Diaz-Rubio, Ana; Tcvetkova, Svetlana; Tretyakov, Sergei

Analytical Models of Reflection and Scattering by Finite-Size Anomalously Reflecting Metasurfaces

Published in:
15th European Conference on Antennas and Propagation, EuCAP 2021

DOI:
10.23919/EuCAP51087.2021.9411336

Published: 22/03/2021

Document Version
Peer-reviewed accepted author manuscript, also known as Final accepted manuscript or Post-print

Please cite the original version:
Diaz-Rubio, A., Tcvetkova, S., & Tretyakov, S. (2021). Analytical Models of Reflection and Scattering by Finite-Size Anomalously Reflecting Metasurfaces. In *15th European Conference on Antennas and Propagation, EuCAP 2021* Article 9411336 EurAAP. https://doi.org/10.23919/EuCAP51087.2021.9411336

Analytical Models of Reflection and Scattering by Finite-Size Anomalously Reflecting Metasurfaces

Ana Díaz-Rubio, Svetlana Tcvetkova, and Sergei Tretyakov
Dept. of Electronics and Nanoengineering, Aalto University, Espoo, Finland, email address: ana.diazrubio@aalto.fi

***Abstract*—The use of tunable metasurfaces for engineering and optimizing propagation environment is actively considered by the telecommunications community. These metasurfaces serve as anomalous reflectors that redirect incident waves into the desired direction. Known models of anomalous reflectors assume that each metasurface element (meta-atom) acts as an independent relay, reflecting the incident waves with a desired phase shift, realizing the phased-array functionality. This assumption, however, is not valid for vast majority of anomalously reflecting metasurfaces. Moreover, the known theories of metasurface reflectors is limited to infinite planar reflectors. In this talk, we will present an approximate analytical model of far-field scattering from anomalously reflecting metasurfaces of a finite size and discuss the features of reflected fields that cannot be found using the known models.**

***Index Terms*—anomalous reflector, reconfigurable intelligent surface (RIS), far-field scattering**

I. Introduction

Current research of anomalously reflecting metasurfaces and their applications (e.g., [1], [2]) is focused on the study and characterization of anomalous reflectors under very specific conditions, usually for a single plane-wave illumination from a specific direction. Moreover, only reflection from anomalous reflectors of an infinite extent are well understood [3].

The most common approach to design metasurfaces for manipulating the direction of reflected waves is to make the reflection phase nonuniform along the metasurface. As it is known from the phased-array antenna theory, in the particular case of anomalous reflection (a plane wave is reflected breaking the reflection law, i.e., the reflection angle is not equal to the incidence angle), the reflection phase should linearly vary compensating the phase mismatch between the incident and reflected waves.

Known approaches to calculation of reflection from finite-sized anomalous reflectors are based on the local reflection coefficient model, that is, on the assumption that at each point of the metasurface the reflected field equals the incident field with the desired phase shift: More advanced models also include correction of the field amplitude based on the power conservation, e.g., [4], [5]. The control of the reflection phase is assumed to be achieved by engineering the local surface impedance of the reflector, using the locally periodic approximation. These models do not account for parasitic reflections due to impedance mismatch [3]. Another limitation is that the known models can be used only for a single plane-wave illumination at the design incidence angle, and they are not useful if the metasurface is illuminated from many directions (the multi-path scenario).

In this presentation we will discuss our recent results on development of analytical models of reflection and scattering from finite-size anomalous reflectors. The developed model is applicable to both phase-gradient reflectors (reflectarrays) [2] and nonlocal or power-flow-conformal anomalous reflectors [3].

II. Theory and Results

We consider reflection and scattering from an anomalously reflecting metasurface illuminated by a plane wave at an arbitrary angle, see Fig. 1.

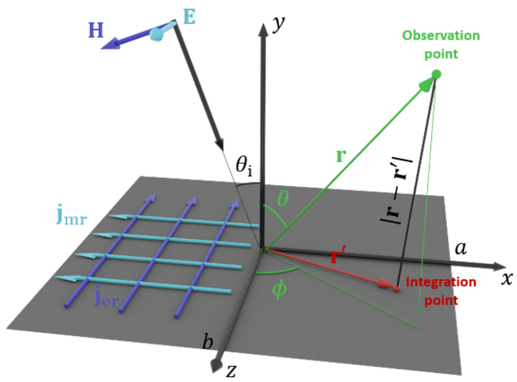

Fig. 1. Geometry of the problem. A finite-size metasurface of rectangular shape (size $2a \times 2b$) is placed at the xz-plane. The surface is illuminated by a TE-polarized plane wave, and the incidence plane is the xy-plane.

