. Thus, finite convergence to optimality may be guaranteed, a measure of the distance to the global optimum may be provided, and commercially available software may be used.
2) As compared to the state of the art of topology-variable-based methods for both radial and radially-operated mesh-designed distribution systems [26]–[30], the modeling capability is substantially extended. First, switching interruptions are considered, thereby significantly improving the accuracy of reliability indices upon those provided by [26] and [30]. In addition, the pre-specification of a particular radial operation condition for mesh-designed grids is not required, unlike [27] and [29]. Note also that both modeling advantages are achieved in a computationally efficient way as the dimensionality issue of [27] and [28] is not featured.
3) Superior computational performance is featured over existing formulations suitable for topology-variable-based reliability assessment also considering switching interruptions [27]–[29]. This behavior is particularly significant for radial networks, for which the proposed approach is between one and three orders of magnitude faster. The computational superiority for both radial and radially-operated meshed networks is a promising result for the subsequent integration of the proposed formulations in reliability-constrained optimization models for distribution systems.
4) System-oriented reliability indices are straightforwardly provided, unlike [27] and [29], which require the calculation of load-node reliability indices to obtain system-level reliability metrics.

The rest of this paper is organized as follows. In Section II, the analytical reliability evaluation of distribution systems and the conventional approach to perform such an assessment are reviewed. Section III presents the proposed models for reliability evaluation of radial and radially-operated meshed distribution grids. Section IV is devoted to a particular instance of reliability-constrained distribution optimization, namely the optimal network reconfiguration problem of meshed grids. In Section V, the proposed formulations are applied to various test grids and the results are analyzed and discussed. Finally, concluding remarks are provided in Section VI.

II. Reliability Assessment of Distribution Systems

In this section, a brief overview of analytical distribution system reliability assessment is provided and the conventional approach is outlined.

A. Problem Description

Distribution system reliability assessment is performed to quantify the amount of customer outages caused by the failure of distribution network components.

A quantitative distribution reliability evaluation is crucial since: 1) it enables regulators to monitor the quality of the services provided by electricity distribution companies, and 2) it helps distribution companies to consider reliability requirements in designing and operating their networks.

In this context, a wide range of reliability indices have been proposed to measure the reliability level of distribution networks [3], [5], [31]. Among these indices, system average interruption frequency index (SAIFI), system average interruption duration index (SAIDI), and energy not served

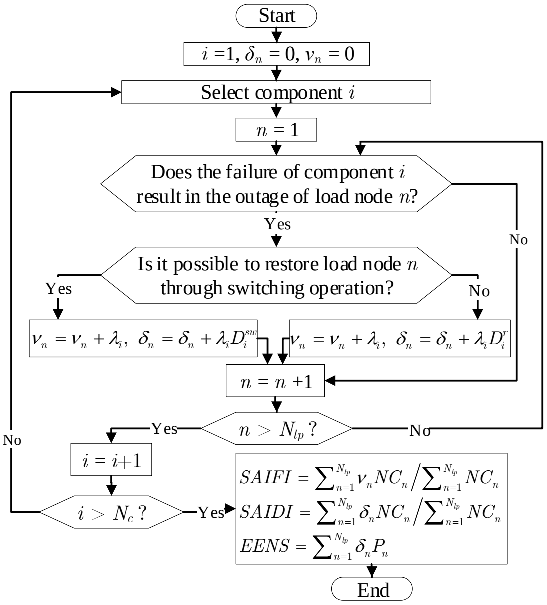

Fig. 1. Flowchart of the conventional approach for distribution system reliability evaluation.

are the most frequently used [3], [5], [17], [20], [32]. Thus, distribution companies need practical techniques to estimate such reliability indices. In the following, a short review of the classic approach for reliability assessment of distribution systems is presented.

B. Traditional Approach for Reliability Evaluation of Distribution Networks

Fig. 1 depicts a flowchart of the conventional approach for the calculation of reliability indices for both radial and radially-operated meshed distribution networks. Note that, for the latter, reliability indices are computed for the radial operational topology determined by the optimization process.

As illustrated, this method comprises two major loops over network components and network load nodes. The former loop aims to quantify the consequences of the failure of each component. Hence, given the failure of a particular component i, all load nodes should be assessed to determine their states, i.e., whether they are affected by the outage of component i, or not. Then, for the affected load nodes, the duration of power interruption is estimated. The load nodes that can be re-energized prior to the repair of the faulty element undergo a switching interruption whose duration is equal to the switching time. For the other affected load nodes, the outage duration is the time required for the repair (or replacement) of the element. Subsequently, the values of average annual frequency and duration of customer outages (ν_n and δ_n, respectively) are updated. Finally, the reliability indices, e.g., SAIFI, SAIDI, and the expected energy not served (EENS), are calculated as shown in the flowchart.

For a better understanding of the application of this method to radial distribution systems, let us consider the simple network depicted in Fig. 2. This network has four load nodes ($n1$–$n4$) and two feeders, each equipped with a circuit breaker (B1 and B2) at the supply node of the feeder (i.e., at the

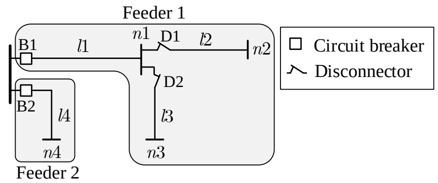

Fig. 2. One-line diagram of a simple illustrative distribution network.

