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Developing a Distributed Robust Energy Management Framework for Active Distribution Systems

Ali Rajaei, Sajjad Fattaheian-Dehkordi, Mahmud Fotuhi-Firuzabad, *Fellow*, *IEEE*, Moein Moeini-Aghtaie, *Member*, *IEEE*, and Matti Lehtonen

***Abstract*—Restructuring and privatization in power systems have resulted in a fundamental transition of conventional distribution systems into modern multi-agent systems. In these structures, each agent of the distribution system would independently operate its local resources. In this regard, uncertainties associated with load demands and renewable energy sources could challenge the operational scheduling conducted by each agent. Therefore, this paper aims to develop a distributed operational management for multi-agent distribution systems taking into account the uncertainties of each agent. The developed framework relies on alternating direction method of multipliers (ADMM) to coordinate the operational scheduling of the agents in a distributed manner. Moreover, a robust optimization technique is employed to consider the worst-case realization associated with the operation of each agent. Finally, the proposed framework is implemented on IEEE 37-bus network to analyze its efficacy in distributed robust operational management of distribution systems with multi-agent structures.**

Index Terms—Energy Management, active distribution systems, distributed management, alternating direction method of multipliers (ADMM), robust optimization.

I. Introduction

Proliferation of conventional distributed generation units, renewable energy sources (RESs), and energy storage units in the forms of electric batteries or electric vehicles, as well as flexible demands has led to development of smart active distribution systems [1]. However, uncertainties associated with the intermittent RESs and end-user demands have caused challenges for utilities from operational and planning perspectives. Furthermore, privatization and development of independently operated resources as well as the concerns associated with complicated analysis of massive amounts of information and raised cyber security issues have led to introduction of distribution systems with multi-agent structures [2]-[3]. In this regard, new procedures should be developed to cope with the multi-agent structure of distribution systems while addressing the uncertainties of the system.

In recent years, various methodologies have been developed by researchers to take into consideration the uncertainty of local resources in operational scheduling procedures. In this regard, stochastic optimization technique is one of the methods that has been employed to model the realization of uncertain parameters. In this methodology, a finite set of scenarios are generated modeling the uncertain parameters, and deployed in the optimization formulation of local resources based upon their probability distribution functions [4]-[5]. Reference [6] develops a framework to manage reactive power in a radial microgrid while taking into account the uncertainty of active power injections of solar panels and household demands via stochastic optimization. The authors in [7] have presented a two-stage stochastic resource scheduling for AC/DC hybrid smart grids. In the first stage, the day-ahead scheduling of DG units is determined in a way that minimizes costs over all scenarios; whereas in the second stage, corrective decisions are made for each possible scenario. Bazrafshan et al. [8] have devised a decentralized active and reactive power management scheme in which the output power of photovoltaic (PV) units can take values from a finite set of scenarios. In [9], a multi-objective framework is proposed to reduce the cost of energy and carbon dioxide emission in a distributed energy system, while utilizing 24-h scenarios to handle the uncertain parameters of supply and demand. Moreover, a decentralized framework is proposed in [10] to operate a multi-area integrated electricity and natural gas systems, while utilizing stochastic programming to address the uncertainty of wind power.

While stochastic optimization algorithm enables operators to address the uncertainty of input data, the procedure of generating scenarios, estimating their probabilities, as well as the associated computational burden to analyze the overall optimization model hinder its application in scheduling of real systems. As a result, other algorithms like robust optimization technique have recently been applied to model uncertain parameters in operational scheduling of local resources. Robust optimization algorithm strives to solve the operational scheduling problem considering the worst-case scenario of uncertain parameters which makes the computational procedure more tractable. In this approach, uncertainty associated with each parameter is modeled by a confidence interval which avoids necessity of defining probability distribution functions [11]-[15].

A. Rajaei, S. Fattaheian-Dehkordi and M. Fotuhi-Firuzabad are with the Department of Electrical Engineering, Sharif University of Technology, Tehran, Iran.
S. Fattaheian-Dehkordi and M. Lehtonen are with Electrical Engineering and Automation Department of Aalto University, Espoo, Finland.
M. Fotuhi-Firuzabad is a visiting professor at Aalto University, Finland.
M. Moeini-Aghtaie is with the Department of Energy Engineering, Sharif University of Technology, Tehran, Iran.
(Emails: alirajaei95@gmail.com, fattaheian@ ee.sharif.edu, fotuhi@sharif.edu, moeini@sharif.edu, matti.lehtonen@aalto.fi).

In [11], energy scheduling of home appliances has been modeled with robust optimization technique to take into account the worst-case operational scenario of all uncertain home appliances. Robust approach has been adopted in [12]-[12] in order to accommodate the worst realization of uncertain variables. Reference [12] aims to optimize the operational scheduling of a microgrid considering islanded and grid-connected modes, while bidding strategy of a microgrid aggregator participating in a pool electricity market is developed in [13]. Authors in [14] have proposed a robust-based two-stage procedure to schedule resources in transmission and distribution systems. The proposed approach strives to address the variability of wind power considering the flexibility provided by the interconnected electricity and natural gas networks. Reference [15] has developed an adaptive robust min-max-min model to conduct unit commitment considering operational constraints of transmission networks. In this regard, robust algorithm is employed to address uncertainties associated with wind power production. Furthermore, the unit commitment model is decomposed into three levels and solved iteratively utilizing primal and dual cuts. While [12]-[15] have studied robust management of power systems from different perspectives, the multi-agent structure of modern power systems is not considered in the developed models.

As mentioned, traditional distribution systems are moving towards multi-agent structures, in which each agent is responsible for managing its own local resources [16]. In this regard, new methodologies are required in order to enable agents to independently operate their resources while considering operational constraints of the underlying distribution grid. In particular, alternating direction method of multipliers (ADMM) has received significant attention in recent research works with the aim of facilitating distributed operation of multi-agent systems. In this method, a central optimization problem is broken down into smaller sub-problems which are iteratively solved by system's agents in a decentralized fashion. The robust co-optimization of electricity and gas systems utilizing ADMM approach is investigated in [17]. In this regard, the operational management of electricity and gas systems are conducted by independent entities while ADMM is taken into account to iteratively coordinate the scheduling of the two systems. In other words, electrical system and gas system are operated in a central manner, while operational coordination of their respected utilities is ensured by ADMM algorithm. Moreover, authors in [18] have developed a decentralized scheme to schedule generation units and tie-line interchanges in multi-area systems, while robust optimization is utilized to handle wind uncertainty. Noted that the developed methods in [17]-[18] rely on DC power flow model which may not lead into reliable results in distribution networks. References [19]-[20] have applied ADMM to efficiently control the flow of reactive power in distribution systems. A closed-form solution for the ADMM-based optimal power flow with SOCP relaxation (SOCP-OPF) in a radial distribution system is proposed in [21]. Moreover, the proposed ADMM-based SOCP-OPF is developed in [3] in order to be deployed on a multi-agent system with the bilateral energy trading structure. In [22], decentralized operational scheduling of multi-microgrid distribution system with energy-hubs is studied. In this respect, robust optimization is used to model uncertain parameters; however, the proposed approach does not consider the distribution network's constraints. Authors in [23] have devised a decentralized robust model in order to coordinate the operation of distribution company and electric vehicle (EV) aggregators. Furthermore, distributed energy resources (DERs) coordination in distribution systems is investigated in [24]. It is noteworthy that the proposed methods in [22]-[24] are not fully distributed and rely on the distribution system operator to coordinate the operation of multiple entities and operate the distribution network. Authors in [25] have utilized robust optimization technique and ADMM algorithm to address the uncertainty of local resources while distributedly managing multi-microgrid distribution systems. In this method, the energy exchanged by each microgrid is considered to be fixed in case of system disturbance; therefore, the operational point of local resources in each microgrid would be exploited to address the net-load deviation in the respected microgrid during the real-time operation. In other words, the uncertainty consequences in the system would merely be resolved locally by each microgrid. Furthermore, the operational characteristics of the grid are dismissed in the proposed methodology in [25]. Table I presents a comprehensive taxonomy table of research works in the context of this paper.