Because the metasurface is a periodical structure, it can scatter into several diffraction lobes, whose directions are defined by the reflection angles $\theta_{{\rm r}n}$, $n = 0, \pm 1, \ldots$. These angles depend on the incident angle $\theta_{\rm i}$, the metasurface structure period D, and the wavenumber k in the following way:

$$\sin\theta_{{\rm r}n} = \sin\theta_{\rm i} + \frac{n2\pi}{kD} \tag{1}$$

Next, we introduce a macroscopic reflection coefficient as a combination of the individual reflection coefficients for each individual reflected mode as

$$R(\theta_{\rm i}, x) = \sum_n r_n(\theta_{\rm i}) e^{-jk(\sin\theta_{{\rm r}n} - \sin\theta_{\rm i})x} e^{-jk(\cos\theta_{{\rm r}n} + \cos\theta_{\rm i})y} \tag{2}$$

where $r_n(\theta_{\rm i}) = |r_n(\theta_{\rm i})| \exp[j\phi_n(\theta_{\rm i})]$ is the individual reflection coefficient for each propagating diffracted mode. For phase-gradient metasurfaces the individual reflection coefficients are found using the mode-matching approach for solving reflection problems for periodically modulated impedance boundaries [6]. For nonlocal metasurfaces they need to be found numerically (for the illumination at the design angle, these metasurfaces realize theoretically perfect response with $r_0 = \sqrt{\cos\theta_{\rm i}/\cos\theta_{\rm r}}$, and all the other individual-mode reflection coefficients are zero).

Now we can write the reflected field as the sum over the propagating harmonics,

$$E_{{\rm r}z} = E_0 \sum_n r_n(\theta_{\rm i}) e^{-jk(\sin\theta_{{\rm r}n} x' + \cos\theta_{{\rm r}n} y')} \tag{3}$$

and calculate the corresponding magnetic field of these plane waves. Next, we introduce the effective equivalent Huygens electric and magnetic surface currents

$$\mathbf{j}_{\rm e} = \hat{\mathbf{y}} \times (\mathbf{H}_{\rm i} + \mathbf{H}_{\rm r}), \qquad \mathbf{j}_{\rm m} = -\hat{\mathbf{y}} \times (\mathbf{E}_{\rm i} + \mathbf{E}_{\rm r}) \tag{4}$$

and calculate the corresponding scattered field as (e.g., [7])

$$\mathbf{E}_{\rm r}(\mathbf{r}) = \frac{1}{j\omega\varepsilon}[\nabla\nabla + k^2] \int_S \mathbf{j}_{\rm e}(\mathbf{r}') G_0(\mathbf{r},\mathbf{r}') dS' \\ - \nabla \times \int_S \mathbf{j}_{\rm m}(\mathbf{r}') G_0(\mathbf{r},\mathbf{r}') dx'dz' \tag{5}$$

where

$$G_0(\mathbf{r},\mathbf{r}') = \frac{1}{4\pi} \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} \tag{6}$$

is the scalar Green function, and the integration is over the metasurface plane.

In the far-field approximation, this integral simplifies to

$$\mathbf{E}_{\rm r}(\mathbf{r}) \approx \frac{jk}{4\pi} \frac{e^{-jk|\mathbf{r}|}}{|\mathbf{r}|} \int_S e^{jk\hat{\mathbf{r}}\cdot\mathbf{r}'} \hat{\mathbf{r}} \times [Z_0 \hat{\mathbf{r}} \times \mathbf{j}_{\rm e}(\mathbf{r}') + \mathbf{j}_{\rm m}(\mathbf{r}')] dx'dz' \tag{7}$$

where Z_0 is the wave impedance of the surrounding space and $\hat{\mathbf{r}}$ is the unit vector along $\mathbf{r}$, pointing from the center of the metasurface plate to the observation point. This approximation is valid under the following assumptions: $|\mathbf{r}| \gg \lambda$, $|\mathbf{r}| \gg L$, and $L^2/|\mathbf{r}| \ll \lambda$. Here, $L = \max(2a, 2b)$ is the largest size of the metasurface plate.

The integration is trivial because for plane-wave illumination the integrand is an exponential function. For the considered polarization and for the observation point in the incidence plane, the result reads

$$E_{{\rm r}z} = \frac{jk}{4\pi} \frac{e^{-jk|\mathbf{r}|}}{|\mathbf{r}|} E_0 S \Big[(\cos\theta - \cos\theta_{\rm i}) {\rm sinc}(k a_{\rm ef}) \\ + \sum_n r_n(\theta_{\rm i})(\cos\theta + \cos\theta_{{\rm r}n}) {\rm sinc}(k a_{{\rm ef}n}) \Big] \tag{8}$$

where $a_{\rm ef} = (\sin\theta - \sin\theta_{\rm i})a$, and $a_{{\rm ef}n} = (\sin\theta - \sin\theta_{{\rm r}n})a$ represents the effective size of the metasurface for each diffracted mode, and $S = 4ab$ is the total surface of the metasurface.

As an example, in Fig. 2 we show the scattering pattern for a phase-gradient anomalous reflector that is designed to reflect normally incident waves to the 70° direction. For the design incident angle (for the normal incidence) we see that the main lobe of the scattered field is indeed tilted by 70°, and there is some modest scattering into other, undesired propagating harmonics. However, when the illumination direction is different, the response changes dramatically: most of the incident power is reflected back toward the illumination source.

