Tedeschi, Nicola; Frezza, Fabrizio; Sihvola, Ari

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- Comparison with a metamaterial absorber proposed in the literature

Correspondence to:
N. Tedeschi,
tedeschi@diet.uniroma1.it

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Electromagnetic interaction with uniaxial metamaterials

N. Tedeschi[1], F. Frezza[1], and A. Sihvola[2]

[1] Department of Information Engineering, Electronics and Telecommunications, "La Sapienza" University of Rome, Rome, Italy, [2] Department of Radio Science and Engineering, Aalto University, Espoo, Finland

Abstract In this paper, the reflection behavior of a particular class of metamaterials, strongly connected with the realization of the DB boundary conditions (so called due to the vanishing of the normal components of the **D** and **B** fluxes) and the soft and hard (SH) boundary conditions, is presented. The metamaterial under study is modeled as an anisotropic, uniaxial, material with both permittivity and permeability dyadics. We consider several characteristics of such medium: presenting the critical angle of total transmission for the SH/DB material, analyzing its behavior as a function of the longitudinal permittivity and permeability, for both positive and negative values, and presenting its applications to the electromagnetic absorbers and to the polarization inverters.

1. Introduction

Electromagnetic metamaterials are widely studied in the literature, and in the last decade several structures have been proposed with characteristics beyond those of natural media. Their applications concern several areas, e.g., enhancing antennas properties, electromagnetic absorbers, frequency filters, guiding surfaces, beam focusing, and many others [*Capolino*, 2009]. An interesting area where the metamaterials seem to be of great importance is the realization of electromagnetic boundary conditions: starting from a condition on the electromagnetic components on an interface, there are several materials that are able to realize such condition, e.g., the perfect electromagnetic conductor (PEMC), or the DB boundary condition [*Lindell and Sihvola*, 2009; *Nayyeri et al.*, 2013]. The former requires the cancelation of the tangential components to the interface of both the electric and magnetic fields. The latter requires the cancelation of the perpendicular components of both the electric flux density, $\boldsymbol{D}$, and the magnetic flux density, $\boldsymbol{B}$:

$$\hat{\boldsymbol{n}} \cdot \boldsymbol{D}; \qquad \hat{\boldsymbol{n}} \cdot \boldsymbol{B} \tag{1}$$

where $\hat{\boldsymbol{n}}$ is the unit vector perpendicular to the interface. It is important to emphasize that given a boundary condition, several materials are able to realize it. For example, the realization of the DB boundary condition has been proposed by means of an anisotropic, uniaxial medium with both permittivity and permeability dyadics, both with the optic axis perpendicular to the interface and with zero axial components. However, another medium that realizes such boundary condition is a certain exotic material labeled as uniaxial skewon-axion medium [*Hehl and Obukhov*, 2003; *Lindell and Sihvola*, 2009]. Other boundary conditions proposed in the literature that are of great interest for both the electromagnetic theory and applications are the so-called soft and hard (SH) boundary conditions [*Kildal*, 1988]. Such conditions require the cancelation of a component tangential to the interface of both the electric and magnetic fields: the condition is either soft or hard, depending on the direction of such component, if it is either perpendicular or parallel to the plane of incidence, respectively. The SH boundary conditions and the DB condition can be merged in a more general boundary condition called SH/DB boundary condition. It requires the cancelation of the electric and magnetic fields along a generic direction, let us call such direction $\hat{\boldsymbol{c}}$:

$$\hat{\boldsymbol{c}} \cdot \boldsymbol{E}; \qquad \hat{\boldsymbol{c}} \cdot \boldsymbol{H} \tag{2}$$

The realization of such boundary condition has been proposed by means of a uniaxial material with the optic axis along $\hat{\boldsymbol{c}}$ and with zero longitudinal components of the permittivity and permeability dyadics [*Tedeschi et al.*, 2013a]. Another possible realization has been proposed by means of a uniaxial skewon-axion medium [*Lindell and Sihvola*, 2013]. These media present many applications [*Elser et al.*, 2006; *Silverinha and Engheta*, 2007; *Alekseyev et al.*, 2010; *Sun and Yu*, 2012; *Engheta and Alú*, 2011]. For example, in electromagnetic cloaking [*Valagiannopoulos and Tsitsas*, 2012; *Engheta and Alù*, 2008] and in imaging [*Culhaoglu et al.*, 2014].

