Rahman, F M Mahafugur; Pirsto, Ville; Kukkola, Jarno; Routimo, Mikko; Hinkkanen, Marko

State-space control for LCL filters: Converter versus grid current measurement

Published in:
IEEE Transactions on Industry Applications

DOI:
10.1109/TIA.2020.3016915

Published: 01/11/2020

Document Version
Peer-reviewed accepted author manuscript, also known as Final accepted manuscript or Post-print

Please cite the original version:
State-Space Control for LCL Filters: Converter Versus Grid Current Measurement

F. M. Mahafugur Rahman, Student Member, IEEE, Ville Pirsto, Jarno Kukkola, Mikko Routimo, Member, IEEE, and Marko Hinkkanen, Senior Member, IEEE

Abstract—This paper deals with discrete-time state-space current control of three-phase converters equipped with an LCL filter. Either the converter or grid current is measured and the unknown states are estimated using a reduced-order observer. The stability and dynamic performance of the control designs based on these two current measurement options are compared by means of analysis and experiments at different sampling frequencies and under varying grid conditions, ranging from strong to very weak. Equal reference-tracking performance under nominal conditions is used as a basis for comparison between these two options. If a strong grid is assumed in the control tuning, the controller based on the grid current measurement (GCM) is found to be more robust against varying grid conditions in a wide range of sampling frequencies than the controller based on the converter current measurement (CCM). The CCM leads to better dynamic performance as compared to the GCM if the resonance frequency of the system falls below the critical resonance frequency.

Index Terms—Feedback, grid converter, LCL filter, state-space current control, weak grid.

Nomenclature

Space Vectors

Space vectors in synchronous dq-coordinates are marked as follows.

- \( e_g \) Grid voltage.
- \( i_c \) Converter current.
- \( i_g \) Grid current.
- \( u_c \) Converter voltage.
- \( u_g \) PCC voltage.

Space vectors in stationary coordinates are marked with the superscript \( s \).

Transfer Functions

- \( F_c, F_g \) Reference prefilters.
- \( G \) Reference-tracking transfer function.
- \( G_c, G_g \) Feedback controllers.
- \( Y \) Closed-loop admittance.

Matrices

- \( C_g \) Output matrix for the GCM.
- \( C_c \) Output matrix for the CCM.
- \( I \) Identity matrix.
- \( K \) State-feedback gain matrix.
- \( K_o \) Observer gain matrix.
- \( \Phi \) State-transition matrix.
- \( \Gamma \) Input matrix for the converter voltage reference.
- \( \Gamma_e \) Input matrix for the grid voltage.

Other Symbols

- \( C_f \) LCL-filter capacitance.
- \( f_c \) Critical frequency.
- \( f_r \) Resonance frequency.
- \( f_s \) Sampling frequency.
- \( k_i \) Integral gain.
- \( k_t \) Feedforward gain.
- \( L_{fc} \) Converter-side inductance of an LCL filter.
- \( L_{fg} \) Grid-side inductance of an LCL filter.
- \( L_g \) Grid inductance.
- \( T_s \) Sampling period.
- \( \alpha_c \) Closed-loop bandwidth of current control.
- \( \zeta_r, \zeta_r \) Damping ratios.
- \( \omega_g \) Grid angular frequency.
- \( \omega_r \) Resonance angular frequency.

Parameter estimates used in the control tuning are marked with hats.

I. INTRODUCTION

Grid converters equipped with an LCL filter are increasingly used to connect distributed and renewable energy sources to the electric grid. The LCL filter is the preferred option to attenuate the switching harmonics because of its compact size and better grid-current quality in comparison with the L filter [1], [2]. Active resonance damping of the LCL filter by means of control [3] improves the system efficiency as compared to passive damping [4]. State-space control provides a straightforward way for active resonance damping and for setting the dominant dynamics of the closed-loop system [3], [5]–[17].

As shown in Fig. 1, the current control can be based on either the CCM or the GCM. In the CCM-based control, the current sensors can be integrated into the converter module, which reduces the amount of external cabling and simplifies signal conditioning hardware. Furthermore, the same sensors can be used for protecting the converter against overcurrent. On the other hand, with the GCM, the power factor and the current injected into the grid can be directly controlled at the
is carried out at different sampling frequencies and under varying grid conditions, ranging from strong to very weak. The grid current is chosen as the controlled variable in both options. Equal reference-tracking performance under nominal conditions is used as a basis for comparison. The design of the controllers is based on the direct pole-placement method with the radial projection for the resonance damping of the LCL filter [11], [18]. Both measurement options are experimentally compared in terms of reference tracking and disturbance rejection using a 12.5-kVA three-phase grid converter under strong and very weak grid conditions.

