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Published in:
IOP Conference Series: Earth and Environmental Science

DOI:
[10.1088/1755-1315/710/1/012043](https://doi.org/10.1088/1755-1315/710/1/012043)

Published: 26/04/2021

Document Version
Publisher's PDF, also known as Version of record

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Please cite the original version:
Korhonen, O., Lojander, M., Löfman, M., Koskinen, M., & Korkiala-Tanttu, L. (2021). The effect of stress path on the dilatation angle of soils. *IOP Conference Series: Earth and Environmental Science*, 710(1), Article 012043. <https://doi.org/10.1088/1755-1315/710/1/012043>

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To cite this article: O Korhonen *et al* 2021 *IOP Conf. Ser.: Earth Environ. Sci.* **710** 012043

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The effect of stress path on the dilatation angle of soils

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Abstract. Modelling the horizontal displacements in subsoil is essential in many geotechnical design situations. Especially dilatation angle should be correctly defined in order to ensure accurate modelling. Hence, this paper studies how the direction of stress path affects the dilatation angle of various soils. The study utilizes numerous anisotropically consolidated and drained compression triaxial tests, and the samples represent normally consolidated soft clay, stiff over-consolidated silt and dry sand. The plastic potential function is derived based on the triaxial test results, using stress values in which the dilatation angle is identical to the direction of stress path. The shape of potential function is assumed to be similar to the yield function, but with different parameters (non-associated modelling of strains). Finally, deformations are calculated by using the concept of potential energy, which is a second order function in mean effective stress - deviatoric stress -space. Simulated values are then compared to triaxial test measurements, and the agreement between values is found to be rather good.

1. Introduction

Modelling the horizontal displacements in subsoil is essential in geotechnical design situations such as sheet pile walls and embankments with low stability. These horizontal displacements are controlled by the changes in the stress state and the mechanical properties of the subsoil. Regarding the mechanical properties, especially dilatation angle should be correctly defined in order to accurately model the horizontal displacements.

Experimental results on Ojakkala sand [1] have shown that the dilatation angle (defined as the ratio between incremental plastic volumetric strains ε_v and shear (deviatoric) strains ε_s) is dependent on the imposed stress path. To be more specific, triaxial tests (common triaxial tests and true triaxial tests) on Ojakkala sand have shown that samples compacted to identical density and in same initial pressure exhibit different dilatation when the direction of stress path is varied. Therefore, the plastic strain increment vector is not always normal to the tangent of potential function curve, as the applied stress path direction can distort the dilatation angle.

This paper studies how the direction of stress path affects the dilatation angle of three soil types (clay, silt and sand), and offers one solution on how to consider the stress path effect by means of energy functions. The study utilizes numerous anisotropically consolidated drained compression triaxial tests. The samples represent normally consolidated, soft Finnish clay, stiff, over-consolidated silt and dry sand.

Deformations of cohesive soils are often calculated using simplified associated soil models, in which the plastic potential function and the yield function are assumed identical. The yield function of the



Finnish soft clay is reported by several authors to be an ellipse in triaxial stress space [2-4]. In this analysis the yield function is not needed and the potential surface is approximated with a parabola. The potential function is derived based on triaxial test results, using stress values in which the dilatation angle is identical to the direction of the stress path. At these stress values, the definition of plastic potential function applies. The form of potential function is assumed to be similar to the yield function, but with different parameters. Finally, deformations are calculated by using the concept of potential energy, which is a second order function in mean effective stress (p') - deviatoric stress (q) -space. The calculated deformations are compared to the experimental results in order to study the performance of the derived model.

2. Determination of the plastic potential function g

In this paper three different types of soil are presented. Otaniemi clay is very soft and normally consolidated clay. The properties of Otaniemi clay has been reported by [2-4]. The tests carried out were mainly triaxial CA-tests (anisotropically consolidated tests) with stress ratios of constant q/p varying in the range of $-0,6 \dots 0,9$. The slope of the normal to the potential function g was found to be:

$$\frac{dq}{dp} = \frac{(k^2 - \alpha^2)}{(k - \alpha M)} \quad (1)$$

where M is the stress ratio q/p in critical state, $\eta = q/p$, $k =$ parameter and $\alpha =$ stress ratio when the slope of the normal to potential is zero. When $k = 2,0$, g is an ellipse. For Otaniemi clay the parameters are $M = 1,1$, $\alpha = 0,4$ and $k = 0,2$.

Lahti silt is slightly over-consolidated and very stiff. The potential function g is the same as for Otaniemi clay. The mechanical properties of Lahti silt have been previously reported by [2,3]. The tests on Lahti silt were mainly ordinary CADC-tests (anisotropically consolidated drained compression triaxial tests) with the slope of the stress path $dq/dp = 3:1$. In addition, the test CAD413B with Lahti silt had a stress path with constant value of q . The derivative of potential function g as a function of q/p is illustrated in figure 4. For Lahti silt the parameters in equation (1) are $M = 1,3$, $\alpha = 0,42$ and $k = 2,0$.