TABLE I
EFFECTS OF FEEDER SECTION FAILURES ON LOAD NODES

Faulty Feeder Section	Frequency				Duration			
	$n1$	$n2$	$n3$	$n4$	$n1$	$n2$	$n3$	$n4$
$l1$	λ_1	λ_1	λ_1	0	$\lambda_1 D_1^r$	$\lambda_1 D_1^r$	$\lambda_1 D_1^r$	0
$l2$	λ_2	λ_2	λ_2	0	$\lambda_2 D_2^{sw}$	$\lambda_2 D_2^r$	$\lambda_2 D_2^{sw}$	0
$l3$	λ_3	λ_3	λ_3	0	$\lambda_3 D_3^{sw}$	$\lambda_3 D_3^{sw}$	$\lambda_3 D_3^r$	0
$l4$	0	0	0	λ_4	0	0	0	$\lambda_4 D_4^r$

sending extremes of feeder sections $l1$ and $l4$). Moreover, there is a disconnector (isolator or disconnect switch), denoted by D1 and D2, at the supply side of each of the other feeder sections (i.e., $l2$ and $l3$). This is the basic switch arrangement of distribution grids.

In order to calculate the above-mentioned reliability indices, the following assumptions are considered: 1) only sustained interruptions are taken into account, 2) annual failure rates as well as repair and switching times of feeder sections are known, and 3) malfunction of switches is negligible. The first two assumptions are in line with the standard definition of the reliability indices of interest [3], [5], whereas the third is typically considered in the literature on distribution system reliability assessment [5]–[7], [9]–[25] including all recent references describing topology-variable-based formulations [26]–[30]. When a failure occurs on a feeder, its circuit breaker trips and disconnects the whole feeder [5]. Subsequently, partial restoration [3] is enabled by post-fault reconfiguration of the radially-operated network topology whereby the service is restored for circuit sections upstream of the fault. To that end, the proper normally-closed disconnector is opened in order to isolate the faulty area and the breaker is then closed to energize the nodes upstream of the faulty section. Once the repair is accomplished and the fault is cleared, the isolated section is also connected.

Accordingly, in order to calculate the reliability indices for the network depicted in Fig. 2, the effects shown in Table I are considered. Since the breakers are assumed to be fully reliable, faults on a feeder have no effects on the customers connected to another feeder. Hence, for instance, load node 4 is not affected by the failure of $l1$–$l3$.

Because the supply paths for nodes $n1$–$n3$ include feeder section $l1$, once a fault occurs on this line, it is not possible to restore any of these load nodes until the repair is completed. Hence, the annual outage durations of those nodes, δ_1–δ_3, must contain the repair time of all failures occurring along feeder section $l1$, which is equal to $\lambda_1 D_1^r$. Likewise, the consequences of the failures of $l2$, $l3$, and $l4$ can be determined, as reported in Table I.

Subsequently, the summation of the values in each frequency column gives the average annual failure rate of the load nodes ν_n. Analogously, the average annual duration of customer outages at load node n, i.e., δ_n, is equal to the summation of the values in the corresponding duration column. Finally, the reliability indices can be obtained using the equations shown in Fig. 1.

Although this method is straightforward, it cannot be incorporated into standard mathematical programming models for distribution system planning and operation. In the following, novel equivalent mathematical formulations are developed to circumvent this issue while overcoming the modeling and computational limitations of previous topology-variable-based works [26]–[30].

III. Proposed Technique

This section describes the proposed formulations for analytical reliability assessment of radial and radially-operated meshed distribution systems.

A. Radial Networks

As explained below, linear formulations are derived for the calculation of reliability indices of radial distribution networks.

1) EENS: The classic equation for the calculation of EENS is as follows (Fig. 1) [5]:

$$EENS = \sum_{n=1}^{N_{lp}} \delta_n P_n. \tag{1}$$

Hence, for the illustrative network depicted in Fig. 2, expression (1) becomes:

$$\begin{aligned} EENS={}&(\lambda_1 D_1^r+\lambda_2 D_2^{sw}+\lambda_3 D_3^{sw})P_1+(\lambda_1 D_1^r+\lambda_2 D_2^r+\lambda_3 D_3^{sw})P_2 \\ &+(\lambda_1 D_1^r+\lambda_2 D_2^{sw}+\lambda_3 D_3^r)P_3+(\lambda_4 D_4^r)P_4 \end{aligned} \tag{2}$$

which can be rewritten as:

$$\begin{aligned} EENS ={}& \lambda_1 D_1^r(P_1+P_2+P_3)+\lambda_2 D_2^r(P_2)+\lambda_3 D_3^r(P_3) \\ &+\lambda_4 D_4^r(P_4)+\lambda_2 D_2^{sw}(P_1+P_3)+\lambda_3 D_3^{sw}(P_1+P_2). \end{aligned} \tag{3}$$

According to the network topology (Fig. 2), it can be inferred that the first four terms in the right-hand side of (3) are the sum over all feeder sections of the annual failure duration of each feeder section multiplied by its downstream demand. This implies that the nodes served through a given feeder section cannot be restored prior to the repair of that feeder section. Moreover, the last two terms of (3) correspond to the sum over all feeder sections without a circuit breaker of the failure rate of each feeder section multiplied by its switching time and the whole demand of the feeder minus the downstream demand of that feeder section. This reflects the fact that load nodes upstream of a given feeder section can be restored by the switching operation prior to the repair being completed. Note that this practical modeling aspect was disregarded in [26] and [30] and thus constitutes a distinctive feature of this work.

