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Published in:
MIPT (PHYSTECH) - QUANT 2020

DOI:
10.1063/5.0055253

Published: 16/06/2021

Document Version
Publisher's PDF, also known as Version of record

Please cite the original version:
Korkalainen, T., Lilja, I., Perelshtein, M. R., Petrovnin, K. V., Paraoanu, G. S., & Hakonen, P. J. (2021). Vacuum-induced correlations in superconducting microwave cavity under multiple pump tones. In G. Lesovik, V. Vinokur, & M. Perelshtein (Eds.), *MIPT (PHYSTECH) - QUANT 2020* Article 030001 (AIP Conference Proceedings; Vol. 2362). American Institute of Physics. https://doi.org/10.1063/5.0055253

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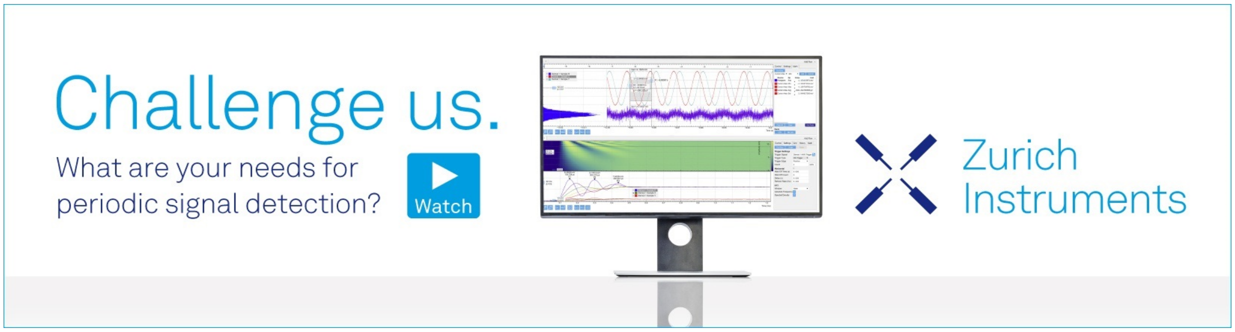

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Vacuum-induced Correlations in Superconducting Microwave Cavity under Multiple Pump Tones

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Abstract. Quantum correlations are an essential resource in advanced information processing based on quantum phenomena. Remarkably, the vacuum state of a quantum field may act as a key element for generation of strong quantum correlations. Besides, superconducting microwave cavities offer an excellent platform for experimental studies of such quantum effects. In this work, we numerically investigate vacuum correlations in a flux-tunable superconducting cavity under multiple pump tones. We consider double and triple pumping cases and explore pairwise correlations between three frequency bands specified within a single cavity resonance. Our work shows that three pumps produce more correlations than two pumps; thus, quantum resources facilitated by the triple pump scheme offers enhanced prospects for quantum data processing using parametric microwave cavities.

INTRODUCTION

One of the most remarkable predictions of quantum theory is that fields, even in the vacuum state, have strong quantum fluctuations that can produce observable effects on different scales. Recently, in addition to the purely physical interest [1, 2], a novel idea has emerged: engineering of the vacuum fluctuations to create quantum devices and protocols [3, 4].

Superconducting microwave cavities have attracted significant attention in the past decade, mainly because of the pressing interest emerging in quantum computing with artificial atoms [5, 6, 7]. However, the possibilities offered by these systems are not limited to quantum computing [8, 9, 10]. For instance, squeezed microwave fields – essential tool in quantum optics – were generated by amplifying the quantum fluctuations in superconducting cavities by flux-tuning the effective energy of a superconducting quantum interference device (SQUID) [11]. Furthermore, squeezed states produced using microwave cavities have been shown to exhibit correlations between photons at separate frequencies [12]. Some of the experiments provide elegant analogies with motion of a mirror in free space [13]: parametric amplification of vacuum fluctuations in a superconducting microwave circuit has been demonstrated to stimulate spontaneous downconversion processes [14, 15, 16], analogous to the dynamical Casimir effect [17]. These correlations can be used as resources for various quantum information processing tasks [18, 19, 20].

