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Effect of Delay Equalization on Loudspeaker Responses

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ABSTRACT

The impulse response of a generalized two-way loudspeaker is modeled and is delay equalized using digital filters. The dominant features of a loudspeaker are low and high corner roll-off characteristics and the behavior at the crossover points. The proposed model characterizes also the main effects of the mass-compliance resonant system. The impulse response, its logarithm and spectrogram, and the magnitude and group delay responses are visualized and compared with those measured from a two-way loudspeaker. The model explains the typical group-delay variations and magnitude-response deviations from a flat response in the passband. The group-delay equalization of the loudspeaker is demonstrated in two different methods. The first method, the time-alignment of the tweeter and woofer elements using a bulk delay, is shown to cause ripple in the magnitude response. The second method, which flattens the group delay of the speaker model in the whole audio range, leads to preringing in the impulse response.

1 Introduction

This paper focuses on the modeling of the linear properties of loudspeakers. The impulse response is measured on the acoustical axis of a loudspeaker (on-axis) as well as in other orientations to understand directional characteristics and to estimate the power response. The impulse response of a loudspeaker, measured in anechoic conditions, contains all the essential information about the time-domain and frequency-domain response characteristics of that loudspeaker but not the effects related to nonlinear characteristics.

Passive and active loudspeakers are systems containing electrical, mechanical, and acoustical subsystems, combining to create the time-frequency characteristics audible in the acoustic output and visible in the impulse response. The fundamental characteristic of a loudspeaker is that it is a bandpass system. The roll-off rate of the low- and high-frequency cut-offs define the global characteristics of such a system. The second main feature in a loudspeaker is its multi-way nature. Loudspeakers are typically two- or three-way systems.

Most loudspeakers are dominantly minimum-phase systems. A minimum-phase system releases its energy with a minimal delay, which leads to an impulse response having large signal values in the beginning and decaying with time. This is a typical behavior in natural causal and stable systems that do not store energy. The phase response of a minimum-phase system is linked to its magnitude response and is the Hilbert transform of the (natural) logarithmic magnitude response [1]. However, it is possible to modify the phase response of a system without affecting its magnitude response.

This paper sheds light on the fundamental characteristics of a two-way loudspeaker system and attempts to increase the understanding of the potential of time-domain system equalization in loudspeakers. One motivation for the present paper is to study how completely the fundamental observable characteristics of a loudspeaker system describe the measurable impulse response. Secondly, this paper demonstrates the effects of time-domain equalization, such as delaying the transducers relative to each other or using allpass delay equalizers [2]. We show how delay equalization affects the length of the impulse response of a loudspeaker.

The use of an appropriately chosen delay has been proposed for synchronizing the elements of a multi-channel speaker system [3]. A more refined method of time-domain equalization is to use frequency-dependent time-domain equalizers either for the whole system or for the individual driver channels. Such time-domain equalizers are relatively easy to implement as digital allpass filters [4, 5] without affecting the magnitude response characteristics of the loudspeaker system, or as additional constraints to the design of a time-frequency equalizer. Greenfield and Hawksford [6] proposed to flatten the loudspeaker magnitude response using a minimum-phase IIR filter and linearize the excess phase using an FIR filter formed by sampling the time-reversed target impulse response. Other authors [7, 8] have proposed using a time-reversed allpass filter to flatten the group delay of a loudspeaker.

This paper is organized as follows. Section 2 introduces the minimum-phase loudspeaker model. Section 3 studies how linear-phase crossover filters affect the impulse response. In Section 4, a measured response of a two-way loudspeaker is compared with the model response. Section 5 shows how delaying one driver channel or applying a delay equalizer to the loudspeaker modify the impulse response. Section 6 concludes this paper.

2 Linear Loudspeaker Model

The loudspeaker is a bandpass device, typically with close to minimum-phase characteristics. In the following, the loudspeaker is modeled as a cascade of minimum-phase filters. A discrete-time system with a sample rate of 100 kHz is created. Even if the modeling is conducted in discrete time, the same principles apply to continuous-time systems, with the main difference being the condensation of the high-frequency part of the spectrum close to the Nyquist limit related to the bilinear transform linking continuous-time to discrete-time representations.

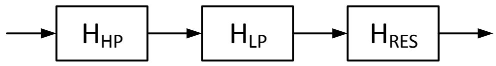

Fig. 1: One-way loudspeaker model.

Two system models are developed next: a one-way loudspeaker having no crossover filters but with the typical high- and low-frequency roll-off bandwidth limitation (see Fig. 1) and a two-way loudspeaker including the effects of a crossover network and a tweeter-woofer system model. Both models have the same low- and high-frequency limitations.