Table I Taxonomy of research works on centralized/decentralized optimization of energy systems considering uncertainty

Ref	Robust Opt.	Stochastic Opt.	ADMM	Network Modeling	Description
[5]-[7]	×	✓	×	SOCP/AC-PF	Central Optimization of the designated system
[8]	×	✓	✓	SOCP	Decentralized power management in distribution grid
[9]	×	✓	×	×	Optimizing the distribution system considering environmental constraints
[10]	×	✓	✓	DC-PF	Decentralized operation of gas and electricity systems
[11]	✓	×	×	×	Central energy scheduling of home appliances
[12]	✓	×	×	×	Central operational scheduling of MGs
[13]	✓	×	×	×	Bidding strategy of a MG aggregator
[14]	✓	×	×	DC-PF	Central coordination of electricity and gas system
[15]	✓	×	×	SOCP	Central unit commitment in transmission network
[17]	✓	×	✓	DC-PF	Decentralized coordination of electricity and gas system
[18]	✓	×	✓	DC-PF	Decentralized generation and tie-line scheduling in multi-area systems
[22]	✓	×	✓	×	Operational optimization of multi-MGs considering a central coordinator
[23]	✓	×	✓	DC-PF	• Operational scheduling of EVs in distribution system • The network constraints are merely considered by system operator while EV aggregators optimize their scheduling
[24]	✓	×	✓	SOCP	• Residential DER coordination in distribution grid • The network's constraints are merely considered by system operator
[25]	✓	×	✓	×	• Distributed operation of multi-MG system • Network's constraints are not considered; Power exchange among MGs is considered to be fixed
This paper	✓	×	✓	SOCP	Distributed operation of distribution system with multi-agent structure

Despite various studies, which have been conducted to address the uncertainty of local resources and distributed management of multi-agent distribution systems; to the best of authors' knowledge, employing robust optimization technique in a fully distributed manner to securely operate multi-agent distribution systems has not yet been fully investigated in previous proposed frameworks. Motivated by the aforementioned points, this paper aims to develop a robust optimal scheduling scheme in the context of the ADMM algorithm to address the uncertainty of system agents, while coping with the multi-agent structure of the system. Note that none of the previous research works have developed an ADMM-based robust optimization technique which complies with multi-agent distribution system, and either rely on a central coordinator to operate the distribution system, or deploy linear power flow algorithm to model the distribution network. Furthermore, the developed approach in [25] overlooks the underlying distribution network which could result in a sub-optimal dispatch solution. In this regard, robust optimization techniques could not be employed in the previously developed ADMM-based mathematical optimization models to schedule the power exchange with the grid, address the uncertainty of local resources, and cope with the distributed nature of multi-agent systems. Consequently, this paper aims to provide a mathematical modeling of an ADMM-based energy management scheme in a multi-agent distribution system; in which each agent independently schedules its resources utilizing robust optimization technique. In this context, in the first step, the centralized robust optimization of the distribution system is presented; while, in the second step, the primary optimization model is modified in a way that copes with the ADMM concept to facilitate distributed optimization of multi-agent systems without a central coordinator. Based on the above discussions, the main contributions of this paper could be pointed out as follows:

- Utilizing the robust optimization approach for operational scheduling of independent agents immunizes day-ahead management of the distribution system against associated uncertainties. Moreover, the proposed approach copes with the distributed nature of multi-agent systems where each agent could employ a robustness budget to adjust the robustness level of its respective operational scheduling.
- The proposed scheme takes into account ADMM algorithm in order to coordinate the day-ahead scheduling of independently operated agents without a central coordinator. In this regard, information exchange between agents is managed in a way that addresses cyber-security and privacy concerns. Respectively, a mathematical formulation is developed based on the ADMM concept to facilitate the combination of robust optimization technique in distributed operation of multi-agent systems considering operational modeling of the grid.

The rest of this paper is structured as follows. In Section II, the centralized framework for robust operational scheduling of distribution systems is formulated. In Section III, ADMM algorithm is taken into account to develop the distributed robust operational management of multi-agent distribution systems. Finally, the results associated with implementing the developed framework on the modified IEEE 37-bus network are presented in Section IV, followed by conclusions in Section V.

II. Centralized Robust Scheduling Framework

A. System Modeling

Distribution systems are undergoing a dramatic modification as a result of privatization and restructuring in power systems. In this regard, future distribution systems would be structured as multi-agent systems, in which each agent is independently operated and controlled. Figure 1 represents the model of a distribution system with a multi-agent structure considered in the proposed scheme for energy management in this paper.

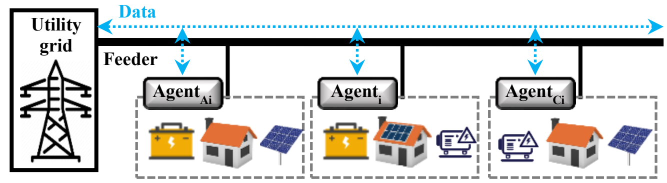

Fig. 1. The model of a distribution system with multi-agent structure.

Traditionally, operational scheduling of local resources in distribution systems is conducted in a central manner where a central coordinator is responsible for collecting and analyzing operational information of all system agents. In this section, robust optimization technique is taken into account to address the uncertainty associated with local resources while operational management of the distribution system is conducted in a central manner.

B. Deterministic Operational Scheduling of Distribution Systems

It is considered that the modeled distribution grid consists of a set of nodes given by $\Omega_0^N := \{0,...,N\}$ and a set of lines shown by Ω^L. Node 0 represents the common coupling point between the transmission and distribution system, while the remaining nodes represent independent agents of the system. In the presented radial distribution system, each node $i \in \Omega^N = \Omega_0^N \setminus \{0\}$ connects to a unique ancestor node shown by A_i and a set of child nodes, represented by C_i. In this regard, line i represents the line between nodes A_i and i [26]. Moreover, reactance and resistance associated with line i are respectively denoted by x_i and r_i. Without loss of generality, it is considered that each node has a conventional distributed generation (DG) unit, photovoltaic (PV) unit, battery energy storage (BES) unit, and load demand indexed by set $i \in \Omega^N$. Finally, the optimal operational scheduling model for the multi-agent distribution system over the operational horizon T is presented as follows:

$$\min \sum_{t\in T} \sum_{i\in\Omega_0^N} OF_{i,t} \tag{1a}$$

s.t.

$$\begin{cases} OF_{i,t} = LMP_t . P_t^{trans} & \text{if } i = 0, t \in T \\ OF_{i,t} = GC_{i,t} + DC_{i,t} & \text{if } i \neq 0, t \in T \end{cases} \tag{1b}$$

$$GC_{i,t} = a_i Pg_{i,t}^2 + b_i Pg_{i,t} + c_{i,t} \quad i \in \Omega^N, t \in T \tag{1c}$$

$$DC_{i,t} = \alpha_i . \left(Pd_{i,t}^{Sch} - Pd_{i,t}^{Ex} \right)^2 \quad i \in \Omega^N, t \in T \tag{1d}$$

$$\left| \left(Pd_{i,t}^{Sch} - Pd_{i,t}^{Ex} \right) \big/ Pd_{i,t}^{Ex} \right| \leq AD_i \quad i \in \Omega^N, t \in T \tag{1e}$$

$$\sum_{t\in T} Pd_{i,t}^{Sch} = \sum_{t\in T} Pd_{i,t}^{Ex} \quad i \in \Omega^N \tag{1f}$$

$$Qd_{i,t}^{Sch} = \phi_{i,t} Pd_{i,t}^{Sch} \quad i \in \Omega^N, t \in T \tag{1g}$$

$$SoC_{i,t} = SoC_{i,t-1} + \frac{eff_i}{Eb_i}\left(Pb_{i,t}^{Ch} - Pb_{i,t}^{DCh}\right) \quad i \in \Omega^N, t \in T \tag{1h}$$

$$p_{i,t} = Pg_{i,t} + Ps_{i,t} - Pd_{i,t}^{Sch} + Pb_{i,t}^{DCh} - Pb_{i,t}^{Ch} \quad i \in \Omega^N, t \in T \tag{1i}$$

$$q_{i,t} = Qg_{i,t} - Qd_{i,t}^{Sch} \quad i \in \Omega^N, t \in T \tag{1j}$$

$$p_{i,t} = P_t^{trans} \quad i = 0, t \in T \tag{1k}$$

$$v_{A_i,t} = v_{i,t} + 2\left(r_i PF_{i,t} + x_i QF_{i,t}\right) + l_{i,t}\left(r_i^2 + x_i^2\right) \quad i \in \Omega^L, t \in T \tag{1l}$$

$$\sum_{j \in C_i}\left(PF_{j,t} + l_{j,t} r_j\right) - p_{i,t} = PF_{i,t} \quad i \in \Omega^N, t \in T \tag{1m}$$

$$\sum_{j \in C_i}\left(QF_{j,t} + l_{j,t} x_j\right) - q_{i,t} = QF_{i,t} \quad i \in \Omega^N, t \in T \tag{1n}$$

$$PF_{i,t}^2 + QF_{i,t}^2 \le v_{i,t} l_{i,t} \quad i \in \Omega^N, t \in T \tag{1o}$$