Ojakkala sand samples were compacted to a very dense state. Tests were carried as CADC-tests with various directions of the stress path. In addition to conventional triaxial tests, true-triaxial tests were carried out as well. The mechanical properties of Ojakkala sand have been studied by Korhonen and Laaksonen [1].

Figure 1 shows, how the direction of dilatation $\delta\varepsilon_v/\delta\varepsilon_s$ changes when the direction of the stress path is varied. This example represents Ojakkala sand. Several samples are compacted in the same initial void ratio $e = 0,54$ and are then consolidated anisotropically to the same initial stress state. The samples are assumed to be identical. The samples are dry, so the water content is zero and the pore pressure does not affect the test results.

To find the potential function g , which is independent of the applied stress path, special points of test data should be utilized. These data points are the points where the dilatation angle $\delta\varepsilon_v/\delta\varepsilon_s$ is identical to the slope of the stress path $\Delta p/\Delta q$. In these points also the normal of potential function g has the same slope in (p, q) -space. In the simplest case these points are supposed to lie on a straight line in $q/p - \delta\varepsilon_v/\delta\varepsilon_s$ -plane. In order to find the potential function $g(p, q)$, equation (2a) is integrated in order to get equation (2b):

$$\frac{dq}{dp} = \frac{d\bar{A}}{dA} \quad (2a)$$

$$\frac{dq}{dp} = \frac{(-\bar{A})}{(\bar{A}+1)} = 0 \quad (2b)$$

In equation (2a), $\delta\varepsilon_v/\delta\varepsilon_s$ is the normal of g , M is the stress ratio q/p when $\delta\varepsilon_v/\delta\varepsilon_s = 0$, k is a parameter and C is constant of integration. If $k = -2$, g represents a parabola as in the example of figure 2a. Of course, other choices than a straight line are possible. For example, equation (1) is suitable for soft soils.

Another way to determine g is to use CA-tests with various ratios of $\Delta q/\Delta p$. By this method only two points of g can be found. The third way to determine g is to use the measured values of $\delta\varepsilon_v/\delta\varepsilon_s$ with the principle shown in figure 2a.

Now that g is determined, we know the two vectors that affect the dilatation angle $\delta\varepsilon_v/\delta\varepsilon_s$. These are the applied incremental stress-vector ($\Delta q, \Delta p$) and the normal of potential function g . In order to find the effect of g we must choose a surface, by which we can select the “length” of the normal vector of g . This can be done by selecting an energy function $E(q, p)$ and $E(\varepsilon_v, \varepsilon_s)$ on which both the applied stress vector and the vector normal to g lie. The principle is illustrated in figure 2a. A suitable and simple assumption of the energy function is a circle with center in origin. This is illustrated in Figure 2b and Figure 2c.

The dilatation angle $\delta\varepsilon_v/\delta\varepsilon_s$ is calculated by the system of following equations:

$$2^2 = 1^2 + 1^2 - 2^2 \quad (3a)$$

$$2 = \frac{1}{(2 - \bar{A}_0)} + \quad (3b)$$

$$= \frac{2 + \bar{A}_1 - 2\bar{A}_0}{2 + \bar{A}_1 - 2\bar{A}_0} \quad (3c)$$

where g is the slope of the normal of potential function, (p_0, q_0) is the initial stress state, (p_1, q_1) is the applied stress state, and (p_2, q_2) represents the same energy than in state (p_1, q_1) but is situated on the normal of g .

In this paper, the incremental potential energy function is simply:

$$\Delta = \Delta(+) = \Delta(2 + 2) \quad (4a)$$

$$\Delta = \Delta(2 + 2) \quad (4b)$$

where a and b are parameters. These represent circles. Kinetic energy is neglected. The energy increment, equation (4a-4b) can be split in components:

$$\Delta = \Delta + \Delta + \Delta + \Delta \quad (5)$$

All components in equation (5) except the last one represents conserve energy and in critical state are zero. The last term represents dissipative energy (friction loss). In figures 2b and 2c estimations of $E(q, p)$ and $E(\varepsilon_v, \varepsilon_s)$ are illustrated. The final dilatation angle is the direction of the sum of vectors ($\Delta p, \Delta q$) and the normal to g as illustrated in figure 2a.

In figure 2d, the observed and calculated dilatation angles $\delta\varepsilon_v/\delta\varepsilon_s$ are shown as a function of stress ratio q/p with various directions of stress paths. The strains are calculated by using the initial volume of the sample; in other words, all the calculated strains presented in this paper are so-called engineering strains.

