Abedi, Mohsen; Wichman, Risto

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Published in:
2020 IEEE WIRELESS COMMUNICATIONS AND NETWORKING CONFERENCE (WCNC)

DOI:
10.1109/WCNC45663.2020.9120591

Published: 01/05/2020

Document Version
Peer reviewed version

Please cite the original version:
Abedi, M., & Wichman, R. (2020). Cellular Network Planning under Variable QoS Requirements Using Voronoi Algorithm. In *2020 IEEE WIRELESS COMMUNICATIONS AND NETWORKING CONFERENCE (WCNC)* [9120591] (IEEE Wireless Communications and Networking Conference). IEEE. https://doi.org/10.1109/WCNC45663.2020.9120591

Cellular Network Planning under Variable QoS Requirements Using Voronoi Algorithm

Mohsen Abedi, Risto Wichman
School of Electrical Engineering, Aalto University
E-mails: {mohsen.abedi, risto.wichman}@aalto.fi

***Abstract*—We study the optimization of base station (BS) localization in cellular networks with non-uniform service demand distribution taking into account a variety of server-side and user-side requirements. To this end, we propose a power Voronoi algorithm (PVA) to obtain uniform traffic volume shares per cell while simultaneously meeting the user-side and server-side requirements of the heterogeneous devices. We describe a numerical method to solve the optimization problem by updating the weights associated to different cells in the PVA. Then, we take into account minimum signal to interference plus noise ratio at all devices as a user-side requirement. It is observed that BSs are concentrated in the areas with condensed demand and high SINR requirements. To show the advantages of the proposed method, the results are compared for different SINR requirement distributions giving insight on the performance of the algorithm.**

***Index terms*—** Cellular network topology, power Voronoi diagram, service demand distribution

I. INTRODUCTION

Cell planning is one of the major tasks in cellular network design depending on multiple parameters like geographical locations of base stations, desired coverage area, expected traffic demand in the target area, antenna configuration, available frequency bands, etc. By ever increasing number of users and expansion of network sizes, tractable tools for cell planning are of high demand.

Cell load coupling is one of the most useful parameters when analyzing the performance of cellular network to guarantee tolerable interference and minimum outage probability. The load coupling model also gives a metric for performance evaluation in terms of resource consumption as well as network feasibility. In [1], the authors characterize a rigorous model for load-coupling in LTE networks as a non-linear and non-trivial system of equations, and discuss the necessary and sufficient conditions for the feasibility of the load coupling system. In [2], the authors focus on transmission energy region and show that the sum power is minimized when the cellular network operates at full-load. In [3], the problem of load balancing is studied in non-orthogonal multiple access (NOMA) scenario. The problem of energy efficiency maximization in load-coupling scenarios is investigated in [4].

The objectives of cellular network planning are to determine the number of BSs and their locations to meet the traffic demand and quality-of-service (QoS) requirements. Optimally, BSs should serve equal amount of traffic while simultaneously guaranteeing various QoS requirements and taking into account the limited radio resources. User traffic is typically spatially non-uniformly distributed in the target area, and analytical methods to optimize BS locations are unavailable in this case [5]. Therefore, achieving all the planning goals, e.g. coverage, balanced traffic, and meeting minimum SINR requirements at the same time is challenging. It is necessary to trade off the planning goals, or to increase the number of BSs such that the goals can be simultaneously satisfied.

Maximization of the efficiency of resource utilization by optimal assignment of resources, slices, and users are studied in [6]. Signal processing issues with dense cellular networks are addressed in [7] wherein revised cell planning methods applicable to heterogeneous networks (HetNets) are proposed to achieve two ultimate goals; coverage and capacity. In [8], the authors suggest a robust optimization problem for cell planning as well as a cutting plane method to relax the complexity of the optimization problem. Cell planning is also performed via SNR and SINR gain [9], mixed integer programming problem [10], and successive elimination of BSs [11]. A literature review of the state of the art in cell planning and its challenges in 5G is given in [12].

Voronoi diagram and related algorithms provide another approach for determining the locations of the base stations in wireless cellular networks. Voronoi diagram divides a space into cells based on the distances to generators inside the space. The points are assigned to exactly one generator which minimizes the distance metrics used. Centroidal Voronoi and its applications are discussed in [13], a primary dynamic algorithm to construct Voronoi diagram is proposed in [14], and generalization of Voronoi diagram and algorithms are extensively discussed in [15]. The area coverage of cellular networks with respect to Voronoi partitioning is studied in [16]. The authors in [17] study BS location optimization using the concept of node function. A joint use of umbrella diagram and Voronoi tessellation in the wireless problems are proposed in [18]. User outage probability taking into account SINR and user selection is studied in [19] wherein cell areas form Voronoi tessellation. The overlapping behavior of Voronoi and SINR diagram in different BS positions scenarios is studied in [20].