The low-corner-frequency model contains two highpass filters, the first modeling the low-corner-frequency roll-off towards low frequencies of the mechanic-acoustic system (H_{HP}) and the second modeling the possible bass reflex or passive resonator resonance effects in the magnitude response (H_{RES}). A bass reflex port creates a Helmholtz resonator with the cabinet volume. A passive resonator also creates a mass-compliance resonance system that can be tuned with the mechanical characteristics of the driver and the compliance of the enclosure volume. These are modeled using a second-order highpass filter H_{RES}. All filters are modeled as minimum-phase maximally flat (Butterworth) filters. The one-way speaker model contains the three filters in cascade (Fig. 1). In the two-way system, these filters are located in the woofer channel.

The tweeter channel contains a fourth-order lowpass filter H_{LP} modeling the acoustic-electronic roll-off at the high corner frequency. The steep high-frequency roll-off is frequently seen when the natural tweeter roll-off is combined with an electronic bandwidth limitation and describes well the typical characteristics seen at the tweeter roll-off. The exact frequency of this roll-off varies depending on the overall system design. The roll-off typically takes place below 30 kHz due to tweeter driver characteristics.

In the loudspeaker model, the driver outputs are first generated independently and then summed together to obtain the total acoustic response. The summation is equivalent to measuring the on-axis system response in anechoic conditions. Ideally, this creates a flat response within the loudspeaker passband (Fig. 2). The −6-dB points of the modeled loudspeaker are set to 43 Hz and 22 kHz. These values describe a typical system behavior.

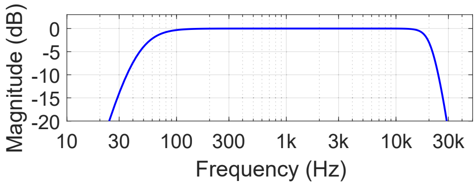

Fig. 2: Magnitude response of the one-way loudspeaker model.

Differences in the loudspeaker responses occur in the impulse and phase responses. We compute the group delay, i.e., the negative derivative of the phase function [1] describing the delay the system causes for an input signal. The group delay of a loudspeaker contains the contributions of the system's low and high corner frequencies [2], the crossover filter responses [2], and the mass-compliance resonant system tuning at the woofer frequencies.

Figures 3 and 4 display the impulse response and group delay, respectively, of a one-way system, defined only by its low and high corner frequencies and the shape of their roll-off. The visible contribution in Fig. 3(a) is created by the highpass corner frequency. The lowpass corner frequency and the mass-compliance resonator contribute little to the impulse response, but they ring over a long period of time (Fig. 3(b)). Inspecting the impulse response on the logarithmic scale (Fig. 3(c)) shows the details of the level variations in the impulse response. The group-delay curve (Fig. 4) has the typical increase in delay towards low frequencies, with a peak around 30 Hz. The mass-compliance resonant system also contributes to the maximum value of the delay.

2.1 Spectrogram on a Log Frequency Scale

The spectrogram of the impulse response offers further understanding of the time-frequency distribution of the energy in a loudspeaker response. Figure 5 presents the spectrogram of the impulse response. This spectrogram has been computed by splitting the impulse response into 5-ms segments (500 samples at the 100-kHz sample rate), which are time-windowed using a 500-sample Blackman window. The hop size in the time domain is 1 sample. The resulting segment data is evaluated using the discrete-time Fourier transform (DTFT) at 256 frequencies spaced logarithmically between 10 Hz and 50 kHz. The DTFT magnitude is normalized to 0 dB, which corresponds to the black color. Smaller levels are depicted in shades of gray on the log scale.

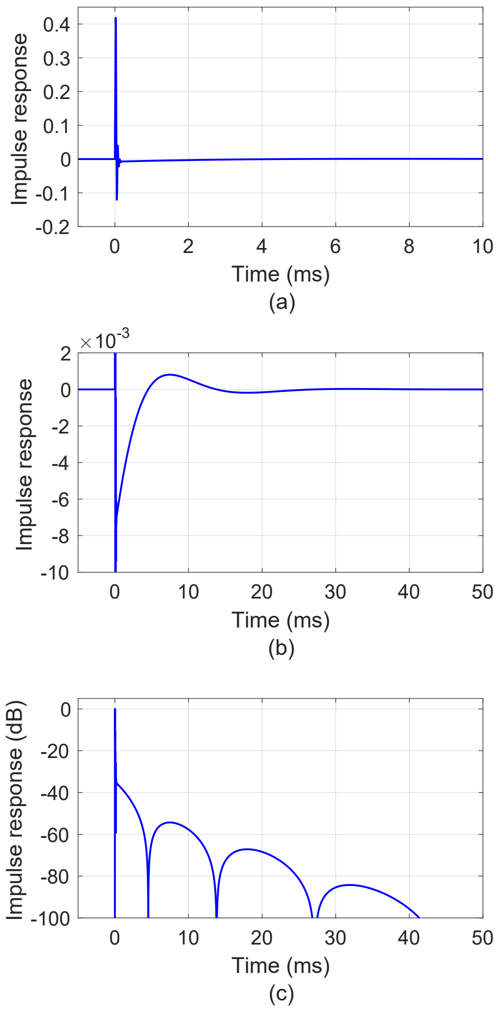

Fig. 3: (a) Impulse response of the one-way loudspeaker model, (b) its expanded low-frequency tail, and (c) the same on the logarithmic scale. Note the different time scales.