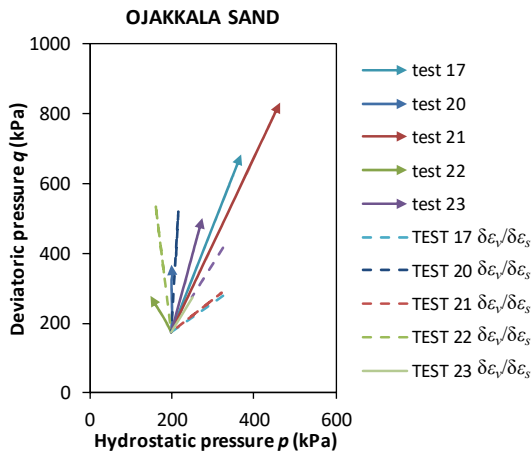


Figure 1. Initial dilatation angles and corresponding stress paths of Ojakkala sand ($e=0,54$)

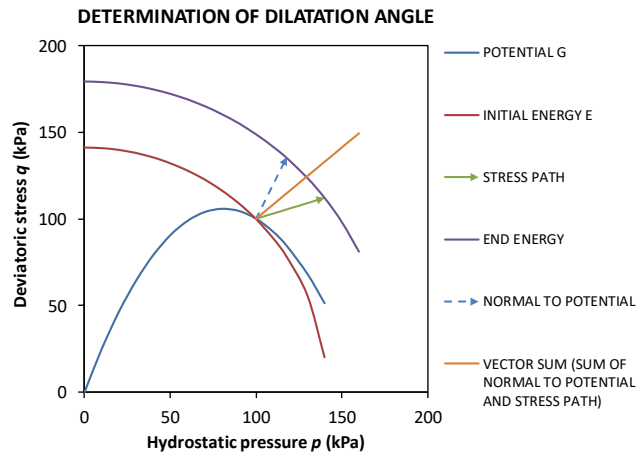


Figure 2a. Determination of the dilatation angle.

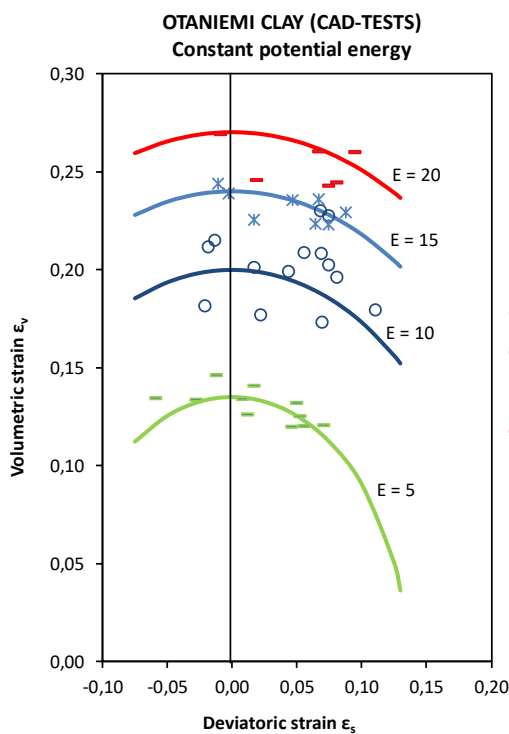


Figure 2b. Constant potential energy ($E = p\epsilon_v + q\epsilon_s$) as a function of strains.

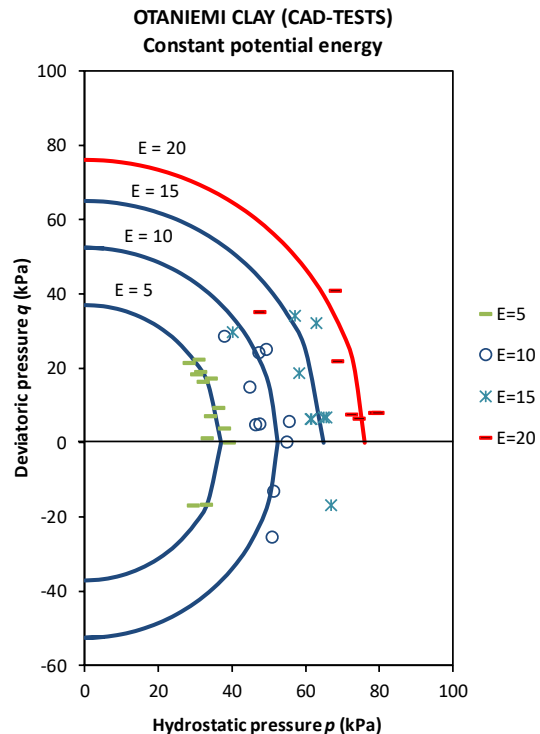


Figure 2c. Constant potential energy ($E = p\epsilon_v + q\epsilon_s$) as a function of stresses.