In case we neglect power losses, the total demand downstream of each feeder section is equal to its power flow. This result stems from Kirchhoff's current law (KCL) and can be cast in a matrix form as:

$$\mathbf{A} \times \mathbf{f} = \mathbf{P} \tag{4}$$

where element $a_{n,l}$ of matrix $\mathbf{A}$ is equal to -1 if load node n is the sending node of branch l, $+1$ if load node n is the receiving node of branch l, and 0 otherwise.

For the illustrative example of Fig. 2:

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},\ \mathbf{f} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix},\ \text{and } \mathbf{P} = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix}.$$

Using (4), expression (3) can be rewritten as:

$$\begin{aligned} EENS ={}& \lambda_1 D_1^r f_1+\lambda_2 D_2^r f_2+\lambda_3 D_3^r f_3+\lambda_4 D_4^r f_4 \\ &+\lambda_2 D_2^{sw}(f_1-f_2)+\lambda_3 D_3^{sw}(f_1-f_3). \end{aligned} \tag{5}$$

Thus, a general form for EENS is given by:

$$EENS = \sum_{l \in L} \left[\lambda_l D_l^r f_l + \lambda_l D_l^{sw} \left(\sum_{\bar{l} \in SL} (\xi_{\bar{l},l} f_{\bar{l}}) - f_l \right) \right] \tag{6}$$

where $\xi_{\bar{l},l}$ determines the first feeder section (i.e., the feeder section at the sending extreme) of the feeder to which feeder section l belongs. In addition, the relationship between f_l and P_n is modeled by (4).

2) SAIDI: The standard equation for SAIDI calculation is (Fig. 1) [5]:

$$SAIDI = \frac{\sum_{n=1}^{N_{lp}} \delta_n NC_n}{\sum_{n=1}^{N_{lp}} NC_n}. \tag{7}$$

In the right-hand side of (7), the denominator is a constant, which is equal to the total number of customers connected to the distribution network. Note also that the numerator has a form analogous to (1), with NC_n playing the role of P_n. Hence, for the illustrative example, following the procedure described for EENS yields:

$$\begin{aligned} \sum_{n=1}^{N_{lp}} \delta_n NC_n ={}& \lambda_1 D_1^r(NC_1+NC_2+NC_3) \\ &+\lambda_2 D_2^r NC_2+\lambda_3 D_3^r NC_3+\lambda_4 D_4^r NC_4 \\ &+\lambda_2 D_2^{sw}(NC_1+NC_3)+\lambda_3 D_3^{sw}(NC_1+NC_2) \end{aligned} \tag{8}$$

which is analogous to (3). Defining h_l as the number of customers connected to the nodes downstream of feeder section l, the following relationship between h_l and NC_n holds:

$$\mathbf{A} \times \mathbf{h} = \mathbf{NC}. \tag{9}$$

For the illustrative example, $\mathbf{h} = [h_1, h_2, h_3, h_4]^T$ and $\mathbf{NC} = [NC_1, NC_2, NC_3, NC_4]^T$.

Note that (9) is identical to (4), where $\mathbf{f}$ and $\mathbf{P}$ are replaced with $\mathbf{h}$ and $\mathbf{NC}$, respectively. Thus, $\mathbf{h}$ can be viewed as the vector of power flows resulting from the application of KCL to a fictitious lossless system with the same topology as the network under consideration and nodal demands equal to the corresponding number of connected customers.

Similar to (5), expression (8) can be rewritten as (10):

$$\begin{aligned} \sum_{n=1}^{N_{lp}} \delta_n NC_n ={}& \lambda_1 D_1^r h_1+\lambda_2 D_2^r h_2+\lambda_3 D_3^r h_3 \\ &+\lambda_4 D_4^r h_4+\lambda_2 D_2^{sw}(h_1-h_2)+\lambda_3 D_3^{sw}(h_1-h_3). \end{aligned} \tag{10}$$

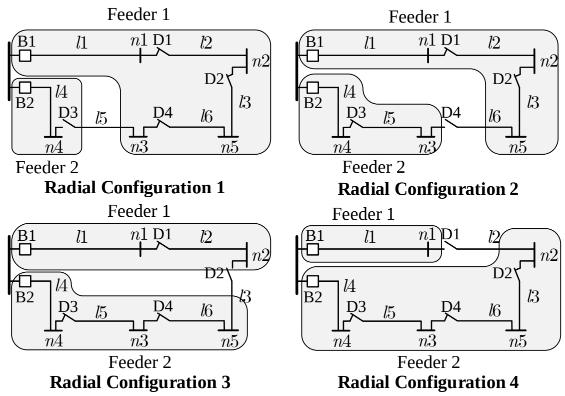

Fig. 3. Radial configurations of a simple meshed distribution network.