This process is similar to the spectrogram computation used in [9].

The spectrogram (Fig. 5) shows that the highest impulse response energy at frequencies above about 100 Hz appears around zero time. The plot is blurred because of time windowing. Increasing the system delay at low frequencies shows as a smearing of the signal energy over a wide range on the time scale. This is also suggested by Fig. 3(c).

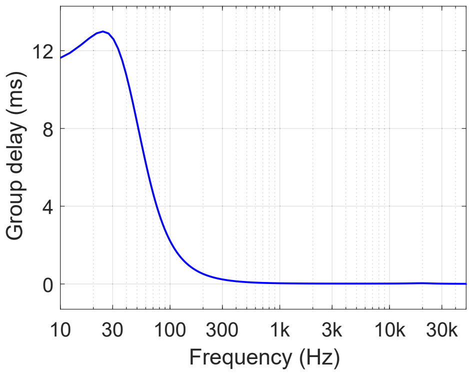

Fig. 4: Group delay of the one-way system.

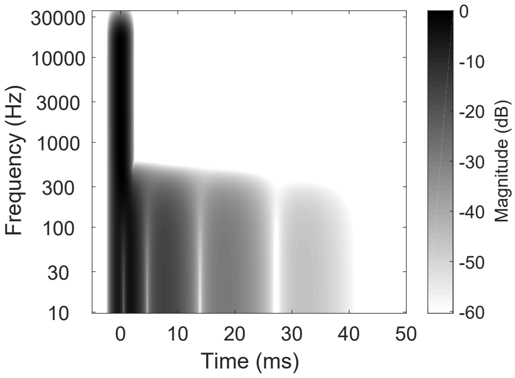

Fig. 5: Spectrogram of the impulse response with a logarithmic frequency axis.

2.2 Modeling Crossover Filters

Next, we add the contribution of the crossover filters. The model structure with one crossover network is presented in Fig. 6. The crossover filters are modeled as the most typical Linkwitz-Riley filters [10, 11]. These are minimum-phase filters with a 6-dB attenuation at the crossover point and an equal group delay for the highpass and lowpass output. Each Linkwitz-Riley filter is created as a cascade of two identical fourth-order Butterworth filters, so that the order of the crossover filters is eight. Typical values for the crossover frequency of a two-way system are between 1 and 3 kHz. Here, the crossover frequency has been chosen as 1.0 kHz. The magnitude responses of the crossover filters together with the highpass and lowpass filters corresponding to the mechanical properties are shown in Fig. 7.

The crossover filters form an allpass system but affect the phase response, and this is visible in the impulse response (see Fig. 8) and group delay: An increase is seen in the group delay in Fig. 9 around the crossover frequency 1 kHz. The lowpass branch delay is larger than the highpass branch. When the crossover frequency increases, the delay variation at the crossover and the delay difference between the lowpass and highpass branches decrease. The tendency for the delay to increase towards low frequencies below 300 Hz remains similar as this is mainly set by the system characteristics at the low corner frequency.

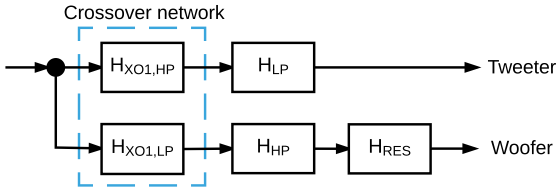

Fig. 6: Two-way loudspeaker model.

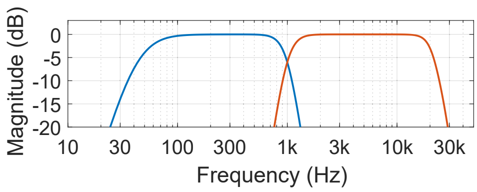

Fig. 7: Magnitude responses of a woofer (blue) and tweeter (red line) using eighth-order crossover filters.

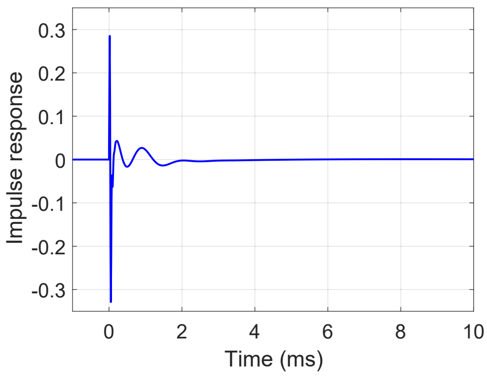

Fig. 8: Impulse response of two-way speaker model.