II. SYSTEM MODEL

A. Open-Loop System

Fig. 1 shows a space-vector circuit model of an LCL filter and a grid in stationary coordinates (vectors marked with the superscript s). The converter voltage is $u_{\text{c}}$, the voltage across the capacitor is $u_i$, the PCC voltage is $u_g$, and the grid voltage is $e_g$. The converter current is denoted by $i_c$ and the grid current by $i_g$. The LCL filter parameters are denoted by $L_{fc}, C_f$, and $L_g$. The grid inductance is $L_g$ and the grid-side inductance seen from the capacitor terminals is $L_{gt} = L_{tg} + L_g$. The undamped resonance angular frequency of the system

$$\omega_r = 2\pi f_r = \sqrt{\frac{L_{fc} + L_g}{L_{fc} C_f L_g}}$$

(1)

depends on the grid inductance $L_g$, i.e., increasing the grid inductance decreases the resonance frequency.

The model of the system shown in Fig. 1, after transformation to synchronous dq-coordinates rotating at the grid angular frequency $\omega_g$, is considered in this paper. The derivation of the model is given in Appendix A. The computational delay of one sampling period exists in standard implementations, i.e., [5], [11]

$$u_c(k + 1) = e^{-js_0 T_s} u_{c, \text{ref}}(k)$$

(2)

where $u_{c, \text{ref}}$ is the converter voltage reference and $T_s = 1/f_s$ is the sampling period.

Including the delay (2), a discrete-time model for the plant shown in Fig. 1 can be expressed as

$$x(k + 1) = \Phi x(k) + \Gamma u_{c, \text{ref}}(k) + \Gamma_e e_g(k)$$

(3)

where $x = [i_c, u_i, i_g, u_c]^T$ is the state vector and the system matrices are defined in Appendix A.

Fig. 2 shows the block diagrams of the open-loop systems (inside the gray blocks). The open-loop transfer functions can be derived from (3) for both measurement options. For example, the transfer functions from the converter voltage

1 As compared to the preliminary study [29], two significant additions have been made. Firstly, two different grid strengths (strong and very weak) are assumed for control tuning to examine the effect of the grid inductance estimate on the stability of current control. Secondly, the robustness against grid strength is analyzed at different sampling frequencies.
degrees-of-freedom (2DOF) control structure. The structure consists of a reference prefilter and a feedback controller. According to Fig. 2(a), the GCM loop gain is $L_g(z) = Y_g(z)G_g(z)$ and the closed-loop response is

$$i_g(z) = \frac{F_g(z)L_g(z)i_\text{g,ref}(z)}{1 + L_g(z)\gamma_g(z)} + \frac{Y_g(z)}{1 + L_g(z)}e_g(z)$$  \hspace{1cm} (9)

where $G(z)$ is the reference-tracking transfer function and $Y(z)$ is the closed-loop output admittance. The output admittance can be shaped only via the controller $G_g(z)$, while reference tracking can also be affected via the prefilter $F_g(z)$.

According to Fig. 2(b), the CCM loop gain is $L_c(z) = Y_c(z)G_c(z)$ and the closed-loop response is

$$i_g(z) = \frac{F_c(z)L_c(z)H_c(z)}{1 + L_c(z)}i_\text{g,ref}(z) + \left[\frac{H_c(z)Y_c(z)}{1 + L_c(z)} + H_g(z)Y_g(z)\right]e_g(z).$$  \hspace{1cm} (10)

By examining (9) and (10), it can be observed that the open-loop transfer functions, cf. (4)–(7), play a significant role in the characterization of the closed-loop system. It is evident from Figs. 2 and 3 that the CCM loop gain becomes unavoidably different from the GCM loop gain, leading to different closed-loop output admittances. Due to the additional degree of freedom provided by the reference prefilter, reference tracking of the CCM case can be designed to equal that of the GCM case under nominal conditions.

