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On the Stability of Volts-per-Hertz Control for Induction Motors

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Abstract—This paper deals with the stability analysis of volts-per-hertz (V/Hz) control for induction motors. The dynamics of the electrical and mechanical subsystems of the induction motor model are nonlinearly coupled by the electromagnetic torque and the back-electromotive force. Under open-loop V/Hz control, the nonlinear interaction is known to give rise to small-signal oscillations while operating at medium speeds under light loads. In this paper, it is shown that the interaction also causes a nonoscillatory unstable mode to appear at low speeds under heavy loads (despite the perfect flux level), manifesting itself as a flux collapse or surge. It is also shown that the electrical subsystem with the rotor speed input and the electromagnetic torque output has nonpassive operating regions, which indicates a risk of detrimental interactions with the mechanical subsystem. Finally, a feedback design is proposed in order to enlarge the passive and stable regions and improve the damping. The theoretical results are validated by means of simulations and experiments on a 45-kW induction motor drive.

Index Terms—Eigenvalues, induction machine, passivity, scalar control, stability, volts-per-hertz (V/Hz) control.

I. INTRODUCTION

THE DEVELOPMENT of power semiconductors enabled the first pulsewidth-modulated (PWM) variable-speed induction motor drives in the 1960s [1]. As the research on field-oriented control then was in its infancy, the control method of choice was open-loop volts-per-hertz (V/Hz) control [2]–[6]. V/Hz control today remains a popular choice for low-cost drives due to its inherent sensorless operation, simplicity, and ease of use: the end user defines only a V/Hz curve and acceleration ramps [7]. Furthermore, V/Hz control is common in high-speed drives due to straightforward utilization of the full inverter voltage [8], [9]. Naturally, as compared to field-oriented control, V/Hz control also has some drawbacks: its reference-tracking response is either oscillating or slow (depending on the selected ramps), torque-production capability is poor at low speeds, and the stator current is difficult to limit.

The dynamics of open-loop V/Hz control equal those of the induction motor alone. The electrical subsystem (cf. Fig. 1 for a typical equivalent circuit model) and the mechanical subsystem of the motor are nonlinearly coupled by the electromagnetic torque and the back-electromotive force. The stability of the induction motor was studied by means of its linearized model in the pioneering works [2]–[6]. It was found out that the interaction between the electrical and mechanical subsystems may lead to an unstable region (appearing typically at medium speeds under light loads), which gives rise to small-signal oscillations. These oscillations may also excite torsional resonances of the drivetrain mechanics [10], [11].

The stator resistance voltage drop (RI) has to be compensated for in order to maintain a desired flux level (and torque production) at low speeds. The RI compensator can be based on the steady-state vectorial voltage equation with the low-pass-filtered, measured stator current [12], [13]. The steady-state speed error due to the slip can also be compensated for [13]. These two compensators alter the operating point, but they do not yet guarantee the stability of the drive system.

Modern versions of V/Hz control aim to stabilize the drive by means of feedback from the stator current [14]–[18], while the RI and slip compensators are also used in order to maintain the desired operating point. Typically, the operating-point component of the measured stator current is first filtered out. The remaining current component (representing the deviation about the operating-point value) is fed back to the stator voltage and (optionally) to the stator frequency via feedback gains [18]. In general, the resulting six gains are difficult to design since the linearized motor model is of the fifth order and depends on the operating point. Therefore, the previous studies resorted to numerical analyses, which allowed taking into account details but prevented finding analytical results.

In this paper, we study the stability of V/Hz control by reviewing the existing results and augmenting them with new findings. It is shown that the interaction between the electrical and mechanical subsystems causes an unstable low-speed region to appear under heavy loads (despite a perfect RI compensator), in addition to the well-known unstable mid-speed region. This nonoscillatory unstable mode manifests itself as a flux collapse or surge. It is also shown that the electrical subsystem with the rotor speed input and the electromagnetic torque output has nonpassive operating regions, indicating a risk of detrimental interactions with the mechanical subsystem. Finally, feedback gains are designed by means of the passivity concept and analytical formulations. The proposed feedback design significantly enlarges the passive and stable regions and improves the damping. The theoretical results are validated by

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means of simulations and experiments on a 45-kW induction motor drive.

II. INDUCTION MOTOR MODEL

The stator current is represented by a real column vector $i_s = [i_{sd}, i_{sq}]^T$, whose elements $i_{sd}$ and $i_{sq}$ are the direct- and quadrature-axis components, respectively, and the superscript $T$ marks the transpose. Other vector quantities are represented similarly. Furthermore, the identity matrix $I = [1, 0, 0, 1]^T$, the orthogonal rotation matrix $J = [0, -1, 0, 0]^T$, and the zero matrix $0$ are frequently used in the following equations.