In the present paper, we want to analyze the behavior of the anisotropic uniaxial medium, able to realize the SH/DB boundary condition, in several situations. We will consider the condition $\overline{\overline{\epsilon}} = \overline{\overline{\mu}}$ for the metamaterial, we call such relation as matching condition. This kind of materials have been widely studied in the literature. First of all, in the isotropic case, they are useful for reducing backscattering reflection in anechoic chambers and from stealthy objects. Furthermore, when anisotropic metamaterials are considered, the matching condition can be obtained with a proper designing of the structure: if the medium is designed with inclusions in Cartesian arrangement, then the eigenaxes of both the permittivity and the permeability are the same. To get also the same eigenvalues for the two dyadics requires balancing between the electric and magnetic moments in the three directions [*Lindell et al.*, 2009; *Karilainen and Tretyakov*, 2012]. The electromagnetic absorbers realized by metamaterials are an example of application where the matching condition must be satisfied. In *Landy et al.* [2008], a perfect metamaterial absorber, working at the microwave frequencies, has been presented. Afterward, several other metamaterial absorbers have been proposed in different frequency ranges [*Grant et al.*, 2011; *Dayal and Ramakrishna*, 2012]. The analysis of such absorbers has always been done in the simplifying assumption of isotropic metamaterials. Under such hypothesis, the permittivity and permeability of the material at the absorbing frequency respect the matching condition $\epsilon = \mu$. However, being structurally stratified metamaterials, they are anisotropic media, either uniaxial or biaxial, depending on the transverse shape of the unit cell with respect to the stratification direction. As a consequence the matching condition for these metamaterials is an equality between dyadics.

First of all, we consider the reflection coefficient for several values of the transverse components of the permittivity and permeability dyadics, obtaining an analytical expression for the angle of total transmission predicted in *Tedeschi et al.* [2013a]. Moreover, the reflection by an interface with a uniaxial medium with negative electromagnetic characteristics is analyzed. Furthermore, we consider the relation between the DB medium and the perfectly matched layer presented in *Tedeschi et al.* [2013b], and we show its behavior with respect to an isotropic matched material, often applied in the literature to model the metamaterial absorbers. Finally, we analyze the reflection at the interface of a particular uniaxial metamaterial able to invert the polarization of the incident wave, i.e., an interface where the reflected wave has an opposite polarization with respect to the one of the incident wave.

In section 2, we briefly summarize the theoretical results for the generic uniaxial medium presented in *Tedeschi et al.* [2013a]. In section 3, we consider the reflection at the interface between an isotropic medium and the uniaxial one in some special cases. In section 4, the conclusions are drawn.

In the following, we assume a time dependence of the form $e^{-i\omega t}$. Moreover, the dyadics $\overline{\overline{\epsilon}}$ and $\overline{\overline{\mu}}$ are the dimensionless, relative electric permittivity and magnetic permeability, respectively. Furthermore, we indicate all the unit vectors by bold letters with a hat, e.g., $\hat{\boldsymbol{c}}$.

2. Theoretical Formulation

In this section, we define the problem of the reflection of a plane wave by a uniaxial medium.

Let us consider a plane wave propagating in an isotropic medium in the half-space $z < 0$, see Figure 1. The half-space $z > 0$ is filled with a uniaxial medium with the following relative permittivity and permeability dyadics:

$$\overline{\overline{\epsilon}} = \epsilon_t \overline{\overline{I}}_t + \epsilon_c \hat{\boldsymbol{c}}\hat{\boldsymbol{c}} \quad \text{and} \quad \overline{\overline{\mu}} = \mu_t \overline{\overline{I}}_t + \mu_c \hat{\boldsymbol{c}}\hat{\boldsymbol{c}} \tag{3}$$

with $\overline{\overline{I}}_t = \overline{\overline{I}} - \hat{\boldsymbol{c}}\hat{\boldsymbol{c}}$ being the two-dimensional unit dyadic. When the longitudinal part of these dyadics is zero, the interface with this medium realizes the boundary conditions in (2). Let us note that this medium is quite different from an ordinary uniaxial medium, in fact, the ordinary uniaxial media have only the electric permittivity, i.e., the permittivity is a dyadic, but the permeability is a scalar quantity. The implementation of the SH/DB with this medium is easily obtained when the longitudinal characteristics approach zero: in fact, in this case, the components of the electric and magnetic field parallel to the optics axis approach zero, too.