In [21], the authors use Voronoi algorithm as a tool for site positioning given an arbitrary non-uniform service demand distribution, and then optimize the transmit powers for load balancing in the network. In [22], the authors propose a simplified model for load-coupling system and propose a cell planning method using Voronoi diagram to balance the loads in cellular networks. However, these contributions do not

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This work was supported in part by Academy of Finland under Grants 287249, 311752

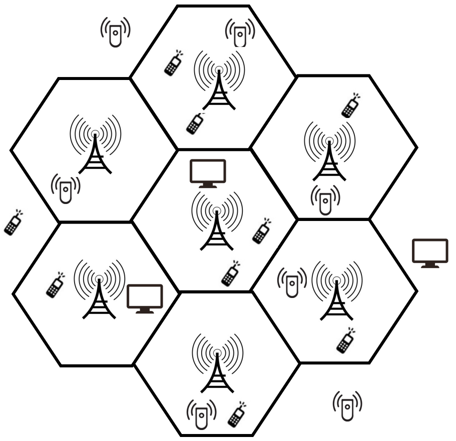

Fig. 1. Wireless cellular network with BSs and devices with different QoS requirements

impose any QoS constraints, and even when traffic volumes are equalized between the cells, users on the cell edges may experience very low SINRs and some cells might be overloaded.

To properly serve devices with non-uniform demand and QoS distributions, deploying regular grid topology such as cells in identical hexagonal tessellation is not optimal. The novelty of this work is to provide a method to find the locations for BSs with a non-uniform spatial distribution of devices to strike a balance in volume shares among the cells taking into account the non-uniform spatial distribution of a variety of requirements imposed by the devices and a set of requirements imposed by the BSs, referred to as user-side and server-side requirements, respectively. The optimization of BS locations is performed via a power Voronoi algorithm to obtain equal balancing functions among the cells. As a result, the proposed method can be applied for cell planning in any kind of network with various constraints whether imposed by the devices or by the BSs. We illustrate the performance of the proposed method to determine the locations of the base stations in case of synthetic service demand and minimum SINR requirement distributions.

The rest of the paper is organized as follows. Section II presents the system model. Section III describes the problem statement and the proposed algorithm. Section IV presents the simulation results and section V provides the conclusion.

II. SYSTEM MODEL

Planning of cellular networks includes a method of determining the BS locations, or the selection of candidate BS sites within the service area to meet the requirements, e.g. balanced volume, balanced loads, coverage, etc. In cellular networks, demand and QoS requirements of the connected devices are typically non-uniformly distributed in the target area. Fig. 1 shows a schematic of wireless cellular network wherein BSs serve different types of devices with various amount of demand and QoS requirements. This kind of hexagonal tessellation would be optimum in case of uniform service demand distribution and uniform QoS requirements. Regardless of the QoS requirements, in a well-planned cellular network, the amount of traffic volume shares at each cell are equal and thus, BSs are condensed in the areas with high service demand requirement, [21].

Volume shares or equivalently traffic volume at each cell is defined as the summation of demand, given in continuous space by

$$V_i = \int_{\mathcal{J}_i} \delta_j dj, \tag{1}$$

where δ_j refers to *demand* and is defined as the rate requirement of a device j. Also, i is the cell index and $\mathcal{J}_i \in \mathcal{J}$ refers to the i^{th} cell area. $\mathcal{J}$ denotes the whole service area which forms a *tessellation*, i.e. $\mathcal{J} = \bigcup_{i=1}^{N} \mathcal{J}_i$ and $\mathcal{J}_i \cap \mathcal{J}_k = \emptyset$ for $i \neq k$.

Power Voronoi diagram associates each point in the area to a generator (here, a BS) and it is formulated as [14], [21]

$$\mathcal{J}_i \triangleq \{j \in \mathcal{J} | \|j - a_i\|^2 - w_i \leq \|j - a_k\|^2 - w_k, \forall i \neq k\}, \tag{2}$$

where $\|.\|$ stands for L_2-norm, and a_i and w_i refer to i^{th} BS location and the associated Voronoi weight, respectively. In particular, $w_i = 0$ for all i in (2) results in *centroidal* Voronoi tessellation where each device j is associated to the closest BS, [13]. By adjusting the weights w_i, one can shrink or enlarge cell sizes to meet the network planning requirements.

