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# Linking socio-economic aspects to power system disruption models

## Supplementary Information

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### Modeling power flow in a looped network

#### Nomenclature

$F$	Power flow (MW)
$l, l'$	Transmission line
$lf$	Loop forming
$L, L'$	Number of transmission lines
$R$	Resistance ( $\Omega$ )
$P$	Power production/consumption (MW)

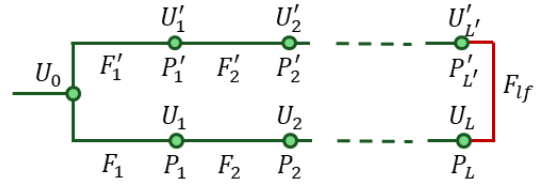


Fig. S1. Power line loop. Red line is a loop-forming line.

The calculation method used to model power flows in a looped transmission grid lines is based on calculation of the power flow in a so-called loop-forming line [1,2]. A loop-forming line represents a line that connects the other two lines to form a loop as illustrated in Fig. S1. The steps of derivation for power flow through such a loop-forming line are as follows.

First, voltage levels for all points are computed for both branches formed when loop-forming line is taken out of the loop:

$$U_1 = U_0 - \Delta U_1 = U_0 - R_1 I_{01} = U_0 - R_1 \frac{F_1}{U} \quad (S1)$$

$$U = \frac{U_0 + U_1}{2} \quad (S2)$$

$$U_1 = U_0 - \frac{2F_1 R_1}{U_0 + U_1} \quad (S3)$$

$$U_1 = \sqrt{U_0^2 - 2F_1 R_1} \quad (S4)$$

$$U_2 = \sqrt{U_1^2 - 2F_2 R_2} = \sqrt{U_0^2 - 2F_1 R_1 - 2F_2 R_2} \quad (S5)$$

$$U_L = \sqrt{U_0^2 - 2 \sum_{l=1}^L F_l R_l} \quad (S6)$$

$$U_{L'} = \sqrt{U_0^2 - 2 \sum_{l'=1}^{L'} F_{l'} R_{l'}} \quad (S7)$$

Since exclusion of loop-forming line is just imaginary for calculation purposes, power flow through that line has to be included:

$$\begin{cases} F_l \Rightarrow F_l + F_{lf}, & l \in [1, L] \\ F_{l'} \Rightarrow F_{l'} - F_{lf}, & l' \in [1, L'] \end{cases} \quad (S8)$$

$$\begin{cases} U_L = \sqrt{U_0^2 - 2 \sum_{l=1}^L (F_l + F_{lf}) R_l} \\ U_{L'} = \sqrt{U_0^2 - 2 \sum_{l'=1}^{L'} (F_{l'} - F_{lf}) R_{l'}} \end{cases} \quad (S9)$$

Then, calculation for one branch is extended though the loop-forming line:

$$U_{L+1} = \sqrt{U_0^2 - 2 \sum_{l=1}^L (F_l + F_{lf}) R_l - 2 F_{lf} R_{lf}} \quad (S10)$$

$U_{L+1}$  and  $U_{L'}$  describes voltage at the same point, therefore they must be equal:

$$U_{L+1} = U_{L'} \quad (S11)$$

$$\sum_{l=1}^L (F_l + F_{lf}) R_l + F_{lf} R_{lf} = \sum_{l'=1}^{L'} (F_{l'} - F_{lf}) R_{l'} \quad (S12)$$

$$\sum_{l=1}^L F_l R_l + \sum_{l=1}^L F_{lf} R_l + F_{lf} R_{lf} = \sum_{l'=1}^{L'} F_{l'} R_{l'} - \sum_{l'=1}^{L'} F_{lf} R_{l'} \quad (S13)$$

$$F_{lf} \left( \sum_{l=1}^L R_l + \sum_{l'=1}^{L'} R_{l'} + R_{lf} \right) = - \sum_{l=1}^L F_l R_l + \sum_{l'=1}^{L'} F_{l'} R_{l'} \quad (S14)$$

$$F_{lf} = - \frac{\sum_{l=1}^L F_l R_l - \sum_{l'=1}^{L'} F_{l'} R_{l'}}{\sum_{l=1}^L R_l + \sum_{l'=1}^{L'} R_{l'} + R_{lf}} \quad (S15)$$

## References

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