The dispersion equation in this uniaxial medium is the following:

$$\left(\boldsymbol{k} \cdot \overline{\overline{\epsilon}} \cdot \boldsymbol{k} - \epsilon_c k_t^2\right)\left(\boldsymbol{k} \cdot \overline{\overline{\mu}} \cdot \boldsymbol{k} - \mu_c k_t^2\right) = 0 \tag{4}$$

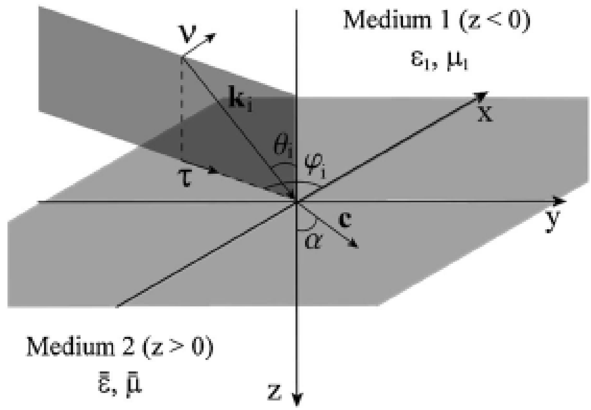

Figure 1. Geometry of the problem.

where $\boldsymbol{k}$ is the propagation vector of the electromagnetic wave and $k_t = \omega\sqrt{\epsilon_t \mu_t}$ is the transverse wave number. From the dispersion equation, the polarization vector of the two waves, allowed to propagate in the medium, can be obtained as a function of the optic axis $\hat{\boldsymbol{c}}$ and of the propagation vector $\boldsymbol{k}$:

$$\boldsymbol{e}_E = \hat{\boldsymbol{k}}_E \times \hat{\boldsymbol{c}} \qquad \boldsymbol{e}_H = \hat{\boldsymbol{c}} - \hat{k}_{H\parallel}\hat{\boldsymbol{k}}_H \tag{5}$$

$$\boldsymbol{h}_E = -\frac{1}{\zeta_t}\left(\hat{\boldsymbol{c}} - \hat{k}_{E\parallel}\hat{\boldsymbol{k}}_E\right) \quad \boldsymbol{h}_H = \frac{1}{\zeta_t}\left(\hat{\boldsymbol{k}}_H \times \hat{\boldsymbol{c}}\right) \tag{6}$$

The polarization vectors of the electric field are dimensionless; on the other hand, the vectors of the magnetic field have the dimensions of an admittance.

Starting from the expressions (5) and (6) and following a procedure similar to that developed in *Chen* [1983], the reflection and transmission coefficients of the interface with this medium can be evaluated. The explicit expression of such coefficients is presented in *Tedeschi et al.* [2013a]. The reflection coefficients are in a matrix form, connecting the incident and the reflected waves in both the polarizations:

$$\begin{pmatrix} E_r^{TE} \\ E_r^{TM} \end{pmatrix} = \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix} \cdot \begin{pmatrix} E_I^{TE} \\ E_I^{TM} \end{pmatrix} \tag{7}$$

The obtained expressions are of particular interest because they allow us to compute the electromagnetic wave reflected by a general uniaxial medium with any values of the parameters (3).

The components of the reflection matrix depend on the transmitted wave vector in the anisotropic medium. The components of such vector parallel to the interface are the same of the incident wave vector, while the component perpendicular to the interface can be computed by the following expression:

$$k_z = -\frac{\boldsymbol{k}_\tau \cdot \bar{\bar{\epsilon}} \cdot \hat{\boldsymbol{u}}_z}{\hat{\boldsymbol{u}}_z \cdot \bar{\bar{\epsilon}} \cdot \hat{\boldsymbol{u}}_z} + \left[\left(\frac{\boldsymbol{k}_\tau \cdot \bar{\bar{\epsilon}} \cdot \hat{\boldsymbol{u}}_z}{\hat{\boldsymbol{u}}_z \cdot \bar{\bar{\epsilon}} \cdot \hat{\boldsymbol{u}}_z}\right)^2 - \frac{\left(\boldsymbol{k}_\tau \cdot \bar{\bar{\epsilon}} \cdot \boldsymbol{k}_\tau\right) - \epsilon_c k_t^2}{\hat{\boldsymbol{u}}_z \cdot \bar{\bar{\epsilon}} \cdot \hat{\boldsymbol{u}}_z}\right]^{1/2}. \tag{8}$$

This expression holds in *TE* polarization; the expression in *TM* polarization can be obtained by the duality principle.