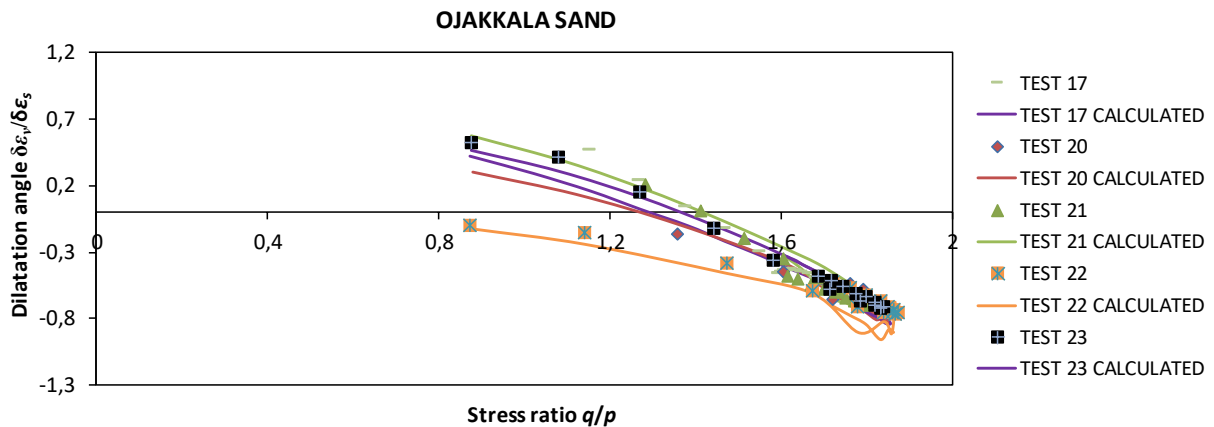


Figure 2d. Ojakkala sand. Calculated and observed values of dilatation angle versus stress ratio q/p .

3. Determination of the deformations by an empirical energy function

Earlier in equation (5), energy was expressed using stress and strain increments. In larger scale one has to develop an expression for energy with both stresses and strains to create a link between stress vectors and strain vectors. These are $\sigma^2 = (p^2 + q^2)$ and $\varepsilon^2 = (\varepsilon_v^2 + \varepsilon_s^2)$, where σ is the length of the stress vector and ε is the length of the strain vector.

Energy $E(\sigma)$ is assumed to be separated to two parts: if stress ratio q/p does not exceed critical value M , energy will increase without limit. If not, the energy will decrease mainly by volumetric growth or softening. The empirical energy function is:

$$(\quad + \quad +) = (\quad_1 + \quad_1 + \quad_1) \tag{6}$$

where C, K, H, K_l, H_l and λ are parameters. Equation (6) presents a solution of $d^2E/d\varepsilon^2 + dE/d\varepsilon + E = 0$, where $\sigma = dE/d\varepsilon$. Energy $E(\sigma)$ can be calculated by iteration from equation (6). In figures 8a and 8b energy is shown with the help of stresses and deformations respectively.

Deformation can be calculated from (an almost linear) Equation (7):

$$= \quad^2 \tag{7}$$

where k is a parameter, which depends on the slope of the stress path.

4. Results

The observed and calculated values of the volumetric strain and the stress ratio for Otaniemi clay are illustrated in figure 3. The slope of the stress path is 3:1, and the calculated results match very well to the observed values. The calculated inclinations of normal vectors of Lahti silt are shown in figure 4. Figures 5, 6 and 7 (Lahti silt) show, that dilatation angle is determined quite well because the shapes of the calculated curves are almost the same as measured. The calculated strains do not match well to observed values.

The results of Ojakkala sand are illustrated in figures 9 to 11. Measured and calculated deformations with different stress paths are shown. The results do not match very well although the dilatation angle has been determined quite well (figure 2d). The main reason for this discrepancy is the difficulties in calculating deformations in softening area.

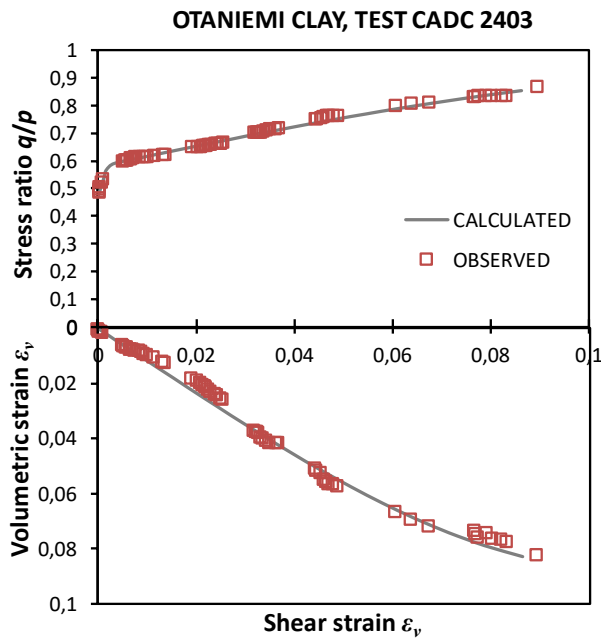


Figure 3. Otaniemi clay. Stress ratio q/p and volumetric strain ε_v as function of shear strain ε_s .