### III. Current Control

Fig. 1 shows the overall block diagram of the current control system. The current controller operates in PCC-voltage coordinates, where $u_\text{g} = u_g + j\omega$. Fig. 4 shows the structures of the two state-space controllers to be compared. The state and parameter estimates are marked with hats. To provide the basis for the comparison, reference tracking of the CCM case is designed to equal that of the GCM case under nominal conditions. The reference-tracking transfer function $G(z)$ becomes equal in (9) and (10) if the following three conditions are met: 1) DC gains are equal; 2) closed-loop poles are equal; and 3) closed-loop zeros are equal.
A. Control Laws

In accordance with Fig. 4, the control law for the GCM is

\[
x_1(k+1) = x_1(k) + i_{g,ref}(k) - i_c(k)
\]

\[
u_{c,ref}(k) = k_t i_{g,ref}(k) - K_\dot{x}(k) + k_i x_1(k)
\]

where \(k_i\) is the integral gain, \(k_t\) is the feedforward gain, \(K\) is the state-feedback gain, \(x_1\) is the integral state, and \(\dot{x}\) is the vector consisting of the measured and estimated states. The states are estimated using a reduced-order observer, as explained in Appendix B. The integral action eliminates the steady-state error between the grid current reference and measured grid current. The current controller and the reduced-order observer is presented in detail in [12].

Similarly, the control law for the CCM is

\[
x_1(k+1) = x_1(k) + i_{c,ref}(k) - i_c(k)
\]

\[
u_{c,ref}(k) = k_t i_{c,ref}(k) - K_\dot{x}(k) + k_i x_1(k).
\]

In this case, the integral action eliminates the steady-state error between the converter current reference and measured converter current. As already mentioned, the grid current is chosen as the controlled variable also in the CCM case. Therefore, the converter current reference \(i_{c,ref}\) has to be expressed as a function of the grid current reference \(i_{g,ref}\). Based on the plant model shown in Fig. 1, the control error becomes zero in the steady state with accurate parameter estimates, if the converter current reference is

\[
i_{c,ref} = (1 - \omega_g^2 \hat{C}_f \hat{L}_g) i_{g,ref} + j \omega_g \hat{C}_f u_{g,ref}.
\]

The PCC voltage reference is

\[
u_{g,ref} = \sqrt{\epsilon_{gN}^2 - (\omega_g \hat{L}_g i_{g,ref})^2 - \omega_g^2 \hat{L}_g i_{g,ref}}
\]

where \(\epsilon_{gN}\) is the rated grid voltage magnitude. In this way, the DC gain of the transfer function \(G(z)\) in the CCM case can be made equal to that of the GCM case.

B. Gain Calculation

1) Design Process: Fig. 5 shows a flowchart for computing the controller gains \(k_i, k_t, K, K_c\) according to the direct pole-placement method [5], [12]. The nominal parameters of the LCL filter are typically known at the design stage, while the grid inductance \(L_g\) is unknown. Typically, the controller gains are calculated assuming a strong grid under nominal conditions [5], [11], [12]. However, the grid inductance can be taken into account in control tuning, if weak grid conditions are to be expected. In this paper, the controller gains are calculated with the assumption of a strong grid \((L_g = 0)\) and a very weak grid \((L_g = 1 \text{ p.u.})\) to examine the effect of the grid inductance estimate on the stability of current control.

The selection of the sampling frequency \(f_s\) also affects the stability of current control. Two distinct stability regions of the LCL filter resonance, below and above the critical resonance frequency \(f_c = f_s/6\), are identified in [26]. The high resonance frequency region occurs if \(f_r > f_c\) and the low resonance frequency region occurs if \(f_r < f_c\). In this paper, the robustness of the current measurement options is examined at two different sampling frequencies for which \(f_c\) becomes lower and higher than \(f_r\). After selecting \(L_g\) and \(f_s\), the matrices \(\Phi\) and \(\Gamma\) are computed. These matrices are used in the observer and in control gain calculation.

2) Desired Closed-Loop Poles: The system has three open-loop poles, located at

\[
p_{ol,1,2} = \exp[-j(\omega_g \pm \omega_r)T_s] \quad p_{ol,3} = \exp(-j\omega_g T_s).
\]

The open-loop poles depend on the grid inductance \(L_g\) and on the sampling frequency \(f_s = 1/T_s\). As an example, Fig. 6 shows the open-loop pole locations for the grid inductance \(L_g = 0\) and the sampling frequency of \(f_s = 10 \text{ kHz}\). The parameters of the LCL filter given in Table I are used.