3. Reflection at the Interface With Uniaxial Metamaterials

In this section, the behavior of the coefficients developed in *Tedeschi et al.* [2013a] is analyzed in some interesting cases, i.e., for some particular anisotropic media.

In the following, we will often consider the condition $\bar{\bar{\epsilon}} = \bar{\bar{\mu}}$ for the metamaterial, we call such relation as matching condition.

If the matching condition holds, the reflection matrix becomes symmetric and the diagonal elements become equal to one another, because the material presents the same response to the electric and the magnetic fields. Moreover, we suppose that the propagation vector of the incident wave, the optic axis, and the vector perpendicular to the interface are coplanar. In this case the nondiagonal coefficients are zero, as usual for uniaxial materials [*Chen*, 1983]. The behavior of the diagonal coefficients is shown in Figure 2 as a function of the

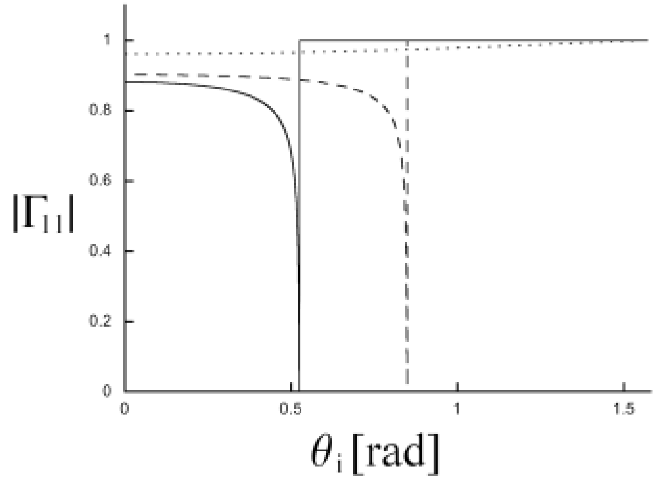

Figure 2. Magnitude of the reflection coefficient Γ_{11} as a function of the azimuthal incident angle, when medium 1 is air and medium 2 has $\alpha = 30°$, longitudinal parameters $\epsilon_c = \mu_c = 0.001$ and for different values of the transverse parameters $\epsilon_t = \mu_t$: 1 (solid line), 1.5 (dashed line), and 10 (dotted line). The incident wave has the propagation vector on the (y, z) plane.

azimuthal incident angle θ_i, when $\alpha = 30°$, $\epsilon_c = \mu_c = 0.001$ and for different values of the transverse parameters. We can see that the reflection coefficient approaches zero in the cases $\epsilon_t = \mu_t = 1$ and $\epsilon_t = \mu_t = 1.5$, for two different values of the incident angle, let us call these values "critical angles," θ_c. We have to emphasize that the function is continuous in a neighborhood of θ_c. It becomes discontinuous only in the limit $\epsilon_c, \mu_c \to 0$. The critical angle can be defined as the incident angle for which the transmitted wave is parallel to the optic axis: in this case, in fact, the wave is totally transmitted [*Tedeschi et al.*, 2013a]. In the limit $\epsilon_c, \mu_c \to 0$, the Snell law holds, and the condition for the critical angle can be written as follows:

$$\sqrt{\epsilon_1 \mu_1} \sin \theta_c = \sqrt{\epsilon_t \mu_t} \sin \alpha \tag{9}$$