3 Linear-Phase Crossover Model

In the digital domain, the crossover filter can also be implemented as an FIR filter, which can have a linear phase response [12]. The lowpass and highpass outputs

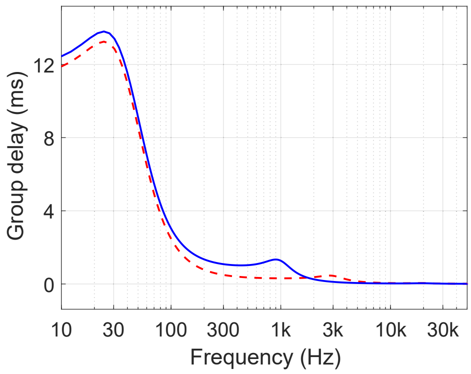

Fig. 9: Group delay of the two-way speaker model, showing a local increase in delay near the crossover frequency at 1 kHz (blue solid line) and at 3 kHz (red dashed line). Cf. Fig. 4.

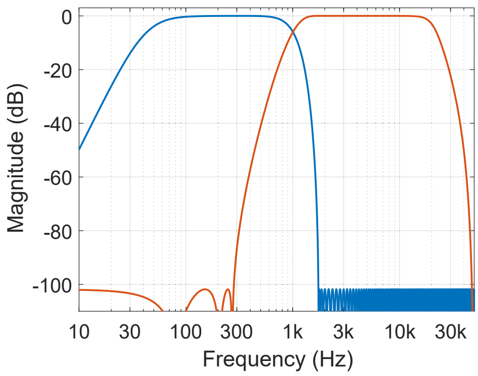

Fig. 10: Magnitude responses of signal paths in a two-way speaker using equiripple FIR crossover filters with a 100-dB stopband rejection.

can be implemented as complementary filters. Such a filter can have also an exactly constant delay and therefore no contribution to the relative input-to-output timing.

The precision of the phase match of the crossover-driver system is in reality affected not only by the crossover filter but also any other component, such as the drivers, having a non-flat magnitude response or filtering effect. For simplicity, in this work we assume the drivers to have a flat magnitude response on the passband of the associated crossover filter.

In the following, a two-way system is modeled with a linear-phase FIR filter. The magnitude response is practically the same for this and the previous examples. The crossover frequency is the same (1 kHz), defined

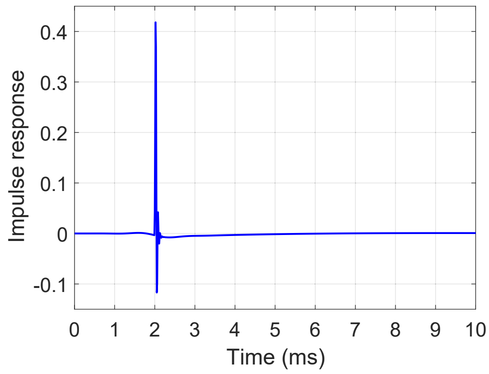

Fig. 11: Impulse response of the loudspeaker model with linear-phase FIR crossover filters showing the preringing.

with a transition ending at the 100-dB stopband attenuation at 1.7 kHz in the lowpass kernel filter. The FIR filters were designed using the Parks-McClellan optimization algorithm, which leads to an equiripple design [1]. Their magnitude responses are shown in Fig. 10. The FIR crossover creates a constant delay in the passband mid frequencies not affected by the low or high corner frequency roll-off. The length of this delay is half of the FIR filter order, because the kernel filter has a linear phase.

The linear-phase FIR crossover model shows the preringing typical of FIR filters. By preringing we mean the non-zero level in the impulse response appearing before the largest magnitude value in the impulse response. Since the filter order N is relatively small ($N = 400$ in this case), the preringing is short and, due to the shape of the filter's magnitude response, decays rapidly when moving to times earlier than the main peak in the impulse response. The contribution of the linear-phase crossover filters is visible in the impulse response of the loudspeaker model, which is delayed and also has some preringing on the left side of the main peak (see Fig. 11). A causal linear-phase FIR filter introduces a constant input-to-output delay. The added group delay of 2.0 ms corresponds to 200 samples ($N/2$) at the 100-kHz sample rate. This is also visible as a time-offset of the main peak of the impulse response in Fig. 11.

The magnitude of preringing becomes more visible when the same impulse response is displayed on a logarithmic scale (Fig. 12). Figure 13 shows the corresponding group-delay curve, which has an approxi-

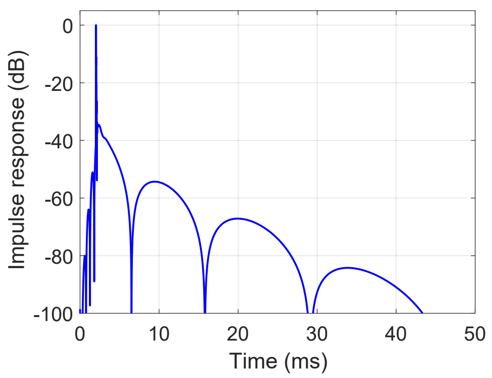

Fig. 12: Impulse response of Fig. 11 on a logarithmic scale. The peak has been normalized to 0 dB.