In addition to the three open-loop poles, the computational delay and the integral action add two more poles to the
closed-loop system. In order to design equal poles for $G(z)$ in (9) and (10), the nominal closed-loop poles are placed at the same locations for both measurement cases. The resonant poles of the LCL filter are damped using the radial projection technique, i.e., the undamped natural frequency of the resonant pole pair is not altered, as shown with the dashed arrows in Fig. 6. Thus, the closed-loop control poles are parametrized as [5]

$$p_{1,2} = \exp \left[ \left( -\zeta_s \pm j \sqrt{1 - \zeta_s^2} \right) \omega_r T_s \right]$$

$$p_{3,4} = \exp (-\alpha_c T_s)$$

$$p_5 = 0$$

(16)

where $\zeta_s$ is the damping ratio of the resonant poles and $\alpha_c$ is the approximate closed-loop bandwidth of current control. The resonance angular frequency estimate $\omega_r$ depends on the grid inductance estimate $\hat{L}_g$ according to (1). The observer poles are parametrized as [5]

$$p_{o1,o2} = \exp \left[ \left( -\zeta_o \pm j \sqrt{1 - \zeta_o^2} \right) \omega_t T_s \right]$$

(17)

where $\zeta_o$ is the damping ratio.

The dominant dynamics are determined by the pair $p_{3,4}$ of double poles [5], [12]. The pole $p_5$ originating from the computational delay is not moved, since it is already in the optimal location. In both measurement cases, the reference-feedforward zero is placed to cancel one of the control poles at $p_{3,4}$. The plant has two resonant open-loop zeros, as explained in [5]. These zeros become inherently equal in both cases through the transfer function $H_c(z)$, cf. (8). Thus, the transfer function $G(z)$ yields equal zeros in both cases. It is worth mentioning that the transfer function $G(z)$ does not depend on the observer under nominal conditions.

The grid inductance estimate $\hat{L}_g$ in the control tuning affects the choice of the current control bandwidth $\alpha_c$. The larger the grid inductance estimate $\hat{L}_g$ is, the larger the controller gains are for a given bandwidth $\alpha_c$. Too large gains amplify the measurement noise and cause the converter output voltage to saturate during a change in the operating point. Therefore, the grid inductance limits the achievable bandwidth. This limitation is of a fundamental nature, and it affects other controller types as well. In this paper, the current control bandwidth is $\alpha_c = 2\pi \cdot 400$ rad/s for $\hat{L}_g = 0$ and $\alpha_c = 2\pi \cdot 100$ rad/s for $\hat{L}_g = 1$ p.u. The damping ratios are $\zeta_s = \zeta_o = 0.7$. After defining the closed-loop poles (16) and (17), the controller and observer gains are calculated numerically, as in [11], [12].

The nominal system parameters, given in Table I, are used in the gain calculation. As an example, the resulting closed-loop poles with the nominal grid inductance of $\hat{L}_g = 0$, the nominal control bandwidth of $\alpha_c = 2\pi \cdot 400$ rad/s, and the sampling frequency of $f_s = 10$ kHz are shown in Fig. 6.

### IV. Robustness Analysis

In this section, first, the robustness of both current measurement options (GCM and CCM) against grid strength variations is examined. Further, the dynamic performance obtained with the GCM and CCM is compared by means of frequency responses under strong ($\hat{L}_g = 0$) and very weak ($\hat{L}_g = 1$ p.u.) grid conditions.

#### A. Closed-Loop Poles

The robustness against grid strength variations is analyzed at the sampling frequencies of 5 kHz and 10 kHz by computing the eigenvalues (poles) of the closed-loop systems (9) and (10). The sampling frequencies are achieved using single-update PWM and double-update PWM at the switching frequency of 5 kHz. The resonance frequency is $f_r = 1.34$ kHz in the strong grid case. Two different grids, strong ($\hat{L}_g = 0$) and very weak ($\hat{L}_g = 1$ p.u.), are assumed in the control tuning for the gain calculation, as explained in Section III-B.

1) **Strong Nominal Grid**: Fig. 7(a) shows the loci of the closed-loop poles for both measurement cases as the actual grid inductance is increased in the range $\hat{L}_g = 0 \ldots 1$ p.u. The sampling frequency is $f_s = 5$ kHz, i.e., $f_c > f_r$. The inductance estimate $\hat{L}_g = 0$ is used in the controller and the nominal control bandwidth is $\alpha_c = 2\pi \cdot 400$ rad/s. The corresponding nominal poles are shown with the green crosses. When the actual grid inductance increases, the poles move toward the unit circle. At $\hat{L}_g = 1$ p.u., the poles remain inside the unit circle only for the GCM case, i.e., it is stable. The
CCM case is more sensitive to the grid inductance variations. The CCM case becomes unstable due to the complex conjugate poles associated with the resonance of the LCL filter. These are also the least damped poles in the GCM case.