Voronoi algorithms divide the service area into cells by moving the BSs to mass centroids (in this case demand centroids) given by

$$c_i \triangleq \frac{\int_{\mathcal{J}_i} j \delta_j dj}{\int_{\mathcal{J}_i} \delta_j dj} \triangleq \frac{\iint_{\mathcal{J}_i} (x,y) \delta(x,y) dxdy}{\iint_{\mathcal{J}_i} \delta(x,y) dxdy}, \tag{3}$$

where x and y refer to the Cartesian coordinates of device j. Cell planning to acquire equal traffic volumes using centroidal and power Voronoi algorithm has been addressed in [21], [22]. In this framework, authors derived a step-by-step algorithm for BS placement to assure that all cells capture the equal amount of volume shares. So, with the purpose of volume share balancing in the cellular networks with non-uniform spatial demand distribution, we formulate the BS placement optimization problem as

$$\underset{\mathcal{J}_1, \mathcal{J}_2, \ldots, \mathcal{J}_N}{\text{minimize}} \quad \max_i V_i \tag{4a}$$

$$\text{s.t.:} \quad \mathcal{J}_i \in \mathcal{J}. \quad 1 \leq i \leq N \tag{4b}$$

The non-convexity of the optimization problem (4) is characterized in Appendix A.

Since (4) is a non-convex optimization problem, the solution is not straightforward. However the optimal solution is found via a combination of Centroidal Voronoi and Power Voronoi algorithms [21]. Centroidal Voronoi algorithm (CVA) has the following steps; I) Initially, BSs are randomly located in the target area, II) tessellation for the service area is derived using (2) with $w_i = 0$ for all i, III) BSs are moved to the mass centroids calculated by (3), IV) the procedure is repeated until the locations of the BSs converge.

CVA moves BSs such that the cells serve the closest users, but since the number of users per cell and the traffic demand are not taken into account, it fails to yield accurately balanced traffic volume shares per cell. In contrast, power Voronoi algorithm (PVA) updates the weights in step (II) based on the volumes calculated by (1). Therefore, PVA is able to change the association of users to cells such that smaller cells are located in the areas with high demand and vice versa. As optimal solution to (4), centroidal and power Voronoi algorithms yield BS locations such that the traffic volumes of the cells are equal i.e. $V_1 = V_2 = \cdots = V_N$, where N denotes the number of cells [21]. In [21], [22], the authors applied PVA in cellular network planning to balance volume shares and cell loads. However, PVA still does not guarantee any QoS requirements the devices may have.

III. PROBLEM STATEMENT

In a well-planned cellular network, BSs are condensed in the areas with high service demand density. However, the devices at the edge of the cell receive the smallest SINR in the cell and the devices at the edge of large cells have the least SINR in the whole network and are most likely to violate minimum SINR requirements. Thus, cell sizes should be small in the areas with devices with high SINR requirements. Therefore, we are motivated to propose a cell planning methodology capturing the main features of the network, including the constraints imposed by the devices and BSs. We assume that both devices and BSs impose some requirements to the network which can be partly or fully met by a suitable network topology. First, we formulate the optimization problem in general case wherein both users and servers impose constraints to the network. Then, we consider the special case where devices impose different minimum SINR requirements.

A. Cell Planning with User-Side and Server-Side Requirements

We tackle the cell planning problem by introducing heterogeneous power Voronoi algorithm (H-PVA) to balance the cell volumes while taking into account two types of requirements, i.e. user side requirements (UR) and server side requirements (SR). UR includes user side constraints such as minimum SINR requirements, latency limits, etc., imposed by the devices in the target area, while server side requirements account for network requirements such as throughput including cell load factor, resource consumption limits, etc. [1]. Therefore, the cell planning optimization problem is given by

$$\underset{\mathcal{J}_1,\mathcal{J}_2,\ldots,\mathcal{J}_N}{\text{minimize}} \quad \max_i V_i \tag{5a}$$

$$\text{s.t.:} \quad \text{UR}_j^{(k)} \geq \gamma_j^{(k)}, \quad j \in \mathcal{J}_i, \quad 1 \leq i \leq N, 1 \leq k \leq K, \tag{5b}$$

$$\text{SR}_i^{(l)} \geq \theta_i^{(l)}, \quad 1 \leq i \leq N, \quad 1 \leq l \leq L, \tag{5c}$$