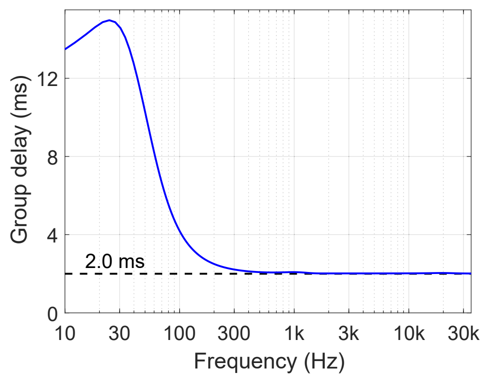

Fig. 13: Group delay of the loudspeaker model with linear-phase crossover filters. The dashed line indicates the group delay caused by the linear-phase crossover filters.

mately constant value of 2.0 ms at frequencies above approximately 300 Hz.

4 Comparison with Loudspeaker Measurements

The validity of the two-way loudspeaker model was tested by comparing it with measurements from a real two-way loudspeaker. The measured loudspeaker is a small-size loudspeaker with a bass reflex port and crossover frequency of 3.2 kHz, according to the manufacturer's specifications. Its impulse response is shown in Fig. 14(a). Similarly to the digital model, the measured impulse response comprises a tall impulse and a long tail caused by the crossover filters. The measured impulse response is also shown on a logarithmic scale in Fig. 14(b).

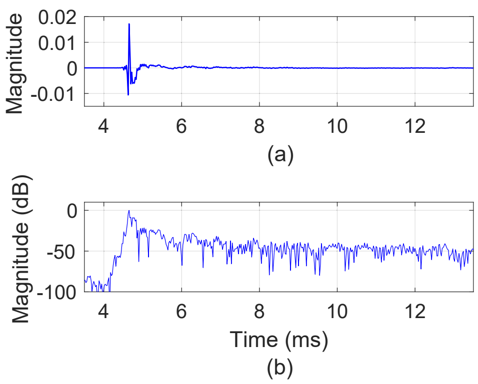

Fig. 14: Impulse response of the measured two-way loudspeaker on (a) a linear magnitude scale and (b) a logarithmic scale.

Next, the parameters of the model were adjusted to match the magnitude response and group delay in the model to the measured. The low-corner-frequency of the model woofer element is adjusted to a value that produces the same −6-dB point of approximately 52 Hz. In addition, the crossover-filter order is set to four, and the crossover frequency to 2.9 kHz to conform to the measured data. A slight alteration is needed in the manufacturer-specified crossover frequency.

The magnitude response of the model and the measured loudspeaker show good agreement (Fig. 15). The measured magnitude response is not perfectly flat. This is typically a result of the characteristics of the drivers. There are small differences at low frequencies between the measured system and the model. A better match of the magnitude responses could be obtained if the low-corner-frequency of the model is increased. However, this would lead to an increasing mismatch in the group-delay responses. Alternatively, the order of the low-frequency highpass model could be increased.

The group delay estimated from the measured impulse response is compared with that of the model in Fig. 16. The group delays of the model and the real loudspeaker are in good agreement everywhere except at low frequencies. This is due to the aforementioned compromise between the ideal parameters for the magnitude-response match and group-delay match. The small increase in the group delay caused by the crossover filters, however, is well modeled. The propagation time in the measured impulse response is accounted for in order to achieve the correct overall level (vertical offset) for the measured group delay.

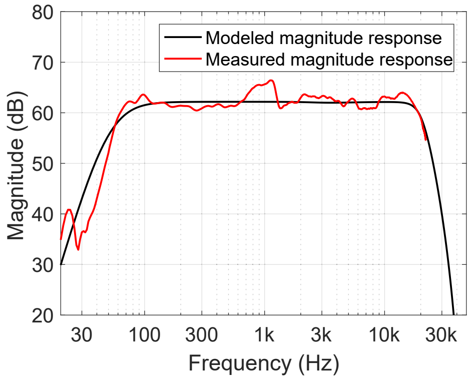

Fig. 15: Magnitude responses of the two-way loudspeaker model and the measured loudspeaker.

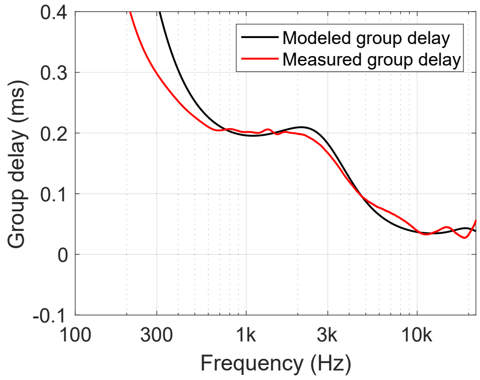

Fig. 16: Group delay of the modeled and measured loudspeakers. Note that the frequency limits are different than in other figures.

