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Assessing the prediction uncertainty in a route optimization model for autonomous maritime logistics

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Abstract

Unmanned operations and automation in modern industry create complex everyday problems, which require algorithmic thinking and creativity. Development of risk assessment methods is critical for the future of this business segment. To provide decision support for the management of an autonomous emission control boat, we begin by proposing a k-Nearest-Neighbors (k-NN) based trajectory prediction method. This is employed in a bi-objective routing problem of finding a Hamiltonian circuit in a dynamic network defined by predicted locations of ships over time. The objectives are maximizing the number of measurement tasks to be done and minimizing the corresponding total travel distance of the emission control boat. To evaluate the impact of trajectory prediction uncertainty among Pareto optimal itineraries, we propose a risk measure in a mean-risk framework. The risk is defined based on an expected shortfall when implementation of an efficient itinerary under the predicted trajectories needs rescheduling based on realized trajectories. The risk measure helps the decision maker to evaluate choice alternatives among efficient itineraries under predicted trajectories and to make a balanced risk-adjusted decision. We show how historical data is employed in integer linear programming for the estimation of such risk measure. Empirical results demonstrate such estimation.

Keywords: mean-risk analysis; maritime autonomous transport; route prediction; nearest neighbour; integer programming;

1. Introduction

Ship movement information is becoming increasingly available with Automatic Identification System (AIS) (see International Maritime Organization (2002)). AIS provides a way of sending information on e.g. position, speed, course, and identity of a vessel in order to help other vessels avoid collision, and maritime authorities to monitor the movements of vessels. The resulting information has made it possible to develop mathematical modelling and robust analysis to avoid human error and enable more efficient operations (see e.g. Meriteollisuus (2016)). According to Gu et al. (2020) research on au-

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tonomous vessels has increased significantly in recent years. However, most of them have focused on navigation control and safety issues. Studies regarding other topics, such as transport and logistics, are very limited. Akbar et al. (2020) investigated the economic advantages of utilizing autonomous over conventional ships and provided evidence that autonomous ships might contribute to considerable cost savings. However, it is a generally agreed concept that prediction models need to be accompanied by discussion and analysis of their uncertainty.

In the field of analysis for maritime transportation systems, the effect of risk and uncertainty is infrequently discussed or quantified. Sormunen et al. (2015) discuss a case study dealing with risk analysis for a chemical spill in the Gulf of Finland and analysis of the related uncertainties based on expected spill frequency and spill volumes caused by collisions in the Gulf of Finland. Klemola et al. (2009) present a prediction model based on Bayesian networks for forecasting maritime traffic in the Gulf of Finland over several years, and evaluate the risk of environmental accidents. Probabilistic risk assessment techniques have been used in other recent studies on maritime transportation systems. Merrick et al. (2005) developed a Bayesian meta-model to examine the impact of uncertainty on policy implications of a study on proposed ferry service expansions in San Francisco Bay. Mean-risk models facilitate the trade-off analysis of mean reward and risk in real decision problems dealing with uncertainty; see e.g., Ruszczyński and Vanderbei (2003) and references therein. Lua et al. (2018) developed a mean-risk based nonlinear mixed integer programming model for transportation network protection. The conditional value at risk, CVaR introduced by Rockafellar and Uryasev (2000), is included as the risk measure for the total system cost. A mean-risk model for traffic assignment problem proposed in Nikolova and Stier-Moses (2014) takes into account the stochastic nature of travel times extending the traditional model of Wardrop equilibrium when uncertainty is present in the network.

In this article we build on mean-risk analysis as well. Our research task concerns an optimization model to support decision-making for an autonomous emission control boat measuring greenhouse gas emissions of ships in the Gulf of Finland. The method evaluates the uncertainty of a predetermined optimal itinerary for a time-dependent moving-target, travelling salesman problem based on two objectives. (see e.g. Maskooki and Nikulin (2018) and Bourjolly et al. (2006) for some real-world application examples). The control boat management needs information about the locations of ships passing through the work area over time many hours ahead in order to plan an efficient itinerary. For optimal routing, bi-criteria optimization framework is employed with the objectives of maximizing the number of ships visited and minimizing the travel distance. For an itinerary of a given day, predicted trajectories of ships are used for finding Pareto-optimal itineraries (for relevant applications of trajectory prediction see e.g. Virjonen et al. (2018) and Mazzarella et al. (2015) for predicting vessel motion in a specific area, Pallotta et al. (2013), Wijaya and Nakamura (2013) for predicting a ship’s route in heavily trafficked fairways, and Ristic et al. (2008) for anomaly detection).

However, optimality of the predetermined itinerary holds only for predicted scenarios which are uncertain. Thus, it is not sufficient to plan an efficient itinerary based on deterministic optimization only. The route prediction model provides predicted locations within minutes’ accuracy in time several hours ahead; however, over the day the predicted locations often have minor or major deviations from actual locations, resulting in a need for revising the itinerary. Therefore, along with trajectory prediction, the uncertainty of the prediction must be assessed. Instead of using probabilistic techniques, we rely on mean-risk analysis to evaluate the quality of Pareto-optimal solutions. Based on expected shortfall, we develop a risk measure to be estimated based on historical observations. Thereby, the risk component
yields a third criterion in the routing problem of choosing from the set of Pareto-optimal itineraries. Hence, the purpose of this study is to evaluate the practicality of the trajectory prediction based itinerary when it is implemented in real situation.

According to Kaplan (1997), there are three elements underlying decision making under uncertainty: a set of options from which to choose, an evaluation of the outcomes of each option, and a value judgement on each outcome. To provide the first element we introduce a model for predicting the future behaviour of ships navigating in the Baltic sea based on k-NN method. This provides the most likely scenario, the predicted trajectories of ships, to be used in an optimization model resulting in a set of options for the decision maker (DM). Each option is a Pareto-optimal itinerary for an autonomous emission control boat, including its costs in terms of travel distance, evaluated by the number of measurements and uncertainty related to trajectory prediction errors. To assess the risk of uncertainty, we propose an evidence-based quantitative risk measure. The risk of implementing an efficient itinerary in real situation is measured by such historical benchmark knowledge. The whole process provides us a problem specific Artificial Intelligence (AI) based tool for supporting decision making. However, to answer the question "what is the best decision option among all efficient itineraries?" the decision maker’s preferences are still needed. Based on her value judgement, first, the trade-off between the number of measurements and the total travel distance needs to be quantified, and second, her risk attitude enters the optimal choice through a weight on the risk measure in a mean-risk model.