$$\mathcal{J}_i \in \mathcal{J}, \quad \forall i, \tag{5d}$$

where $\text{UR}_j^{(k)}$ and $\text{SR}_i^{(l)}$ refer to the k^{th} requirement of user j and l^{th} requirement of server i, respectively. Using the same argument as with (4) one can conclude that (5) is a non-convex optimization problem. Therefore, a solution to (5) is not derived through traditional methods. Moreover, in some cases, meeting all user-side and server side requirements might be impossible at the same time. Moving the constraints into objective function, one can reformulate (5) as

$$\underset{\mathcal{J}_1,\mathcal{J}_2,\ldots,\mathcal{J}_N}{\text{minimize}} \quad \max_i f_i(\boldsymbol{\delta}, \text{UR}_1^{(1)}, ..., \text{UR}_{M_i}^{(K)}, \text{SR}_1^{(1)}, ..., \text{SR}_N^{(L)}) \tag{6a}$$

$$\text{s.t.:} \quad \mathcal{J}_i \in \mathcal{J}, \quad 1 \leq i \leq N, \tag{6b}$$

where M_i denotes the number of devices in cell i and balancing function is defined as

$$f_i(\boldsymbol{\delta}, \text{UR}_1^{(1)}, ..., \text{UR}_M^{(K)}, \text{SR}_1^{(1)}, ..., \text{SR}_N^{(L)}) \tag{7a}$$

$$= \int_{\mathcal{J}_i} \delta_j + \sum_{k=1}^{K} \kappa_k (e^{[\gamma_j^{(k)} - UR_j^{(k)}]^+} - 1) dj + \sum_{l=1}^{L} \lambda_l (e^{[\theta_i^{(l)} - SR_i^{(l)}]^+} - 1) \tag{7b}$$

where

$$[x]^+ = \begin{cases} x, & x \geq 0 \\ 0. & x < 0 \end{cases}$$

The performance of the optimization problem (6) depends on the constraint weighting factors $\kappa_k \geq 0$ and $\lambda_l \geq 0$ for all k and l, which are derived empirically.

In (6), the user-side and server-side constraints are moved to the objective function which transforms the optimization problem to be similar to (4). Therefore, to solve (6), H-PVA in Table I is proposed which takes similar algorithmic steps as in [21, Algoritm 1] and [22, TABLE 1] with a new balancing function f_i. As f_i is defined in (7), the objective function of (6) increases drastically with a minor violation of the user-side or server-side constraints, (5b) and (5c), respectively. Defining the locations of users by $j \triangleq (x, y)$, we rewrite the demand centroid of a cell as

$$c_i' \triangleq \frac{\iint_{\mathcal{J}_i} (x, y) \Big(\delta(x, y) + \sum_{k=1}^{K} \kappa_k (e^{[\gamma^{(k)}(x,y) - UR^{(k)}(x,y)]^+} - 1) \Big) dx dy}{\iint_{\mathcal{J}_i} \delta(x, y) + \sum_{k=1}^{K} \kappa_k (e^{[\gamma^{(k)}(x,y) - UR^{(k)}(x,y)]^+} - 1) dx dy}, \tag{8}$$

where the mass centroid is only a function of demand and the user-side constraints.

The cell planning steps of H-PVA are as follows: First, random locations for N BSs are chosen in the target area. Second, Voronoi tessellation associated to the current BSs are derived using (2) with the current weights. Third, with the current BS locations and area tessellation, UR and SR are calculated for all users and servers, respectively. Forth, BS are moved to the mass centroid calculated by (8) and new tessellation is derived using (2). Fifth, with the current BS location and area tessellation, the balancing functions f_i are calculated for every i. Sixth, the Voronoi weights are calculated such that the cells with higher f_i have higher weights w_i and vice versa. Finally, by updating UR and SR for all users and servers, respectively, the BSs are moved to new centroids using (8) and new Voronoi tessellation is derived using (2) with updated weights. This procedure is repeated until the BS locations converge.

As a result of H-PVA, we have a cell planning with BSs at the locations wherein f_is are minimized at $f_1 = f_2 = ... = f_N$. Ideally, the equality $V_i = f_k$ for all i and k indicates that all the user-side and server-side constraints are accurately met and the volume shares at cells are balanced.