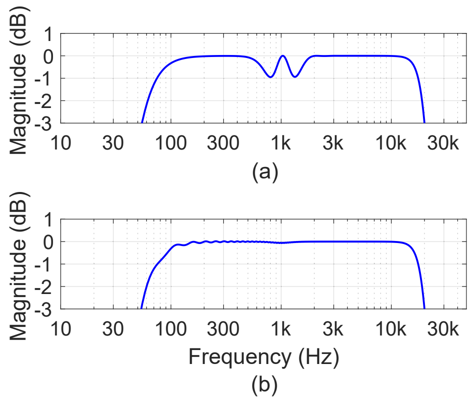

Fig. 17: Magnitude response of (a) the bulk-delay aligned model and (b) the delay-equalized model of the two-way speaker.

5 Delay Equalization

In this section, we study two delay equalization methods that can be applied to a two-way speaker. The main motivation for creating a loudspeaker response with constant input-to-acoustic output delay is to maintain the electronic signal waveforms accurately in the acoustic output pressure variation.

5.1 Time-Aligning using Bulk Delay

The easiest approach to delay equalization is to delay the different outputs of a multi-way loudspeaker crossover such that the delay through all the outputs becomes equal. This leads to a system response in which the input-to-output latency is constant in most parts of the passband, excluding the frequency regions around the crossover points that show increased latency in the case where the crossover filter has minimum-phase characteristics. When this bulk-delay alignment method is used, achieving the same phase in the complementary highpass and lowpass output branches may be difficult, resulting in a non-flat system response across the crossover frequency region.

Figures 17(a) and 18 show the effect of aligning the system branches using bulk delay in the magnitude response and group delay, respectively, of the two-way loudspeaker model. The crossover frequency is set to 1 kHz. A bulk delay of 0.93 ms is added to the tweeter channel to time-align its latency across most of the audible frequency range. As is seen in Fig. 17(a), the bulk delay causes a ripple in the magnitude spectrum around the crossover frequency. In addition, as is seen in Fig. 18, the group delay is not precisely constant: At low frequencies, the bass reflex port still causes a large increase in latency, and there is still an increase in the group delay around the crossover frequency.

The impulse response of the bulk-delay modified loudspeaker model on a logarithmic scale is shown in Fig. 19, which resembles the impulse response of the original system. The obvious difference in the responses is the increase in the latency due to the bulk delay. However, when the impulse responses are compared in dB, one can see some differences in the impulse response lengths. Here, we have defined the length using a level of -60 dB in comparison to the maximum value of the impulse response. The length decreases by 41 samples or 0.41 ms, which corresponds to only about 3% of the original impulse response length.

As the phase responses of the highpass and lowpass

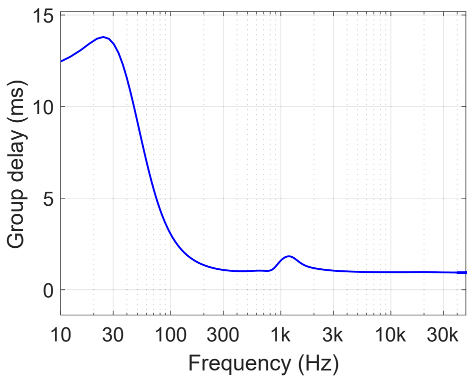

Fig. 18: Group delay of the bulk-delay aligned model of the two-way speaker. Cf. Fig. 16.

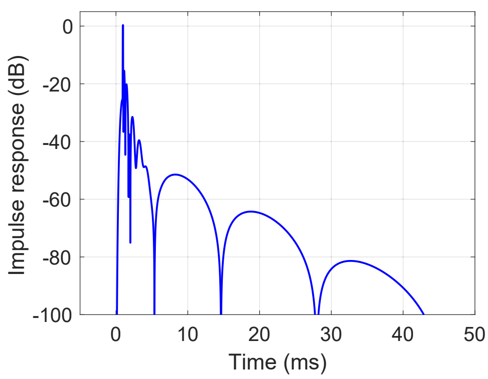

Fig. 19: Impulse response on a logarithmic scale of the bulk-delay aligned model of the two-way speaker.

branches cannot be matched across the crossover region using this method, a separate filter would be needed to flatten the magnitude-response ripple in the on-axis response. Even if this is done, there will be frequency-dependent variations in the on-axis magnitude response across the crossover region due to the problems in matching the phases of the low- and highpass branches. In order to alleviate both of these problems, a more precise method to compensate the delay in the different system output branches, for example, using optimized allpass filters or complementary FIR filters, is needed.

5.2 Time-Aligning using a Delay Equalizer

Other approaches for creating constant-latency system responses with better control at the crossover regions are available, such as the symmetrical FIR filter design within the main audible range, or one or more allpass filters [5] used as time-domain equalizers for traditional minimum-phase crossover designs. Our FIR crossover filter is an example of the first design approach.