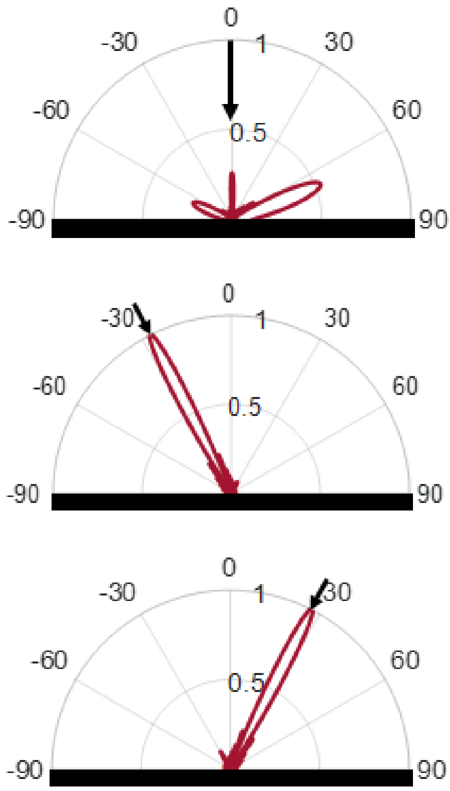

Fig. 2. Scattering directivity pattern for a phase-gradient anomalous reflector designed to reflect normally incident plane waves into the 70° degree direction, for different directions of the incident plane wave. The size of the metasurface is $10\lambda \times 10\lambda$. The scattering amplitude is normalized to the maximum value for the PEC plate of the same size. That is, $F_{\rm sc}(\theta) = F_{\rm r}(\theta) + F_{\rm sh}(\theta)$, where $F_{\rm r}(\theta) = \frac{1}{2\cos\theta_{\rm i}} \sum_n r_n(\theta_{\rm i})(\cos\theta_{{\rm r}n} + \cos\theta){\rm sinc}(k a_{{\rm ef}n})$ and $F_{\rm sh}(\theta) = \frac{1}{2\cos\theta_{\rm i}}(\cos\theta - \cos\theta_{\rm i}){\rm sinc}(k a_{\rm ef})$.

In the presentation, we plan to show also comparison with full-wave simulations and measured results, and discuss the differences between the far-zone scattering from phase-gradient metasurfaces and from nonlocal and power-flow-conformal surfaces.

III. Conclusion

In summary, we have presented an analytical model of reflection and scattering from finite-size anomalously reflecting metasurfaces. The developed method is based on a synthetic use of the physical optics and the theory of diffraction by gratings. In contrast to earlier works (e.g., [4], [5]), we take into account the presence of multiple propagating harmonics of the reflected field and do not assume locally specular reflection, as in the conventional physical-optics models of scattering from impedance bodies with slowly varying surface impedance. The results show that the response of anomalous reflectors dramatically depends on the illumination direction. We hope that the presented analytical approach will be useful in creation of models of wave propagation in complex environments that include intelligent reconfigurable metasurfaces (RIS).

Acknowledgment

This work was supported in part by the European Commission through the H2020 ARIADNE project under grant 871464.

References

[1] C. Liaskos, S. Nie, A. Tsioliaridou, A. Pitsillides, S. Ioannidis, and I. Akyildiz, A new wireless communication paradigm through softwarecontrolled metasurfaces, *IEEE Commun. Mag.,* vol. 56, no. 9, pp. 162-169, 2018.

[2] M. Di Renzo, M. Debbah, D.-T. Phan-Huy, A. Zappone, M.-S.Alouini, C. Yuen, V. Sciancalepore, G. C. Alexandropoulos, J. Hoydis, H. Gacanin, et al., Smart radio environments empowered by reconfigurable AI metasurfaces: An idea whose time has come, *EURASIP Journal on Wireless Communications and Networking,* vol. 2019, no. 1, pp. 1–20, 2019.

[3] A. Díaz-Rubio, V. Asadchy, A. Elsakka, and S. Tretyakov, From the generalized reflection law to the realization of perfect anomalous reflectors, *Science Advances,* vol. 3, no. 8, p. e1602714, 2017.

[4] Ö. Özdogan, E. Björnson, and E. G. Larsson, Intelligent reflecting surfaces: Physics, propagation, and pathloss modeling, *IEEE Wireless Commun. Lett.,* vol. 9, no. 5, pp. 581–585, 2019.

[5] M. Di Renzo, A. Zappone, M. Debbah, M.-S. Alouini, C. Yuen, J. D. Rosny, and S. Tretyakov, Smart radio environments empowered by reconfigurable intelligent surfaces: How it works, state of research, and road ahead, *IEEE Journal on Selected Areas in Communications,* vol. 38, no. 11, pp. 2450-2525, 2020.

[6] X. Wang, A. Díaz-Rubio, V. S. Asadchy, G. Ptitcyn, A. A. Generalov, J. Ala-Laurinaho, and S. A. Tretyakov, Extreme asymmetry in metasurfaces via evanescent fields engineering: Angular-asymmetric absorption, *Phys. Rev. Lett.,* vol. 121, no. 25, p. 256802, 2018.

[7] A. Osipov and S. Tretyakov, *Modern electromagnetic scattering theory with applications,* John Wiley & Sons, 2017.