In this work, we consider the dynamics of a parametrically pumped superconducting cavity and numerically study correlations generated in such a system when modulated by several tones. The ensuing several parametric downconversion processes are found to form mutual vacuum-induced correlations within the frequency span of a single microwave resonance mode. Here we concentrate on pairwise correlations between three frequency bands in the double and triple pump tone cases and find that triple pumping, i.e. the use of three pumps, leads to stronger correlations between photons than double pumping.

METHODS

The studied system consists of a superconducting flux-tunable microwave resonator as described in [21]; a schematic experimental realization of the parametric device and the measurement system is illustrated in Fig. 1. The dynamics of such a pumped circuit is governed by the semiclassical Hamiltonian

$$H = \hbar\omega_r a^\dagger a + \frac{\hbar}{2i}\sum_{p=1}^{d}[\alpha_p^* e^{i\omega_r t} + \alpha_p e^{-i\omega_r t}](a+a^\dagger)^2, \tag{1}$$

MIPT (PhysTech) - QUANT 2020
AIP Conf. Proc. 2362, 030001-1–030001-5; https://doi.org/10.1063/5.0055253
Published by AIP Publishing. 978-0-7354-4102-6/$30.00

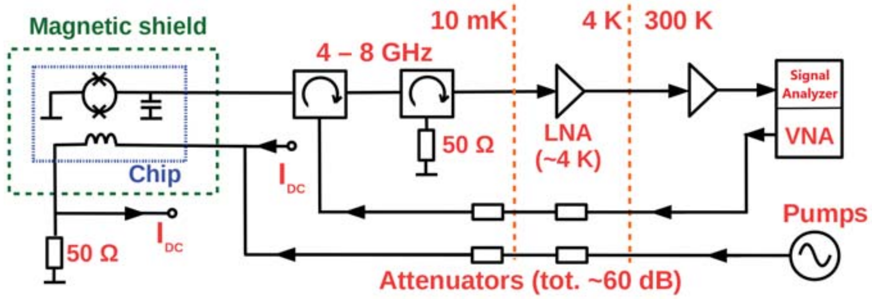

FIGURE 1. Schematics of the measurement system used in characterization of the flux-tunable microwave resonator (adapted from Ref. 21).

where ω_r and ω_p denote the resonance and pump frequency, respectively, and the final index in the sum, d, is equal to either 2 or 3, corresponding to the double and triple pumping setup, respectively. The coupling strength of the pump indexed by p into the cavity is given by the parameter α, which is proportional to the pumping power as described in the supplement of Ref. [16].

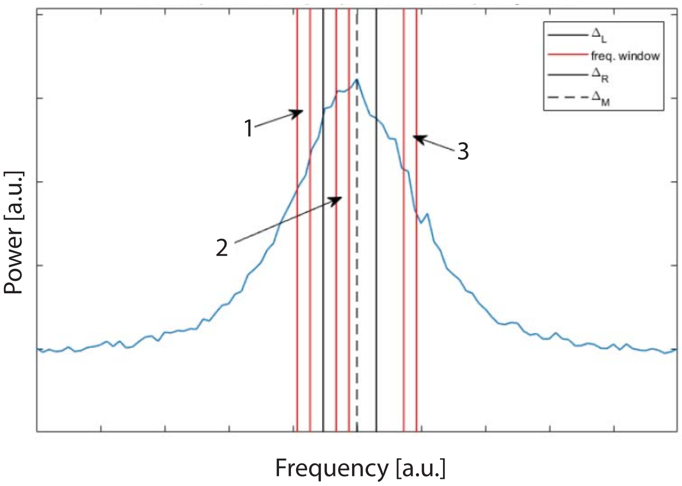

FIGURE 2. Illustration of the simulated power spectrum at a moderate pumping power. The solid black lines represent the extremal pump offsets (labeled Δ_L and Δ_R) while the dashed black line represents the pump operating at the cavity frequency (labeled Δ_M). The red lines show the frequency bands, labeled from left to right as 1, 2 and 3, which are selected symmetrically about the corresponding pump offsets (either Δ_L "1-2", Δ_R "2-3", or Δ_M "1-3").