Hence, a general form for SAIDI is given by:

$$SAIDI = \frac{\sum\limits_{l \in L} \lambda_l \left[D_l^r h_l + D_l^{sw} \left(\sum\limits_{\bar{l} \in SL} (\xi_{\bar{l},l} h_{\bar{l}}) - h_l \right) \right]}{\sum\limits_{n=1}^{N_{lp}} NC_n} \tag{11}$$

where the relationship between h_l and NC_n is modeled by (9).

3) SAIFI: As shown in Fig. 1, this index is usually expressed as [5]:

$$SAIFI = \frac{\sum\limits_{n=1}^{N_{lp}} \nu_n NC_n}{\sum\limits_{n=1}^{N_{lp}} NC_n}. \tag{12}$$

The right-hand side of (12) is identical to that of (7) except for the fact that δ_n is replaced with ν_n. Moreover, as per Table I, ν_n results from dropping D_l^r and D_l^{sw} in the corresponding expressions of δ_n. Accordingly, by removing all D_l^r and D_l^{sw} from (11), a general form for SAIFI is given by:

$$SAIFI = \frac{\sum\limits_{l \in L} \left(\lambda_l \sum\limits_{\bar{l} \in SL} \xi_{\bar{l},l} h_{\bar{l}} \right)}{\sum\limits_{n=1}^{N_{lp}} NC_n} \tag{13}$$

where the relationship between h_l and NC_n is modeled by (9).

B. Radially-Operated Meshed Networks

Similar to the models presented in [27] and [29], the expressions proposed in Section III-A are not readily applicable to most of the optimization problems associated with distribution network planning and operation. This is due to the fact that, based on industry practice, in almost all these problems, the network has a meshed structure which is radially operated by opening some of the disconnecting switches. Unfortunately, the topology of the radial operation is initially unknown as the states of the disconnecting switches defining such a configuration are the optimal values of the binary decision variables of those optimization problems.

As an example, let us consider the simple meshed network depicted in Fig. 3. As can be observed, this network can be operated under four radial configurations, according to the states of the disconnectors D1–D4. Thus, for a given feeder section li, the first branch of the feeder to which li belongs is initially unknown. For instance, in Configuration 1 of Fig. 3, $l6$ is part of Feeder 1, whose first branch is $l1$. On the other hand, in Configurations 3 and 4, feeder section $l6$ is in Feeder 2, which starts with branch $l4$. Hence, the values of $\xi_{\bar{l},l}$ in (6), (11), and (13) cannot be determined *a priori*. In fact, $\xi_{\bar{l},l}$ is a function of the switch states. Likewise, the KCL equations also depend on the radial network topology associated with the switch states.

In the following subsections, novel topology-variable-based mixed-integer linear expressions are derived to calculate the reliability indices of a radially-operated mesh-designed distribution network. As a major distinctive aspect, switching interruptions, which were disregarded in [26] and [30], are effectively accommodated without featuring the dimensionality issue of [27] and [28]. It is worth mentioning that, in addition to the assumptions described in Section II-B, the following considerations characterize the proposed model:

1) The state of the disconnector of each feeder section is modeled by the corresponding binary variable y_l, which is 1 if the disconnector is closed, being 0 otherwise. Note that y_l are decision variables of the reliability-constrained optimization problem in which the proposed reliability assessment model is embedded.
2) The reliability-constrained optimization problem minimizes an objective function, which is monotonically increasing with respect to the reliability indices.
3) As done in all references on topology-variable-based reliability assessment [26]–[30], post-fault network reconfiguration is implemented to restore the service for load nodes upstream of the fault. Thus, we neglect the impact on reliability of additional post-fault reconfiguration by operating normally-open tie switches. In other words, it is assumed that if a tie switch is open under normal operation, it will not be closed during the switching actions after fault occurrence.

Admittedly, a complete assessment of reliability of meshed networks should consider 1) additional post-fault network reconfiguration to restore the service for load nodes downstream of the fault, and 2) non-fully reliable switches. This generalization would, however, render the problem essentially intractable through optimization. These modeling limitations notwithstanding, addressing operational and planning models considering reliability, albeit ignoring additional post-fault network reconfiguration and non-fully reliable switches, is relevant to the decision maker as it provides a first estimate of a cost-effective and reliable solution [4], [17], [19], [25]–[30].

1) EENS: According to the aforementioned considerations, the model for EENS devised in Section III-A can be extended to handle a radially-operated meshed network as follows:

$$EENS = \sum_{l \in L} \left(\lambda_l D_l^r \left| f_l \right| + \lambda_l D_l^{sw} UP_l \right) \tag{14}$$

$$\mathbf{A} \times \mathbf{f} = \mathbf{P} \tag{15}$$

$$-M^F y_l \leq f_l \leq M^F y_l; \quad \forall l \in L. \tag{16}$$

Expression (14) is the general form of (6) in which the absolute value of f_l is utilized and a new variable, UP_l, is used to designate the total demand of the nodes upstream of each feeder section l with its switch closed. It is worth noting that the absolute value of f_l is required since the flow directions of feeder sections depend on the states of the switches. For instance, feeder section $l3$ has different flow directions in Configurations 2 and 4 of Fig. 3. Analogous to (4), expression (15) represents KCL. Matrix $\mathbf{A}$ is built in the same way as presented in Section III-A considering an arbitrary choice for the sending and receiving nodes of each branch. Note that matrix $\mathbf{A}$ is analogous to that used in the dc load flow model adopted for transmission networks. As an example, for the meshed network depicted in Fig. 3, this matrix can be written as follows:

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

where it is assumed that $n1$–$n4$ are the destination nodes of branches $l1$, $l2$, $l5$, and $l4$, respectively, whereas $n5$ is the destination node of both $l3$ and $l6$.