The rest of the paper is organized as follows. Section 2 introduces the approach for trajectory prediction of ships. Such predictions are employed in the model of Section 3 for finding Pareto optimal itineraries. In the spirit of mean-risk framework, Section 4 proposes a risk measure to help the decision maker evaluate trajectory prediction uncertainty among Pareto optimal itineraries for making a balanced risk-adjusted choice. We also show how historical data is employed for the estimation of such risk measure. An empirical case study in Section 5 demonstrates assessment of route prediction uncertainty in a route optimization model. Section 6 concludes.

2. Trajectory prediction

For route optimization of the emission control boat, consider a given day and a time horizon $[0, T]$ of $T$ hours. The goal of trajectory prediction is to provide estimates for trajectories over $[0, T]$ of the ships present in the measurement work area some time during the planning horizon. Online data of ships present at the sea are available at some time preceding initial stage of the time horizon. The prediction model produces the trajectories of the ships through the work area and these data is an input for the control boat’s route optimization model.

The prediction of the trajectories in our study is based on the k-nearest neighbour method, which is described in Virjonen et al. (2018). For each ship, k nearest neighbours are searched from historical training data, and the trajectory of the ship within the work area is predicted as the median of the trajectories of the k nearest neighbour. The distance measure is a convex combination of two parts with weight coefficients $\nu$ and $1 - \nu$. The parts consist of the speed difference between the ships being compared, and the average distance between their trajectories. A 30 minutes start window preceding the begin-

\footnote{Time-ordered set of states of a dynamical system}
ning of the time horizon $[0, T]$ is used for such comparison of the ships. The hyper-parameters $(\nu, k)$ of the trajectory prediction model are optimized using leave-one-out cross-validation (LOO CV); see e.g., Virjonen et al. (2018). The number of nearest neighbour $k$ is chosen from the range of 1 to 30, and $\nu$ from the a discrete set between 0 and 1 with increments 0.05. All the calculations for ship trajectory prediction were made with Matlab 2017b.

2.1. Data preparing

AIS data used in the present study were gathered during December 2017 to December 2018. The data were fetched from the open interface of Finnish Transport Infrastructure Agency (2019) every 10 minutes, and saved into JSON format. The data consists of vessels equipped with A-class AIS transmitter. In the trajectory prediction model, only the MMSI (Maritime Mobile Service Identity) number of the ship, the location in latitude and longitude, and the time were employed. Here we use May, June and July 2018 data for training data, consisting of 3894 ships.

The data are further processed into passages, i.e. into segments of continuous movement. In theory, one passage consists of one journey from one harbour to another. In practice, there exist gaps in the AIS data, and some unpredictable deceleration due to e.g. optimizing the arrival time, which resulted into shorter passages in some cases. A passage is started when the speed of a ship exceeds 4 knots, and stops as it falls below 4 knots. Only passages with duration at least 2 hours are accepted. If there is a gap longer than 1 hour in the AIS data for a particular ship, a new passage is started.

Given these conditions and removing ships with no valid passages, there are 35833 passages in total for 3016 different ships in our three month training data. After subdividing the ship trajectories into passages, each passage is linearly interpolated with 10 minutes uniform time sampling beginning at midnight. For each passage, the location of the ship over time determines data points in the passage.

The set of all passages are further arranged into subsets using a special space partitioning algorithm. In space partitioning, the sea area is divided into a finite number of nodes defined by a uniform rectangular grid with 10 km spacing in both east-west and north-south direction. The closest node of the data points in each passage is found, and the passage is assigned to the its closest node. This partitioning procedure reduces the time needed for finding the $k$ nearest neighbors. For each node, the percentage of ships entering the work area after being close to the node, is calculated.

2.2. Optimizing the hyper-parameters

For the test day in concern, the start window is defined from 8:00 (UTC+3) to 8:30. For this day, the ships present at the sea during the start window are sought. For each ship, the closest node to its first data point within the start window is found; this node is the start node of the ship. The historical passages assigned to the start node are further filtered to acquire the training data. The conditions for passages to be accepted into the training data are:

1. The passage enters the measurement work area after being nearest to the start node
2. The passage has at least 5 data points before the data point, which is closest to the start node
3. The passage has at least 1 data point after exiting the measurement work area
The first condition ensures that the training ships enter the work area after being in the start node. It was assumed that there exists prior information on the ships, e.g. on the destination according to the AIS metadata so obvious ships not entering the work area could be excluded. If there is no information on the destination harbour, the node information can be exploited: If a passage is assigned to a node with a very low percentage of the ships going to the work area, then the passage (ship) can be excluded.

The second condition ensures that the determination of the distance measure is possible. If there exist at least 5 data points before the point, which is closest to the start node, then the speed difference and trajectory distance between the ships being compared can be evaluated rather adequately.

The third condition ensures that the ship leaves the measurement work area, i.e. odd behavior is excluded (no ship usually stops within the work area), and there are no incomplete trajectories in the training data.

The \((k, \nu)\) parameter optimization is conducted for the start node as follows. For each \((k, \nu)\) pair, a LOO CV is conducted: one training data passage is taken as the validation data, and the rest remain as the training data. The training data were used to predict the entering time to the work area, and it is compared with the actual entering time, then the error is calculated for the validation ship. This procedure is repeated for each training data passage, and the mean absolute error is calculated. The combination of \((k, \nu)\) producing the smallest mean absolute error is chosen as the optimal parameters. In this case study, the trajectories of the ships are rather concentrated into ship lanes, so the error of time had more detrimental effect than the error of location.

![Error of entering time](image1)

![Error of entering location (km)](image2)

Fig. 1. The error of the time (left) and location (right) at the point of entering the work area for each test day; the ships of seven test days are shown in different colors.
2.3. Test data

For testing in Section 5 the combined performance of the trajectory prediction model and the emission control boat route optimization, one week of August 2018 data is used. For training data, historical data of May-July 2018 are first divided into passages, as explained above. For a given test day of August 2018, and for each ship present within the start window, the optimized \((k, \nu)\) and the training passages are used to determine the predicted trajectory for the ship within the work area. Each predicted trajectory is interpolated with 5 minutes time spacing to facilitate the optimal itinerary calculation for the emission control boat in Section 5.

The error of time and location at the entering point to the work area are presented in Fig. 1 for each ship for the seven test days. The median absolute error of time is 9 minutes. 55% of the ships have absolute error of time less than 10 minutes. The median error of location is 1 km. 67% of the ships have error of location less than 2 km. Above mentioned values apply for prediction accuracy for 12 hours ahead. The prediction accuracy improves if a shorter time horizon were applied. For 6 hours ahead, the median absolute error of time is 3 minutes. 73% of the ships have absolute error of time less than 10 minutes. The median error of location is 1 km. 67% of the ships have error of location less than 2 km.