Fig. 7(b) shows the loci of the closed-loop poles with the sampling frequency of $f_s = 10$ kHz, i.e., $f_s < f_c$. The corresponding nominal pole locations are shown with the green crosses. At $L_g = 1$ p.u., all the closed-loop poles remain inside the unit circle in both measurement cases, i.e., they are stable in the whole range of the grid inductance variation. However, the CCM provides better damping for the resonance-frequency poles.

2) Very Weak Nominal Grid: Fig. 8 shows the loci of the closed-loop poles as the actual grid inductance is decreased in the range $L_g = 1 \ldots 0$ p.u. The inductance estimate $\hat{L}_g = 1$ p.u. is used in the controller and the nominal control bandwidth is $\alpha_c = 2\pi \cdot 100$ rad/s. The corresponding nominal poles are shown with the green crosses. Fig. 8(a) shows the loci for the sampling frequency of $f_s = 5$ kHz and Fig. 8(b) for the sampling frequency of $f_s = 10$ kHz. As expected, the closed-loop poles are well damped under weak grid conditions. Both measurement cases become unstable if the actual grid is strong ($L_g = 0$).

3) Effect of the Sampling Frequency: Fig. 9 shows the stability map of the closed-loop system for both measurement cases when the sampling frequency is varied in the range $f_s = 2.5 \ldots 10$ kHz and the actual grid inductance is varied in the range $L_g = 0 \ldots 1$ p.u. The stability map is obtained by numerical evaluation of the eigenvalues of the closed-loop system. In the stable region, all the eigenvalues of the closed-loop system are inside the unit circle.

In Fig. 9(a), the strong grid ($\hat{L}_g = 0$) is assumed in the control tuning and the nominal control bandwidth is $\alpha_c = 2\pi \cdot 400$ rad/s. In this case, a higher sampling frequency is required for the CCM case to provide stable operation for the whole range of the grid inductance variation. The GCM case is robust against the grid inductance in a wider range of sampling frequencies.

In Fig. 9(b), the very weak grid ($\hat{L}_g = 1$ p.u.) is assumed in the control tuning and the nominal control bandwidth is $\alpha_c = 2\pi \cdot 100$ rad/s. Neither of the measurement options provides stable operation for the whole range of the grid inductance variation. However, these tuning assumptions guarantee stable operation in very weak grids for a wide range of sampling frequencies. As shown in Fig. 9(b), the CCM case provides a larger stability range for the grid inductance variation as
Fig. 9. Stability maps as a function of the actual grid inductance $L_g$ and the sampling frequency $f_s$: (a) $\hat{L}_g = 0$ and $\alpha_c = 2\pi \cdot 400$ rad/s; (b) $\hat{L}_g = 1$ p.u. and $\alpha_c = 2\pi \cdot 100$ rad/s. The black crosses correspond to the nominal conditions in Figs. 7 and 8. The cyan circles refer to the conditions used in the frequency response analysis in Figs. 10 and 11 and in the experiments in Figs. 13 and 14. The green circles refer to the conditions used for the experimental validation shown in Fig. 15.

compared to the GCM case.

It is worth noticing that stable operation is ensured independently of the sampling frequency and of the grid assumption ($\hat{L}_g = 0$ or $\hat{L}_g = 1$ p.u.) under nominal conditions, marked with the black crosses in Fig. 9. This shows that the discrete-time state-space controller can stabilize the system irrespective of the ratio of the filter resonance frequency to the sampling frequency, which is not possible with the PI or PR controllers [25]–[28]. Furthermore, both measurement cases provide stable operation for actual grid inductances of 0..1 p.u. when the resonance frequency of the system falls below the critical resonance frequency.

B. Frequency Responses

Reference tracking and disturbance rejection are analyzed by means of frequency responses under strong and very weak grid conditions. The sampling frequency of $f_s = 10$ kHz and the grid inductance of $\hat{L}_g = 0$ are used in the control tuning, since only this combination of parameters (out of the four examined combinations shown with the black crosses in Fig. 9) provides stable operation for both measurement cases for the whole range of grid strength variations.