B. Special case: Planning with minimum SINR requirements

To illustrate the performance of H-PVA in Table I and providing the framework for the numerical results in section IV, we take into account only one user-side requirement in the network, minimum SINR requirement, i.e., $K = 1$ and $L = 0$. To this end we consider cell planning to balance the cell volumes while taking into account the SINR requirements imposed by the devices in the target area. The optimization problem is formulated as

$$\underset{\mathcal{J}_1,\mathcal{J}_2,...,\mathcal{J}_N}{\text{minimize}} \quad \max_i f_i(\boldsymbol{\delta}, SINR_1, ..., SINR_M) \tag{9a}$$

$$\text{s.t.:} \quad \frac{P_i h_{ij}}{\sum_{k\neq i}^{N} P_k h_{kj} + \sigma^2} \triangleq SINR_j \geq \gamma_j, \quad j \in \mathcal{J}_i, \ \ \forall i, \tag{9b}$$

$$\mathcal{J}_i \in \mathcal{J}, \ \ \forall i, \tag{9c}$$

where from (7), the balancing function f_i is defined as

$$f_i = \int_{\mathcal{J}_i} \delta_j + \kappa(e^{[\gamma_j - SINR_j]^+} - 1)dj, \tag{10}$$

and (9b) refers to SINR constraints of the devices. Moreover, P_i and h_{ij} denote the power of the i^{th} BS and the channel power gain between the device j and the i^{th} BS. We assume that all BSs are working at full load [1] resulting in the worst case interference scenario such that the devices receive interference from all BSs. At the same time, this implies that there are enough radio resources available in the network such that the cells are not overloaded. In (10), the parameter κ strikes a balance between demand and SINR requirement through all the cells. So, we rewrite the centroid of a cell as

$$c_i' \triangleq \frac{\iint_{\mathcal{J}_i}(x,y)\Big(\delta(x,y) + \kappa(e^{[\gamma(x,y) - SINR(x,y)]^+} - 1)\Big)dxdy}{\iint_{\mathcal{J}_i}\delta(x,y) + \kappa(e^{[\gamma(x,y) - SINR(x,y)]^+} - 1)dxdy}, \tag{11}$$

Now, by applying (11) at each iteration in H-PVA, BSs are moved to the new mass centroids calculated based on demand distribution and SINR values. In cell planning for the network with minimum SINR requirements, mass centroid (11) is calculated in line 2, SINR values for all devices are calculated in line 4, and balancing functions (10) are calculated in line 6 of H-PVA in Table I. As a result of H-PVA, the balancing functions satisfy $f_1 = f_2 = ... = f_N$ and ideally, the base stations are located in the sites that make the second term of the integrand in (10) zero meaning that all SINR constraints are satisfied.

Suppose a group of devices with large minimum SINR requirement are accumulated at a small area $\mathcal{A}$ within the cell k in the target area. If the serving BS is not close enough to the area $\mathcal{A}$, the received SINRs will be smaller than the minimum requirement which increases the balancing function f_k and therefore, by applying H-PVA, shrinks the k^{th} cell size. The mass centroid calculated by (11) veers toward the area $\mathcal{A}$ which moves the location of the k^{th} BS closer to $\mathcal{A}$. Now, as soon as the k^{th} BS is close to $\mathcal{A}$, the second term of the integral in (10) converges to zero and the optimization problem (9) strikes a balance among the balancing functions (here, traffic volume shares). As a result, the cell planning is performed satisfying the non-uniform minimum SINR distribution as well as balancing the traffic volumes of the cells at the same time.