3. Pareto optimal itineraries based on predicted trajectories

An optimization model is presented in Maskooki and Nikulin (2018) to support decision-making for finding an optimal routing for a boat measuring greenhouse gas emissions of ships in a specific work area in the Gulf of Finland. In any feasible itinerary over a time horizon during a day, in general it is not possible to visit all targets appearing in the work area. Therefore, a bi-criteria optimization framework is adopted. The two objectives are to maximize the number of targets visited and to minimize the total travel distance. The chosen itinerary of a given day is based on trajectories of ships in the work area predicted at the beginning of the planning horizon using the method described in Section 2. Given \(\alpha\), the number of ships visited, the model finds an optimal sequence of \(\alpha\) ships by minimizing the total travel distance denoted by \(z\). Thus, optimal plans for \(\alpha = 1, 2, \ldots\), determine the entire Pareto optimal set of itinerary plans.

Next, we present a reformulation of the model in Maskooki and Nikulin (2018). In addition to serving for the computation of Pareto set under predicted trajectories, the model is also an essential starting point in Section 4 for developing an estimation procedure of our risk measure related to uncertainty in predicted locations. Formally, the problem of finding a Pareto optimal plan leads to an MILP problem as follows.

Consider a time horizon \([0, T]\) where time \(t = 0\) refers, for instance, to 8:30 a.m. of a given day. According to the prediction, \(n\) ships appear in the work area during \([0, T]\). The time horizon is discretized into \(m\) time slots \(k\) of length \(w\); \(k = 1, 2, \ldots, m\); time slot \(k = 0\) refers to the initial time stage 0. For \(k > 0\), we assume the location of ships remain unchanged during time slot \([(k - 1)w, kw)\). Let \(v^k_i \in \mathbb{R}^2\) be the predicted coordinate vector of ship \(i\) in time slot \(k\). Such predicted locations of \(n\) ships within each time slot in \([0, T]\) are given. Let \(d^k_{ij}\) be the distance to travel from the predicted location \(v^k_i\) of ship \(i\) at time slot \(k\) to the location \(v^l_j\) of ship \(j \neq i\) at a time slot \(l > k\), and define \(d^k_{ij} = \| v^l_j - v^k_i \|\). Let \(\delta^k_{ij} = (1/c)d^k_{ij}\) be the travel time with a constant speed \(c\) of the control boat and \(p\) be the processing time
for each ship. The time slots $w < 2p$ are chosen short enough not to allow more than one ship processed in a single slot. For notational convenience, we let ships $i = 0$ and $i = n + 1$ refer to the depot. In order to find the minimum travel distance $z$ for visiting $\alpha$ ships, the optimization employs binary variables $x_{kl}^{ij}$, which is equal to 1 if ship $i$ is visited in location $v_{i}^{k}$ at time slot $k$, and subsequently ship $j$ is visited in location $v_{j}^{l}$ at time slot $l$; otherwise $x_{kl}^{ij} = 0$. An illustration in Figure 2 shows an example of optimal itinerary in August 9, 2018. The itinerary includes the exact locations ordered by their time slots. The control boat reaches the first ship ($i=29$) at time slot $k=13$. Each binary variable $x_{kl}^{ij}$ is defined only if $i$ and $j$ are in the work area in slots $k$ and $l$, respectively; in other words, if ship $i$ is not in the work area at time slot $k$ or ship $j$ is not in the work area at time slot $l$ then we require $x_{kl}^{ij} = 0$. For notational simplicity, we suppress such feasibility requirements. The problem is formulated as follows.

The objective is to minimize the total travel distance:

\[ \min z = \sum_{i,j=0}^{n+1} \sum_{k,l=0}^{m} d_{ij}^{kl} x_{ij}^{kl}. \]  (1)
The following constraint ensures exactly one exit from depot \( i = 0 \):

\[
\sum_{j=1}^{n} \sum_{k,l=0}^{m} x_{0j}^{kl} = 1. \tag{2}
\]

For \( 1 \leq j \leq n \), if ship \( j \) is visited in time slot \( l, 0 < l < m \), then the immediate predecessor ship \( i \neq j, i \geq 0 \), is visited at time slot \( k < l \), and the immediate successor ship \( i \neq j, i \leq n + 1 \), is visited at time slot \( k > l \). This is ensured by the following flow conservation constraints:

\[
\sum_{i=0}^{n} \sum_{k=0}^{l-1} x_{ij}^{kl} = \sum_{i=0}^{n+1} \sum_{k=l+1}^{m} x_{ji}^{lk} \quad \forall 1 \leq j \leq n, 0 < l < m. \tag{3}
\]

The second objective is to maximize the number of visits. Let integer parameter \( \alpha \) indicate the number of visits. The Pareto frontier could then be generated in terms of \( z \) versus \( \alpha \) by varying \( \alpha \) over the set \( 1, \ldots, n \). Therefore, the following constraint is added to ensure that exactly \( \alpha \) ships are visited,

\[
\sum_{i,j=1}^{n+1} \sum_{k,l=1}^{m} x_{ij}^{kl} = \alpha. \tag{4}
\]

Note in (4), \( x_{ij}^{kl} = 1 \) indicates departure after visiting ship \( i > 0 \) to visit next ship \( j \geq 0 \) with \( j = n + 1 \) meaning return to the depot. Condition (5) below requires that each ship \( i > 0 \) is visited at most once. Thus the left side expression in (4) is the number of ships \( i > 0 \) visited.