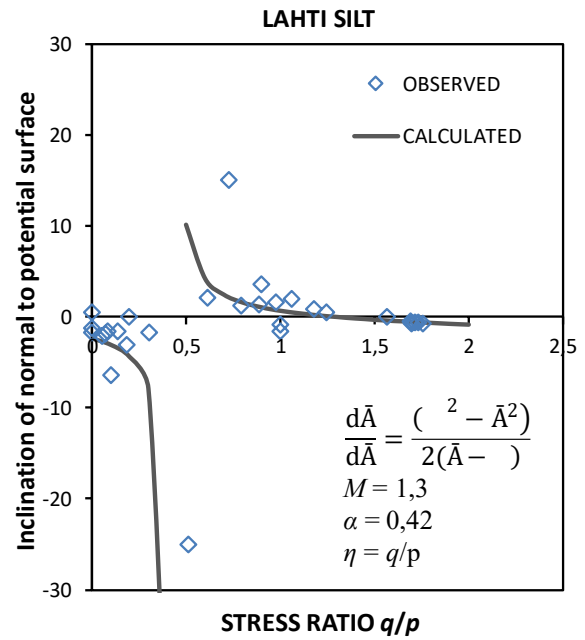


Figure 4. Lahti silt. Determination of plastic potential.

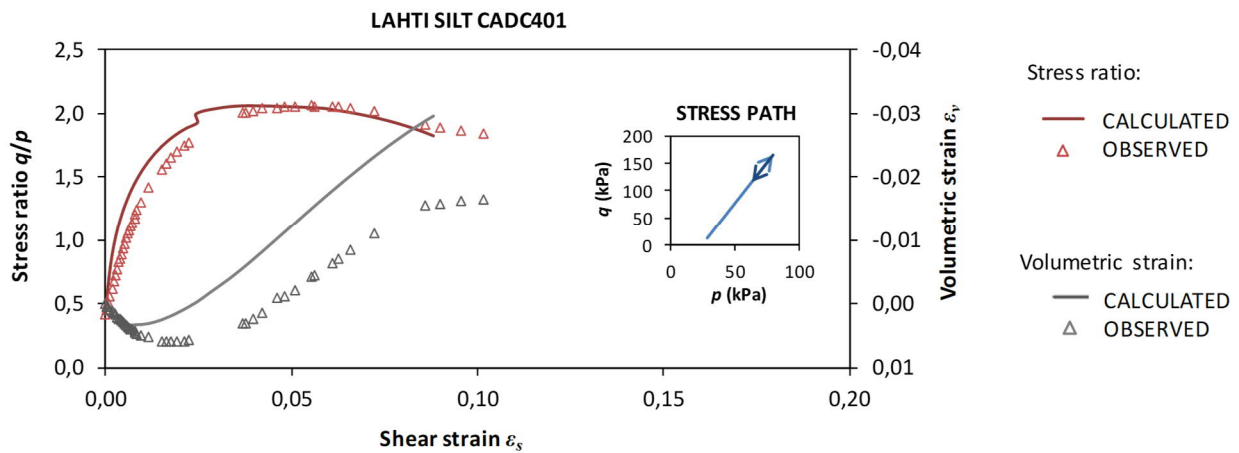


Figure 5. Lahti silt. Observed and calculated volumetric strain ε_v and stress ratio q/p versus shear strain ε_s .

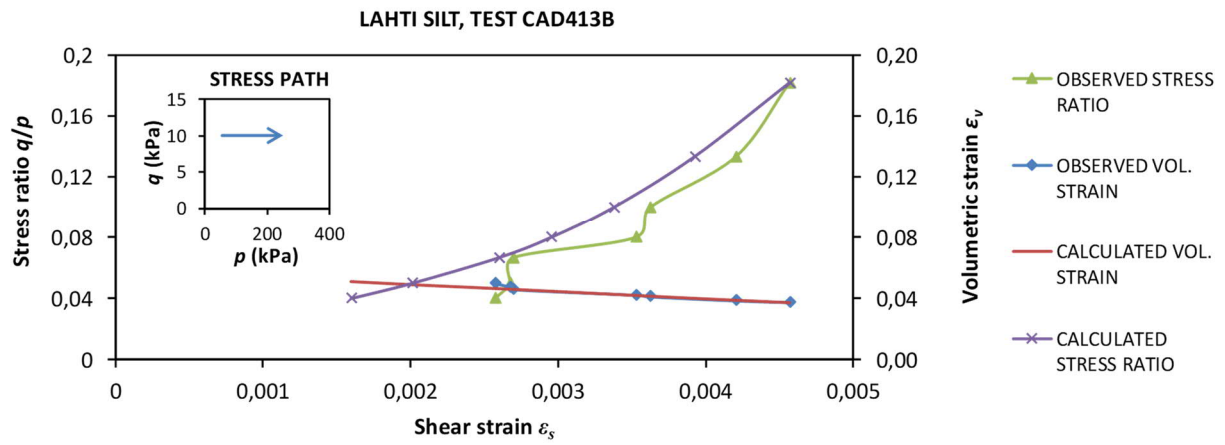


Figure 6. Lahti silt. Observed and calculated volumetric strain ϵ_v and stress ratio q/p versus shear strain ϵ_s .