$$v_i^{Min} \le v_{i,t} \le v_i^{Max} \quad i \in \Omega^N, t \in T \tag{1p}$$

$$0 \le Pg_{i,t} \le Pg_i^{Max} \quad i \in \Omega^N, t \in T \tag{1q}$$

$$Q_i^{G,Min} \le Q_{i,t}^G \le Q_i^{G,Max} \quad i \in \Omega^N, t \in T \tag{1r}$$

$$SoC_i^{Min} \le SoC_{i,t} \le SoC_i^{Max} \quad i \in \Omega^N, t \in T \tag{1s}$$

$$0 \le Pb_{i,t}^{Ch} \le Pb_i^{Ch,Max} \quad i \in \Omega^N, t \in T \tag{1t}$$

$$0 \le Pb_{i,t}^{DCh} \le Pb_i^{DCh,Max} \quad i \in \Omega^N, t \in T \tag{1u}$$

where the objective function (1a) aims to minimize the operational cost of the multi-agent distribution system over the scheduling horizon T. In this regard, the active power exchange between distribution and transmission level is denoted by P_t^{trans}; whereas the cost associated with this exchange is represented by LMP_t. Moreover, the operational cost of each agent i is modeled considering the generation cost of conventional DG units represented by $GC_{i,t}$, and discomfort cost of flexible loads represented by $DC_{i,t}$. Equation (1c) models the generation cost of the conventional DG unit, where $Pg_{i,t}$ and $Qg_{i,t}$ represent the active and reactive power generation of the conventional DG unit in node i. Equation (1d) formulates the discomfort cost of each agent i, which is associated with the deviation of the scheduled demand $Pd_{i,t}^{Sch}$ from the expected value $Pd_{i,t}^{Ex}$, and a unit discomfort cost α_i. In (1e), the demand deviation of flexible load is limited to an allowable deviation (AD_i) limit. Constraint (1f) forces that the sum of scheduled demand and expected demand are equal, ensuring that the primary energy consumption of each agent over the operational horizon is supplied. In (1g), it is considered that the reactive demand $Qd_{i,t}^{Sch}$ is proportional to scheduled active demand based on an input parameter $\phi_{i,t}$ for each agent i. The operational model of BES unit is presented in (1h), where state of charge (SoC), energy capacity, charging rate, discharging rate, and efficiency are denoted by $SoC_{i,t}, Eb_i, Pb_{i,t}^{Ch}, Pb_{i,t}^{DCh}, eff_i$, respectively. Also, the power production of PV units is presented by $Ps_{i,t}$. Furthermore, PF_i, QF_i, and l_i represent active power flow, reactive power flow, and squared current magnitude of line i, while v_i, p_i, and q_i are used to show the squared voltage magnitude, active power injection, and reactive power injection of node i, respectively. The nodal active and reactive power balance are enforced in (1i)-(1j), while operational constraints of SOCP-relaxed DistFlow model [26] are formulated in (1l)-(1o). Equation (1k) shows that active power injection of node 0 is equal to power exchange from the transmission network. Finally, constraints (1p)-(1u) enforce lower and upper bounds on voltage magnitude of nodes, active/reactive power generation of conventional DG units, and SoC, charging/discharging rate of BES units, respectively.

C. *Uncertainty Modeling*

In this paper, the uncertainty associated with load demands and power production of PV units are taken into consideration. In this regard, the expected load demands and power production of PV units could be modeled as follows:

$$Pd_{i,t}^{Ex} \in \left[\overline{Pd_{i,t}^{Ex}} - \Delta Pd_{i,t}^{Ex}, \overline{Pd_{i,t}^{Ex}} + \Delta Pd_{i,t}^{Ex}\right] \quad i \in \Omega^N, t \in T \tag{2a}$$

$$Ps_{i,t}^{Ex} \in \left[\overline{Ps_{i,t}^{Ex}} - \Delta Ps_{i,t}^{Ex}, \overline{Ps_{i,t}^{Ex}} + \Delta Ps_{i,t}^{Ex}\right] \quad i \in \Omega^N, t \in T \tag{2b}$$

where $\overline{Pd_{i,t}^{Ex}}$ is the forecasted value of load demand $Pd_{i,t}^{Ex}$, and $\Delta Pd_{i,t}^{Ex}$ shows its maximum forecast error. Similarly, $\overline{Ps_{i,t}^{Ex}}$ represent the forecasted value of PV power production $Ps_{i,t}^{Ex}$, and $\Delta Ps_{i,t}^{Ex}$ shows its maximum forecast error.

In this respect, robust optimization technique is taken into consideration to address the worst-case realization of uncertainty with the operation of each agent in the developed energy management of the system.

D. *Robust Operational Scheduling of Distribution Systems*

The robust operational scheduling of the multi-agent system with the aim of minimizing the total cost is formulated as follows:

$$\max_{\tilde{y}} \min_{x} \sum_{t \in T} \sum_{i \in \Omega^N} OF_{i,t} \tag{3a}$$

s.t.

$$Pd_{i,t}^{Ex} = \overline{Pd_{i,t}^{Ex}} + \varphi_{i,t}^{d+} \Delta Pd_{i,t}^{Ex} - \varphi_{i,t}^{d-} \Delta Pd_{i,t}^{Ex} \quad i \in \Omega^N, t \in T \tag{3b}$$

$$Ps_{i,t}^{Ex} = \overline{Ps_{i,t}^{Ex}} + \varphi_{i,t}^{s+} \Delta Ps_{i,t}^{Ex} - \varphi_{i,t}^{s-} \Delta Ps_{i,t}^{Ex} \quad i \in \Omega^N, t \in T \tag{3c}$$

$$0 \le \varphi_{i,t}^{d-}, \varphi_{i,t}^{d+}, \varphi_{i,t}^{s-}, \varphi_{i,t}^{s+} \le 1 \quad i \in \Omega^N, t \in T \tag{3d}$$

$$\sum_{t \in T}\left(\varphi_{i,t}^{d-} + \varphi_{i,t}^{d+}\right) \le \Gamma_i^D \quad i \in \Omega^N \tag{3e}$$

$$\sum_{t \in T}\left(\varphi_{i,t}^{s-} + \varphi_{i,t}^{s+}\right) \le \Gamma_i^S \quad i \in \Omega^N \tag{3f}$$

and (1b)-(1u).

where $\tilde{y}$ represents the uncertain variables (i.e., $Pd_{i,t}^{Ex}, Ps_{i,t}^{Ex}$) and x represents the decision variables in (1). Moreover, $\varphi_{i,t}^{d+}, \varphi_{i,t}^{d-}, \varphi_{i,t}^{s+}, \varphi_{i,t}^{s-}$ are auxiliary variables taken into account to model the forecast error of expected values. The objective function (3a) minimizes the operational cost of agents over their respective worst-case scenarios. In addition, it is improbable that expected values by each agent over all time periods in the scheduling horizon dramatically deviate from their forecasted value. In this regard, robustness budgets for demands and PV units (i.e., Γ_i^D, Γ_i^S) are utilized to control the trade-off between robustness and optimality of the solution by enforcing limitation over demand deviation and PV production of each agent during the scheduling horizon. It is noteworthy that the robustness budget would be set based upon each agent's perspective regarding its associated risk. In other words, a risk -seeker agent selects a higher value for the robustness budget while a risk-averse agent considers lower values for the designated budget of uncertainty. In this context, the minimum and maximum values of robustness budget are correspondingly equal to zero and the total number of uncertain parameters.

III. Distributed Robust Scheduling Framework

The proposed operational management model could be implemented on a distribution system using a central coordinator that collects and analyzes information of all entities in a centralized fashion. However, this approach requires transmission of the detailed operational characteristics of local resources to the central operator, which could cause privacy and cyber-security concerns. As a result, ADMM approach [27] is employed to distributedly conduct the robust scheduling model in a multi-agent distribution system.