Finally, expression (16) is considered to set the flows of switched-off feeder sections to 0. Note that a suitable value for the big-M parameter M^F is $\sum_{n=1}^{N_{lp}} P_n$.

The value of UP_l can be determined by subtracting f_l from the flow of the first section of the associated feeder. However, the feeder to which a given feeder section l belongs is a function of the states of the switches. Hence, we consider binary-valued continuous decision variables $z_{l,\bar{l}}$ in such a way that $z_{l,\bar{l}}$ becomes 1 if feeder section $\bar{l}$ is the first branch of the feeder in which feeder section l is located and the switch of feeder section l is closed. Thus, UP_l can be expressed as:

$$UP_l = \sum_{\bar{l} \in SL} z_{l,\bar{l}}(|f_{\bar{l}}| - |f_l|); \forall l \in L \setminus SL \tag{17}$$

$$UP_l = 0; \forall l \in SL. \tag{18}$$

Moreover, $z_{l,\bar{l}}$ is constrained as follows:

$$\sum_{\bar{l} \in SL} z_{l,\bar{l}} = y_l; \forall l \in L \setminus SL \tag{19}$$

$$z_{l,\bar{l}} \geq 1 + \sum_{l' \in L} \chi_{l',m}(y_{l'} - 1); \forall l \in L \setminus SL, \forall \bar{l} \in SL, \forall m \in \Psi_{l,\bar{l}} \tag{20}$$

$$z_{l,\bar{l}} \geq 0; \forall l \in L \setminus SL, \forall \bar{l} \in SL. \tag{21}$$

Expression (19) indicates that if feeder section l is in service, i.e., y_l is equal to 1, the summation of variables $z_{l,\bar{l}}$ over $\bar{l} \in SL$ is 1, since feeder section l must have a source branch. Then, among all the possible paths between feeder sections l and $\bar{l}$, a single $z_{l,\bar{l}}$ is equal to 1 as per (20). This is done by setting the minimum value of target variable $z_{l,\bar{l}}$ to 1. The non-negativity of $z_{l,\bar{l}}$ is imposed in (21). Note that although variables $z_{l,\bar{l}}$ are continuous variables, they are binary valued as per expressions (19)–(21).

As an example, let us consider branch $l3$ in the illustrative network depicted in Fig. 3. As there are two source branches, $l1$ and $l4$, two continuous variables, denoted by $z_{l3,l1}$ and $z_{l3,l4}$, are used to determine the source feeder section of branch $l3$. Therefore, using (19), we have:

$$z_{l3,l1} + z_{l3,l4} = y_{l3}. \tag{22}$$

According to the network topology, there is only one path from branch $l3$ to branch $l1$, i.e., $l1$-$l2$-$l3$. Thus, $\Psi_{l3,l1}$ includes only one path, namely $m1$, for which the χ values are:

$$\chi_{l1,m1} = \chi_{l2,m1} = \chi_{l3,m1} = 1, \chi_{l4,m1} = \chi_{l5,m1} = \chi_{l6,m1} = 0. \tag{23}$$

Hence, expression (20) becomes:

$$z_{l3,l1} \geq 1 + (y_{l1} - 1) + (y_{l2} - 1) + (y_{l3} - 1). \tag{24}$$

Analogously, for the path $m2$ from $l3$ to $l4$, namely $l4$-$l5$-$l6$-$l3$, we have:

$$\chi_{l1,m2} = \chi_{l2,m2} = 0, \chi_{l3,m2} = \chi_{l4,m2} = \chi_{l5,m2} = \chi_{l6,m2} = 1 \tag{25}$$

$$z_{l3,l4} \geq 1 + (y_{l3} - 1) + (y_{l4} - 1) + (y_{l5} - 1) + (y_{l6} - 1). \tag{26}$$

Note that (22), (24), and (26) consistently determine the source branch of $l3$ based on the states of the switches. As an example, for Configuration 1, using $y_{l1} = y_{l2} = y_{l3} = y_{l4} = y_{l6} = 1$ and $y_{l5} = 0$ in (22), (24), and (26) gives:

$$z_{l3,l1} + z_{l3,l4} = 1$$

$$z_{l3,l1} \geq 1 + (1-1) + (1-1) + (1-1) \Rightarrow z_{l3,l1} \geq 1$$

$$z_{l3,l4} \geq 1 + (1-1) + (1-1) + (0-1) + (1-1) \Rightarrow z_{l3,l4} \geq 0$$

which results in $z_{l3,l1} = 1$ and $z_{l3,l4} = 0$, as desired.

The model for EENS (14)–(20) features two sources of nonlinearity, namely the absolute value operator (i.e., $|f_l|$ and $|f_{\bar{l}}|$ terms) in (14) and (17) as well as the product terms $z_{l,\bar{l}}|f_l|$ and $z_{l,\bar{l}}|f_{\bar{l}}|$ in (17).