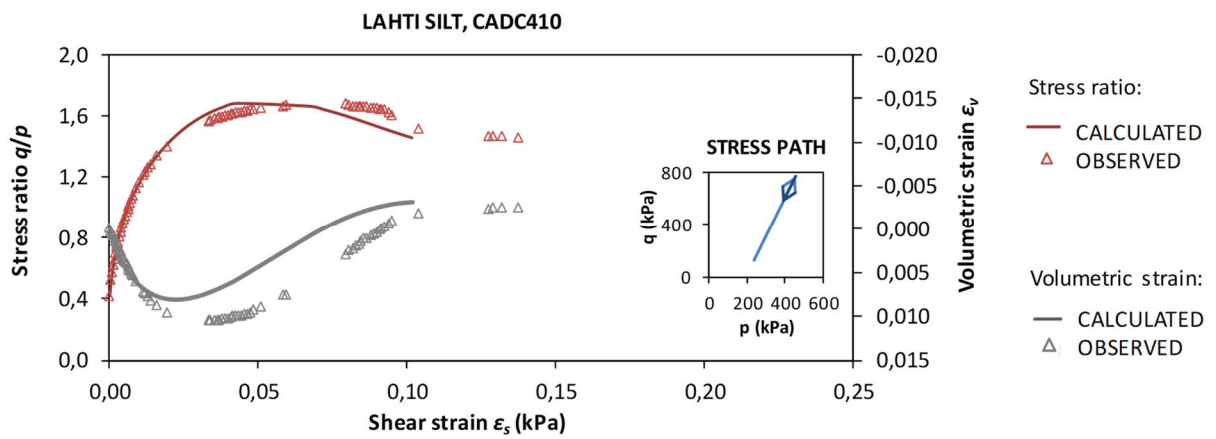


Figure 7. Lahti silt. Observed and calculated volumetric strain ϵ_v and stress ratio q/p versus shear strain ϵ_s .

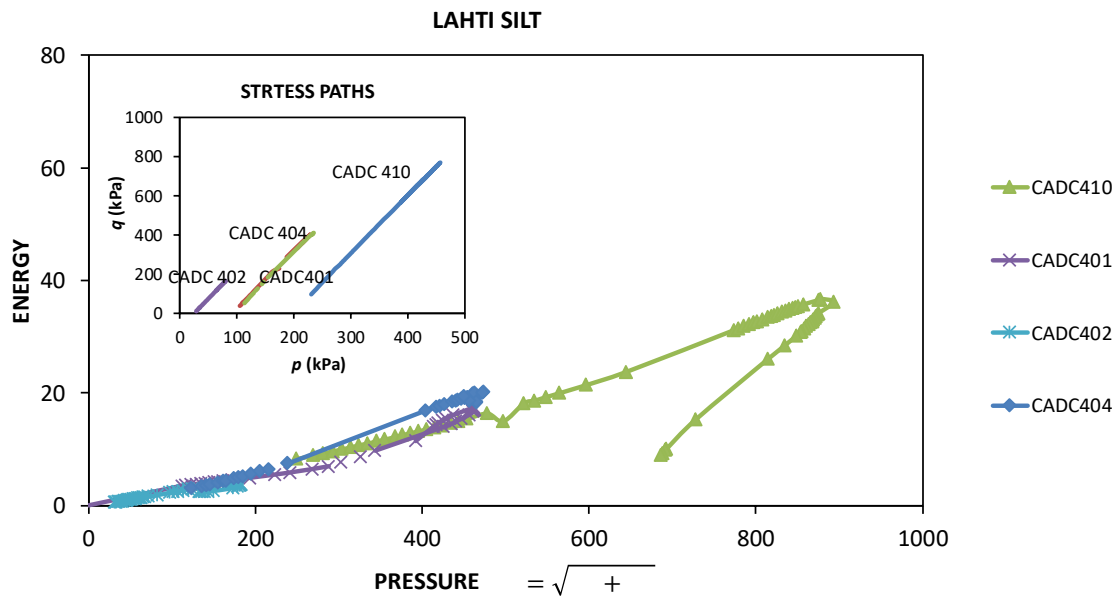


Figure 8a. Lahti silt. Energy versus pressure.