A. ADMM Approach

Consider a convex optimization problem as follows:

$$\min_{x\in K_x, z\in K_z} f(x)+g(z) \quad \text{s.t.} \quad Ax+Bz=c \,. \tag{4}$$

The augmented Lagrangian model of the convex optimization problem is derived using a Lagrangian multiplier λ and a positive constant ρ as follows:

$$L_\rho(x,z,\lambda) := f(x)+g(z)+\lambda^T(Ax+Bz-c) + \frac{\rho}{2}\|Ax+Bz-c\|_2^2 \,. \tag{5}$$

In order to apply the ADMM model, a three-step procedure could be defined in which the variables would be iteratively updated as below:

$$x^{k+1} = \arg\min_{x\in K_x} L_\rho(x,z^k,\lambda^k) \tag{6a}$$

$$z^{k+1} = \arg\min_{z\in K_z} L_\rho(x^k,z,\lambda^k) \tag{6b}$$

$$\lambda^{k+1} = \lambda^k + \rho(Ax^{k+1}+Bz^{k+1}-c) \tag{6c}$$

where (6a)-(6c) are respectively considered as x-update, z-update, and dual-update. In this context, x and z could be regarded as local and global variables, respectively. Finally, the following criteria are defined in order to ensure convergence of the approach.

$$r^k := \|Ax+Bz-c\| \tag{7a}$$

$$s^k := \rho\left\|A^T B(z^k - z^{k-1})\right\| \tag{7b}$$

where r^k represents primal residual, and s^k shows dual residual of the ADMM model.

B. ADMM-Based Robust Scheduling Model

The power flow modeling equations in the distribution grid, (1l)-(1o), couple variables associated with operational state of adjacent neighbors; which hinders the robust scheduling model (3) to be conducted in a distributed manner. Consequently, in order to decompose the optimization problem, agents need to consider copies of coupling variables in order to modify (3) into an ADMM model. Therefore, each agent runs a sub-problem considering its local variables and global variables that stand as adjacent agents' variables. Furthermore, global variables could be taken into account to ensure the consensus of sub-problems conducted by adjacent agents. Finally, ADMM model could be deployed by deriving Augmented Lagrangian model and decomposing it into x-update and z-update steps to be solved by each agent. In this context, $x_i := [v_i, l_i, PF_i, QF_i]$ are local variables of node i, $x_{j,i} := [l_{j,i}, PF_{j,i}, QF_{j,i}]$ denote the duplicated local variables of child nodes, $v_{A_i,i}$ shows the copy of the ancestor's voltage magnitude, and $z_i = [v_i^z, l_i^z, PF_i^z, QF_i^z]$ represent the global variables considered by agent i. In this regard, coupling constraints associated with ADMM model could be defined as shown in (8). Moreover, the Lagrangian multipliers associated with coupling constraints (8) are listed in Table I.

$$v_{i,t} = v_{i,t}^z \,,\; l_{i,t} = l_{i,t}^z \,,\; PF_{i,t} = PF_{i,t}^z \,,\; QF_{i,t} = QF_{i,t}^z \quad i\in\Omega^N \tag{8a}$$

$$l_{j,i,t} = l_{j,t}^z \,,\; PF_{j,i,t} = PF_{j,t}^z \,,\; QF_{j,i,t} = QF_{j,t}^z \quad i\in\Omega^N, j\in C_n \tag{8b}$$

$$v_{A_i,i,t} = v_{A_i,t}^z \quad i\in\Omega^N \tag{8c}$$

Table II Lagrangian Multipliers

$\lambda_{i,t}^1$:	$v_{i,t} = v_{i,t}^z$		$\lambda_{i,t}^2$:	$l_{i,t} = l_{i,t}^z$
$\lambda_{i,t}^3$:	$PF_{i,t} = PF_{i,t}^z$		$\lambda_{i,t}^4$:	$QF_{i,t} = QF_{i,t}^z$
$\mu_{j,t}^1$:	$l_{j,i,t} = l_{j,t}^z$		$\mu_{j,t}^2$:	$PF_{j,i,t} = PF_{j,t}^z$
$\mu_{j,t}^3$:	$QF_{j,i,t} = QF_{j,t}^z$		$\gamma_{i,t}$:	$v_{A_i,i,t} = v_{A_i,t}^z$

1) *x-update*

The x-update optimization in the ADMM procedure could be formulated as in (9). It is noteworthy that the generation cost of conventional DG units and the discomfort cost of flexible demands are linearized utilizing piece-wise linear functions.

$$\max_{\tilde{y}} \min_{x} \sum_{t\in T}\sum_{i\in\Omega_0^N} \Big(OF_{i,t} + \lambda_{i,t}^T\left(x_{i,t}-z_{i,t}\right) + \sum_{j\in C_i}\mu_j^T\left(x_{j,i,t}-z_{j,t}\right) + \gamma_i\left(v_{A_i,i,t}-v_{A_i,t}^z\right) + \frac{\rho}{2}\left(\left\|x_{i,t}-z_{i,t}\right\|_2^2 + \sum_{j\in C_i}\left\|x_{j,i,t}-z_{j,t}\right\|_2^2 + \left\|v_{Ai,i,t}-v_{A_i,t}^z\right\|_2^2\right)\Big) \tag{9a}$$

s.t.

$$\begin{cases} OF_{i,t} = LMP_t . P_t^{trans} & \text{if } i=0, t\in T : f_{i,t}^0 \\ OF_{i,t} = GC_{i,t} + DC_{i,t} & \text{if } i\neq 0, t\in T : f_{i,t} \end{cases} \tag{9b}$$

$$GC_{i,t} = \sum_{sg\in\Omega_i^G} \pi_{i,t,sg}^g p_{i,t,sg}^g \quad i\in\Omega^N, t\in T : \varsigma_{i,t} \tag{9c}$$

$$Pg_{i,t} = \sum_{sg\in\Omega_i^G} p_{i,t,sg}^g \quad i\in\Omega^N, t\in T : \delta_{i,t} \tag{9d}$$

$$DC_{i,t} = \sum_{sd\in\Omega_i^D} \pi_{i,t,sd}^{d+} p_{i,t,sd}^{d+} + \sum_{sd\in\Omega_i^D} \pi_{i,t,sd}^{d-} p_{i,t,sd}^{d-} \quad i\in\Omega^N, t\in T : \omega_{i,t} \tag{9e}$$

$$Pd_{i,t}^{Sch} - Pd_{i,t}^{Ex} = \sum_{sd\in\Omega_i^D} p_{i,t,sd}^{d+} - \sum_{sd\in\Omega_i^D} p_{i,t,sd}^{d-} \quad i\in\Omega^N, t\in T : \beta_{i,t} \tag{9f}$$

$$\sum_{t\in T}\sum_{sd\in\Omega_i^D} p_{i,t,sd}^{d+} = \sum_{t\in T}\sum_{sd\in\Omega_i^D} p_{i,t,sd}^{d-} \quad i\in\Omega^N : \eta_i \tag{9g}$$

$$Qd_{i,t}^{Sch} = \phi_{i,t} Pd_{i,t}^{Sch} \quad i\in\Omega^N, t\in T : \iota_{i,t} \tag{9h}$$

$$SoC_{i,t} = SoC_{i,t-1} + \frac{eff_i}{Eb_i}\left(Pb_{i,t}^{Ch} - Pb_{i,t}^{DCh}\right) \quad i\in\Omega^N, t\in T : \theta_{i,t}^B \tag{9i}$$

$$p_{i,t} = Pg_{i,t} + Ps_{i,t} - Pd_{i,t}^{Sch} + Pb_{i,t}^{DCh} - Pb_{i,t}^{Ch} \quad i \in \Omega^N, t \in T : \psi_{i,t} \tag{9j}$$

$$p_{i,t} = P_t^{trans} \quad i = 0, t \in T : \tau_{i,t} \tag{9k}$$

$$q_{i,t} = Qg_{i,t} - Qd_{i,t}^{Sch} \quad i \in \Omega^N, t \in T : \zeta_{i,t} \tag{9l}$$

$$v_{A_i,i,t} = v_{i,t} + 2\left(r_i PF_{i,t} + x_i QF_{i,t}\right) + l_{i,t}\left(r_i^2 + x_i^2\right) \quad i \in \Omega^N, t \in T : O_{i,t} \tag{9m}$$

$$\sum_{j \in C_i}\left(PF_{j,i,t} + l_{j,i,t} r_j\right) - p_{i,t} = PF_{i,t} \quad i \in \Omega^N, t \in T : m_{i,t} \tag{9n}$$

$$\sum_{j \in C_i}\left(QF_{j,i,t} + l_{j,i,t} x_j\right) - q_{i,t} = QF_{i,t} \quad i \in \Omega^N, t \in T : n_{i,t} \tag{9o}$$

$$0 \le p_{i,t,sg}^g \le p_{i,sg}^{g,Max} \quad i \in \Omega^N, t \in T, sg \in \Omega_i^G : \upsilon_{i,t,sg}^g \tag{9p}$$

$$Q_i^{G,Min} \le Q_{i,t}^G \le Q_i^{G,Max} \quad i \in \Omega^N, t \in T : 0 \le \sigma_{i,t}^{G-}, \sigma_{i,t}^{G+} \tag{9q}$$

$$0 \le p_{i,t,sd}^{d-}, p_{i,t,sd}^{d+} \le p_{i,sd}^{d,Max} \quad i \in \Omega^N, t \in T, sd \in \Omega_i^D : 0 \le \xi_{i,t,sd}^{d-}, \xi_{i,t,sd}^{d+} \tag{9r}$$

$$0 \le Pb_{i,t}^{Ch} \le Pb_i^{Ch,Max} \quad i \in \Omega^N, t \in T : 0 \le \theta_{i,t}^{Ch+} \tag{9s}$$

$$0 \le Pb_{i,t}^{DCh} \le Pb_i^{DCh,Max} \quad i \in \Omega^N, t \in T : 0 \le \theta_{i,t}^{DCh+} \tag{9t}$$

$$SoC_i^{Min} \le SoC_{i,t} \le SoC_i^{Max} \quad i \in \Omega^N, t \in T : 0 \le \theta_{i,t}^{B-}, \theta_{i,t}^{B+} \tag{9u}$$

and (3b)-(3f).