We want to emphasize that photon pairs are generated by parametric downconversion processes on frequency bands located symmetrically about the half of the pump frequency [22, 23]. Our analysis of the correlations is based on direct numerical integration for solving the Heisenberg-Langevin equation in the rotating frame of the cavity. The rotating wave approximation, furthermore, allows us to neglect higher frequency components of the field, which was also assumed in Ref. [12], for example. After these approximations, the Heisenberg-Langevin equation is of the form

$$\frac{da}{dt} = \sum_{p=1}^{d} \alpha_p e^{2i\Delta_p t} a^\dagger - \frac{\kappa}{2} a - \sqrt{\kappa} a_{in}, \tag{2}$$

where κ is a cavity decay rate and a_{in} is an input field operator for quantum noise. In our numerical integration, the quantum noise is emulated by randomly generated Gaussian white noise. The output field in the time-domain is obtained using input-output formalism $a_{out}(t) = \sqrt{\kappa} a(t) - a_{in}(t)$. The detuning of the halved pump frequency $\omega_p/2$ with respect to the cavity frequency is denoted by $\Delta_p = \omega_p/2 - \omega_r$ and addressed as pump offset for short. For interfrequency correlation analysis, we transform the output field to frequency domain via Fourier transform with zero frequency in the center (the rotating frame frequency).

The simulations were performed with an asymmetric frequency configuration illustrated in Fig. 2. Two pumps were operating at small positive and negative pump offsets, asymmetrically located with respect to the cavity frequency in the double pumping setup. A third pump was introduced at the cavity resonance frequency in the triple pumping configuration. The frequency bands were chosen symmetrically about each pump offset frequency, as illustrated in Fig. 2 where the pump offsets starting from the negative frequency in ascending order are denoted by Δ_L, Δ_M, and Δ_R, while the frequency bands are labeled by 1, 2, and 3.

RESULTS AND DISCUSSION

We consider here two types of correlations between the frequency bands: squeezing (SQ) correlations and beamsplitter (BS) type of correlations. Physically, squeezing correlations correspond to $\langle a_i a_j \rangle$ and beamsplitter correlations correspond to $\langle a_i^\dagger a_j \rangle$, where i, j refer to the frequency bands 1, 2, and 3 specified above. Beamsplitter correlations are formally analogous to those obtained using an optical beam-splitter component, except that here the correlation is in the frequency space instead of the real space. These correlations can be analyzed using $a_{out}(\omega)$ amplitudes on a discrete array of k frequency points resulting in the following representation:

$$\mathrm{SQ} = (f \star \tilde{g})[n] = \sum_{m=1}^{k} f^*[m] g[n-m], \tag{3}$$

$$\mathrm{BS} = (f \star g)[n] = \sum_{m=1}^{k} f^*[m] g[n+m]. \tag{4}$$

The vectors f and g are governed by the complex Fourier transform components of a_{out} on a selected frequency range; they hold the information of the frequency bands being correlated and are indexed in ascending order such that the first index corresponds to the lowest frequency point on each band. $\tilde{g}$ denotes that the ordering of the frequency array components is reversed.

