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Linking socio-economic aspects to power system disruption models

Supplementary Information

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Modeling power flow in a looped network

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>Power flow</td>
<td>MW</td>
</tr>
<tr>
<td>l, l'</td>
<td>Transmission line</td>
<td></td>
</tr>
<tr>
<td>L, L'</td>
<td>Number of transmission lines</td>
<td></td>
</tr>
<tr>
<td>R</td>
<td>Resistance</td>
<td>Ω</td>
</tr>
<tr>
<td>P</td>
<td>Power production/consumption</td>
<td>MW</td>
</tr>
</tbody>
</table>

The calculation method used to model power flows in a looped transmission grid lines is based on calculation of the power flow in a so-called loop-forming line [1,2]. A loop-forming line represents a line that connects the other two lines to form a loop as illustrated in Fig. S1. The steps of derivation for power flow through such a loop-forming line are as follows.

First, voltage levels for all points are computed for both branches formed when loop-forming line is taken out of the loop:

\[ U_1 = U_0 - \Delta U_1 = U_0 - R_1 I_{01} = U_0 - R_1 \frac{F_1}{U} \]  
\[ U = \frac{U_0 + U_1}{2} \]  
\[ U_1 = U_0 - \frac{2F_1 R_1}{U_0 + U_1} \]  
\[ U_1 = \sqrt{U_0^2 - 2F_1 R_1} \]  
\[ U_2 = \sqrt{U_1^2 - 2F_2 R_2} = \sqrt{U_0^2 - 2F_1 R_1 - 2F_2 R_2} \]  
\[ U_L = \sqrt{U_0^2 - 2 \sum_{l=1}^{L} F_l R_l} \]

Fig. S1. Power line loop. Red line is a loop-forming line.
\[ U'_{L'} = \sqrt{U_0^2 - 2 \sum_{l=1}^{L'} F'_l R'_l} \]  

(S7)

Since exclusion of loop-forming line is just imaginary for calculation purposes, power flow through that line has to be included:

\[
\begin{align*}
F_l &\Rightarrow F_l + F_{lf}, \ l \in [1, L] \\
F'_l &\Rightarrow F'_l - F_{lf}, \ l' \in [1, L']
\end{align*}
\]  

(S8)

\[
\begin{align*}
U_L &= \sqrt{U_0^2 - 2 \sum_{l=1}^{L} (F_l + F_{lf}) R_l} \\
U'_L &= \sqrt{U_0^2 - 2 \sum_{l'=1}^{L'} (F'_l - F_{lf}) R'_l}
\end{align*}
\]  

(S9)

Then, calculation for one branch is extended though the loop-forming line:

\[ U_{L+1} = \sqrt{U_0^2 - 2 \sum_{l=1}^{L} (F_l + F_{lf}) R_l - 2F_{lf} R_{lf}} \]  

(S10)

\[ U_{L+1} \] and \[ U'_L \] describes voltage at the same point, therefore they must be equal:

\[ U_{L+1} = U'_L \]  

(S11)

\[ \sum_{l=1}^{L} (F_l + F_{lf}) R_l + F_{lf} R_{lf} = \sum_{l'=1}^{L'} (F'_l - F_{lf}) R'_l \]  

(S12)

\[ \sum_{l=1}^{L} F_l R_l + \sum_{l=1}^{L} F_{lf} R_l + F_{lf} R_{lf} = \sum_{l'=1}^{L'} F'_l R'_l - \sum_{l'=1}^{L'} F_{lf} R_{lf}' \]  

(S13)

\[ F_{lf} \left( \sum_{l=1}^{L} R_l + \sum_{l'=1}^{L'} R'_l + R_{lf} \right) = - \sum_{l=1}^{L} F_l R_l + \sum_{l'=1}^{L'} F'_l R'_l \]  

(S14)

\[ F_{lf} = \frac{\sum_{l=1}^{L} F_l R_l - \sum_{l'=1}^{L'} F'_l R'_l}{\sum_{l=1}^{L} R_l + \sum_{l'=1}^{L'} R'_l + R_{lf}} \]  

(S15)

References