The objective function (9a) aims to minimize the operational cost of each agent over the worst-case scenario, while reducing the gap associated with global and local variables as a consensus measure. Equation (9c) represents the total generation cost of the DG unit, where $\pi_{i,t,sg}^g$ and $p_{i,t,sg}^g$ respectively present the cost and active power generation in each segment $sg \in \Omega_i^G$. Furthermore, $p_{i,t,sd}^{d+}$ and $p_{i,t,sd}^{d-}$ show the positive and negative deviation of load demand in each segment $sd \in \Omega_i^D$. Moreover, $\{f_{i,t}^0, f_{i,t}, \varsigma_{i,t}, \delta_{i,t}, \omega_{i,t}, \beta_{i,t}, \eta_i, \iota_{i,t}, \psi_{i,t}, \tau_{i,t}, \zeta_{i,t}, O_{i,t}, m_{i,t}, n_{i,t}, \upsilon_{i,t,sg}^g, \sigma_{i,t,sg}^{G-}, \sigma_{i,t,sg}^{G+}, \xi_{i,t,sd}^{d-}, \xi_{i,t,sd}^{d+}, \theta_{i,t}^B, \theta_{i,t}^{B-}, \theta_{i,t}^{B+}, \theta_{i,t}^{Ch+}, \theta_{i,t}^{DCh+}\}$ are the Lagrangian multipliers associated with (9b)-(9u).

The developed max-min optimization model (9) could be reformulated into a single optimization utilizing duality theory. In this regard, the inner minimization problem of (9) has the following form:

$$\begin{aligned} &\min_x \ \frac{1}{2} x^T C x + p^T x \\ &\text{s.t} \ \ Ax \ge b \quad ; \lambda \\ &\qquad x \ge 0 \end{aligned} \tag{10}$$

where λ is the Lagrangian multiplier and C is a positive semidefinite matrix based on its diagonal form with positive elements. It is noteworthy that the positive semi-definiteness feature of the C matrix assures that (10) is convex; therefore, it's respective dual model would converge to a global optimum [28]. In this respect, the dual model could be formulated as follows:

$$\begin{aligned} &\max_\lambda \ -\frac{1}{2} u^T C u + b^T \lambda \\ &\text{s.t} \ \ A^T \lambda - Cu \le p \quad ; x \\ &\qquad \lambda \ge 0 \end{aligned} \tag{11}$$

In this regard, the max-min optimization model (9) could be recast into a max optimization model as presented in (12).

$$\begin{aligned} \max \ & \sum_t \Big(-\frac{\rho}{2}\Big(u_{x_{i,t}}^2 + \sum_{j \in C_i} u_{x_{j,i,t}}^2 + u_{v_{A_i,i,t}}^2\Big) - Pd_{i,t}^{Ex}\beta_{i,t} - Ps_{i,t}^{Ex}\psi_{i,t} \\ & - \sum_{sd \in \Omega_i^D} p_{i,t,sd}^{d,Max}\left(\xi_{i,t,sd}^{d-} + \xi_{i,t,sd}^{d+}\right) - \sum_{sg \in \Omega_i^G} p_{i,sg}^{g,Max}\upsilon_{i,t,sg} + Q_i^{G,Min}\sigma_{i,t}^{G-} \\ & - Q_i^{G,Max}\sigma_{i,t}^{G+} - Pb_i^{Ch,Max}\theta_{i,t}^{Ch+} - Pb_i^{DCh,Max}\theta_{i,t}^{Ch-} + SoC_i^{Min}\theta_{i,t}^{B-} \\ & - SoC_i^{Max}\theta_{i,t}^{B+}\Big) \end{aligned} \tag{12a}$$

s.t.

$$\begin{cases} f_{i,t}^0 \le 1 & \text{if } i = 0, t \in T \ : OF_{i=0,t} \\ f_{i,t} \le 1 & \text{if } i \ne 0, t \in T \ : OF_{i \ne 0,t} \end{cases} \tag{12b}$$

$$-\varsigma_{i,t} - f_{i,t} \le 0 \quad i \in \Omega^N, t \in T : GC_{i,t} \tag{12b}$$

$$\psi_{i,t} + \delta_{i,t} \le 0 \quad i \in \Omega^N, t \in T : Pg_{i,t} \tag{12c}$$

$$-\delta_{i,t} + \pi_{i,t,sg}^g \varsigma_{i,t} - p_{i,t,sg}^{g,Max}\upsilon_{i,t,sg}^g \le 0 \quad i \in \Omega^N, t \in T, sg \in \Omega_i^G : p_{i,t,sg}^g \tag{12d}$$

$$\zeta_{i,t} + \sigma_{i,t}^- - \sigma_{i,t}^+ \le 0 \quad i \in \Omega^N, t \in T : Qg_{i,t} \tag{12e}$$

$$-\omega_{i,t} - f_{i,t} \le 0 \quad i \in \Omega^N, t \in T : DC_{i,t} \tag{12f}$$

$$-\beta_{i,t} - \psi_{i,t} - \phi_{i,t}\iota_{i,t} \le 0 \quad i \in \Omega^N, t \in T : Pd_{i,t}^{Sch} \tag{12g}$$

$$\beta_{i,t} + \eta_i - \xi_{i,t}^{d+} + \pi_{i,t}^{d+}\omega_{i,t} \le 0 \ \ i \in \Omega^N, t \in T, sd \in \Omega_i^D : p_{i,t,sd}^{d+} \tag{12h}$$

$$-\beta_{i,t} - \eta_i - \xi_{i,t}^{d-} + \pi_{i,t}^{d-}\omega_{i,t} \le 0 \ \ i \in \Omega^N, t \in T, sd \in \Omega_i^D : p_{i,t,sd}^{d-} \tag{12i}$$

$$\iota_{i,t} - \zeta_{i,t} \le 0 \quad i = 0, t \in T \ : Qd_{i,t}^{Sch} \tag{12j}$$

$$\tau_t - LMP_t f_{i=0,t}^0 = 0 \quad i = 0, t \in T \ : P_t^{trans} \tag{12k}$$

$$m_{i,t} - \psi_{i,t} - \tau_{i=0,t} = 0 \quad i \in \Omega^N, t \in T \ : p_{i,t} \tag{12l}$$

$$n_{i,t} - \zeta_{i,t} = 0 \quad i \in \Omega^N, t \in T \ : q_{i,t} \tag{12m}$$

$$-\theta_{i,t}^B + \theta_{i,t+1}^B + \theta_{i,t}^{B-} - \theta_{i,t}^{B+} \le 0 \quad i \in \Omega^N, t \in T : SoC_{i,t} \tag{12n}$$

$$\frac{eff_i}{Eb_i}\theta_{i,t}^B - \psi_{i,t} - \theta_{i,t}^{Ch+} \le 0 \quad i \in \Omega^N, t \in T : Pb_{i,t}^{Ch} \tag{12o}$$

$$-\frac{eff_i}{Eb_i}\theta_{i,t}^B + \psi_{i,t} - \theta_{i,t}^{DCh+} \le 0 \quad i \in \Omega^N, t \in T : Pb_{i,t}^{DCh} \tag{12p}$$

$$O_{i,t} - \rho u_{v_{i,t}} \le \lambda_{i,t}^1 - \rho v_{i,t}^z \quad i \in \Omega^N, t \in T \ : v_{i,t} \tag{12q}$$

$$\left(r_i^2 + x_i^2\right)O_{i,t} - \rho u_{l_{i,t}} \le \lambda_{i,t}^2 - \rho l_{i,t}^z \quad i \in \Omega^N, t \in T \ : l_{i,t} \tag{12r}$$