1) Reference Tracking: The reference-tracking transfer function in (9) and (10) can be expressed as

$$\frac{i_{g}(z)}{i_{g,\text{ref}}(z)} = G(z) = G_{dd}(z) + jG_{qd}(z).$$

When the grid-voltage disturbances are omitted, the $d$-component of the grid current is

$$i_{gd}(z) = G_{dd}(z)i_{g,\text{ref}}(z) - G_{qd}(z)i_{gq,\text{ref}}(z).$$

Fig. 10(a) shows the frequency responses of $G_{dd}(z)$ for both measurement cases under strong grid (nominal) conditions.

The frequency responses are identical and critically damped. Fig. 10(b) shows the frequency responses of $G_{dd}(z)$ under very weak grid conditions. It can be seen that the closed-loop systems become poorly damped, as expected based on Fig. 7(b). Fig. 10(b) also shows that the CCM provides a slightly larger bandwidth and better resonance damping.

2) Disturbance Rejection: The closed-loop admittance in (9) and (10) can be expressed as

$$\frac{i_{g}(z)}{e_{g}(z)} = Y(z) = Y_{dd}(z) + jY_{qd}(z).$$

When only the grid-voltage disturbance is considered, the $d$-component of the grid current can be formulated as

$$i_{gd}(z) = Y_{dd}(z)e_{gd}(z) - Y_{qd}(z)e_{gq}(z).$$
Fig. 12. (a) Photograph of the experimental setup; and (b) external inductors used for emulating the different grid conditions.

![Image](image_url)

Fig. 13. Measured step responses of the grid current components $i_{gd}$ and $i_{gq}$ with $f_s = 10$ kHz, $\alpha_c = 2\pi \cdot 400$ rad/s, and $L_g = 0$: (a) strong grid $L_g = 0$; (b) very weak grid $L_g = 1$ p.u.

Fig. 14. Measured responses of $i_{gd}$ and $i_{gq}$ when a grid-voltage dip of 0.5 p.u. is applied at 10 ms: (a) strong grid $L_g = 0$; (b) very weak grid $L_g = 1$ p.u.

V. EXPERIMENTAL RESULTS

Both current measurement cases are compared by means of experiments using a three-phase 12.5-kVA 50-Hz grid converter system (Table I). Fig. 12(a) shows a photograph of the experimental setup. The switching frequency of the test converter is 5 kHz and synchronous sampling (twice per carrier, $f_s = 10$ kHz) is used. The design parameters are the same as in the previous analyses ($\zeta_r = \zeta_o = 0.7$, $\alpha_c = 2\pi \cdot 400$ rad/s for $\hat{L}_g = 0$, and $\alpha_c = 2\pi \cdot 100$ rad/s for $\hat{L}_g = 1$ p.u.).

A slow PLL having the bandwidth of $2\pi \cdot 2$ rad/s is used in order to avoid the coupling between the current control and PLL dynamics [21]. Different grid conditions, such as $L_g = 0$ (strong grid), $L_g = 0.45$ p.u., $L_g = 0.85$ p.u., and $L_g = 1$ p.u. (very weak grid), are emulated by adding external inductors shown in Fig. 12(b) between the PCC and the grid.

The experiments were carried out first assuming the strong grid ($\hat{L}_g = 0$) in the control tuning [cf. Fig. 9(a)]. Figs. 13 and 14 show the corresponding measured responses of the grid current components $i_{gd}$ and $i_{gq}$. Fig. 13 shows the measured responses of $i_{gd}$ and $i_{gq}$, when a step of 0.2 p.u. is applied to the reference $i_{gd,ref}$. Fig. 13(a) shows the responses under strong grid (nominal) conditions. Both measurement cases provide similar dynamics, since they have equal closed-loop poles and equal reference-tracking performance, cf. Figs. 7(b) and 10(a). The responses are critically damped and very close to the desired dynamics. Fig. 13(b) shows the measured responses under very weak grid conditions ($L_g = 1$ p.u.). As expected, the responses are underdamped and slower than the specified nominal responses, cf. Figs. 7(b) and 10(b). Furthermore, the response for the CCM is better damped.

Fig. 11(a) shows the frequency responses of $Y_{dd}(z)$ for both measurement cases under strong grid conditions and Fig. 11(b) under very weak grid conditions. It can be seen that the CCM provides better disturbance rejection at low frequencies as compared to the GCM.
Fig. 15. Measured responses of the grid current components and the converter voltage reference, when a stepwise change is applied in the grid inductance $L_g$ at 0.05 s: (a) 1 p.u. $\rightarrow$ 0.85 p.u.; (b) 1 p.u. $\rightarrow$ 0.45 p.u.; and (c) 0.45 p.u. $\rightarrow$ 0. The nominal grid inductance is $\hat{L}_g = 1$ p.u., the nominal control bandwidth is $\alpha_c = 2\pi \cdot 100$ rad/s, and the sampling frequency is $f_s = 10$ kHz. The black line in the fourth subplots shows the maximum available voltage $u_{dc}/\sqrt{3}$.