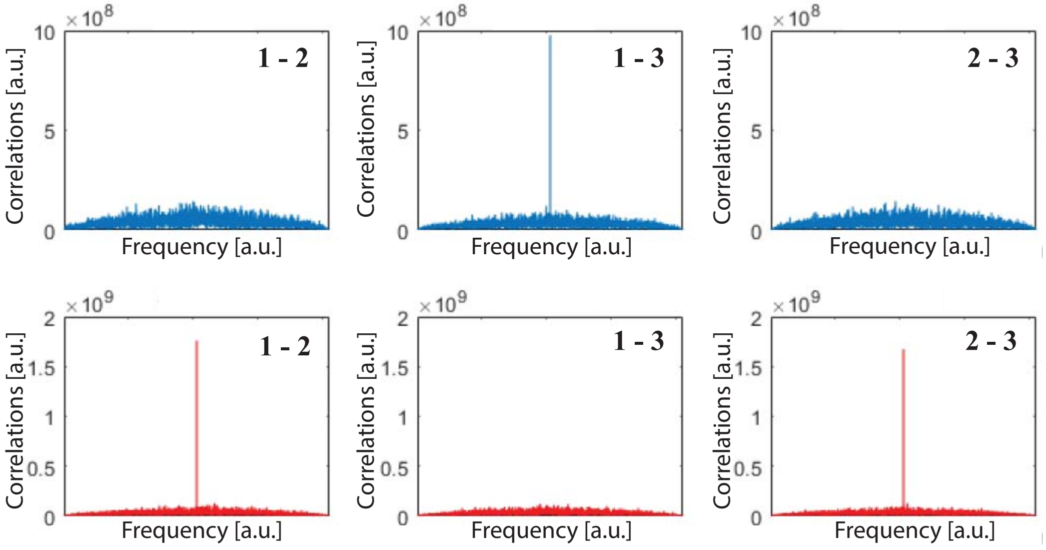

FIGURE 3. Beamsplitter (first row in blue) and squeezing (second row in red) correlations between the frequency bands 1, 2 and 3 under double parametric pumping. Correlations were calculated using Eq. 3 and Eq. 4 and are shown as a function of a frequency shift n on a grid of $2k$ points. Two pumps generated squeezing correlations between the inner band and an extremal frequency pairs, "1-2" and "2-3", while beamsplitter correlations arise between the extremal band configuration "1-3".

The configuration employing two pumps generates squeezing correlated photons between frequency pairs "1-2", and "2-3". Curiously, the interaction between two squeezing correlations also results in beamsplitter correlations between the extremal bands 1 and 3. These correlations are induced by the vacuum as the same quantum fluctuation plays a role in the downconversion of both photon pairs [12]. Correlations between frequency bands which were calculated using Eq. 3 and Eq. 4 are depicted in Fig. 3 as a function of frequency shift: it is clear that at certain frequencies, the SQ and BS values are significantly higher than the noise level indicating the emergence of correlations.

The power dependence of the correlations was determined by calculating the maximum numerical cross-correlation value of the real part of the squeezing and beamsplitter correlations at each pump power level. We find that the correlations increase first approximately linearly with low pumping amplitudes, but then exponentially at larger modulation strengths.

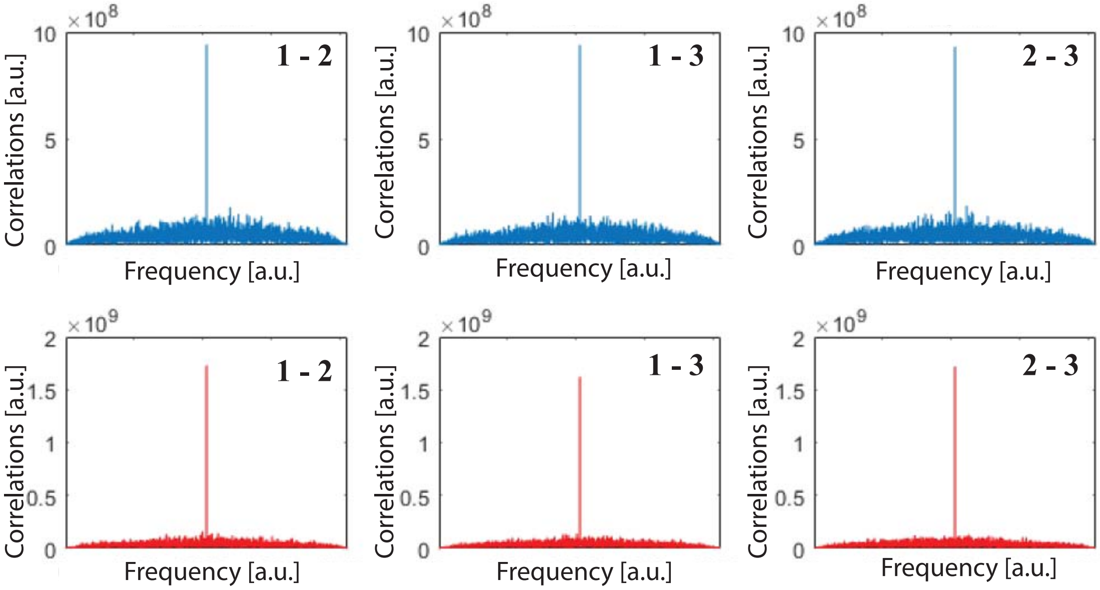

FIGURE 4. Beamsplitter (first row in blue) and squeezing (second row in red) correlations between the frequency bands 1, 2, and 3 under triple parametric pumping. Correlations were calculated using Eq. 3 and Eq. 4 and are shown as a function of a frequency shift n on a grid of $2k$ points. The third pump generates additional beamsplitter correlations between band configurations "1-2" and "2-3". Squeezing correlations are found now to emerge also between the extremal band configuration "1-3".