TABLE I
NETWORK PLANNING USING HETEROGENEOUS POWER VORONOI ALGORITHM

```
Definitions
$\mathcal{J} = [\mathcal{J}_1, \mathcal{J}_2, ..., \mathcal{J}_N]^T$,    % cell areas.
$\mathcal{C}^p = [\mathcal{C}_1^p, \mathcal{C}_2^p, ..., \mathcal{C}_N^p]^T$,    % AP locations
$\delta = [\delta_1, \delta_2, ..., \delta_N]^T$,    % demands
$\text{UR}^{(k)} = [\text{UR}_1^{(k)}, ..., \text{UR}_M^{(k)}]^T$,    % User-side requirements
$\text{SR}^{(l)} = [\text{SR}_1^{(l)}, ..., \text{SR}_N^{(l)}]^T$,    % Server-side requirements
$f = [f_1, ..., f_N]^T$,    % objective value calculated by (7)
$\mathcal{W} = [w_1, w_2, ..., w_N]^T$,    % weights.

Initialization
$N$,    % the number of cells
$M$,    % the number of users
$\epsilon$,    % small enough value
$p \leftarrow 1$,    % weight updating step counter
$\mathcal{C}^0$,    % random AP locations
$\kappa = [\kappa_1, ..., \kappa_K]$    % weighting factor
$\lambda = [\lambda_1, ..., \lambda_L]$    % weighting factor
$\gamma^{(k)} = [\gamma_1^{(k)}, ..., \gamma_M^{(k)}]$    % User-side constraints
$\theta^{(l)} = [\theta_1^{(l)}, ..., \theta_N^{(l)}]$    % server-side constraints
$CC = [\gamma^{(l)}, ..., \gamma^{(K)}, \theta^{(1)}, ..., \theta^{(L)}]$
                                        % Compact form of constraints
$\mathcal{W}^0 \leftarrow \mathbf{0}$,
$S$,    % scale factor

H-PVA: Heterogeneous Power Voronoi Algorithm
1   While $\|\mathcal{C}^{p-1} - \mathcal{C}^{p-2}\| > \epsilon$ do
2       $\mathcal{C}^p \leftarrow \mathsf{Centroid}(\mathcal{J}, \delta, \text{UR}^{(1)}, ..., \text{UR}^{(K)}, \kappa, \gamma)$
            % mass centroid (8)
3       $(\mathcal{J}, \boldsymbol{\delta}) \leftarrow \mathsf{PowerVoronoi}(\mathcal{C}^p, \mathcal{W})$,
4       $[\text{UR}^{(1)}, ..., \text{UR}^{(K)}, \text{SR}^{(1)}, ..., \text{SR}^{(L)}] \leftarrow \mathsf{ServiceCal}(\mathcal{J}, \mathcal{C}^p)$,
5       $CS \leftarrow [\text{UR}^{(1)}, ..., \text{UR}^{(K)}, \text{SR}^{(1)}, ..., \text{SR}^{(L)}]$
                                        % Compact form of service values
6       $f \leftarrow \mathsf{fcal}(\mathcal{J}, \delta, CC, CS, \kappa, \gamma)$
            % $f_i$ calculation (7)
7       $\mathcal{W} \leftarrow \mathcal{W} + S \times \frac{\mathbf{tr}(\mathbf{diag}(f)\mathbf{1} - Nf}{\mathbf{tr}(\mathbf{diag}(f))}$,
8       $p \leftarrow p + 1$
9   End While
10  Return
```

IV. NUMERICAL RESULTS

In this section, we provide numerical simulations to show the performance of H-PVA for cell planning in a synthetic target area when devices have different QoS requirements. To this end, we optimize BS site locations using H-PVA in Table I for the synthetic spatial demand distribution with I) without minimum SINR requirement, II) uniform spatial minimum SINR requirement $\gamma(x,y) = 0$[dB] at all locations, III) a synthetic spatial distribution for minimum SINR requirement. To provide a fair comparison between our results and the prior art, the same assumptions as in [21] are made as

- The number of BSs: $N = 30$.
- Total volume: $V = 90\text{s}/130\text{ms} = 692.3$.
- Minimum user rate: $R_{min} = 1\text{Mbps}$.
- System bandwidth: 25 MHz.
- Channel gain: $h_{ij} = d_{ij}^{-3}$, where d_{ij} refers to the distance between BS i and device j.
- Noise power: $\sigma^2 = 0$ such that the system is interference limited.
- Uniform and normalized BS powers: $\mathbf{P} = \mathbf{1}$.