In the case $\epsilon_t = \mu_t = 1$, the critical angle is $\theta_c = \alpha = 30°$: being the transverse parameters of the uniaxial medium equal to unity, when the incident wave is parallel to the optic axis, it does not "see" differences between the two media. Therefore, the reflection coefficient is zero at this angle. In the second case, when $\epsilon_t = \mu_t = 1.5$, the total transmission occurs for a larger value of the incident angle: in fact, because of the difference between the refractive indexes, the transmission angle is parallel to the optic axis for an incident angle $\theta_i > \alpha$; applying equation (9), we find the value $\theta_c \approx 0.85$ rad. In the third case, when $\epsilon_t = \mu_t = 10$, the transmitted wave vector is never parallel to the optic axis and the reflection coefficient is never zero, because the relation (9) returns a value of $\sin \theta_c$ greater than unity. The amplitude of the reflection coefficient for angles greater than the critical angles is equal to unity because of the total reflection effect, due to the small values of the longitudinal permittivity and permeability. On the other hand, for values of the incident angle lower than the critical angle, the reflection coefficient is close, but not equal, to unity. It can be shown that, when the longitudinal parameters approach zero, the reflection coefficient around zero approaches unity.

In order to show the behavior for the case of zero axial permittivity and permeability, in Figure 3 the magnitude of the reflection coefficient $|\Gamma_{11}|$ is shown at normal incidence, $\theta_i = 0$, as a function of the anisotropy, i.e., as a function of the longitudinal properties, ϵ_c and μ_c, of the dyadics. The optics axis forms an angle $\alpha = 30°$ and the transverse properties of the dyadics have been considered equal to unity, $\epsilon_t = \mu_t = 1$. From Figure 3, it is clear that the interface behaves as a perfect reflector when the longitudinal properties approach zero, i.e., when the interface behaves as a SH/DB boundary. The reflection coefficient approaches zero when the longitudinal characteristics become equal to unity, because the uniaxial medium becomes a vacuum. For greater values the reflection coefficient grows slowly. If we consider negative values of ϵ_c, we find that the material behaves as a perfect reflector until a certain critical value $\epsilon_c^* < 0$, and for lower values, the magnitude of the reflection

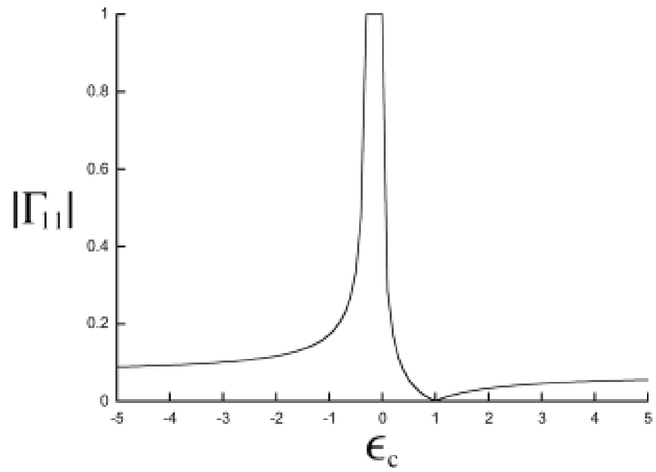

Figure 3. Magnitude of the reflection coefficient Γ_{11} as a function of the longitudinal parameters of the second medium, $\epsilon_c = \mu_c$, when medium 1 is air and medium 2 has $\alpha = 30°$, transverse parameters $\epsilon_t = \mu_t = 1$. The incident wave has the propagation vector on the (y, z) plane, i.e., $\varphi_I = 90°$, with $\theta_I = 0°$.

coefficient starts to decrease. To understand such behavior, we must consider the transmitted wave vector at normal incidence, which can be obtained from the expression (8), by imposing $\boldsymbol{k}_s = 0$:

$$k_z = k_t \sqrt{\frac{\epsilon_c}{\epsilon_t \sin^2 \alpha + \epsilon_c \cos^2 \alpha}}. \tag{10}$$

From this expression, we can see that when $\epsilon_c < 0$, the k_z becomes purely imaginary, i.e., no wave can propagate in the metamaterial. This condition holds until the denominator under the square root remains positive, i.e., for $\epsilon_c/\epsilon_t > -\tan^2 \alpha$. When the ratio ϵ_c/ϵ_t becomes less than such critical value, the transmitted wave vector comes back to be positive and the transmitted wave starts to propagate again. As a consequence the metamaterial behaves as a perfect reflector for $\epsilon_c^* < \epsilon_c < 0$, with $\epsilon_c^* = -\epsilon_t \tan^2 \alpha$.