The triple pumping configuration involves a third pump, which can be set to induce simultaneous formation of squeezing and beamsplitter correlations. This is done in the configuration of Fig. 2 by setting the third pump offset Δ_M to zero. Fig. 4 displays all the additional correlations generated by the third pump: squeezing correlations emerge for pair "1-3" across Δ_M and beamsplitter correlations are generated between pairs of frequencies "1-2" and "2-3". We find that all of the emerging correlations increase as a function of pumping power.

CONCLUSION

We have studied the dynamics of a superconducting microwave resonator under parametric amplification of quantum noise by numerically solving the Heisenberg-Langevin equation obtained from a semiclassical Hamiltonian. Pumping the cavity resulted in correlated frequency bands across the pump offset frequencies, which were analyzed in terms of squeezing and beamsplitter (coherence) type of correlations. Also, we showed that the triple pumping scheme with pump "redundancy" produces more pairwise correlations between vacuum photons than the double pumping setting across three frequencies. The increased correlation is due to the generation of simultaneous beamsplitter and squeezing correlations owing to the extra pump. Thus, due to the additional beam splitter correlations, parametrically pumped microwave systems with fully pairwise squeezed frequencies can potentially provide excellent quantum resources for future quantum information processing utilizing a parametric superconducting cavity [24].

ACKNOWLEDGMENTS

This work was supported by the Academy of Finland grants No. 314448 (BOLOSE), No. 312295, and No. 312296 (CoE, Quantum Technology Finland). Our research was also funded in part by ERC (QuDeT, No. 670743), COST Action CA16218 (NANOCOHYBRI), and the European Microkelvin Platform (EMP, No. 824109). MRP acknowledges financial support via visiting fellowships granted by the Centre for Quantum Engineering at Aalto University. KP and GSP would like to acknowledge the EU project QUARTET (grant agreement No. 862644), and projects under the Scientific Advisory Board for Defence of Finland and Saab-Aalto collaboration.