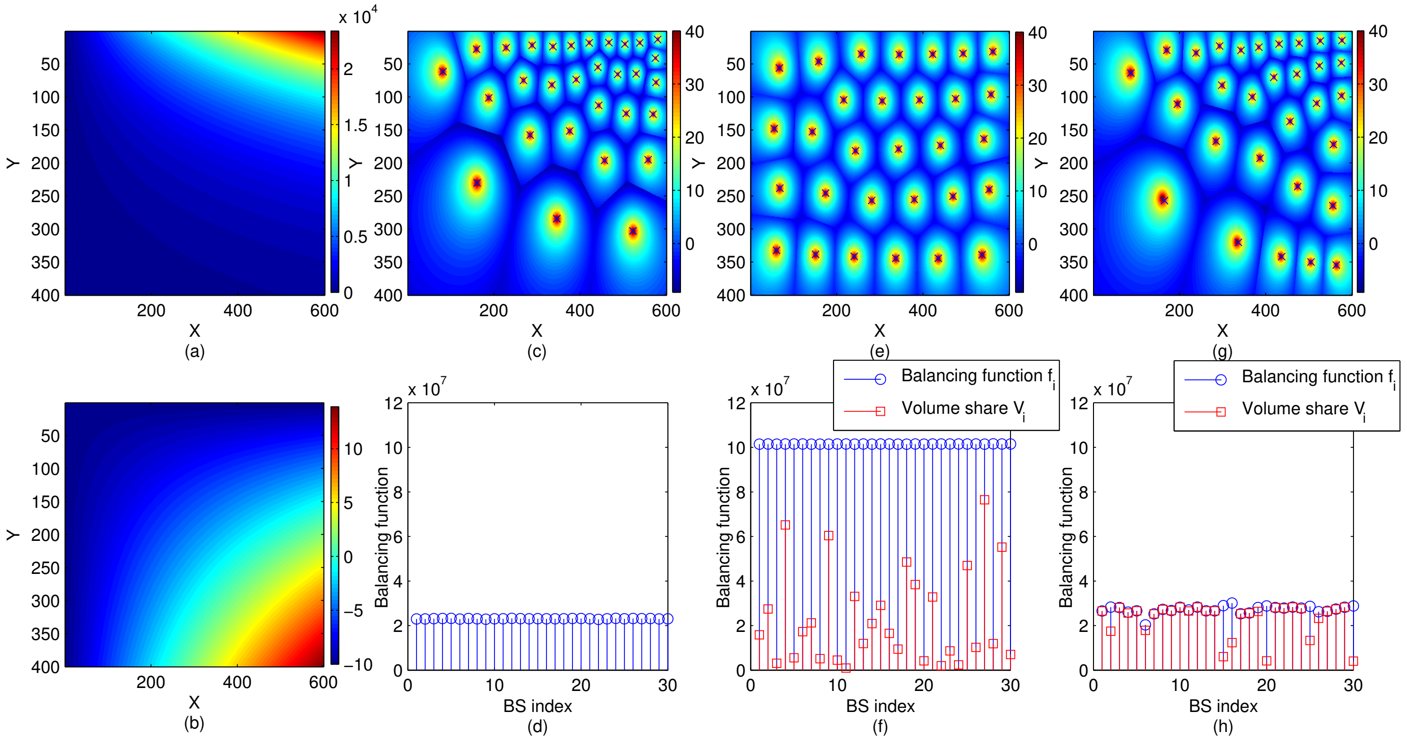

Fig. 2. (a): Target area with spatial demand distribution $\delta^*(x,y) = \frac{x}{100}e^{-\frac{y}{100}}$, (b): target area with spatial minimum SINR requirement distribution $\gamma^*(x,y) = -10 + 10^{-4}xy$[dB], (c): cell planning using H-PVA in Table I and resulting SINR for demand distribution δ^* without minimum SINR requirement, i.e. $\kappa = 0$, (d): resulting balancing function f_i in the cell planning for demand distribution δ^* without minimum SINR requirement, i.e. $\kappa = 0$ shown in Fig. 2. c, (e): cell planning using H-PVA in Table I and resulting SINR distribution for demand distribution δ^* and minimum SINR requirement $\gamma(x,y) = 0$[dB] for all x and y, (f): resulting volume and balancing function in cell planning for demand distribution δ^* and minimum SINR requirement $\gamma(x,y) = 0$[dB] for all x and y, shown in Fig. 2. e, (g) cell planning using H-PVA in Table I and resulting SINR distribution for demand distribution δ^* and minimum SINR requirement γ^*[dB], (h): resulting volume shares and the balancing function f_i in the cell planning for demand distribution δ^* and minimum SINR requirement distribution γ^* shown in Fig. 2. g.

The performance of the proposed method is illustrated in Fig. 2 (a)-(h). Fig. 2 a) shows a synthetic spatial demand distribution of the devices given by $\delta^*(x,y) \triangleq \delta^* = \frac{x}{100}e^{-\frac{y}{100}}$. It is worth noting that the total volume here is normalized such that $V = \int_{\mathcal{J}} \delta^*(x,y)dxdy = 692.3$[Mbps]. Figure 2. b) illustrates a synthetic spatial distribution of the minimum SINR requirements with $\gamma^*(x,y) \triangleq \gamma^* = -10 + 10^{-4}xy$[dB]. As it can be seen, the highest SINR requirement is concentrated in the lower right part of the target area.

Fig. 2. c) shows the locations of the base stations resulting from the optimization problem (9) using H-PVA in TABLE I when $\kappa = 0$. Thus, the cell planning is performed to balance the volume shares regardless of the SINR requirements. As it can be seen in Fig. 2. c), the BSs are concentrated in the upper right part to serve the high traffic zone, while large cells are located in the lower left part where the demand is low. Fig. 2. d) shows the respective volume shares per cell balanced around 23 Mbps [21], [22]. It is observed that with the cell planning in Fig. 2. c), approximately 38% and 7.5% of the target area receive SINR less than 0[dB] and -4 [dB], respectively.