We also restrict the number of visits to each ship \( i \) to be at most one:

\[
\sum_{j=1}^{n+1} \sum_{k,l=1}^{m} x_{ij}^{kl} \leq 1 \quad \forall i = 1, 2, \ldots, n. \tag{5}
\]

Given ship \( i \) is processed at time slot \( k \), and is succeeded by ship \( j \neq i \) processed at some time slot \( l \). Let non-negative variable \( s_k \) and \( s_l \) denote the start time of processing at time slots \( k \) and \( l \), respectively. Then we need to satisfy the following requirement:

\[
s_k + (p_i + \delta_{ij}) x_{ij}^{kl} \leq s_l \quad \forall i, j \in \{0, \ldots, n + 1\}, i \neq j, k, l \in \{0, \ldots, m\}, k < l \tag{6}
\]

where the processing time \( p_i = p \), for all \( 1 \leq i \leq n \), \( p_0 = 0 \).
We also ensure that, the start time of processing in 6 falls into its correct time slot by adding the following limits to each start time $s_k$,

$$s_0 = 0 \text{ and } w(k-1) \leq s_k \leq w_k \quad \forall \; k \in \{1, \ldots, m\}. \quad (7)$$

Note that if in a solution of the model $x^{kl}_{ij} = 1$ and there is slack in inequality (6), then the control boat waits in location $v^t_{ij}$. The waiting time is the slack $s_l - \lfloor s_k + (p_i + \delta^{kl}_{ij})x^{kl}_{ij} \rfloor$. To justify the second objective of minimizing the total travel distance, we assume that fuel consumption is negligible during the waiting time.

4. Mean-risk analysis for uncertainty of trajectory prediction

As mentioned earlier, predicted locations often have minor or major deviations from actual locations, resulting in tardiness or earliness when the control boat is following an itinerary found by the route optimization model of Section 3. At time $t = 0$ (the time of decision) the Pareto frontier based on predicted locations is available, but actual locations of ships are not known. Our goal is to evaluate statistically the expected loss (to be specified below) in real situation by following the optimal schedule given in advance based on the predicted trajectories. We will point out that loss here can be defined as the increase in travel distance due to the predetermined sequence of ships to be visited.

In Section 4.1 we discuss the decision problem of DM in the spirit of mean-risk framework. We introduce a risk measure to help DM to evaluate possible choices within the range of Pareto optimal solutions at hand, and to make a balanced risk-adjusted decision. Section 4.2 shows how historical data can be employed for estimation of the risk measure.

As a brief account of mean-risk modeling appearing in the literature, we discuss in parallel some finance applications. A situation similar to ours appears, for instance, in financial decision making: at the time of financial investment, realizations of future risks are not known; however risks are most commonly modeled based on observations from the history of financial markets. The classical case is the mean-risk model of Markowitz where the criterion for portfolio choice is to maximize $e - \mu \rho$, where $e$ is the expected return, $\rho$ is the variance of return and $\mu$ is a weight reflecting individual risk preferences. Although this mean-variance approach is criticized for its risk measure (penalizing also favorable outcomes), it still is much used in practice, and today, various mean-risk approaches are widely used for decision under uncertainty; see e.g., [Ruszczynski and Vanderbei (2003)] for mean-risk analysis employing coherent measures of risk. As in the Markowitz model, the objective in the mean-risk model is to maximize $e - \mu \rho$, where $e$ is the mean reward, $\rho$ is a risk measure and $\mu$ is a subjective weight for risk preferences. One among coherent measures of risk is expected shortfall: for a random reward $\tilde{r}$ with expected value $e$, the expected shortfall is $E[(e - \tilde{r})_+]$ where $(e - \tilde{r})_+$ denotes projection on the non-negative axis. CVaR proposed in [Rockafellar and Uryasev (2000)] is another commonly used coherent measure of risk.
4.1. A mean-risk framework for the routing problem

In this subsection we discuss the decision problem faced by the manager (DM) while choosing the itinerary. This also provides a natural basis to see how the various uncertainties enter DM’s problem. The presentation involves multiple items:

1. Pareto optimal set of itinerary based on predicted trajectories for choosing the preferred plan
2. definition of reward (pay-off) used by DM in evaluating realized tours of the control boat
3. adopting a mean-risk model accounting for the prediction errors
4. principles of estimating the risk measure for our mean-risk model
5. optimal routing under prediction uncertainty summarizing the decision problem

In the sequel, we discuss each one of the items.

4.1.1. Pareto set of tours
The chosen itinerary is based on the deterministic optimization model. Given predicted locations of the n ships during the time interval [0, T], the Pareto optimal itinerary can be found using a MILP model as described in Section 3. Given the number of ships α to be visited, z is the minimum travel distance.

4.1.2. Reward of realized tour
At the beginning of the day, at time t=0, DM chooses an itinerary for the control boat over the day until time T (e.g., ten hours later). DM’s preferences are based the realized reward (pay-off) observed at time T, at the end of the horizon after the control boat has completed its task. In our bi-criteria framework, such reward depends on the levels of two criteria: first, \( \hat{\alpha} \) which is the realized number of ships visited for emissions measurements, and second, \( \hat{z} \) which is the realized total travel distance (km) from the harbor, visiting \( \hat{\alpha} \) ships and returning to harbor. We employ a linear value function \( v(\hat{\alpha}, \hat{z}) = \hat{\alpha} - \lambda \hat{z} \) defining the realized reward \( \hat{v} \).

We assume that the weight \( \lambda \) reflecting DM’s trade-off preferences is a known parameter. It may be estimated by a simple trade-off question: how many extra kilometres \( \Delta z \) is accepted by DM at most in return to an extra ship visited. The parameter \( \lambda \) is then determined by indifference equation \( (\hat{\alpha} + 1) - \lambda(\hat{z} + \Delta z) = \hat{\alpha} - \lambda \hat{z} \) and thus \( \lambda = 1/\Delta z \) with a dimension 1/km.

At time \( t = 0 \) when DM chooses a routing plan for the vessel, trajectories of n ships are predicted over the time horizon of T hours (as explained in Section 2). Thus, the predicted locations of ships \( \{v_i^k\}, i = 1, \ldots, n, k = 0, \ldots, m \) for possible visits as well as the target values \( (\alpha, z) \) are given. However, predicted locations have deviations from actual locations during \( [0, T] \), resulting in tardiness or earliness when the control boat tends to visit the ships in the sequence of the chosen itinerary. Hence, given the realized trajectories of ships, the timing of visiting the predetermined sequence of ships may need to be revised and possibly some visits need to be omitted resulting in realized levels \( \hat{\alpha} \) and \( \hat{z} \) of the two criteria and a realized reward \( \hat{v} = \hat{\alpha} - \lambda \hat{z} \).

At the time of the decision \( (t = 0) \) of route optimization, given an itinerary plan, realized trajectories are not known wherefore the number of ships to be visited, the total travel distance and the reward are a random variables \( \hat{\alpha}, \hat{z} \) and \( \hat{v} \), respectively, and their realizations are denoted by \( \hat{\alpha}, \hat{z} \) and \( \hat{v} \).
4.1.3. The mean-risk model

In principle, given a stochastic model for the trajectories of the $n$ ships, a possibility to deal with the uncertainty of the positions of the ships is to employ stochastic optimization and formulate DM’s problem by maximizing expected utility of the reward; for instance, maximizing expected reward. However, even the deterministic model based on predicted location data is hard due to a large number of binary variables\(^2\). Therefore, we have chosen a framework to provide decision support for DM based on the Pareto optimal tours of the deterministic model combined with a risk measure to judge the risk related to prediction errors.