VI. CONCLUSION

This paper presented the stability analysis of the converter and grid current measurement options for discrete-time state-space control at different sampling frequencies and under varying grid conditions, ranging from strong to very weak. Equal reference-tracking performance is designed for both cases under nominal conditions based on the direct pole-placement method. Both measurement cases stabilize the system independently of the ratio of the filter resonance frequency to the sampling frequency under nominal conditions, i.e., the state-space controller eliminates the fundamental limitation of PI and PR controllers. Furthermore, with the assumption of strong grid in the control tuning, both measurement cases can provide stable operation for actual grid inductances of 0 to 1 p.u. when the resonance frequency of the system falls below the critical resonance frequency. With high sampling frequencies, the CCM case leads to better disturbance rejection in comparison with the GCM case under both strong and weak grid conditions. Furthermore, the CCM case provides faster
reference tracking and better resonance damping in weak grids. However, the GCM case is found out to be robust against grid strength variations for a wider range of sampling frequencies as compared to the CCM case.

APPENDIX A
DISCRETE-TIME PLANT MODEL

In synchronous $dq$-coordinates rotating at the grid angular frequency $\omega_g$, a continuous-time model of the plant is

$$\frac{dx_p}{dt} = \begin{bmatrix} -\frac{j\omega_g}{C_i} & \frac{1}{C_i} & 0 \\ \frac{1}{L_e} & -\frac{j\omega_g}{L_e} & \frac{1}{L_e} \\ -\frac{j\omega_g}{L_c} & 0 & -\frac{1}{L_c} \end{bmatrix} x_p + \begin{bmatrix} \frac{1}{L_e} \\ 0 \\ 0 \end{bmatrix} u_c + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} e_g$$

(22)

where $x_p = [i_c, u_t, i_g]^T$ is the state vector. The PWM is modeled as the zero-order hold (ZOH) in stationary coordinates, and the currents are sampled synchronously with the ZOH. Under these assumptions, the model (22) can be converted to an exact hold-equivalent discrete-time model, whose system matrices are [5]

$$\Phi_p = e^{A_p T_s}, \quad \Gamma_p = \int_0^{T_s} e^{A_p \tau} e^{j\omega_g (T_s - \tau)} d\tau \cdot B_p$$

$$\Gamma_{ep} = \int_0^{T_s} e^{A_p \tau} d\tau \cdot B_{ep}.$$ (23)

The closed-form expressions for these system matrices are available in [5]. With the computational delay (2), the discrete-time model for the plant shown in Fig. 1 is

$$x(k+1) = \begin{bmatrix} \Phi_p & \Gamma_p \\ 0 & 0 \end{bmatrix} x(k) + \begin{bmatrix} \Gamma_{ep} \\ 0 \end{bmatrix} e_g(k)$$

$$i_c(k) = \begin{bmatrix} 0 & 0 & 1 \\ 0 \end{bmatrix} x(k)$$

$$i_g(k) = \begin{bmatrix} 1 & 0 & 0 \\ 0 \end{bmatrix} x(k)$$

(24)

where $x = [i_c, u_t, i_g, u_c]^T$ is the state vector and $\gamma = e^{-j\omega_g T_s}$ is the nonzero element in the input matrix $\Gamma$.

APPENDIX B
REDUCED-ORDER OBSERVER

The unknown states are estimated using a reduced-order observer. Due to the integral action of the controller, the grid voltage $e_g$ is considered as an unknown disturbance. Following the standard approach [18], the reduced-order observer for the GCM case becomes [12]

$$\dot{i}_c(k) = \begin{bmatrix} \hat{\phi}_{11} & \hat{\phi}_{12} & \hat{i}_c(k-1) \\ \hat{\phi}_{21} & \hat{\phi}_{22} & \hat{u}_t(k-1) \end{bmatrix} i_c(k) + \begin{bmatrix} \hat{\phi}_{13} & \hat{\phi}_{23} \\ 0 & 0 \end{bmatrix} i_g(k-1)$$

$$+ \begin{bmatrix} \hat{\phi}_{14} & \hat{\phi}_{24} \end{bmatrix} u_e(k-1) + K_o e_o(k)$$

$$e_o(k) = i_g(k-1) - \hat{i}_g(k) - \hat{i}_g(k-1) - \hat{\phi}_{33} u_e(k-1) - \hat{\phi}_{34} u_e(k-1)$$

(25)

where $K_o$ is the observer gain, $e_o$ is the estimation error of the current, the elements $\hat{\phi}_{ij}$ refer to those of $\dot{\Phi}$, and the converter voltage is obtained from $u_e(k) = e^{-j\omega_g T_s} u_{c,ref}(k-1)$. The state estimate for the state feedback is $\dot{x} = [\dot{i}_c, \dot{u}_t, \dot{i}_g, \dot{u}_c]^T$.