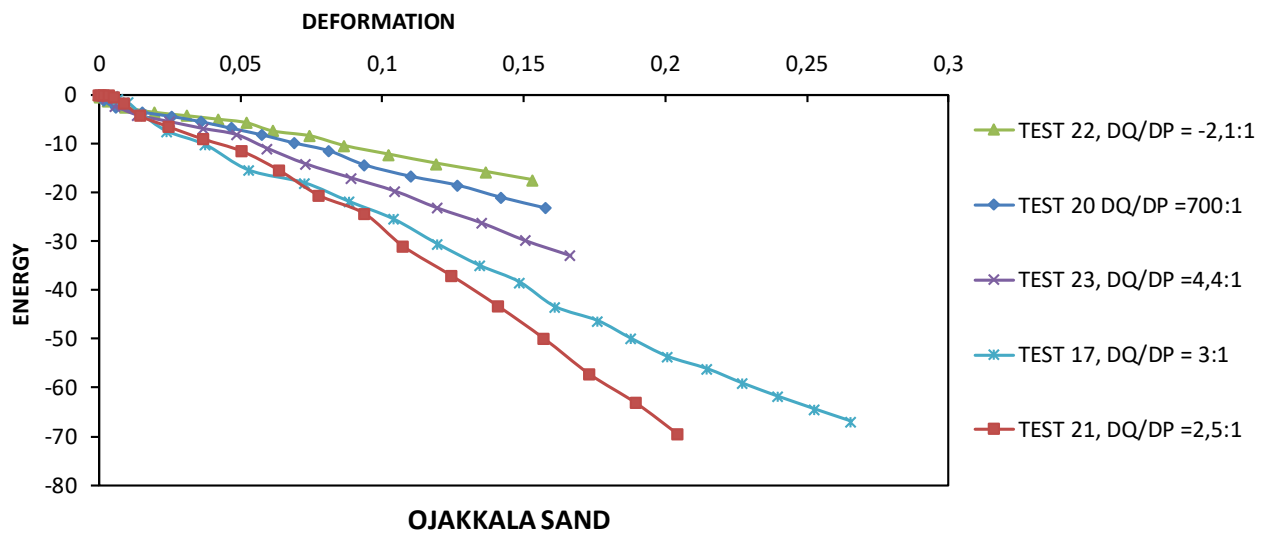


Figure 8b. Ojakkala sand. Energy versus strain.

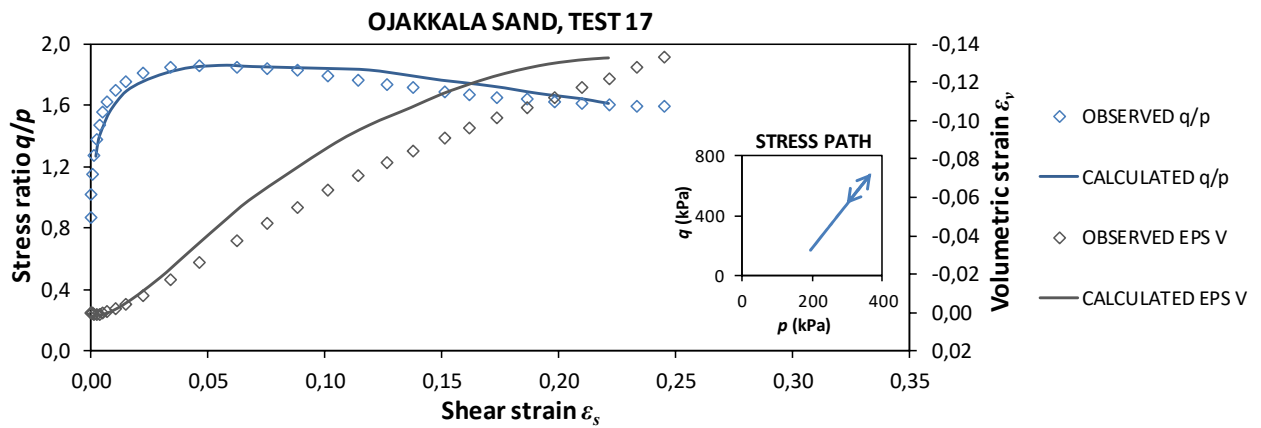


Figure 9. Ojakkala sand. Observed and calculated volumetric strain ϵ_v and stress ratio q/p versus shear strain ϵ_s ($dq/dp = 3:1$).