The chosen itinerary with a Pareto optimal point $(\alpha, z)$ yields target levels of criteria $\alpha$ and $z$ and the target reward $e = e(\alpha) = v(\alpha, z) = \alpha - \lambda z$. In other words, level $e$ is the anticipated reward calculated based on a Pareto optimal point $(\alpha, z)$ and the realized reward depends on the success of implementing the chosen itinerary. To account for such uncertainty, a risk measure $\rho(\alpha)$ introduces a third criterion in DM’s decision problem. The objective of DM in our mean-risk framework is to maximize the target reward $e$ (serving as a substitute of expected reward) penalized by a risk component accounting for the uncertainty in implementing the chosen itinerary; i.e., the problem of DM is to find a Pareto optimal solution $(\alpha, z)$ to

$$\max_{\alpha} e(\alpha) - \mu \rho(\alpha),$$

where $\rho(\alpha)$ is a risk measure and $\mu$ is a positive weight reflecting DM’s risk preferences. Our intention is not to solve the problem (8) for DM. Instead we just provide decision support by evaluating the risk measure $\rho(\alpha)$ of the random reward $\tilde{v}$ for any fixed Pareto optimal solution $(\alpha, z)$ determined by the MILP problem in (1)–(7). Additionally, we may provide average value of realized reward $\hat{v}$ as an estimator of the expected reward $E[\tilde{v}]$ for any given Pareto point $(\alpha, z)$; for an illustration, see Section 5.2.

In our framework, the risk measure is the expected shortfall from the target reward\(^3\). Thus, for a random reward $\tilde{v}$ with a target reward $e$, our risk measure $\rho$ is given by the expected shortfall:

$$\rho(\alpha) = E[(e - \tilde{v})_+] = E\left[\frac{\left| e - \tilde{v} \right|}{\left| e - \tilde{v} \right|} \right].$$

Here $(\cdot)_+$ denotes projection on the non-negative axis. Given the chosen routing plan $(\alpha, z)$, the target reward $e = \alpha - \lambda z$, and the resulting realized reward $\hat{v} = \hat{\alpha} - \lambda \hat{z}$, the realized risk measure is $\hat{\rho}(\alpha) = (e - \hat{v})_+$. An estimate of the risk measure $\rho$ is the sample mean of such realizations $\hat{\rho}$ in a sample of historical observations.

4.1.4. Estimating the risk measure

There is a large number of sources resulting in uncertainty, such as weather conditions, missing or wrong data, human-originated source, etc. These are among many features affecting the deviation (of predicted locations) from actual locations of ships at certain points in time of interest. The prediction

\(^2\) Maskooki et al. \cite{Maskooki2013} proposed heuristics to speed up computations.

\(^3\) A coherent measure of risk in \cite{Ruszczynski2003} is defined as expected shortfall from the expected reward.
model (Section 2) uses a k-NN-approach. The idea is to predict the future trajectory of a ship based on historical data, by finding k nearby matching routes, whose behaviour resembles the behaviour of the given ship. Such model is able to predict several hours ahead with a reasonable accuracy.

Dealing with uncertainties using traditional mean-risk models is similar to our case. For example, in portfolio optimization there is a vast number of uncertainties whose realizations determine the realized reward (return); see e.g., Vuolteenaho (2002) where the main uncertainties driving stock return realizations are studied. However, the sources of uncertainties in financial mean-risk analysis are rarely discussed. The risk measure is directly estimated based on historical realized returns (reward) extracted from the market price series. We follow such tradition as well, and thereby, various sources of uncertainty are mixed together in a simple manner, and the risk measure $\rho$, the expected shortfall of reward $\hat{v}$ from the target reward $e$, is estimated by a sample mean based realized reward $\hat{v}$ on historical observations with features similar to those prevailing during the day of route optimization.

Based on historical data we evaluate statistically the risk measure, the expected shortfall from the target reward. The true realized reward is only observed for one historical plan which were chosen to be implemented in practice. However, we can exploit the available vast information on realized trajectories of ships for any other routing plan which were not chosen to be followed. Such a reasonable proxy of realized reward is obtained by optimal adjustments in the plan under real ship trajectories as described in Section 4.2 below.

To clarify the use of data for estimating the risk measure, consider a day of historical observations. From the beginning of this day, based on predicted routes of ships in the work area during the day, we have the resulting Pareto-optimal routing plans with target levels $(\alpha, z)$ of objectives and target reward $e = \alpha - \lambda z$ for each level of $\alpha$ as well as the chosen plan for the emission control boat. From the end of the day we have realized trajectories of all ships appearing in the work area during the day as well as the realized plans of the control boat. Based on this realized plan we have realized levels of objectives $(\hat{\alpha}, \hat{z})$, the realized reward $\hat{v} = \hat{\alpha} - \lambda \hat{z}$, and the realized shortfall $(e - \hat{v})_+$ from the target reward $e$ of the Pareto optimal plan $(\alpha, z)$ chosen for that day. However, as mentioned earlier, such true realized reward is only observed for the Pareto plan $(\hat{\alpha}, \hat{z})$ which was chosen to be implemented. To extract reasonable proxy of other realized rewards (which were not implemented in practice), we exploit the information represented by the realized trajectories of all ships. For all other Pareto-optimal plans, using the optimization model presented in Section 4.2 we obtain optimal adjustments in the initial plan yielding levels $\hat{\alpha}$ and $\hat{z}$ of the two objectives and the reward $\hat{v} = \hat{\alpha} - \lambda \hat{z}$. Because $\hat{v}$ is based on realized trajectories we consider it as a reasonable proxy of realized reward at the end of the day. Therefore, for any Pareto optimal plan $(\alpha, z)$, for convenience we say $\hat{\alpha}$ is the realized number of ships, $\hat{z}$ is the realized total travel distance, $\hat{v} = \hat{\alpha} - \lambda \hat{z}$ is the realized reward (pay-off) and $\hat{\rho} = (e - \hat{v})_+$ is the realized shortfall. Given all these measures are based on real trajectories of the ships, we believe our notation is not confusing.