A. Large-Signal Model

The induction motor is modeled using the standard inverse- $\Gamma$ model [19], whose equivalent circuit is shown in Fig. 1. The parameters of the model are defined in Table I. With the stator current $i_s$ and the rotor flux linkage $\psi_R$ as the state variables, the nonlinear state equations in synchronous coordinates rotating at the angular speed $\omega_s$ are

$$
L_s \frac{di_s}{dt} = -(R_s I + \omega_s L_d J) i_s + (\alpha I - \omega_m J) \psi_R + u_s \tag{1a}
$$

$$
\frac{d\psi_R}{dt} = R_s i_s - (\alpha I + \omega_s J) \psi_R \tag{1b}
$$

where $u_s$ is the stator voltage, $R_s = R_s + R_R$ is the total resistance, $\alpha = R_R/L_M$ is the inverse rotor time constant, $\omega_m$ is the electrical angular speed of the rotor, and $\omega_r = \omega_m - \omega_m$ is the slip angular frequency. For future reference, the stator flux linkage is $\psi_s = L_d i_s + \psi_R$, cf. Fig. 1. The electromagnetic torque is nonlinear in the state variables,

$$
\tau_m = i_s^T J \psi_R \tag{1c}
$$

where per-unit (p.u.) quantities are assumed.

The mechanical subsystem is governed by

$$
J_m \frac{d\omega_m}{dt} = \tau_m - \tau_L \tag{2}
$$

B. Steady-State Operating Point

The stability of the nonlinear model is to be studied by means of small-signal linearization. The first step is to solve the steady-state operating point of (1) and (2) by substituting $d/dt = 0$. The operating-point quantities are marked with the subscript 0. From (1b), the stator current in the steady state is

$$
i_{s0} = \frac{\alpha I + \omega_m J}{R_s} \psi_{R0}. \tag{3}
$$

Further, since the stator flux linkage is $\psi_{s0} = L_d i_{s0} + \psi_{R0}$, the rotor flux linkage can be expressed as

$$
\psi_{R0} = \frac{R_s}{L_d} (\omega_{t0} i_s + \omega_m J)^{-1} \psi_{s0} \tag{4}
$$

where $\omega_{t0} = \alpha/\sigma$ is the breakdown slip frequency and $\sigma = L_M/(L_M + L_\sigma)$ is the leakage factor. Applying (1c), (3), and (4), the steady-state torque can be expressed as

$$
\tau_{m0} = \frac{\psi_{s0}^2}{R_s} \frac{\omega_{t0}}{\omega_{t0} + \omega_m} \tag{5}
$$

where $\psi_{R0} = \|\psi_{R0}\|$ is the rotor-flux magnitude and other vector magnitudes are marked similarly. The breakdown torque depends on the stator-flux magnitude

$$
\tau_{b0} = \frac{L_M}{L_M + L_\sigma} \frac{\psi_{s0}^2}{2 L_\sigma} \tag{6}
$$

The breakdown torque (6) defines the large-signal stability limit of the induction motor for a given stator flux magnitude.

Fig. 3 shows the feasible operating region originating from the breakdown torque for a 45-kW four-pole induction motor, whose parameters are given in Table I. The stator flux magnitude is constant ($\psi_{s0} = 1$ p.u.) in the base-speed region and decreases inversely proportional to the stator frequency $\omega_{s0}$.
in the field-weakening region. Fig. 3 also shows the steady-state torque characteristics (5) plotted at selected values of the stator frequency. The rated torque of the 45-kW motor is 43% of the breakdown torque in the base-speed region, assuming the constant parameters given in Table I.

The slip angular frequency is obtained from (5) as a function of the torque

$$\omega_s = \frac{\tau_{m0}}{\tau_{r0}} \left( 1 - \sqrt{1 - \frac{\tau^2_{m0}}{\tau^2_{r0}}} \right) \omega_{rb}$$

(7)

where \([\omega_s] \leq \omega_{rb}\) is assumed, corresponding to the torque loci drawn with solid lines in Fig. 3. The operating point of the induction motor can be uniquely defined with three scalar quantities, e.g., stator-flux magnitude \(\psi_s\), stator frequency \(\omega_s\), and torque \(\tau_{m0}\). Using these three quantities, the slip frequency (7), rotor flux (4), stator current (3), and other operating-point quantities can be calculated.