At this point, we note a strange effect: we saw before that the interface acts as a perfect reflector when $\epsilon_c \to 0$. Let us consider now the case in which $\epsilon_t = \epsilon_c^{-1}$ and $\alpha = 0$. In this case the interface becomes totally transparent to the radiation for any incident angle. This fact was predicted in the Perfectly Matched Layer (PML) theory [*Gedney*, 1996]. In Figure 4, the behavior of the reflection coefficient Γ_{11} is shown as a function of the longitudinal properties of the dyadics in a neighborhood of zero. In this case, we consider $\epsilon_t = \mu_t = 100$ and $\theta_I = 60°$. As we predicted, the reflection coefficient becomes zero when $\epsilon_c = 0.01 = \epsilon_t^{-1}$. However, when ϵ_c becomes smaller, the interface behaves again as a perfect reflector. We can recognize two different zones: when $\epsilon_c > 0.01$, the reflection coefficient grows with the longitudinal characteristics, as usual, because of the mismatch between the two media grows. When $0 < \epsilon_c < 0.01$, the reflection coefficient approaches unity: this is the total reflection zone.

As we previously said, in the past few years, the possible realization of a perfect metamaterial absorber has been widely investigated in the literature [*Landy et al.*, 2008; *Grant et al.*, 2011; *Dayal and Ramakrishna*, 2012]. The proposed metamaterials in such studies are considered isotropic and designed to be matched with a vacuum. The previous discussion suggests that enhanced absorbing performances can be obtained by designing the transverse and longitudinal parameters of a uniaxial metamaterial, as proposed in *Tedeschi et al.* [2013b]. In fact, an ideal lossless, matched uniaxial medium can behave as a PML, though the absence of dissipation makes it useless. Moreover, under particular conditions on the transverse and longitudinal permittivities, the total transmittivity of the interface can be kept also in the presence of losses. Just to show an example, we consider a comparison between the reflection coefficient, as a function of the incident angle, for the interface with three different materials: an isotropic metamaterial absorber, of the kind proposed in *Landy et al.* [2008], and two different uniaxial metamaterial absorbers of the kind proposed in *Tedeschi et al.* [2013b]. In all the three cases the media are considered matched with a vacuum, i.e., $\overline{\overline{\epsilon}} = \overline{\overline{\mu}}$. In Figure 5, the magnitude of the reflection coefficient Γ_{11} in these three cases is shown. The relative permittivity of the isotropic absorber is $\epsilon = 2 + i0.01$. The longitudinal and transverse relative permittivities of the two uniaxial media are (let us call

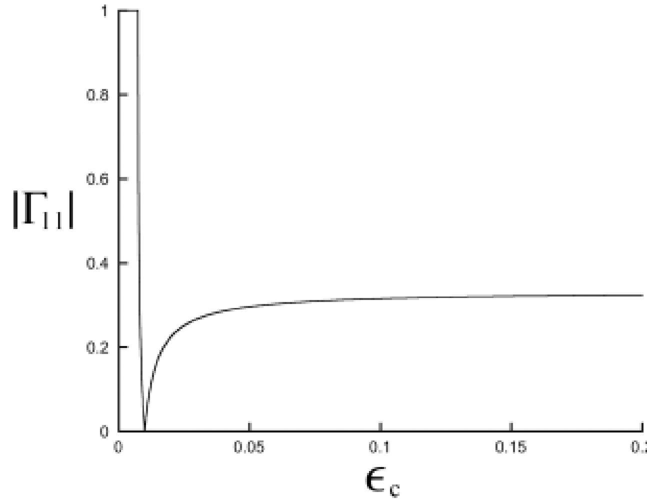

Figure 4. Magnitude of the reflection coefficient Γ_{11} as a function of the longitudinal parameters of the second medium, when medium 1 is air and medium 2 has $\alpha = 0°$, transverse parameters $\epsilon_t = \mu_t = 100$. The incident wave has the propagation vector on the (y, z) plane, i.e., $\varphi_i = 90°$, with $\theta_i = 60°$.

them cases 1 and 2): $\epsilon_{1c} = 0.01$, $\epsilon_{1t} = 100 + i$ and $\epsilon_{2c} = 0.01 + i0.001$, $\epsilon_{2t} = 100 + i$, respectively. We can see that, in case 1, the losses are only in the transverse permittivity, while in the case 2, the losses are both in the transverse and in the longitudinal permittivities. It is interesting that both the uniaxial materials have a reflection coefficient smaller than the isotropic absorber for any incident angle. It can be seen that by increasing the losses in the longitudinal permittivity of the uniaxial material, the behavior of the reflection coefficient tends to be equal to that of the isotropic absorber. This result proves how the analysis of uniaxial materials gives good chances to enhance the characteristics of the metamaterial absorbers. Moreover, it is important to analyze the effect of the losses on the reflection coefficient for a generic uniaxial medium.