To estimate the risk measure $\rho$, we propose the following methodology. For each level of $\alpha$ we consider a target reward $e = \alpha - \lambda z$ given a point $(\alpha, z)$ on the Pareto frontier based on predicted locations of ships. We expect the shortfall to be dependent on the number of visits $\alpha$ relative to all targets $n$; in particular, we assume the realized risk measure $\hat{\rho}$ depends on the ratio $\alpha/n$, the share of ships visited among all ships in the work area during the time horizon $(0,T]$. The risk measure $\rho(\alpha/n)$ is estimated as a sample mean of realized risk measures based on historical data. However, given scarce data for any fixed level of $\alpha/n$, for varying levels of $\alpha/n$, such sample means are conveniently approximated using a linear regression for the relation of $\rho = \rho(\alpha/n)$ based on realized shortfalls $\hat{\rho}$ over all levels of $\alpha/n$. 

In other words, in order to estimate $\rho(\alpha/n)$ for any given level of $\alpha/n$, using a regression model we ‘borrow’ data associated with other levels of $\alpha/n$. This removes the dependency of the realized shortfall to different values of $n$, related to different days. As mentioned above, Section 4.2 shows how historical data is employed for estimation of the realized risk measure.

Given a set of Pareto optimal points $(\alpha, z)$ in the efficient frontier, for any level $\alpha/n$ the risk measure $\rho(\alpha/n)$ is obtained from the regression line estimating the expected shortfall. However, employing the trade-off coefficient $\lambda$ we may equivalently express by $\rho/\lambda$ the risk measure in terms of additional travel distance (km). Such expression is less abstract and may be preferred by the DM. Such distance measure of risk is demonstrated in Figure 6 of the empirical study in Section 5.

4.1.5. Optimal routing under prediction uncertainty

Consider a day with $n$ ships appearing in the work area. Suppose the Pareto frontier with points $(\alpha, z)$ is at hand at time $t=0$ using the model in Section 3 based on predicted locations, and the risk measure $\rho(\alpha/n)$ has been estimated based on historical observations. According to the mean-risk framework, DM has a risk attitude parameter $\mu$ such that the most preferred choice according to (8) maximizes $e - \mu \rho$, where $e = \alpha - \lambda z$ is the target reward and $\rho$ is the risk measure given as a function of $\alpha/n$ by the linear regression model. As already mentioned, we do not intend to solve the problem (8) for DM; instead we provide decision support for DM’s subjective choice of a Pareto optimal routing plan which is most favourable in terms of the conceptual model in (8).

4.2. Evaluating the realized risk measure $\hat{\rho}$

In this subsection we show how reasonable proxies of realized risk measures $\hat{\rho}$ based on realized ship trajectories are obtained from historical observations of Pareto efficient routing plans which not necessarily have been chosen for implementation. Consider one such day of the history. At time $t = 0$ of the day, a decision was made by selecting an itinerary within the available Pareto frontier based on optimization under predicted data. For the chosen plan, the realized reward is observed at the end of the day. Any Pareto solution defines a sequence of ships with predicted start time of their processing. Given a fixed sequence, the following MILP model adjusts the timing of visits according to the actual locations of ships. If no such revised scheduling exists, then one or more ships in the sequence need to be omitted.

Based on predicted locations, suppose an optimal solution for (1)–(7) with Pareto point $(\alpha, z)$ defines the following sequence of ships $\{1, 2, ..., \alpha\}$ for visits within the time horizon $[0, T]$. For notational convenience, here the ships are renumbered: if ship $i_q$ is the $q$th ship in the sequence, then ship $i_q$ is relabelled by $q$. The set of AIS data for the actual locations of $\alpha$ ships within $[0, T]$ are collected to be used for revised scheduling. The time horizon is discretized the same way as in Section 3. Parameters $d_{ij}^{kl}$ and $\delta_{ij}^{kl}$ again denote travel distance and travel time receptively, but calculated based on actual (realized) locations of the same day we predicted. We let ships $i = 0$ and $i = \alpha + 1$ refer to the depot.

A revised schedule is found by solving an MILP problem of maximizing the value function $\hat{v} = \hat{\alpha} - \lambda \hat{z}$ with the total number $\hat{\alpha} \leq \alpha$ of ships visited and distance $\hat{z}$ travelled. For $j > i$ and $l > k$, the optimization employs binary variables $x_{ij}^{kl}$ defined as in Section 3 and again for simplicity, we suppress feasibility requirements based on time slots of ships appearing in the work area.

The rescheduling problem resembles the MILP problem in (1)–(7). However, there are several differ-
ences. First, only $\alpha$ ships are considered for possible visits; second, if ship $i$ is to be visited before ship $j$ in the sequence, then we must have $i < j$ for preserving order of visits in $\{1, 2, ..., \alpha\}$; third, the number of visits $\hat{\alpha}$ is at most $\alpha$. Therefore, constraint (4) is not needed and $\hat{\alpha}$ is only appearing in the objective function; fourth, each ship appears at most once in $\{1, 2, ..., \alpha\}$ so that condition (5) is no longer needed. Hence, the problem is formulated as follows:

The objective is to maximize the realized value which is $\hat{v} = \hat{\alpha} - \lambda \hat{z}$:

$$\max \hat{v} = \sum_{i,j=0}^{\alpha} \sum_{k,l=0}^{m} x_{ij}^{kl} - \lambda \sum_{i=0}^{\alpha+1} \sum_{j=i+1}^{\alpha} \sum_{k,l=0}^{m} d_{ij}^{kl} x_{ij}^{kl}. \quad (10)$$

Similarly as in (2), the following constraint ensures exactly one exit from depot:

$$\sum_{j=1}^{\alpha} \sum_{k,l=0}^{m} x_{0lj}^{kl} = 1. \quad (11)$$

For $1 \leq j \leq \alpha$, if ship $j$ is visited in time slot $l$, $0 < l < m$, then the immediate predecessor ship $i$ is visited at time slot $k < l$ with $0 \leq i < j$, and the immediate successor ship $i$ is visited at time slot $k > l$ with $j < i \leq \alpha + 1$. As in (3), this is ensured by the following flow conservation constraints:

$$\sum_{i=0}^{j-1} \sum_{k=0}^{l-1} x_{ij}^{kl} = \sum_{i=j+1}^{\alpha+1} \sum_{k=l+1}^{m} x_{ij}^{kl} \quad \forall \ 1 \leq j \leq \alpha, \ 0 < l < m. \quad (12)$$

If ship $i$ is processed at time slot $k$, it is succeeded by ship $j > i$ processed at time slot $l$. Non-negative variables $s_k$ and $s_l$ denote the start time of processing at time slots $k$ and $l$, respectively. then following (6), we need to satisfy the following requirement

$$s_k + (p_i + \delta_{ij}^{kl}) x_{ij}^{kl} \leq s_l \quad \forall \ i, j \in \{0, ..., \alpha+1\}, \ i < j, \ k, l \in \{0, ..., m\}, \ k < l \quad (13)$$

where the processing start times $s_k$ satisfy the previous conditions (7), that is, $s_0 = 0$ and for each $k \in \{1, ..., m\}$ we assume $w(k-1) \leq s_k \leq wk$.