$$2r_i O_{i,t} + m_{i,t} - \rho u_{PF_{i,t}} \le \lambda_{i,t}^3 - \rho PF_{i,t}^z \quad i \in \Omega^N, t \in T : PF_{i,t} \tag{12s}$$

$$2x_i O_{i,t} + n_{i,t} - \rho u_{QF_{i,t}} \le \lambda_{i,t}^4 - \rho QF_{i,t}^z \quad i \in \Omega^N, t \in T : QF_{i,t} \tag{12t}$$

$$-O_{i,t} - \rho u_{v_{A_i,t}} \le \gamma_{i,t} - \rho v_{A_i,t}^z \quad i \in \Omega^N, t \in T \ : v_{A_i,i,t} \tag{12u}$$

$$-r_i m_{i,t} - x_i n_{i,t} - \rho u_{l_{i,i,t}} \le \mu_{i,t}^1 - \rho l_{j,t}^z \quad i \in \Omega^N, j \in C_i, t \in T : l_{j,i,t} \tag{12v}$$

$$-m_i - \rho u_{PF_{i,i,t}} = \mu_{i,t}^2 - \rho PF_{j,t}^z \quad i \in \Omega^N, j \in C_i, t \in T : PF_{j,i,t} \tag{12w}$$

$$-n_i - \rho u_{QF_{i,i,t}} = \mu_{i,t}^3 - \rho QF_{j,t}^z \quad i \in \Omega^N, j \in C_i, t \in T : QF_{j,i,t} \tag{12x}$$

and (3b)-(3f).

Where the primary variables associated with each constraint are represented in the developed dual formulation (12). The optimization model (12) is a non-linear problem regarding the product of $Pd_{i,t}^{Ex}.\beta_{i,t}$ and $Ps_{i,t}^{Ex}.\psi_{i,t}$ in (12a). However, the solution of the robust optimization model would be located in extreme points of the uncertainty set; therefore, new binary variables, i.e. $I_{i,t}^{d+}, I_{i,t}^{d-}, I_{i,t}^{s+}, I_{i,t}^{s-}$, as shown in (13) are defined to explore the optimal solution on the vertices of the uncertainty set [15].

$$Pd_{i,t}^{Ex} = \overline{Pd_{i,t}^{Ex}} + I_{i,t}^{d+}\Delta Pd_{i,t}^{Ex} - I_{i,t}^{d-}\Delta Pd_{i,t}^{Ex} \quad i \in \Omega^N, t \in T \tag{13a}$$

$$Ps_{i,t}^{Ex} = \overline{Ps_{i,t}^{Ex}} + I_{i,t}^{s+}\Delta Ps_{i,t}^{Ex} - I_{i,t}^{s-}\Delta Ps_{i,t}^{Ex} \quad i \in \Omega^N, t \in T \tag{13b}$$

$$I_{i,t}^{d+}, I_{i,t}^{d-}, I_{i,t}^{s+}, I_{i,t}^{s-} \in \{0,1\} \quad i \in \Omega^N, t \in T \tag{13c}$$

$$\sum_t I_{i,t}^{d+} + I_{i,t}^{d-} \le \Gamma_i^D \quad i \in \Omega^N \tag{13d}$$

$$\sum_t I_{i,t}^{s+} + I_{i,t}^{s-} \le \Gamma_i^S \quad i \in \Omega^N \tag{13e}$$

In this regard, the nonlinear terms in (12a) could be reformulated as (14) considering (13a).

$$Pd_{i,t}^{Ex}\beta_{i,t} = \overline{Pd_{i,t}^{Ex}}.\beta_{i,t} + \Delta Pd_{i,t}^{Ex}.(I_{i,t}^{d+}\beta_{i,t} - I_{i,t}^{d-}\beta_{i,t}) \ i \in \Omega^N, t \in T \tag{14a}$$

$$Ps_{i,t}^{Ex}\psi_{i,t} = \overline{Ps_{i,t}^{Ex}}.\psi_{i,t} + \Delta Ps_{i,t}^{Ex}.(I_{i,t}^{s+}\psi_{i,t} - I_{i,t}^{s-}\psi_{i,t}) \ i \in \Omega^N, t \in T \tag{14b}$$

The nonlinear terms $I_{i,t}^{d+}\beta_{i,t}, I_{i,t}^{d-}\beta_{i,t}, I_{i,t}^{s+}\psi_{i,t}, I_{i,t}^{s-}\psi_{i,t}$ in (14) originated from the product of continuous and binary variables, which cause the overall problem to be nonlinear. In this respect, big-M method is deployed to recast these nonlinear terms into linear terms as follows:

$$Pd_{i,t}^{Ex}.\beta_{i,t} = \overline{Pd_{i,t}^{Ex}}.\beta_{i,t} + \Delta Pd_{i,t}^{Ex}.(\beta_{i,t}^{+} - \beta_{i,t}^{-}) \tag{15a}$$

$$Ps_{i,t}^{Ex}.\psi_{i,t} = \overline{Ps_{i,t}^{Ex}}.\psi_{i,t} + \Delta Ps_{i,t}^{Ex}.(\psi_{i,t}^{+} - \psi_{i,t}^{-}) \tag{15b}$$

where $\beta_{i,t}^{+}, \beta_{i,t}^{-}, \psi_{i,t}^{+}, \psi_{i,t}^{-}$ are:

$$-\mathrm{MI}.I_{i,t}^{d+} \le \beta_{i,t}^{+} \le \mathrm{MI}.I_{i,t}^{d+} \tag{16a}$$

$$-\mathrm{MI}.I_{i,t}^{d-} \le \beta_{i,t}^{-} \le \mathrm{MI}.I_{i,t}^{d-} \tag{16b}$$

$$-\mathrm{MI}.I_{i,t}^{s+} \le \psi_{i,t}^{+} \le \mathrm{MI}.I_{i,t}^{s+} \tag{16c}$$

$$-\mathrm{MI}.I_{i,t}^{s-} \le \psi_{i,t}^{-} \le \mathrm{MI}.I_{i,t}^{s-} \tag{16d}$$

$$\beta_{i,t} - \mathrm{MI}.(1 - I_{i,t}^{d+}) \le \beta_{i,t}^{+} \le \beta_{i,t} + \mathrm{MI}.(1 - I_{i,t}^{d+}) \tag{16e}$$

$$\beta_{i,t} - \mathrm{MI}.(1 - I_{i,t}^{d-}) \le \beta_{i,t}^{-} \le \beta_{i,t} + \mathrm{MI}.(1 - I_{i,t}^{d-}) \tag{16f}$$

$$\psi_{i,t} - \mathrm{MI}.(1 - I_{i,t}^{s+}) \le \psi_{i,t}^{+} \le \psi_{i,t} + \mathrm{MI}.(1 - I_{i,t}^{s+}) \tag{16g}$$

$$\psi_{i,t} - \mathrm{MI}.(1 - I_{i,t}^{s-}) \le \psi_{i,t}^{-} \le \psi_{i,t} + \mathrm{MI}.(1 - I_{i,t}^{s-}) \tag{16h}$$

It is noteworthy that the MI value in the big-M method should be selected larger than $|\beta_{i,t}|, |\psi_{i,t}|$. In this regard, merging (15) and (16) with (12) results the complete x-update optimization model which would be conducted by each agent in the ADMM procedure.

2) z-update

The formulation of z-update optimization that is run by each agent in the ADMM procedure could be modeled as below:

$$\min_{z_i} \ -\lambda_i^T z_i - \mu_i^T z_i - \sum_{j \in C_i} \gamma_j v_i^z + \frac{\rho}{2}\left(\|x_i - z_i\|_2^2 + \|x_{i,A_i} - z_i\|_2^2 + \sum_{j \in C_i} \|v_{i,j} - v_i^z\|_2^2 \right) \tag{17a}$$

s.t.

$$(PF_{i,t}^z)^2 + (QF_{i,t}^z)^2 \le v_{i,t}^z l_{i,t}^z \qquad i \in \Omega^N, t \in T \tag{17b}$$

$$\underline{v_i} \le v_{i,t}^z \le \overline{v_i} \qquad i \in \Omega^N, t \in T \tag{17c}$$

The objective function (17a) strives to determine global variables taking into account the results of x-update optimization and the inequality power flow constraints of the distribution grid (17b)-(17c). Note that the z-update optimization has a max-min form similar to x-update optimization; however, the z-update optimization model is independent of the uncertain variable of the robust scheduling model, i.e., $p_{i,t}^e$. In this regard, the max-min optimization model is recast into a single min optimization model (17). In other words, the proposed distributed energy management framework and the corresponding formulation have facilitated the integration of robust optimization concept into the ADMM algorithm in order to enable distributed energy management of a multi-agent distribution system considering uncertainty associated with its respective agents. It is noteworthy that the proposed framework copes with the independent operation of agents and so addresses the privacy concerns in distribution systems with multi-agent structures.