REFERENCES

1. P. D. Nation, J. R. Johansson, M. P. Blencowe, and F. Nori, "Colloquium: Stimulating uncertainty: Amplifying the quantum vacuum with superconducting circuits," Reviews of Modern Physics **84**, 1–24 (2012).
2. G. S. Paraoanu, *The quantum vacuum*, edited by I. Parvu, G. Sandu, and I. D. Toader, Vol. 313 (Springer International Publishing, Switzerland, 2015) Chap. 12, p. 181–197.
3. C. Sabín and G. Adesso, "Generation of quantum steering and interferometric power in the dynamical Casimir effect," Physical Review A **92**, 042107 (2015).
4. D. E. Bruschi, C. Sabín, P. Kok, G. Johansson, P. Delsing, and I. Fuentes, "Towards universal quantum computation through relativistic motion," Scientific Reports **6**, 18349 (2016).
5. A. Wallraff, D. I. Schuster, A. Blais, L. Frunzio, R.-S. Huang, J. Majer, S. Kumar, S. M. Girvin, and R. J. Schoelkopf, "Strong coupling of a single photon to a superconducting qubit using circuit quantum electrodynamics," Nature **431**, 162–167 (2004).
6. F. Arute, K. Arya, R. Babbush, D. Bacon, J. C. Bardin, R. Barends, R. Biswas, S. Boixo, F. G. S. L. Brandao, D. A. Buell, B. Burkett, Y. Chen, Z. Chen, B. Chiaro, R. Collins, W. Courtney, A. Dunsworth, E. Farhi, B. Foxen, A. Fowler, C. Gidney, M. Giustina, R. Graff, K. Guerin, S. Habegger, M. P. Harrigan, M. J. Hartmann, A. Ho, M. Hoffmann, T. Huang, T. S. Humble, S. V. Isakov, E. Jeffrey, Z. Jiang, D. Kafri, K. Kechedzhi, J. Kelly, P. V. Klimov, S. Knysh, A. Korotkov, F. Kostritsa, D. Landhuis, M. Lindmark, E. Lucero, D. Lyakh, S. Mandrà, J. R. McClean, M. McEwen, A. Megrant, X. Mi, K. Michielsen, M. Mohseni, J. Mutus, O. Naaman, M. Neeley, C. Neill, M. Y. Niu, E. Ostby, A. Petukhov, J. C. Platt, C. Quintana, E. G. Rieffel, P. Roushan, N. C. Rubin, D. Sank, K. J. Satzinger, V. Smelyanskiy, K. J. Sung, M. D. Trevithick, A. Vainsencher, B. Villalonga, T. White, Z. J. Yao, P. Yeh, A. Zalcman, H. Neven, and J. M. Martinis, "Quantum supremacy using a programmable superconducting processor," Nature **574**, 505–510 (2019).
7. M. R. Perelshtein, A. I. Pakhomchik, A. A. Melnikov, A. A. Novikov, A. Glatz, G. S. Paraoanu, V. M. Vinokur, and G. B. Lesovik, "Advanced quantum supremacy using a hybrid algorithm for linear systems of equations," (2020), arXiv:2003.12770.
8. J. Q. You and F. Nori, "Atomic physics and quantum optics using superconducting circuits," Nature **474**, 589–597 (2011).
9. Y. Hochberg, Y. Zhao, and K. M. Zurek, "Superconducting detectors for superlight dark matter," Physical Review Letters **116** (2016).
10. N. S. Kirsanov, Z. B. Tan, D. S. Golubev, P. J. Hakonen, and G. B. Lesovik, "Heat switch and thermoelectric effects based on Cooper-pair splitting and elastic cotunneling," Physical Review B **99** (2019).
11. B. Yurke, "Squeezed-state generation using a Josephson parametric amplifier," Journal of the Optical Society of America B **4**, 1551 (1987).
12. P. Lähteenmäki, G. S. Paraoanu, J. Hassel, and P. J. Hakonen, "Coherence and multimode correlations from vacuum fluctuations in a microwave superconducting cavity," Nature Communications **7**, 12548 (2016).
13. G. S. Paraoanu and G. Johansson, "Listening to the quantum vacuum: a perspective on the dynamical Casimir effect," Europhysics News **51**, 18–20 (2020).
14. J. R. Johansson, G. Johansson, C. M. Wilson, and F. Nori, "Dynamical Casimir effect in superconducting microwave circuits," Phys. Rev. A **82**, 052509 (2010).
15. C. M. Wilson, G. Johansson, A. Pourkabirian, M. Simoen, J. R. Johansson, T. Duty, F. Nori, and P. Delsing, "Observation of the dynamical Casimir effect in a superconducting circuit," Nature **479**, 376–379 (2011).
16. P. Lähteenmäki, G. S. Paraoanu, J. Hassel, and P. J. Hakonen, "Dynamical Casimir effect in a Josephson metamaterial," Proceedings of the National Academy of Sciences **110**, 4234–4238 (2013).
17. G. T. Moore, "Quantum theory of the electromagnetic field in a variable-length one-dimensional cavity," Journal of Mathematical Physics **11**, 2679–2691 (1970).
18. D. E. Bruschi, C. Sabín, and G. S. Paraoanu, "Entanglement, coherence, and redistribution of quantum resources in double spontaneous down-conversion processes," Physical Review A **95**, 062324 (2017).
19. J. Steinhauer, "Observation of quantum Hawking radiation and its entanglement in an analogue black hole," Nature Physics **12**, 959–965 (2016).
20. J. R. M. de Nova, K. Golubkov, V. I. Kolobov, and J. Steinhauer, "Observation of thermal Hawking radiation and its temperature in an analogue black hole," Nature **569**, 688–691 (2019).
21. T. Elo, T. S. Abhilash, M. R. Perelshtein, I. Lilja, E. V. Korostylev, and P. J. Hakonen, "Broadband lumped-element Josephson parametric amplifier with single-step lithography," Applied Physics Letters **114**, 152601 (2019).
22. D. Walls and G. J. Milburn, eds., *Quantum Optics* (Springer Berlin Heidelberg, 2008).
23. P. Lähteenmäki, V. Vesterinen, J. Hassel, G. S. Paraoanu, H. Seppä, and P. Hakonen, "Advanced concepts in Josephson junction reflection amplifiers," Journal of Low Temperature Physics **175**, 868–876 (2014).
24. G. Adesso, T. R. Bromley, and M. Cianciaruso, "Measures and applications of quantum correlations," Journal of Physics A: Mathematical and Theoretical **49**, 473001 (2016).