III. CONTROL SYSTEM

Fig. 4 shows the V/Hz control system considered in this paper. The operating-point stator current is approximated by means of low-pass filtering the measured current,

$$\frac{d\tilde{i}_{s0}}{dt} = \alpha_f (\tilde{i}_s - \tilde{i}_{s0})$$

(8)

where \(\alpha_f\) is the bandwidth of the filter. If the bandwidth is selected low enough (clearly below the breakdown slip frequency), the low-pass-filtered current \(\tilde{i}_{s0}\) represents the operating-point current. For notational simplicity, quasi-constant quantities appearing in the V/Hz control algorithm are marked with the subscript 0, even though they in general change slowly with time.

IV. LINEARIZED SMALL-SIGNAL MODEL

A. System Matrices

The local stability of any operating point can be analyzed by means of the linearized model. The small-signal deviation of the stator current about the operating point is denoted by \(\delta i_s = \tilde{i}_s - \tilde{i}_{s0}\) and other small-signal variables are marked
\[ \frac{d}{dt} \begin{bmatrix} \delta i_s \\ \delta \psi_R \end{bmatrix} = \begin{bmatrix} -\frac{R_s}{L_s} I - \omega_m J & \frac{1}{\omega_m} \left( \alpha I - \omega_m J \right) \\ -\alpha I - \omega_i J \end{bmatrix} \begin{bmatrix} \delta i_s \\ \delta \psi_R \end{bmatrix} + \begin{bmatrix} \frac{1}{\omega_m} I \\ 0 \end{bmatrix} \delta u_s + \begin{bmatrix} -J i_{s0} \\ -J \psi_{RO} \end{bmatrix} \delta \omega_s + \begin{bmatrix} -\frac{1}{J} J \psi_{RO} \\ J \psi_{RO} \end{bmatrix} \delta \omega_m \] (12a)

\[ \delta \tau_m = \begin{bmatrix} -\psi_{RO}^T J \\ t_{s0}^T J \end{bmatrix} \delta \psi_R + \begin{bmatrix} 0 \\ c_m \end{bmatrix} (12b)

\[ \delta \tau_m = \begin{bmatrix} -\psi_{RO}^T J \\ t_{s0}^T J \end{bmatrix} \delta \psi_R \] (12c)

similarly. As an example, linearization of the torque expression (1c) yields

\[ \delta \tau_m = -\psi_{RO}^T J \delta i_s + t_{s0}^T J \delta \psi_R \] (11)

where the first term \( \delta \tau_{m1} \) originates from the stator current deviation and the second term \( \delta \tau_{m2} \) originates from the rotor-flux deviation.

Linearizing the whole electrical subsystem in (1) yields (12) given at the top of this page. It is worth noticing that the operating-point current \( i_{s0} \) appearing in the matrices \( b_s \) and \( c_m \) depends on the operating-point slip frequency \( \omega_m \) through (3). The control law defined by (9) and (10) is also linearized, \(^2\)

\[ \delta u_s = -\left( K + J \psi_{s0} k^T \right) \delta i_s - k^T \delta \omega_s = -k^T \delta i_s. \] (13)

The low-pass filter in (8) could be easily included in the linearized model. However, its effect on the stability is minor, if the bandwidth \( \alpha_f \) is set low enough as compared to the motor dynamics. Furthermore, the resulting increase in the system order would hinder deriving analytical results. Therefore, the dynamics of the filter (8) are omitted.

The closed-loop system matrix \( A_c \) for the electrical subsystem is obtained by inserting the control law (13) into (12),

\[ A_c = A - \begin{bmatrix} B_s \left( K + J \psi_{s0} k^T \right) + b_s k^T \end{bmatrix} C_s. \] (14)

Furthermore, interconnecting the electrical subsystem (14) with the mechanical subsystem (2) results in the overall system matrix

\[ A_t = \begin{bmatrix} A_c \\ \frac{c_m}{J_m} b_m \\ 0 \end{bmatrix}. \] (15)

The eigenvalues of \( A_t \) define the local stability of the V/Hz-controlled induction motor. Naturally, the model for open-loop V/Hz control is obtained as a special case by substituting \( K = 0 \) and \( k = 0 \) in (14).

\section{B. Transfer Functions}

In order to have an insight into the linearized model, let us consider its transfer-function representation, cf. Fig. 2(b). The electrical subsystem (12) with the control law (13) corresponds to a single-input single-output transfer function from the rotor speed to the electromagnetic torque,

\[ G(s) = -\frac{\delta \tau_m(s)}{\delta \omega_m(s)} = -c_m(sI_4 - A_c)^{-1}b_m = \frac{N(s)}{D(s)} \] (16)

where \( I_4 \) is the 4×4 identity matrix and the negative sign convention is selected for being able to use the standard negative feedback structure in the following analysis. The numerator polynomial \( N(s) \) is of the third order, and the denominator (characteristic) polynomial \( D(s) \) is monic and of the fourth order. The closed-form expressions for these polynomials are given in Appendix A.