Finally, let us consider the case when the incident propagation vector, the optic axis, and the perpendicular direction to the interface are not coplanar. In this case, when the ratio $\epsilon_t/\epsilon_c \rightarrow \infty$, the interface behaves as a polarization inverter [*Tedeschi et al.*, 2013a]. In Figure 6, the amplitude of the cross-reflection coefficient Γ_{12} is shown as a function of $\epsilon_t = \mu_t$, when $\epsilon_c = \mu_c = \epsilon_t^{-1}$. The optic axis, on the plane (y, z), forms an

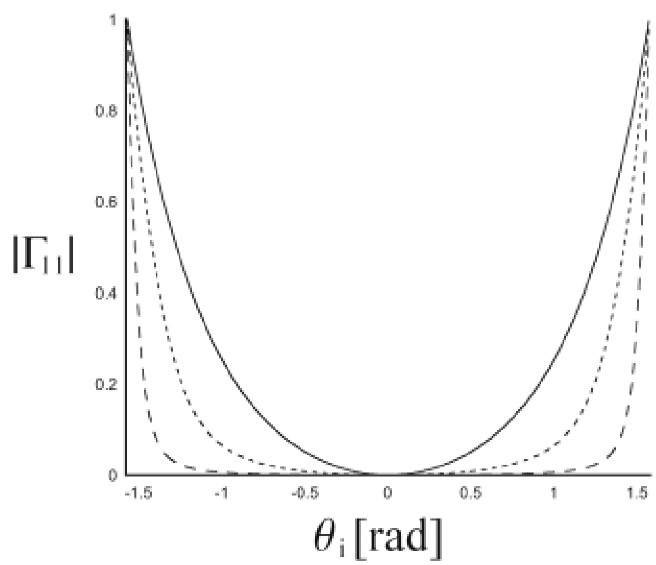

Figure 5. Magnitude of the reflection coefficient Γ_{11}, as a function of the incident angle, for three different interfaces: with an isotropic matched medium with relative permittivity $\epsilon = 2 + i0.01$ (solid line), with a uniaxial medium with relative permittivities $\epsilon_{1c} = 0.01$, $\epsilon_{1t} = 100 + i$ (dashed line), and with a uniaxial medium with relative permittivities $\epsilon_{2c} = 0.01 + i0.001$, $\epsilon_{2t} = 100 + i$ (dotted line).

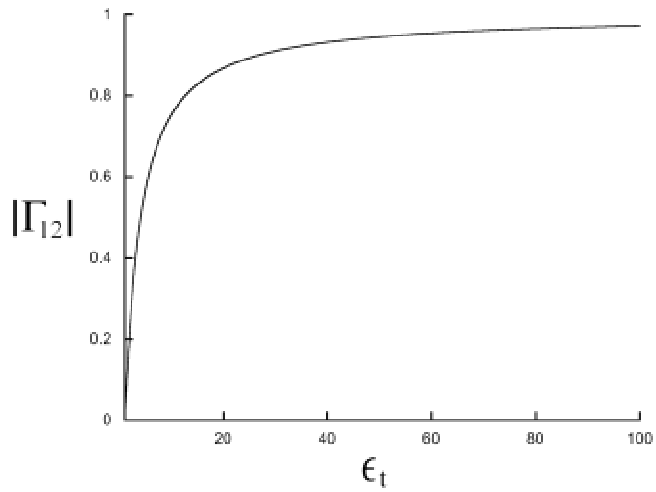

Figure 6. Magnitude of the reflection coefficient Γ_{12}, when medium 1 is air and medium 2 has $\alpha = 45°$, longitudinal parameters $\epsilon_c = \mu_c = \epsilon_t^{-1}$, as a function of the transverse parameters. The incident wave is supposed at normal incidence.