The absolute value operator can be equivalently characterized by introducing two non-negative variables per feeder section indicating the corresponding flow in each direction. Accordingly, f_l and its absolute value can be modeled as follows:

$$f_l = f_l^+ - f_l^-; \forall l \in L \tag{27}$$

$$|f_l| = f_l^+ + f_l^-; \forall l \in L \tag{28}$$

$$f_l^+ \geq 0; \forall l \in L \tag{29}$$

$$f_l^- \geq 0; \forall l \in L. \tag{30}$$

Both non-negative variables f_l^+ and f_l^- cannot simultaneously take a non-zero value, since the power cannot flow in both directions at the same time. This result is attained without imposing any additional constraint due to the fact that the objective function being minimized is monotonically increasing with respect to the reliability indices, and, hence, with respect to $f_l^+ + f_l^-$. Thus, by replacing $|f_l|$ with $f_l^+ + f_l^-$ and adding (27), (29), and (30), all the absolute value operators are eliminated.

Finally, expression (17) can be linearized as follows:

$$UP_l \geq (f_{\bar{l}}^+ + f_{\bar{l}}^-) - (f_l^+ + f_l^-) - M^F(1 - z_{l,\bar{l}}); \forall l \in L \setminus SL, \forall \bar{l} \in SL \tag{31}$$

$$UP_l \geq 0; \forall l \in L \setminus SL. \tag{32}$$

The first two terms in parentheses in the right-hand side of (31) represent $|f_{\bar{l}}|$ and $|f_l|$, respectively. As can be inferred, when $z_{l,\bar{l}}$ is equal to 0, the last term of (31) becomes a big negative number, which relaxes the constraint as UP_l is non-negative (32). If $z_{l,\bar{l}}$ equals 1, expression (31) sets the minimum value of UP_l to $|f_{\bar{l}}| - |f_l|$. As the value of the objective function being minimized increases with UP_l, UP_l is set to its maximum lower bound given by (31) and (32). Hence, the effect of the nonlinear expression (17) is equivalently modeled.

2) SAIDI: Similar to the method described for EENS, we can start with reformulating (9) and (11) as below:

$$SAIDI = \frac{\sum_{l \in L} \lambda_l \left(D_l^r |h_l| + D_l^{sw}\, Uh_l \right)}{\sum_{n=1}^{N_{lp}} NC_n} \tag{33}$$

$$\mathbf{A} \times \mathbf{h} = \mathbf{NC} \tag{34}$$

$$-M^H y_l \leq h_l \leq M^H y_l; \quad \forall l \in L \tag{35}$$

where matrix $\mathbf{A}$ is identical to that of (15) and Uh_l represents the total number of customers connected to the nodes upstream of each feeder section l with its switch closed. Note that a suitable value for the big-M parameter M^H is $\sum_{n=1}^{N_{lp}} NC_n$. Uh_l is modeled using (36) and (37), which are similar to (17) and (18), respectively:

$$Uh_l = \sum_{\bar{l} \in SL} z_{l,\bar{l}} (|h_{\bar{l}}| - |h_l|); \forall l \in L \setminus SL \tag{36}$$

$$Uh_l = 0; \forall l \in SL. \tag{37}$$

The structural similarity of (33)–(37) to (14)–(18) allows utilizing the above-described procedure to yield a linear equivalent. Thus, (33) is linearized by 1) defining two non-negative variables h_l^+ and h_l^-, 2) replacing $|h_l|$ with $h_l^+ + h_l^-$, and 3) incorporating (38)–(40) into the model:

$$h_l = h_l^+ - h_l^-; \forall l \in L \tag{38}$$

$$h_l^+ \geq 0; \forall l \in L \tag{39}$$

$$h_l^- \geq 0; \forall l \in L. \tag{40}$$

Moreover, (36) can be expressed in a linear form as follows:

$$Uh_l \geq (h_{\bar{l}}^+ + h_{\bar{l}}^-) - (h_l^+ + h_l^-) - M^H (1 - z_{l,\bar{l}}); \quad \forall l \in L \setminus SL, \forall \bar{l} \in SL \tag{41}$$

$$Uh_l \geq 0; \forall l \in L \setminus SL. \tag{42}$$

3) SAIFI: As mentioned above, SAIFI can be expressed by eliminating all D_l^r and D_l^{sw} terms in the SAIDI formula. Hence, based on (33), SAIFI can be cast by (43):

$$SAIFI = \frac{\sum_{l \in L} \lambda_l \left(|h_l| + Uh_l \right)}{\sum_{n=1}^{N_{lp}} NC_n} \tag{43}$$

which can be linearized in a similar fashion.

IV. Application

As described in Section III, reliability indices can be cast using alternative linear expressions. Hence, their incorporation into the optimization problems associated with the planning and operation of distribution networks allows the use of standard mathematical programming techniques. To illustrate such an application, we now consider the optimal network reconfiguration problem for radially-operated meshed networks [3], for which the previous models presented in [27], [29], and [30] are not readily suitable.