The block diagram in Fig. 2(b) represents the linearized model of the induction motor, including the mechanical subsystem. The transfer function from the load torque to the electromagnetic torque is

\[ \frac{\delta \tau_m(s)}{\delta \tau_L(s)} = \frac{G(s)}{sJ_m + G(s)} = \frac{1}{J_m D_t(s)} \] (17)

where the characteristic polynomial of the overall system is

\[ D_t(s) = sD(s) + N(s)/J_m = \det(sI_5 - A_l) \] (18)

where \( I_5 \) is the 5×5 identity matrix. The last form provides a link to the corresponding system matrix \( A_l \) given in (15). Naturally, the roots of the characteristic polynomial \( D_t(s) \) equal the eigenvalues of \( A_t \). From (18), it can be realized that the numerator \( N(s) \) affects the stability of the overall system, in addition to the denominator \( D(s) \).\(^3\)

\section{V. OPEN-LOOP CHARACTERISTICS}

In this section, the characteristics of an induction motor are analyzed in open loop, i.e., the feedback gains are set to zero, \( K = 0 \) and \( k = 0 \). The perfect RI and slip compensators are assumed, resulting in accurate operating points. Therefore, possible unstable regions originate solely from the interaction of the electrical and mechanical dynamics. This analysis also applies to conventional V/Hz control [13], where the RI and slip compensators are based on the low-pass-filtered stator current and no additional stabilizing feedback is used, if the filter bandwidth \( \alpha_f \) in (8) is set low enough.

\(^3\)Not surprisingly, the linearized dynamics of the induction motor closely resemble the linearized estimation-error dynamics of a speed-adaptive flux observer, cf. e.g. [21], [22]. In the observer, the mechanical subsystem is replaced with a proportional–integral mechanism for speed estimation, and the electrical subsystem is modified using a nonzero observer gain.
To provide more comprehensive results, the passivity of \( G(s) \) was studied numerically using the parameters of the 45-kW motor. Fig. 3 shows the passive region in the speed–torque plane. In accordance with the above-mentioned analytical result, there is a nonpassive region at low speeds under heavy loads, located symmetrically around the zero-frequency steady-state torque locus. Therefore, this nonpassive low-speed region is larger in the regenerating mode than in the motoring mode. The passive region appears in between very low and medium speeds (for the given motor, in the speed range from 0 to 0.2 p.u. under light loads).

In terms of frequencies normalized by the breakdown slip \( \omega_{rb} \), the shape and size of the passive regions of other motors are similar to those of the 45-kW motor. The breakdown slip of smaller motors can be much larger, which increases the absolute width (in electrical rad/s) of both the nonpassive low-speed region and the passive region.

B. Overall System

The stability of the overall system is studied by means of the eigenvalues of \( A_0 \) in (15), or, equivalently, the poles of the overall system, cf. (18). Fig. 6 shows the poles as the stator frequency \( \omega_{s0} \) varies. The operating-point flux is \( \psi_{s0} = 1 \) p.u. and the total inertia equals the rotor inertia, i.e., \( J_m = J_r \). If the total inertia \( J_m \) approached infinity, the complex conjugate poles in Fig. 6 would approach the poles of the electrical subsystem shown in Fig. 5 and the real pole would approach the origin.

Fig. 6(a) reveals that there are unstable poles at medium speeds (around \( \omega_{s0} \approx 5\omega_{rb} \approx 0.2 \) p.u.) in the no-load condition. If the total inertia were increased such that \( J_m > 2.1J_r \), this unstable mid-speed region would disappear, but the damping would still be poor. Increasing the mechanical damping would also shrink the unstable region. Fig. 6(b) shows that there are unstable real poles at low speeds in the heavy-load condition. If the torque were further increased, this unstable low-speed region would expand. Increasing the total inertia \( J_m \) does not remove this nonoscillatory unstable mode but makes it slower, i.e., the unstable poles move closer to the origin.

Fig. 3 shows the resulting stable and unstable regions in the speed–torque plane. The unstable mid-speed region appears under light loads, originates from the complex-conjugate unstable poles [cf. Fig. 6(a)], gives rise to the current and torque oscillations, and becomes smaller with the increasing total inertia \( J_m \). The unstable low-speed region appears under heavy loads, originates from the unstable real pole [cf. Fig. 6(b)], and causes the flux level to collapse or surge. Its stability limit essentially equals the passivity limit of \( G(s) \), i.e., it does not depend on the total inertia. It can also be seen that the maximum stable torque at very low speeds is significantly less than the breakdown torque, corresponding to the maximum slip of \( |\omega_{s0}| = \alpha \) at \( \omega_{s0} = 0 \).