angle $\alpha = 45°$ with the perpendicular to the interface and the incident plane wave is supposed at normal incidence. Since the matching condition holds, we find that the metamaterial is polarization insensitive, i.e., $\Gamma_{11} = \Gamma_{22}$ and $\Gamma_{21} = -\Gamma_{12}$, where the minus sign in the second equality depends on the choice of the signs of the electric field in the two polarizations. For the metamaterial presented, we see that at normal incidence, the diagonal reflection coefficients are both zero, while the nondiagonal coefficients tend to unity when the ratio ϵ_t/ϵ_c grows. We can understand this behavior investigating the reflection matrix. It can be proved that when the matching condition holds and the optic axis forms an angle 45° with the perpendicular direction to the interface, then the eigenvectors of the reflection matrix are $\boldsymbol{\gamma}_{1,2} = (\pm i, 1)$ [*Tedeschi et al.*, 2013a]. Such eigenvectors represent the polarizations that are not changed by reflection on the interface. We can see that these polarizations are the circular ones. This is because the circular polarization can be seen as a superposition of two linear polarizations with the same magnitude. Each linear polarization is inverted, and the reflected wave is still circularly polarized. As a consequence, if we consider linear polarized waves, an incident wave in the *TE* (or *TM*) polarization generates a reflected wave with *TM* (or *TE*) polarization. Therefore, this particular implementation of the SH/DB boundary has the property to invert the polarization of the incident wave.

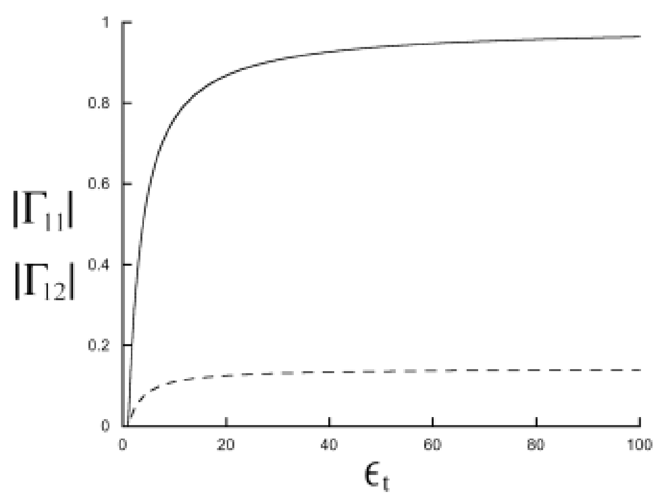

Figure 7. Magnitude of the reflection coefficients Γ_{11} (dashed line) and Γ_{12} (solid line) in the same scenario of Figure 4, but with an azimuthal incident angle $\theta_i = 30°$.

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This behavior is similar in the oblique incidence case, too. In this case the diagonal coefficients Γ_{11} and Γ_{12} are shown in magnitude as functions of ϵ_t in the same conditions as in Figure 7. In this case the diagonal coefficients are not exactly zero, but they are close to zero.

4. Conclusion

In the present paper, we analyzed the behavior of a particular class of anisotropic uniaxial materials. Such materials are strongly connected with the realization of the DB boundary condition and the soft and hard boundary conditions. We studied the reflection coefficients between air and such anisotropic material in several scenarios. First of all, we considered the reflection coefficients for different transverse components of the permittivity and permeability dyadics, finding that the angle of total transmission depends on such components and giving an analytical expression of such angle. Afterward, we analyzed the material as a function of the axial components of the dyadics, considering both positive and negative values. We find that the material behaves as a perfect reflector when the axial permittivity and permeability are either zero or negative but greater than a negative critical value. An analytical expression of such critical value has been found by studying the perpendicular component to the interface of the transmitted wave vector in the anisotropic material. Finally, we presented two important applications of the material to enhance the absorbing performance of a metamaterial absorber and to realize a polarization inverter.

In future studies, the role of the losses in the behavior of the reflection coefficient needs to be analyzed. Furthermore, possible realizations of the uniaxial medium in terms of composite metamaterials have to be studied.

Acknowledgments
All the data shown in the present paper can be obtained by a numerical implementation of the mathematical formulation in *Tedeschi et al.* [2013a]. In order to have access to the data, one can contact Nicola Tedeschi by email: nicola.tedeschi@uniroma1.it.

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