The design of the observer in the CCM case is analogous to that in the GCM case,

$$\begin{bmatrix} \dot{u}_t(k) \\ \dot{i}_g(k) \end{bmatrix} = \begin{bmatrix} \hat{\phi}_{22} & \hat{\phi}_{23} \\ \hat{\phi}_{32} & \hat{\phi}_{33} \end{bmatrix} \begin{bmatrix} u_t(k-1) \\ i_g(k-1) \end{bmatrix} + \begin{bmatrix} \hat{\phi}_{21} \\ \hat{\phi}_{31} \end{bmatrix} i_c(k-1)$$

$$+ \begin{bmatrix} \hat{\phi}_{24} & \hat{\phi}_{34} \end{bmatrix} u_e(k-1) + K_o e_o(k)$$

$$e_o(k) = i_c(k) - \hat{i}_c(k-1) - \hat{\phi}_{12} \dot{i}_c(k-1) - \hat{\phi}_{13} \dot{i}_g(k-1) - \hat{\phi}_{14} \dot{u}_c(k-1).$$

(26)

The corresponding state estimate is $\hat{x} = [\hat{i}_c, \hat{u}_t, \hat{i}_g, \hat{u}_c]^T$.

REFERENCES


**F. M. Mahafugur Rahman** (Student Member, IEEE) received the B.Sc.(Tech.) degree in electrical and electronic engineering from the Chittagong University of Engineering and Technology, Chittagong, Bangladesh, in 2011. He received the M.Sc.(Tech.) degree in electronics and electrical engineering from the Aalto University, Espoo, Finland, in 2016, where he is currently working toward the D.Sc.(Tech.) degree in electrical engineering. His research interests include control of grid-connected converters.

**Ville Pirist** received the B.Sc.(Tech.) and M.Sc.(Tech.) degrees in electrical engineering from the Aalto University, Espoo, Finland, in 2017 and 2019, respectively, where he is currently working toward the D.Sc.(Tech.) in electrical engineering. His current research focuses on grid-connected converters.

**Jarno Kukkola** received the B.Sc.(Tech.), M.Sc.(Tech.), and D.Sc.(Tech.) degrees in electrical engineering from the Aalto University, Espoo, Finland, in 2010, 2012, and 2017, respectively. He is currently a Postdoctoral Researcher with the School of Electrical Engineering, Aalto University, Espoo, Finland. His research interests include control systems and grid-connected converters.

**Mikko Routimo** (Member, IEEE) received the M.Sc.(Eng.), Lic.(Tech.), and D.Sc.(Tech.) degrees in electrical engineering from Tampere University of Technology, Tampere, Finland, in 2002, 2005, and 2009, respectively. Since 2008, he has been with ABB Oy, Drives, Helsinki, Finland, where he is currently a Principal Engineer. Since 2017, he has also been a Professor of Practice with the School of Electrical Engineering, Aalto University, Espoo, Finland. His research interests include power electronics in power systems, control of grid-connected converters, and electric drives.

**Marko Hinkkanen** (Senior Member, IEEE) received the M.Sc.(Eng.) and D.Sc.(Tech.) degrees in electrical engineering from the Helsinki University of Technology, Espoo, Finland, in 2000 and 2004, respectively. He is currently an Associate Professor with the School of Electrical Engineering, Aalto University, Espoo, Finland. His research interests include control systems, electric drives, and power converters.

Dr. Hinkkanen was the corecipient of the 2016 International Conference on Electrical Machines (ICEM) Brian J. Chalmers Best Paper Award, the 2016 and 2018 IEEE Industry Applications Society Industrial Drives Committee Best Paper Awards, and the 2020 SEMIKRON Innovation Award. He is an Associate Editor for the IEEE TRANSACTIONS ON ENERGY CONVERSION and the *IET Electric Power Applications*. 