Given an optimal solution for (10)–(13) and (7) (limits to starting times $s_k$), the proxy of the realized risk measure (RM) based on actual ship locations is defined by the shortfall:

$$\hat{\rho} = (e - \hat{v})_+ = [(\alpha - \lambda z) - (\hat{\alpha} - \lambda \hat{z})]_+ \quad (14)$$

where $z$ is the optimal travel distance for the sequence of ships $\{1, 2, ..., \alpha\}$ based on predicted location data.
5. Experimental results: Assessing the prediction uncertainty

We use one week of August 2018 data for empirical demonstration of trajectory prediction in Section 2 and route optimization based on Pareto-efficient solutions in Section 3 combined with estimation of the risk measure for the mean-risk approach in Section 4. At the end of Section 6, we discuss practical applications where risk estimation is based on historical data associated to conditions similar to the day of predictions. For trajectory prediction, August 2018 data were first divided into passages, as explained in Section 2. The optimized hyper-parameters $k$ and $\nu$ and the training passages saved in that particular start node were used to determine the predicted trajectory within the measurement work area for each ship in question. The predicted data was interpolated with $w = 5$ minutes time spacing to facilitate the optimal itinerary calculation. For each day, $n$ denotes the number of ships appearing in the work area, and given the trajectory predictions for each $n$ ships, the route optimization model of Section 3 is used to find the Pareto optimal points $(\alpha, z)$, and given the realized trajectories for ships, an optimal route revision is found using the model of Section 4.2. Thereby we obtain for each day and for each $\alpha$ a realized risk measures $\hat{\rho}$ in (14) which are employed for regression analysis to obtain the risk measure $\rho(\alpha/n)$ for all levels of $\alpha/n$.

All experiments are done using a time horizon of $T = 12$ hours subdivided into $m = 144$ time slots of $w = 5$ minutes. The speed limit $c$ of the control boat is 25 knots (46.3 km/h). The processing time is $p = 3$ min for all ships. For the DM’s trade-off between the number of ships visited and the total travel distance (km), we use $\lambda = 0.05$ (1/km) for demonstration. For August 6–12, 2018, Table 1 shows the Pareto optimal points $(\alpha, z)$ under predicted trajectories, the number $n$ of ships appearing in the work area during the time span $[0, T]$ as well as dimensions of the MILP problem (1)–(7). The number of ships $n$ ranges from 17 to 30, and the maximum number $\alpha$ (ships visited in Pareto optimal itineraries) ranges from 16 for August 6 to 29 for August 9. An illustration in Figure 2 of August 9 with $\alpha = 25$ shows a part of the Pareto optimal itinerary for the control boat. It also shows the trajectory of the first ship visited and when the control boat reaches each ship. Numbers near each point indicate the timeslot when the ship was located in that place.

The problem dimensions in Table 1 show a very large number of binary variables $x_{IJ}^{kl}$. The number of positive variables $s_k$, start times of processing at time slots $k$, is relatively smaller. The number is at most $m = T/w = 144$.

The computing time for optimizing a single Pareto point using MOSEK with default settings in a standard HP-Z230 work station with 32 Gb RAM varies in a wide range. For example, For August 6, for the 13 levels of $\alpha$ in Table 5 the CPU time ranges from 11 seconds to one hour; for other days the range is wider. Evaluating the realized risk measures $\hat{\rho}$ in historical observations involves optimization as proposed in Section 4.2. However, these problems are easy compared to finding Pareto optimal itineraries, because the sequence of ships to be visited is given. Therefore, we omit the discussion of the computational effort for realized risk measures.

5.1. Realized risk measures $\hat{\rho}$

For each of the seven days of August 2018, to obtain the initial Pareto optimal itineraries in a time horizon $T = 12$ hours, the predicted trajectory data for each ship is provided and computations are based on the
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Table 1

Pareto optimal travel distance $z$ (km) for various levels of $\alpha$ in August 6.–12, 2018, using parameters $m = 144$, $w = 5$ minutes, speed limit of the control boat $c = 25$ knots (46.3 km/h) and processing time $p = 3$ min for all ships. Blank entries refer to infeasibility; $n$ is the total number of ships in the work area during the time horizon $T = 12$ hours. Problem dimensions refer to the problem (1)–(7) for computing Pareto optimal itineraries.

The optimization model stated in (1)–(7). Given $\alpha$, the number of ships visited, an optimal travel distance $z$ is obtained. Such Pareto points $(\alpha, z)$ are computed for all feasible $\alpha$; for $\alpha$ large enough, no feasible solution for (1)–(7) exists. The target reward $e(\alpha, z) = \alpha - \lambda z$ for different $\alpha$ values are shown by red points in Figure 3 where there are 136 points $(\alpha, e)$ for the seven days in total.

For any given day and level of $\alpha$, the optimal sequence under predicted trajectories is revised by
For August 6–12, 2018, the target reward $e = \alpha - \lambda z$ with $\alpha$ (the number of ships visited) and $z$ (the total travel distance, km) for the DM’s preference $\lambda = 0.05/\text{km}$ based on predicted locations is shown in red. For each $\alpha$, the revised routing based on realized locations results in $\hat{\alpha} \leq \alpha$ ships visited and total travel distance $\hat{z}$ for which realized reward $\hat{v} = \hat{\alpha} - \lambda \hat{z}$ is shown in green.

Solving the optimization model in (7)–(13) resulting in $\hat{\alpha} \leq \alpha$ ships visited and an optimal travel distance $\hat{z}$. The resulted realized reward $\hat{v} = \hat{\alpha} - \lambda \hat{z}$ are shown in Figure 3 by green points. Note that $\hat{\alpha}$ and $\hat{z}$ are only visible in the figure through their joint contribution to the realized reward. For any given day and level of $\alpha$, we have $e \geq \hat{v}$; i.e., the red point is never below the green point in Figure 3. Hence, in all of these cases the realized shortfall, the risk measure $\hat{\rho}$ in (14), is the difference $(e - \hat{v})_+ = e - \hat{v}$.