VI. PASSIVITY-BASED FEEDBACK DESIGN

In this section, the feedback gains are designed for the control law given in (9) and (10) by means of the passivity

\[
\operatorname{Re}\{G(j\omega)\} \geq 0 \quad \text{for all} \quad \omega \in [-\infty, \infty]. \quad (19)
\]

However, the electrical subsystem \( G(s) \) fulfills the condition (19) only in limited operating regions. The general analytical expression for \( \operatorname{Re}\{G(j\omega)\} \) is complicated, but some operating points can be analytically treated. For \( \omega_{s0} = 0 \), it can be shown that \( G(s) \) is passive if the slip frequency \( |\omega_{s0}| \leq \alpha \), i.e., if the angle between the current vector \( I_{s0} \) and the rotor flux vector \( \psi_{R0} \) is not more than 45°.
concept and analytical formulations. This approach complements existing numerical design methods, such as [18].

A. Voltage Injection

The case without frequency injection, i.e., $k = 0$, is first considered. The stabilizing feedback signal is injected into the stator voltage according to (9) via the gain matrix

$$K = -R_s I + k_u L_\sigma (\alpha I + \omega_{m0} J)$$  (20)

where $k_u$ is a positive design parameter and the operating-point speed $\omega_{m0}$ corresponds to the (rate-limited) speed reference in the actual control algorithm. As shown in Appendix B, the gain matrix (20) passesivate the transfer function from the speed deviation $\delta \omega_m$ to the current-induced term $\delta \tau_m$ of the electromagnetic torque deviation, cf. (11), and guarantees the internal stability of $G(s)$. The complete passivity of $G(s)$ in every operating point is not guaranteed since the rotor-flux deviation also affects the passivity. Nonetheless, complete passivation does not seem possible by means of simple V/Hz control due to the underlying non-minimum phase system [24].

Fig. 7 shows the poles of the overall system with the gain (20) where $k_u = 0.6$. Fig. 7(a) shows that the system is stabilized in the no-load condition, while Fig. 7(b) shows that there still is an unstable region at low speeds in the heavy-load condition. Fig. 8 shows the resultings stable and unstable regions in the speed–torque plane for $k_u = 0.2$ and $k_u = 0.6$. The stable region covers almost the whole feasible operating region, while narrow unstable regions appear in the vicinity of the breakdown torque and at very low speeds in the regenerating mode. The passive region of $G(s)$ matches with the stable region of the overall system, which indicates robustness against the total inertia (and passive mechanical subsystems in general). Decreasing $k_u$ expands the stable region but decreases the damping. Choosing $k_u = 0$ makes the system marginally stable.

The sensitivity to parameters $L_\sigma$ and $\alpha$ appearing in (20) is not critical, and their rough estimates can be used. A significantly overestimated value for $L_\sigma$ or a too large value for $k_u$ shrinks the stable operating region. The effect of the parameter $\alpha$ on the stability is minimal, and it could be even set to zero in practice. However, for operating at very low speeds under heavy loads, an accurate estimate of the stator resistance $R_s$ is required, not only for the gain (20), but especially for the RI compensation term (9) in order to maintain the flux level. A similar sensitivity to $R_s$ at low speeds is a well-known problem in observer-based sensorless

The gain matrix (20) can be rewritten as $K = k_1 I + k_2 \tau_{m0} J$, where $k_1 = k_u L_\sigma$ and $k_2 = k_u \alpha - R_s$ are constants. It is worth noticing that $L_\sigma$ appears only as a scaling factor and that $k_1 \approx -R_s$.

---

Fig. 6. Poles of the overall system in open loop as the stator frequency $\omega_0$ varies: (a) no load $\tau_{m0} = 0$; (b) heavy load $\tau_{m0} = 0.8 \tau_{b0}$. The inertia is $J_m = J_r$. The unstable poles are shown in magenta color.

Fig. 7. Poles of the overall system with the proposed feedback gains ($k_u = 0.6$, $k_u = 4$) as the stator frequency $\omega_0$ varies: (a) no load $\tau_{m0} = 0$; (b) heavy load $\tau_{m0} = 0.8 \tau_{b0}$. The gray loci correspond to $k_u = 0.6$ and $k_u = 0$. The inertia is $J_m = J_r$.