Whereas design methods for linear-phase FIR filters are well known, the system design usually requires accounting for the frequency-dependent delay effects in the acoustical responses of the transducers, the electronic signal processing, and the amplifier. In order to do this, optimization methods can be used to bring data on these effects into the delay-equalizer design.

Typically, time-domain equalizer designs exclude very low frequencies, because delay effects are known to be less audible at low frequencies. Latency effects are created by modal resonances in listening rooms as well, and additionally, delaying the system output much to enable equalization of latency down to low frequencies is impractical. However, when using the two-way loudspeaker model presented in this paper as a starting point, the effects of the group-delay equalization of the whole frequency band can be easily demonstrated.

The group-delay equalization is achieved with an FIR filter having inverted group-delay characteristics with respect to the two-way loudspeaker model with the crossover frequency of 1 kHz (cf. Fig. 9). The total group delay becomes approximately constant, corresponding to the maximum value of the original group delay plus the delay of the equalizer. The equalizer is obtained with frequency sampling [13], so first the magnitude and the phase of the frequency response must be determined.

The magnitude response of the FIR filter is approximately allpass in the passband of the loudspeaker, but above 30 kHz the response is defined by a raised cosine function similarly as in [14]. The phase is obtained from the target group delay as its antiderivative. When the magnitude and phase responses are specified, they must be mirrored to the negative frequencies except at the zero and the Nyquist frequency (the phase is the negative of the phase at positive frequencies). Finally, the FIR filter coefficients are obtained with the inverse discrete Fourier transform. The length of the filter is 4096 samples, which corresponds to approximately 41 ms. The FIR filter is placed before the crossover filter, and thus it processes the input signal of the loudspeaker.

Figure 17(b) shows that the magnitude response remains almost unchanged. An approximately constant

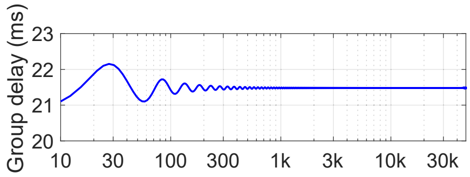

Fig. 20: Group delay of the delay-equalized two-way speaker model.

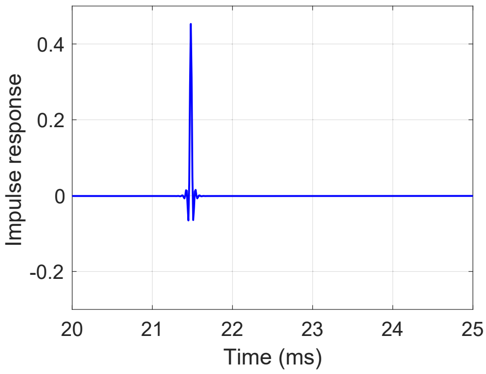

Fig. 21: Impulse response of the delay-equalized model of the two-way speaker.

system group delay is achieved (Fig. 20). However, this leads to dramatic changes in the system impulse response, as can be seen in Fig. 21. (The original impulse response is shown in Fig. 8).

The equalized system impulse response of Fig. 21 resembles an ideal delayed unit impulse. The response has essentially a linear phase. The symmetry is also evident on the logarithmic scale in Fig. 22. With the ideal loudspeaker model used here, such group-delay equalization works extremely well. However, with real loudspeakers, the result would be worse, as the resulting system would have an excessive input-to-output delay and considerable preringing in the impulse response. In reality, the constant delay characteristics would have to be limited in frequency in order to keep the increase in the latency at a reasonable level and to reduce the extent of preringing in the time domain.

5.3 Considering the Audibility of Impulse Response Characteristics

A constant-latency design can reduce the latency deviation down to any accuracy in theory, but the relevant goal for such an equalizer is to limit the delay variation to at least below the limit of audibility. The impulse

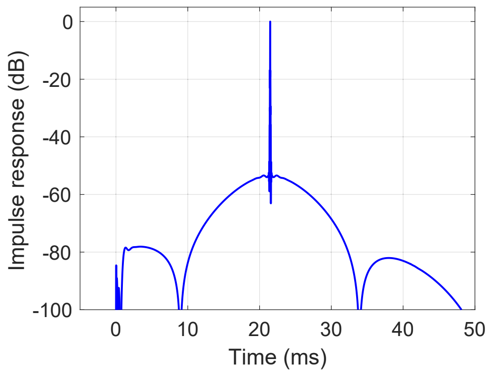

Fig. 22: Log-scale impulse response of the delay-equalized two-way speaker model.

response of a constant-latency system tends to have energy before the largest peak in the impulse response. Increasing the filter order of such an equalizer, for example, with the aim to reduce the delay variation or to increase the crossover filter roll-off rate, tends to increase the time-domain extent of the impulse response. This preringing in the impulse response has resulted in discussions about the possibility of a "pre-echo". This is related to the speculation that parts of the system impulse response occurring early enough before the largest peak in the impulse response might become audible as separate auditory events, constituting an unwanted change in the character of the loudspeaker response where more than one auditory events are recognized.