Figure 4 shows the realized risk measure (RM) values $\hat{\rho}$ for different ratios $\alpha/n$ for the seven days of August. The values $\hat{\rho}$ are calculated based on (14) as described in Section 4.2. Values are consistent with the results illustrated in Figure 3. The realized risk measures $\hat{\rho}$ is the difference of the two rewards (red and green points) for any given day with the ratio $\alpha/n$.

To gain further intuition of what RM values $\hat{\rho}$ mean, we refer to Figure 5. The points show the relation of RM values $\hat{\rho}$ to different levels of extra distance travelled $\Delta z = \hat{z} - z$ at this value. For instance, for RM=3, there are four alternatives for extra travel distance: first, on the top $\Delta z = 60$ km, and fourth, on the bottom $\Delta z$ is zero. The difference is explained by the difference $\Delta \alpha = \alpha - \hat{\alpha}$: in the first case, $\Delta \alpha = 0$ and $\Delta z = 60$, and in the fourth case, $\Delta \alpha = 3$ and $\Delta z = 0$, both resulting in RM=3. Given the trade-off $\lambda = 1/20$, the DM is in fact indifferent among the four cases. In Figure 5 there are several layers of points; the layer on the top includes cases where $\Delta \alpha = 0$, on the next layer below, $\Delta \alpha = 1$, etc. As explained at the end of Section 4.1 employing the trade-off coefficient $\lambda$ we may equivalently express RM in terms of additional travel distance (km). Rescaling the realized risk measure $\hat{\rho}$ to $\hat{\rho}/\lambda$ yields an equivalent risk measure expressed in extra travel distance (km). In Figure 5 this is depicted by the top layer of points. Such expression can be more comprehensible to the DM compared with the expected shortfall in value.
5.2. Estimating the risk measure $\rho$ and expected reward

As can be seen in Figure 4, there is an obvious linear increase in the RM value when the ratio $\alpha/n$ is increased. Even taking into account the data by day (data points by each color in Figure 4), the plot does not support exponential increase. Therefore, to estimate the risk measure $\rho = \rho(\alpha/n)$, the expected shortfall, we fit a linear regression model using data points of Figure 4 on realized risk measures. The estimated regression line is shown in Figure 6. There are totally 136 data points illustrate the realized
Fig. 6. An estimated regression line $\rho = \rho(\alpha/n) = 3.72(\alpha/n) - 0.245$ defines the risk measure $\rho(\alpha/n)$ (left axis) as an approximation of expected shortfall; scaling by $1/\lambda$ yields an equivalent risk measure in km (right axis).

shortfalls (mapped to the left axis) and the value $\rho/\lambda$ (mapped to the right axis) which counts for extra distance including a loss of 20 per ship in kilometres (i.e. $20\Delta\alpha - \Delta z$). The data points yield a linear regression line $\rho = 3.72(\alpha/n) - 0.245$; see the solid red line in Figure 6. The root mean squared error is 1.04. The p-value for the coefficient of the ratio $\alpha/n$ is negligible ($< 10^{-8}$) which is against the null hypothesis of slope equal to zero.

Another interesting information that can be provided to the DM is the estimator of the realized reward for each Pareto point $(\alpha, z)$. Similarly as in Figure 6, the estimated regression line for the conditional expectation realized rewards $\hat{v}$, given $\alpha/n$, is shown in Figure 7.

6. Conclusions

In this paper, we address the problem of route prediction and optimization for an autonomous boat measuring greenhouse gas emissions of ships in Gulf of Finland. For each day, the two objectives are to maximize the number of ships treated and minimize the travel distance of the control boat. We introduced a model for predicting the future movements of ships navigating in the Baltic sea. The trajectory prediction is made using the observed trajectory of a new ship prior to the planned time horizon, earlier collected historical trajectory data, and the k-NN method. Based on the predicted routes, mixed integer linear programming is used to provide a set of Pareto efficient solutions. At the beginning of the time horizon before starting measurement operations, such Pareto solutions form the set of possible candidates for itinerary plans to be followed by the emission control boat. However, the travel times are uncertain due to imprecise trajectory predictions for ships. To assess the impact of uncertainty on the quality of each efficient itinerary choice, we propose a risk measure based on historical benchmark data.
providing realizations of risk. The risk measure (as an expectation of realized measures) is approximated by linear regression in terms of the share of ships treated; the slope of the regression line is positive with a p-value very close to zero.

Although it is not surprising that the risk level increases as we plan to include more tasks to be completed within a given time horizon, but the rate of increase in risk measure is shown to be linear even for many hours (12 hours) ahead of starting the operation. For shorter time horizon of 6 to 8 hours, which is often the case in practice, the risk level is smaller compared to our demonstrative results of the risk assessment (as it is illustrated in Section 5), since the prediction accuracy is higher for shorter horizon. However for a long time horizon, our proposed approach and experimental results present interesting theoretical viewpoint for risk assessment analysis of the case study problem.

A possible subject of future research is whether similar linear characteristic of the risk measure holds true for marine traffic in other sea areas as well. Furthermore, additional factors may be found to explain the level of risk measure, other than the share of ships inspected. The proposed method has no underlying probabilistic basis; however, future research can be based on probabilistic risk assessment techniques, such as Bayesian networks. Finally, observing that target locations in early hours of a day are known with a high precision, unlike for later hours, two-stage stochastic programming seems a worthwhile direction of future research.

Finally, we discuss some concerns for practical implementation of the risk measure for assessing the prediction uncertainty in a route optimization model. Risk estimation needs to be based on historical data related to similar conditions of wind, current, ice, etc. prevailing during the day of prediction. To minimize the computational effort at the beginning of the day, it is convenient to set up a data base, where realized risk measures $\hat{\rho}$ are stored for varying levels of $\alpha$, the number of ships visited and $n$, the number of ships in the work area. At the end of each day, Pareto optimal trajectories based on predicted locations as well as the realized trajectories of the $n$ ships are known so that the realized risk measures $\hat{\rho}$ can be easily computed and stored in the data base together with the prevailing weather and other conditions.
relevant conditions. Thereby, our quantitative risk measure leads in real itinerary planning situations to a practical implementation of the three stage procedure: i) trajectory prediction (Section 2), ii) route optimization (Section 3), and iii) estimation of the risk measure (Section 4). The risk measure helps DM to evaluate efficient choice alternatives among itineraries for a new day of operation and make a balanced risk-adjusted decision.

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