The communication requirements of the ADMM procedure are depicted in Fig. 2. In this regard, prior to conducting x-update optimization of node *i*, $v_{A_i}^z$ and γ_i are taken from the ancestor node, whereas z_j and μ_j are taken from child nodes. Moreover, prior to the z-update optimization, $x_{i.A_i}$ and μ_i are collected from the ancestor node, while $v_{i,j}$ and γ_j are taken from child nodes.

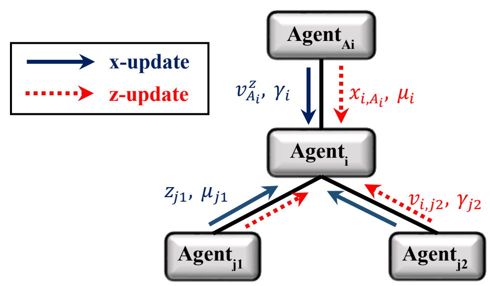

Fig. 2 Communication requirements of the ADMM procedure.

IV. Numerical Results

In this section, the proposed ADMM-based robust day-ahead scheduling scheme is implemented on the modified IEEE 37-bus distribution system [29] in order to analyze the effectiveness of the framework on the distributed management of multi-agent systems. The topology of the 37-bus distribution system is shown in Fig. 3, in which each node of the system is considered to be an independent agent. The operational data of the load profile, LMP, conventional DG units, the discomfort cost, PV units, and BES units are adopted from [3], [30]-[31] and presented in [32]. Figure 4 depicts the price of purchased power from the transmission network, i.e. LMP, and the per unit load profile associated with a workday in the winter utilized in the simulation process. The operational horizon *T* is assumed to be 24 hours with 1-hour time slots. It is considered that load demands are allowed to deviate up to 20% from their expected value, i.e., AD_i=20%. This would improve the flexibility of each agent in order to minimize its respective operational cost. Additionally, the robustness budget $\Gamma_i^D = \Gamma_i^S = 6$ as well as the maximum forecast error $\Delta Pd_{i,t}^{Ex} = 10\%$ and $\Delta Ps_{i,t}^{Ex} = 20\%$ are considered as base values. Finally, a sensitivity analysis is taken into account to investigate the effects of values of robustness budget, maximum allowed deviation, and maximum forecast error on the operational scheduling of the multi-agent system.

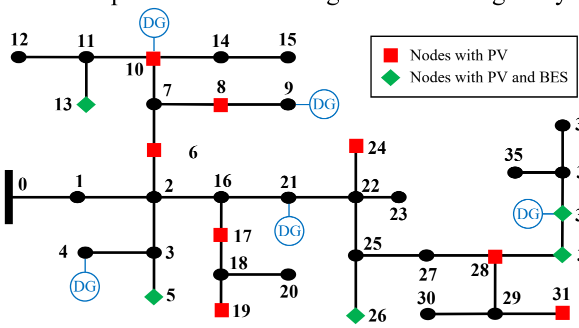

Fig. 3. The modified IEEE 37-bus distribution network.

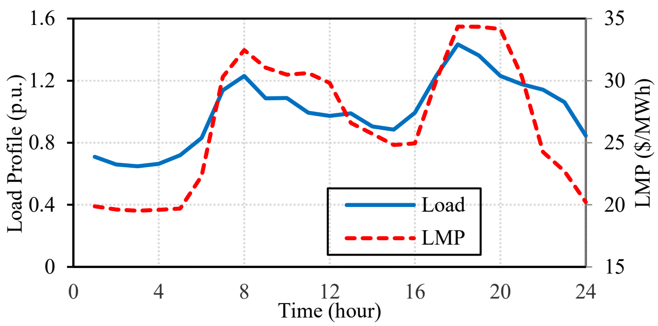

Fig. 4 Load profile and locational marginal price.

As mentioned, each node of the system is considered as an independent agent that participates in the proposed distributed robust scheme to schedule its local resources while communicating non-critical information with its neighbors in order to reach a consensus. As a result, power injection of nodes, i.e. $p_{i,t}$, and the power purchased from transmission level, i.e. P_t^{trans}, are iteratively determined taking into account the worst-case realization of uncertain parameters. In this regard, the amount of power injection of node 32 per iteration at 10:00 and 20:00 is presented in Fig 5. The acquired results indicate that implementing the proposed approach enables the agents to determine their optimal robust scheduling in an iterative manner without a central coordinator. Moreover, Fig. 6 shows the power purchased from transmission network per iteration at 10:00, 18:00, and 20:00. The primal and dual residuals per iteration, i.e. r^k and s^k, are depicted in Fig. 7. The obtained results show that the power purchased from transmission system converges to its optimum value and the residuals reach insignificant values as the framework proceeds; which demonstrates a strong performance of the proposed ADMM-based scheme.

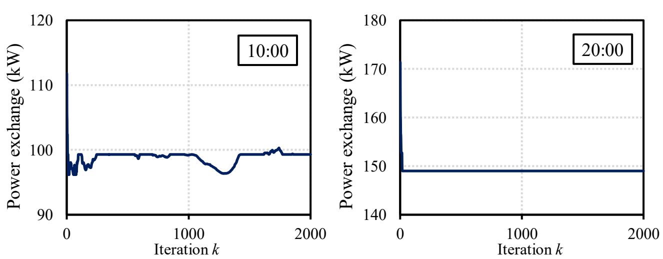

Fig. 5 Amount of power exchange of node 32 per iteration at 10:00 and 20:00.

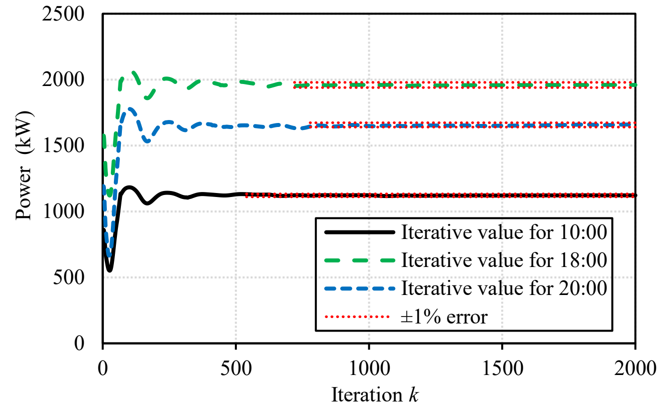

Fig. 6 Power purchased from transmission system per iteration at 20:00.

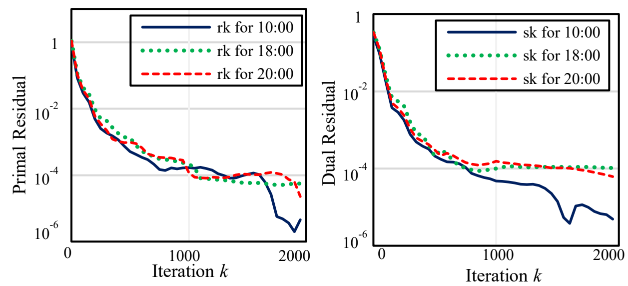

Fig. 7 Primal and dual residual of the ADMM process at 10:00, 20:00, and 22:00.

Figure 8 presents the operational scheduling of agent 32 over the time horizon. In this respect, the expected amount of demand ($\overline{Pd_{i,t}^{Ex}}$), robust expectation of demand ($Pd_{i,t}^{Ex}$), scheduled demand ($Pd_{i,t}^{Sch}$), the expected power production by PV units ($\overline{Ps_{i,t}^{Ex}}$) and robust expectation of PV production ($Ps_{i,t}^{Ex}$), as well as the energy stored by BES unit are shown in Fig. 8. Note that the difference between $Pd_{i,t}^{Sch}$ and $Pd_{i,t}^{Ex}$ is based on the agent's flexibility which would finally minimize its respective cost. The robustness budget $\Gamma_{i32}^{D}=6,\ \Gamma_{i32}^{S}=4$, maximum forecast error $\Delta Pd_{i32,t}^{Ex}=10\%,\ \ \Delta Ps_{i32,t}^{Ex}=20\%$, and maximum allowed deviation AD_{i32}=20% are considered by agent 32 to determine its scheduling. As can be traced in this figure, the worst-case realization for demand is determined in time slots 8:00, 12:00 and 17:00-20:00; when the amount of load demand as well as the price of purchasing power from the transmission system, i.e. LMP, reach their respective maximum values. The scheduled demand by the agent is shifted from time periods with high LMP values to periods with lower LMP values in order to minimize the total operational cost of the agent. Furthermore, the worst-case scenario for PV power production is determined in time slots 11:00-14:00; when the amount of expected amount of PV production and LMP value are relatively higher. In addition, the scheduling results for agent 9 is represented in Fig. 9. It is noteworthy that each agent independently adjusts the robustness of its scheduling by selecting the robustness budget and the forecast error interval. In this respect, agent 9 is considered to be more conservative than agent 32 with robustness budget $\Gamma_{i9}^{D}=8$, maximum forecast error $\Delta Pd_{i9,t}^{Ex}=15\%$, and maximum allowed deviation

AD_{i9}=15%. It could be seen that the worst-case realization for load demand occurs in time slots 7:00-9:00 and 17:00-21:00. Moreover, the DG unit of node 9 is not committed during 0:00—5:00 due to low LMP values, and reaches its maximum generation during 7:00-12:00 and 17:00-21:00 due to high LMP values.