The human auditory system presents a certain amount of masking before and after an auditory event. This is called temporal masking. The level of temporal masking before the auditory event (premasking) is particularly relevant for the potential to recognize a pre-echo. Unfortunately, while it has been shown to exist, the agreement on the level of premasking as a function of time is not unanimous [15, 16].

6 Conclusion

This paper has demonstrated how the salient features of the two-way loudspeaker impulse response can be described by cascading a highpass filter modeling the system characteristics at the low corner frequency and includes the effect of any mass-compliance resonant systems used to enhance the output at low frequencies, a lowpass filter modeling the cutoff of the tweeter driver and the amplifier bandwidth, and a crossover filter. It was demonstrated that the effects of both the group delay of the conventional Linkwitz-Riley crossover filters and the linear-phase FIR crossover filters can be shown with the modeling method.

The effect of delay equalization on the system impulse response was demonstrated using two methods. The first method uses bulk delay to align the crossover output branches in time, and the second uses a general FIR group-delay equalizer to flatten the entire group-delay response. The preringing caused in the system response in the latter case was demonstrated.

The proposed digital modeling principle can be extended to more complex multi-way systems, such as three-way loudspeakers. A companion paper reports on listening tests to verify the audibility of the group-delay variations in a loudspeaker response [17].

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References

[1] Oppenheim, A. V. and Schafer, R. W., *Discrete-Time Signal Processing*, Pearson Education, 3rd edition, 2010.

[2] Thiele, N., "Phase considerations in loudspeaker systems," in *Proc. AES 110th Conv.*, Amsterdam, The Netherlands, May 2001.

[3] Hosoi, S., Hamada, H., and Kameyama, N., "An improvement in sound quality of LFE by flattening group delay," in *Proc. AES 116th Conv.*, Berlin, Germany, May 2004.

[4] Quélhas, M. F., Petraglia, A., and Petraglia, M. R., "Efficient group delay equalization of discrete-time IIR filters," in *Proc. European Signal Process. Conf.*, pp. 125–128, Vienna, Austria, Sept. 2004.

[5] Bharitkar, S., Kyriakakis, C., and Holman, T., "Time-alignment of multi-way speakers with group delay equalization—I," in *Proc. AES 124th Conv.*, Amsterdam, The Netherlands, May 2008.

[6] Greenfield, R. and Hawksford, M., "Efficient filter design for loudspeaker equalization," *J. Audio Eng. Soc.*, 39(10), pp. 739–751, Oct. 1991.

[7] Adam, V. and Benz, S., "Correction of crossover phase distortion using reversed time all-pass IIR filter," in *Proc. AES 122nd Conv.*, Vienna, Austria, May 2007.

[8] Herzog, S. and Hilsamer, M., "Low frequency group delay equalization of vented boxes using digital correction filters," in *Proc. Int. Conf. Digital Audio Effects (DAFX)*, pp. 57–64, Erlangen, Germany, Sept. 2014.

[9] Välimäki, V., Abel, J. S., and Smith, J. O., "Spectral delay filters," *J. Audio Eng. Soc.*, 57(7/8), pp. 521–531, July/Aug. 2009.

[10] Linkwitz, S. H., "Active crossover networks for noncoincident drivers," *J. Audio Eng. Soc.*, 24(1/2), pp. 2–8, Jan. 1976.

[11] Mäkivirta, A. V., "Loudspeaker design and performance evaluation," in D. Havelock, S. Kuwano, and M. Vorländer, editors, *Handbook of Signal Processing in Acoustics*, pp. 649–667, Springer, New York, NY, USA, 2008.

[12] Wilson, R., Adams, G., and Scott, J., "Application of digital filters to loudspeaker crossover networks," *J. Audio Eng. Soc.*, 37(6), pp. 455–464, June 1989.

[13] Parks, T. W. and Burrus, C. S., *Digital Filter Design*, John Wiley and Sons, Inc., New York, USA, 1987.

[14] Lehtonen, H.-M., "Analysis of piano tones using an inharmonic inverse comb filter," in *Proc. Int. Conf. Digital Audio Effects (DAFX)*, pp. 333–340, Espoo, Finland, Sept. 2008.

[15] Dolan, T. G. and Small, A. M., "Frequency effects in backward masking," *J. Acoust. Soc. Am.*, 75(3), pp. 932–936, Mar. 1984.

[16] Fastl, H. and Zwicker, E., *Psychoacoustics: Facts and Models*, Springer, 3rd edition, 2006.

[17] Liski, J., Mäkivirta, A. V., and Välimäki, V., "Audibility of loudspeaker group-delay characteristics," in *Proc. AES 144th Conv.*, Milan, Italy, May 2018.