Fig. 8. Stability and passivity maps with the feedback gain (20). The red solid line shows the stability limit, the blue solid line shows the passivity limit, and the shaded area indicates the passive (stable) region, all for $k_u = 0.6$. The stability limit practically overlaps with the corresponding passivity limit. Due to the passivity, the stable region is independent of the total inertia. This map is obtained with $k_u = 0$ in (21), but it remains essentially the same with $k_u = 4$. Furthermore, the red dashed line shows the stability limit for $k_u = 0.2$, almost overlapping with the breakdown torque (blue dashed line).
control as well.

B. Frequency Injection

The damping can be further improved, if the synchronous frequency used in the coordinate transformations is deviated about the operating-point value. The gain matrix in (10) is selected as (cf. Appendix B)

\[ k = \frac{k_w R_R J \psi_{RO}}{\psi_{RO}} \]  

(21)

where \( k_w \) is a positive design parameter and the operating-point flux \( \psi_{RO} = \psi_{s0} - L_s i_{s0} \) is obtained from the stator flux reference and the low-pass-filtered current. In the linearized model, the gain matrix (21) leads to \( \delta \omega_s = (k_w R_R / \psi_{RO}^2) \delta \tau_{m1} \), i.e., the frequency deviation \( \delta \omega_s \) is proportional to the current-induced torque deviation \( \delta \tau_{m1} \). This frequency injection effectively increases the apparent rotor resistance in some elements of the closed-loop system matrix.

Fig. 7 shows the poles of the overall system with \( k_u = 0.6 \) and \( k_w = 4 \). The frequency injection improves damping, while it does not essentially affect the stability limits, i.e., Fig. 8 is approximately valid for \( k_u = 0.6 \) and \( k_w = 4 \) as well. Consequently, the stable operating region is relatively insensitive to the parameters in (21). If a significantly overestimated value for \( R_R \) or a very large value for \( k_w \) is set, the stable region starts to shrink. It is also to be noted that a slightly simpler variant of (21)—based on the stator-flux reference \( \psi_{s0} \) instead of \( \psi_{RO} \)—could be used. It would provide similar characteristics, but the passive region would be smaller.

VII. Results

The stability of the 45-kW induction motor (cf. Table I) under V/Hz control is studied by means simulations and experiments. Fig. 9 shows a photo of the motor test bench used in the experiments. The total inertia is \( J_m = 1.66 J_r \). The V/Hz control algorithm, illustrated in Fig. 4 and given in detail in Appendix C, was implemented on a dSPACE MicroLabBox prototyping system. The switching frequency of the PWM inverter is 2 kHz, and the sampling frequency is 4 kHz. Inverter nonlinearities were not compensated for. For monitoring purposes, the rotor speed was measured and the electromagnetic torque was estimated using the current-model flux estimator that takes the magnetic saturation into account.

First, the existence of the unstable low-speed region is demonstrated by means of a simulation example, where open-loop V/Hz control with feedforward RI compensation is used.

Fig. 10 shows the simulation result, where the rated load torque is applied and the frequency reference is slowly reversed. The operating-point current \( k_{s0} \), needed for the RI compensation term in (9), is calculated from the known load torque using (3), (4), and (7). In this example sequence, the RI compensation term remains constant since the load torque is constant (and the accelerating torque is negligible). As expected based on the analysis, the drive becomes unstable in the vicinity of zero frequency, resulting in a flux surge after \( t = 10 \) s and collapse after \( t = 28 \) s. The size of the unstable low-speed region can be reduced using the stator current feedback (such as the proposed feedback design), but it seems impossible to completely stabilize the drive at low speeds under heavy loads by means of simple V/Hz control. Inclusion of the magnetic saturation in the simulation model would diminish the flux surge seen in Fig. 10 around \( t = 10 \) s, but the flux collapse around \( t = 28 \) s would be similar in the saturated motor.

Next, the practical version of V/Hz control, where RI compensation is based on the low-pass-filtered measured current, is considered. In the following examples, the bandwidth of the low-pass filter is \( \alpha_f = 0.1 \omega_{rb} \). Fig. 11 shows acceleration, operation at the 0.2-p.u. speed (in the unstable mid-speed region), and deceleration in the no-load condition, while the load torque is zero. Figs. 11(a) and 11(b) show the simulation and experimental results, respectively. Initially, zero feedback gains are used, and the motor drive becomes increasingly unstable after \( t = 2 \) s. Then, the proposed feedback design is enabled at \( t = 4 \) s, which stabilizes the system. In the experimental results, the magnitude of the oscillations increases even faster than in the simulation results. This difference is not surprising since the simulation model assumes a rigid mechanical system and an ideal inverter, while these components are nonideal in the actual drive system.