TABLE II
Radial Networks–Reliability Indices Resulting from the Proposed Approach and the State-of-the-Art Methods [27] and [29]

Test Grid	EENS (MWh/year)	SAIDI (hours/customer/year)	SAIFI (failures/customer/year)
37 Nodes	69.516	1.531	1.805
85 Nodes	49.633	2.427	1.971
137 Nodes	48.136	1.650	1.792
417 Nodes	91.147	0.987	1.669
1,080 Nodes	106.117	1.203	1.998

This problem consists in determining the radial topology that optimizes network operation. The minimization of customer interruption costs, which are expressed in terms of the aforementioned reliability indices, is widely adopted as the optimization goal. Thus, reliability-oriented costs are typically part of the objective function. Here, for the sake of simplicity, reliability costs are replaced in the objective function with a measure of the network reliability level. For quick reference, the network reconfiguration problem is cast as:

$$\min\ w_N EENS + w_D SAIDI + w_F SAIFI \tag{44}$$

Subject to:

$$\text{Linearized version of (14) with } |f_l| \text{ replaced with } f_l^+ + f_l^- \tag{45}$$

$$\text{Linearized versions of (33) and (43) with } |h_l| \text{ replaced with } h_l^+ + h_l^- \tag{46}$$

$$\text{Expressions (15), (16), (18)} - \text{(21), (27), (29)} - \text{(32), (34), (35), and (37)} - \text{(42)} \tag{47}$$

$$\sum_{l \in L} y_l = N_{lp} \tag{48}$$

where the decision variable set comprises y_l, $z_{l,\bar{l}}$, f_l, f_l^+, f_l^-, h_l, h_l^+, h_l^-, Uh_l, UP_l, $EENS$, $SAIDI$, and $SAIFI$.

The objective function (44) represents the reliability level, which is formulated as the weighted sum of the reliability indices EENS, SAIDI, and SAIFI. Linear expressions for such indices are included in (45)–(47). Finally, based on the findings of Lavorato *et al.* [33], expression (48), together with (15) and (34) in (47), guarantees radial operation. Problem (44)–(48) is an instance of MILP. Note that mixed-integer linear programming guarantees finite convergence to the optimum while providing a measure of the distance to optimality along the solution process [34]. Additionally, effective off-the-shelf software based on the branch-and-cut algorithm is readily available [35].

V. Numerical Results

In this section, results from several case studies are presented. For reproducibility of the results, test networks data can be downloaded from [36]. All cases have been run to optimality using GAMS 24.9 and CPLEX 12.6 on a Fujitsu CELSIUS W530 Power PC with a Quad 3.30 GHz Intel Xeon E3-1230 processor and 32 GB of RAM.

TABLE III
RADIAL NETWORKS–COMPUTATIONAL ASSESSMENT

Test Grid	Approach	Number of Decision Variables	Number of Constraints	Simulation Time (per unit)
37 Nodes	Proposed	75	75	1.00
	[27]	2,664	6,624	96.00
	[29]	714	839	11.00
85 Nodes	Proposed	169	169	1.00
	[27]	14,193	35,358	219.95
	[29]	1,632	1,914	11.75
137 Nodes	Proposed	273	273	1.00
	[27]	37,125	92,610	375.47
	[29]	2,684	3,186	13.59
417 Nodes	Proposed	831	831	1.00
	[27]	346,932	866,088	1,133.27
	[29]	8,250	9,845	12.45
1,080 Nodes	Proposed	2,161	2,161	1.00
	[27]	2,332,798	5,829,837	3,103.27
	[29]	21,586	25,895	13.57

TABLE IV
RADIAL NETWORKS–RESULTS WITHOUT SWITCHING INTERRUPTIONS [30]

Test Grid	Reliability Index			Error (%)		
	EENS	SAIDI	SAIFI	EENS	SAIDI	SAIFI
37 Nodes	56.974	1.245	0.654	18.04	18.68	63.77
85 Nodes	44.017	2.218	1.107	11.32	8.61	43.84
137 Nodes	39.794	1.366	0.678	17.33	17.21	62.17
417 Nodes	60.235	0.653	0.323	33.91	33.84	80.65
1,080 Nodes	31.253	0.358	0.121	70.55	70.24	93.94

EENS: MWh/year SAIDI: hours/customer/year
SAIFI: failures/customer/year

A. Reliability Assessment for Radial Distribution Networks

In order to demonstrate its applicability and scalability, the model described in Section III-A is applied to five radial test networks with 37, 85, 137, 417, and 1,080 nodes [27]. Table II presents the reliability indices resulting from the proposed approach and the state-of-the-art methods described in [27] and [29], which have been used for assessment purposes. As expected, identical results are attained by the three methods because they are equivalent from a modeling perspective, i.e., in terms of solution quality.

For the three approaches, Table III presents information on the computational performance associated with the results reported in Table II. Columns 3 and 4 of Table III summarize the dimension of the resulting problems, whereas column 5 lists the relative computational effort normalized about the running time for the model proposed in Section III-A. The simulation times for the proposed approach are respectively 0.219, 0.488, 0.780, 2.414, and 5.975 milliseconds for the test grids with 37, 85, 137, 417, and 1,080 nodes. As can be seen in Table III, the proposed approach is between two and three orders of magnitude faster than the topology-variable-based method described in [27]. More importantly, the computational effort required by the proposed approach is one order of magnitude lower than that associated with the most computationally-efficient technique available in the literature on topology-variable-based distribution reliability assessment [29]. This relevant result stems from the significant reduction in the number of variables and constraints, which paves the way for the applicability of this formulation to reliability-constrained optimization models for distribution systems. More specifically, it can be observed in Table III that the computational time savings are particularly similar to the factors by which the number of constraints is decreased.