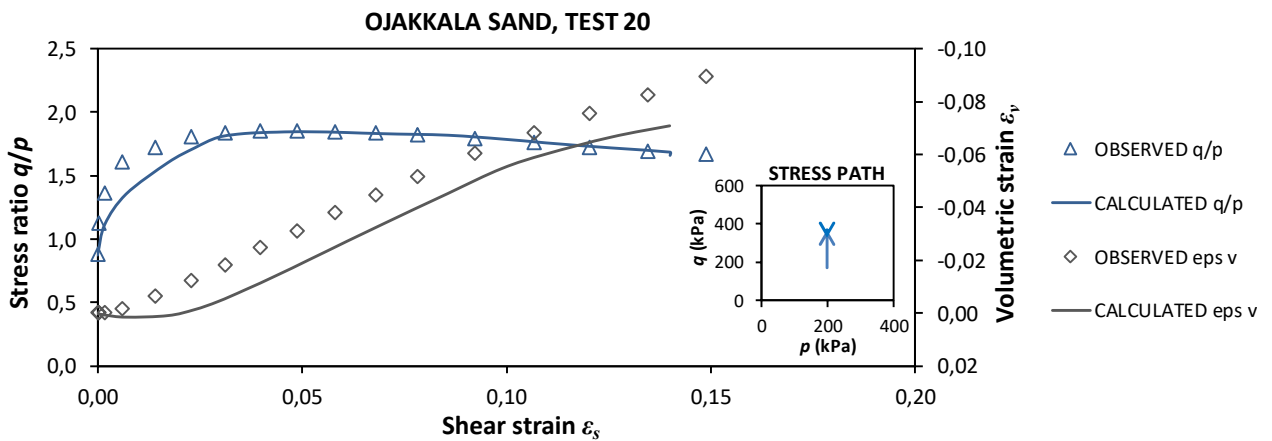


Figure 10. Ojakkala sand. Observed and calculated volumetric strain ϵ_v and stress ratio q/p versus shear strain ϵ_s , ($dq/dp = 700:1$).

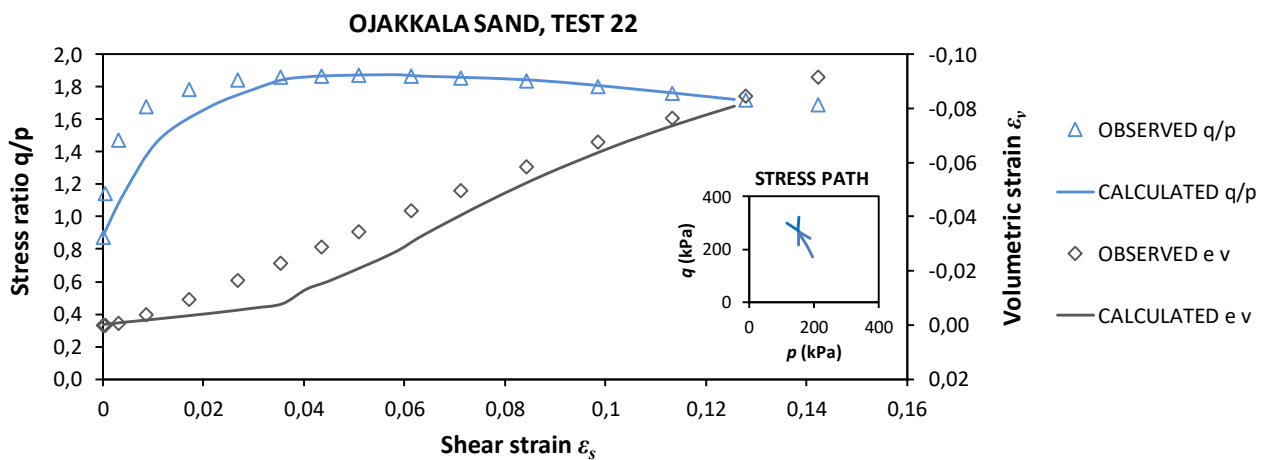


Figure 11. Ojakkala sand. Observed and calculated volumetric strain ϵ_v and stress ratio q/p versus shear strain ϵ_s , ($dq/dp = -2,1:1$).

5. Discussion

The determination of plastic potential needs a lot of different triaxial tests. The direction of the stress path should be the same as the normal vector to plastic potential and the same for dilatation angle. By ordinary stress path ($dq/dp = 3:1$) it is possible to find only one point, by series of CA-tests with various stress paths only two points can be determined. Of course one can calculate the dilatation angle from every stress point (p, q) when the dilatation angle is observed by the method shown in figure 2a. One possibility is to carry out both a set of CA-tests with variable stress ratio q/p and ordinary CADC-tests ($dq/dp = \pm 3:1$) and combine the results of these tests. All possible stress paths can thus be modelled.

6. Conclusions

The experimental results have shown that the dilatation angle of Ojakkala sand is dependent on the imposed stress path. Consequently, in order to calculate volumetric strain ε_v and shear strain ε_s , a reliable method to model the dilatation angle ($\delta\varepsilon_v/\delta\varepsilon_s$) must be developed. This paper has offered one solution on how to consider the effect of stress path direction on the dilatation angle. Using simple equations of energy, it is possible to find a calculation method which is valid also for softening soil.

On the other hand, in this paper the kinetic energy is neglected because all the triaxial tests were carried out using a low rate of strain, less than 1,5 % per hour. By taking account the rate of strain, one could determine its effect on the strength of the samples. Hence, the method of using energy functions for calculating deformations should be further developed.

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