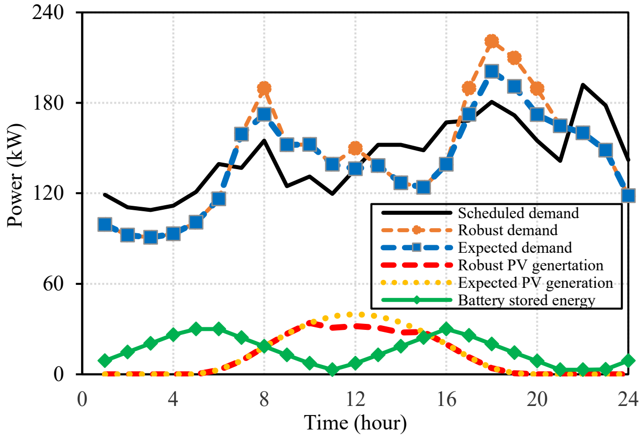

Fig. 8 Power scheduling of node 32 over time horizon.

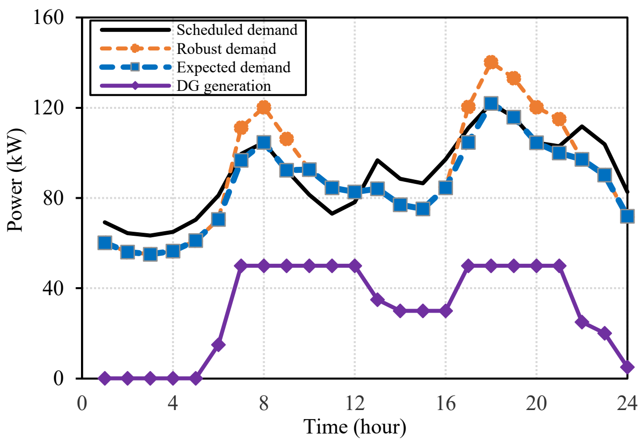

Fig. 9 Power scheduling of node 9 over time horizon.

To investigate the effects of the robustness budget, maximum allowable deviation of power injections, and conceived forecast errors on the operational costs of the system, sensitivity analysis is conducted as shown in Figs. 10-13. Note that in the case studies stated as "without framework", the agents' flexibility is not taken into account in the optimization of the multi-agent system. In this regard, Fig. 10 presents the total operational cost of the system in case of increasing the robustness budget from 0 to 12. The results show that implementing the proposed framework would decrease the operational cost of the multi-agent system. Moreover, the results presented in Fig. 11 indicate the decrease in the total operational cost of the system by increasing the maximum allowed deviation from 0% to 30%. In addition, the increase of the total operational cost associated with the distribution system by increasing the forecast error from 0% to 30% could be traced in Fig. 12. Regarding Fig. 11, employing the proposed framework would result in decreasing the total operational cost of the system in the studied forecast errors. It is noteworthy that while the increase in maximum allowed deviation approximately decreases the operational cost of the system by 5.5%; the increase in forecast error increases the total operational cost by 15.4%. Finally, the effect of scheduling parameters in operational cost of the system is summarized in Fig. 13.

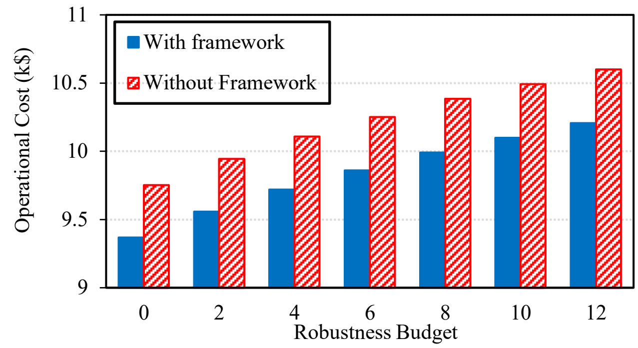

Fig. 10 Total operational cost of the distribution system considering different robustness budgets.

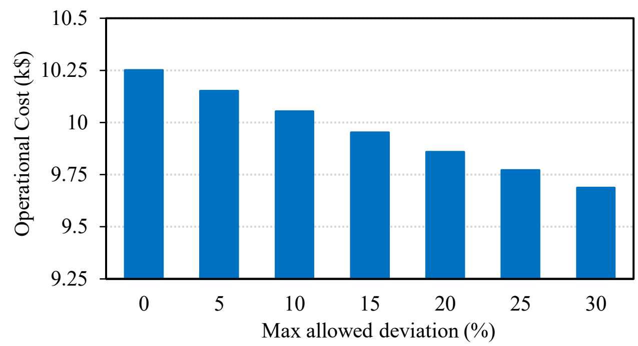

Fig. 11 Total operational cost of the distribution system considering different amounts of allowed load deviation.

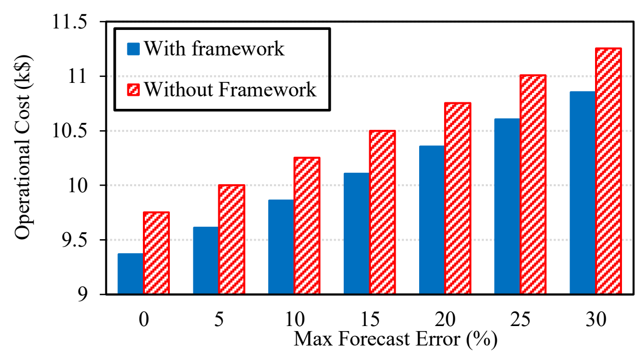

Fig. 12 Total operational cost of the distribution system considering different forecast errors.

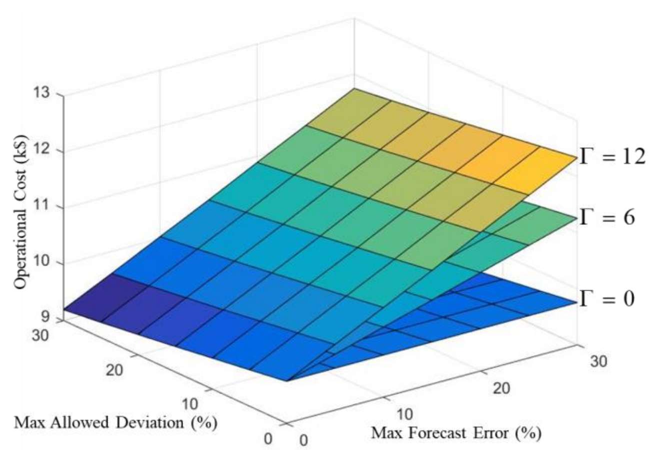

Fig. 13 The sensitivity analysis for total operational cost of the distribution system considering different scheduling parameters.

V. Conclusion

This paper provides an efficient distributed robust energy management framework for distribution systems with multi-agent structures. The proposed scheme relies on ADMM approach to coordinate the operational scheduling of independent agents in a distributed manner, while considering operational constraints associated with distribution networks. In this regard, the communication of each agent is limited to non-critical information with its adjacent neighbors to address the privacy and cyber-security concerns in a multi-agent system. Moreover, integrating the robust optimization concept in the ADMM algorithm enables the agents to conduct their operational scheduling considering the worst-case realization of uncertain parameters. In this context, a robustness budget parameter is employed to model the robustness level in operational scheduling of the agents. The developed framework is implemented on the modified IEEE 37-bus network to investigate the effectiveness of the scheme in distributed robust operational scheduling of multi-agent distribution systems. In addition, sensitivity analysis is taken into consideration to analyze the effects of input parameters on the operational scheduling of the system.

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