TABLE V
MESH-DESIGNED NETWORKS–OPTIMAL RESULTS

	24-Node System	54-Node System	136-Node System
EENS	5.784	11.249	13.245
SAIDI	0.153	0.784	0.713
SAIFI	0.260	0.610	0.769
Switched-Off Feeder Sections	1, 3, 5, 8, 11–13, 18–21, 25, 29	6, 11–13, 15–17, 21, 28, 30, 54, 58, 62	9, 35, 53, 58, 70, 83, 92, 113, 137, 138

EENS: MWh/year SAIDI: hours/customer/year
SAIFI: failures/customer/year

TABLE VI
MESH-DESIGNED NETWORKS–RESULTS WITHOUT SWITCHING INTERRUPTIONS [30]

Test Grid	Actual Reliability Index			Index Difference (%)		
	EENS	SAIDI	SAIFI	EENS	SAIDI	SAIFI
24 Nodes	6.001	0.156	0.327	3.75	1.96	25.77
54 Nodes	11.393	0.793	0.644	1.28	1.15	5.57
136 Nodes	13.325	0.717	0.782	0.60	0.56	1.69

EENS: MWh/year SAIDI: hours/customer/year
SAIFI: failures/customer/year

In order to assess the impact of switching interruptions on the reliability indices, the model proposed in [30] is applied to the investigated radial test networks. As per Table IV, disregarding switching interruptions results in significantly underestimating the reliability indices, with errors ranging between 8.61% and 93.94%. In addition, the inaccuracy incurred by disregarding switching interruptions is stressed for the networks with longer feeders, i.e., the 417- and 1,080-node systems.

B. Radially-Operated Meshed Networks

The reliability-constrained optimization model presented in Section IV is applied to three meshed networks with 24, 54, and 136 nodes. Table V summarizes the optimal results corresponding to all weighting coefficients equal to 1. The attainment of optimality required 0.80 s, 1.04 s, and 862.71 s, respectively, thereby revealing the scalability of the proposed approach.

In order to illustrate the benefits of the proposed approach in terms of computational performance, we have implemented a modified version of the network reconfiguration problem (44)–(48) wherein the proposed reliability assessment model has been replaced with an equivalent albeit computationally expensive formulation based on that described in [28]. As expected, the reliability indices and network topologies achieved by this equivalent approach were identical to those identified by the proposed method and reported in Table V. By contrast, the computational effort required by the benchmark model to attain such results significantly exceeded that featured by the proposed approach by 211%, 832%, and 77% for the 24-, 54-, and 136-node systems, respectively. Thus, the computational effectiveness of the proposed approach is empirically backed.

The advantage of the proposed approach in terms of solution quality has also been illustrated by implementing a modified version of the network reconfiguration problem (44)–(48) wherein switching interruptions are disregarded. To that end, the proposed reliability assessment model has been replaced with that of [30]. Table VI lists the actual reliability indices of the solutions provided by the second benchmark model. For the sake of a fair comparison, such reliability indices result from solving the proposed model (44)–(48) with network reconfiguration decisions fixed to the optimal values provided by the second benchmark model. Table VI also shows the respective relative differences of the resulting reliability indices with respect to those attained by the proposed approach (Table V). As can be observed, for the three test systems, disregarding switching interruptions led to solutions that were outperformed by those achieved by the proposed model as larger values for all reliability indices were identified, with increase factors up to 25.77%.

VI. Conclusion

This paper has presented two novel formulations for distribution network reliability assessment. Due to their reduced dimension as compared with previously reported formulations, the proposed models can be efficiently employed to compute widely-used system-level reliability indices, such as SAIFI, SAIDI, and EENS, while precisely accounting for switching interruptions. The first model consists of a set of linear expressions and is suitable for radial networks. The second model relies on mixed-integer linear programming and allows handling not only radial networks but also radially-operated meshed grids.

For several radial benchmarks with up to 1,080 nodes, numerical simulations show that the first model is between one and three orders of magnitude faster than state-of-the-art approaches to attain identical results. Similarly, numerical experience with a network reconfiguration problem for three meshed systems comprising 24, 54, and 136 nodes illustrates the computational superiority of the proposed MILP-based model. Finally, the comparison with existing topology-variable-based reliability assessment models disregarding switching interruptions reveals the substantially improved accuracy provided by the proposed formulations.

Ongoing research is focused on the application of the proposed models to other optimization problems for distribution network planning and operation. Further research will address the consideration of distributed energy resources with uncertain outputs as well as more complex instances of reliability-constrained distribution operation and planning. Another interesting avenue of research is the extension of the approach to consider practical modeling aspects such as post-fault network reconfiguration to restore the service for load nodes downstream of the fault and non-fully reliable switches. We recognize that such extensions need further research effort and numerical studies.

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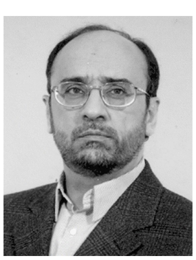

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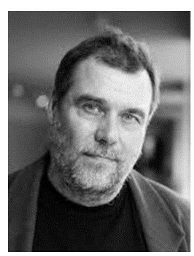

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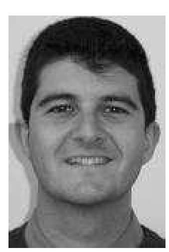

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