\( ^2 \)The feedforward RI compensator applied in the simulation of Fig. 10 is not of practical interest since the required torque (or slip) is generally unknown. However, here it allows demonstrating the stability problem in open loop with the correct flux level.
Fig. 11. Acceleration, constant-speed operation, and deceleration at no load: (a) simulation; (b) experiment. The gains are initially zero ($K = 0, k = 0$). At $t = 4$ s, the proposed feedback design is enabled ($k_u = 0.6, k_\omega = 4$).

Fig. 12 shows a rated load torque step and its removal, while the speed reference is kept at 0.2 p.u. and the proposed feedback design is enabled. Figs. 12(a) and 12(b) show the simulation and experimental results, respectively. It can be seen that the slip compensator corrects the speed error and that the transient response is well damped. The noise in the waveforms originates from the PWM inverter of the load machine. It is also worth mentioning that the proposed feedback design is independent of the slip compensator: the stabilizing feedback could be used even if the slip compensator were disabled.

**VIII. CONCLUSIONS**

In addition to the well-known *unstable mid-speed region*, open-loop V/Hz control of the induction motor has an *unstable low-speed region*, where the flux level tends to collapse or surge under heavy loads, even if the RI compensation is perfect. This phenomenon complicates producing large starting torques or reversing the rotor speed under heavy loads. Similar unstable regions are typical to many sensorless field-oriented control methods. The passivity of the electrical subsystem (with the rotor speed input and the electromagnetic output) can be related to the robustness of V/Hz control against unknown mechanical subsystems. Using the proposed feedback design, the passive and stable regions of V/Hz control can be significantly enlarged and the damping can be improved. However, a narrow unstable region still remains at very low speeds under heavy regenerative loads.

**APPENDIX A**

**ANALYTICAL EXPRESSION FOR $G(s)$**

Assuming a skew-symmetric gain matrix $K = k_1 I + k_2 J$ and a gain matrix $k$ of the form given in (21), the numerator
of the transfer function $G(s)$ in (16) is

$$
N(s) = \frac{\omega_{r0}^2 \omega_{ib}}{R_{r0}} \left\{ s^3 + \left[ (1 + a + a\sigma) \omega_{ib} - \frac{\omega_{m0}^2}{\omega_{rb}} \right] s^2 + \left[ a\sigma(2 + a) \omega_{rb}^2 + \omega_{m0} \omega_{ib} - 2 a \omega_{r0} \right] s + a^2 \omega_{rb}^2 s + \left( \frac{\omega_{m0}^2}{\omega_{rb}} - a^2 \omega_{rb} \omega_{r0} \right) \right\}
$$

where $a = (1 - \sigma)(R_u + k_1)/R_r$, $\omega_{m0} = \omega_{m0} + k_2/L_r$, and $\omega_{m0} = \omega_{m0} + \sigma k_2/L_r$. The denominator is

$$
D(s) = \left[ s^3 + (1 + a + a\sigma) \omega_{rb}^2 + \omega_{m0} \omega_{ib} - 2 a \omega_{r0} \right]^2 + \left( \omega_{m0} + \omega_{ib} \right) s + \left( \omega_{m0} + a \omega_{ib} \right) \omega_{rb}^2 + D_\omega(s)
$$

where the last term

$$
D_\omega(s) = \frac{k_\omega R_{r0} \omega_{rb}}{L_r} \left\{ \left[ s + a \frac{\omega_{m0}}{\omega_{rb}} + (1 + a - \sigma) \right] (s^2 + \omega_{m0}^2) + \left[ a\sigma + \frac{L_r}{R_{r0}} (\omega_{m0} - \omega_{m0}) \omega_{ib} \right] s + \left[ a\omega_{m0} - \frac{L_r}{R_{r0}} (\omega_{m0} - \omega_{m0}) \right] \omega_{m0} \right\}
$$

appears only if $k_\omega$ is nonzero. In this case, the system $G(s)$ is not skew-symmetric anymore. In open loop ($K = 0, k = 0$), the denominator (23) reduces to $D(s) = \det(s I_4 - A) = \det((s - s_1)(s - s_2))$, whose roots, normalized by the breakdown slip, are

$$
s_{1,2} = \frac{\omega_{r0}}{\omega_{rb}} \left[ 1 + \frac{a - \omega_{m0}}{\omega_{rb}} \pm \sqrt{\left( 1 + a \right)^2 - 4 a \sigma - \frac{\omega_{m0}^2}{\omega_{rb}^2} - \frac{1}{2} (a - 1) \omega_{m0}^2 \omega_{rb}^2} \right].
$$

It can be seen that the normalized slip angular frequency $\omega_{r0}/\omega_{rb}$ simply shifts the poles along the imaginary axis. The shape of the root loci as a function of $\omega_{m0}/\omega_{rb}$ depends only on the two parameters, $\sigma$ and $a$ [5].