Figure 2. e) presents the cell planning for the demand distribution in Fig. 2 a) with $\kappa = 10000$ and a minimum SINR requirement $\gamma(x,y) = 0$[dB] for all x and y using H-PVA in Table I. Figure 2. f) shows the respective traffic volume shares V and balancing function f_i. As it can be seen in Fig. 2. e), the BSs are located almost regularly and only 22% and 0.7% of the target area is served by SINR less than 0[dB] and -4[dB], respectively. Therefore, Fig. 2. e) in comparison to Fig. 2. c) results in 42% and 90% decrease in the areas with SINR less than 0 and -4[dB], respectively. Although the cell planning in Fig. 2. e) fails to yield a network topology to serve all the users with SINR higher than 0[dB], it decreases the number of users that violate the minimum SINR requirement. Although the network in Fig. 2. e) is planned based on traffic demand and SINR requirement distributions, the minimum SINR requirement is not satisfied for all devices. As a result, to fully meet minimum SINR requirement in the target area, more sophisticated methods such as power control is needed.

Finally, Fig. 2. g) shows the cell planning in the target area with demand and minimum SINR requirement distributions $\delta^*(x,y)$ and $\gamma^*(x,y)$ presented in Fig. 2. a) and b), respectively, with $\kappa = 5000$. Furthermore, Fig. 2. h) shows the respective traffic volume and balancing function. As shown in Fig. 2. g), the BSs are aggregated to the upper right and lower right regions because of the concentration of demand and higher minimum SINR requirements. Ideally, in the optimum point, $V_i = f_i$ and $V_i = V_j$ for all i and $j \neq i$ which would imply that all SINR requirements are met and the volume shares are equally distributed.

V. CONCLUSION

We studied the planning of wireless cellular networks with the goal of obtaining uniform traffic volume shares among the cells in case of non-uniform traffic demand and user-side (such as SINR, latency, etc.), and server-side requirements (such as load and resource consumption, etc.) by minimizing a set of balancing functions. Then, we introduced a heterogeneous power Voronoi algorithm to strike a balance between the two cell planning goals; balanced traffic volumes per cell and meeting user-side and server-side requirements. Specifically, we apply the proposed method for cell planning in an area with non-uniform traffic and minimum SINR requirement distributions. It was observed that cells have smaller sizes in the areas with high minimum SINR requirement and high traffic demand, while high and uniform SINR requirement distribution yields a regular network topology. The proposed method paves the way to cellular network planning taking into account diverse performance requirements characteristic to wireless communication systems.

APPENDIX

APPENDIX A

To prove the non-convexity of the optimization problem (4), it is sufficient to prove that the objective function V_i is a non-convex function.

The terms V_i are functions in terms of the set of BS locations $\boldsymbol{C}$, [23]. Assume that $\mathcal{J}_i'$, $\mathcal{J}_i''$, and $\mathcal{J}_i'''$ refer to the i^{th} cell area derived by Voronoi tessellation originated (with zero weights) from the set of N BSs $\boldsymbol{C'}$, $\boldsymbol{C''}$, and $\dfrac{\boldsymbol{C'}+\boldsymbol{C''}}{2}$, respectively. Generally, the non-empty area $\mathcal{J}_i''' - \mathcal{J}_i' \cup \mathcal{J}_i'' = \mathfrak{J} \neq \emptyset$ shows that the area $\mathfrak{J}$ is a subset of $\mathcal{J}_i'''$ and has no common area with $\mathcal{J}_i'$ and $\mathcal{J}_i''$. So, using the fact that the demand distribution is arbitrarily non-uniform, it is assumed that the demand distribution in $\mathfrak{J}$ is very large. Also from (1), the volume share at each cell is a function of demand distribution as well as the network topology. Then, we have

$$\begin{aligned} V_i(\frac{\boldsymbol{C}'+\boldsymbol{C}''}{2}, \boldsymbol{\delta}) = \int_{\mathcal{J}_i'''} \delta_j dj \geq \int_{\mathfrak{J}} \delta_j dj > \frac{\int_{\mathcal{J}_i'} \delta_j dj + \int_{\mathcal{J}_i''} \delta_j dj}{2} \\ = \frac{V_i^{(t)}(\boldsymbol{C}', \boldsymbol{\delta}) + V_i(\boldsymbol{C}'', \boldsymbol{\delta})}{2}, \end{aligned} \tag{12}$$

which shows that $V_i(\boldsymbol{C}, \boldsymbol{\delta})$ is a non-convex function of $\boldsymbol{C}$ or equivalently $\mathcal{J}$.

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