**APPENDIX B**

**PASSIVITY-BASED GAIN DESIGN**

According to (11), the torque deviation consists of two terms: $\delta r_{m1}$ originates from the stator current deviation and $\delta r_{m2}$ originates from the rotor-flux deviation. Correspondingly, the electrical subsystem can be split into two parallel-connected systems, $G(s) = G_1(s) + G_2(s)$, where

$$
G_1(s) = -\delta r_{m1}(s) \delta r_{m2}(s) = -c_{m1} (s I_4 - A_c)^{-1} b_m = N_1(s) D(s)
$$

with $c_{m1} = [\psi_{r0}^T, 0, 0, 0]$ and

$$
N_1(s) = \frac{\omega_{r0}^2}{L_r} \left\{ s^3 + (1 + a) \omega_{rb} s^2 + \left( a\sigma \omega_{rb} + \omega_{m0} \omega_{r0} + \omega_{m0} \omega_{ib} \right) s + \omega_{m0} \omega_{m0} \omega_{rb} \right\}.
$$

Unlike $G(s)$, the transfer function $G_1(s)$ can be completely passivated. According to the Kalman–Yakubovich–Popov lemma [23], $G_1(s)$ is passive if there is a symmetric positive definite matrix $P$ and a symmetric positive semidefinite matrix $Q$ such that

$$
P b_m = \psi_{m1}^T \quad P A_c + A_c^T P = -Q.
$$

The matrix

$$
P = \frac{1}{k_u} \left[ \begin{array}{cc} (1 + k_u) L_r & 0 \\ 0 & (1/L_r) \end{array} \right]
$$

is positive definite for positive $k_u$ and satisfies the first condition in (28). The matrix $K$ in (20) and the vector $k$ in (21) satisfy the second condition in (28), resulting in

$$
Q = \left[ \begin{array}{cc} R_{r0} + (1 + k_u) \alpha L_r & 0 \\ 0 & (1 + k_u) R_{r0} + (1 + k_u) \alpha L_r \end{array} \right]
$$

that is positive semidefinite for positive $k_u$ and $k_\omega$. Therefore, $G_1(s)$ is stable and passive at every operating point. Furthermore, the internal stability of $G(s)$ is also guaranteed since it has the same characteristic polynomial $D(s)$. The resulting gain $K$ is similar to a passivity-based observer gain [22].

**APPENDIX C**

**DISCRETE-TIME ALGORITHM**

A discrete-time implementation of V/f Hz control shown in Fig. 4 is described in detail. For the purposes of slip compensation and frequency injection, the operating-point rotor flux is first computed, $\psi_{r0}(k) = \psi_{r0} - L_r i_{s0}(k)$, where $\psi_{r0} = [\psi_{r0}, 0]^T$ is the constant flux reference, $i_{s0}(k)$ is the low-pass-filtered current signal, and $k$ is the discrete-time index. Based on the first form in (5), the operating-point slip is computed using

$$
\omega_{r0}(k) = \frac{R_{r0} \psi_{s0}(k)}{\psi_{r0}(k)}
$$

which is slightly simpler to implement than (7). The measured stator current is transformed to synchronous coordinates

$$
i_s(k) = \exp[-\vartheta_s(k) J] i_s^0(k).
$$

The current deviation $\delta i_s(k) = i_s(k) - i_{s0}(k)$ is computed as an auxiliary signal. With (21), the frequency reference (10) can be expressed as

$$
\omega_s(k) = \omega_{m0}(k) + \frac{k_{R0} \delta r(k)}{\psi_{r0}(k)}
$$

(33a) where $\delta r$ is the current-induced torque deviation signal and $\omega_{m0}(k)$ is the rate-limited speed reference signal, cf. Fig. 4. If the slip compensation is not needed, it can be disabled by setting $\omega_{r0}(k) = 0$ in (33a).

According to (9), the voltage reference is

$$
u_{s, ref}(k) = R_s i_{s0}(k) + \omega_s(k) J \psi_{s0} - K(k) \delta i_s(k)
$$

which is transformed to stator coordinates,

$$
u_s^*(k) = \exp[\vartheta_s(k) J] u_{s, ref}(k).
$$
Finally, the controller state variables are updated for the next sampling instant: the new angle is

\[
\vartheta_s(k+1) = \vartheta_s(k) + T_s \omega_s(k)
\]

(36)

and the new low-pass-filtered current is

\[
i_s0(k+1) = i_s0(k) + T_s \alpha^i \delta i_s(k)
\]

(37)

where \(T_s\) is the sampling period.

REFERENCES


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