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Coarse mesh finite element model for cruise ship global and local vibration analysis

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ABSTRACT

This paper presents a practical procedure for creating finite element (FE) model for vibration analysis of cruise ships. The most preferable FE modelling approaches are studied and discussed through case analysis of common ship structure, which covers the range from low to high frequencies. The application of homogenized equivalent single layer (ESL) theory based equivalent element for stiffened panel is extended to local forced vibration analysis, where inertia induced interaction between plate and stiffener occurs. Modal method is used with an energy-based correction for accounting the plate-stiffener interaction into modal properties. Case study results reveal that mesh density of one 4-node element per web frame is suitable for global FE-model when vibration analysis is limited to global hull girder modes. For such modes it is sufficient to only include membrane stiffness of the stiffened panels. For investigating the response of higher frequencies, bending properties of stiffened panel should be included and mesh density should be at least two elements per web frame. Then forced vibration analysis can be performed with excellent accuracy up to frequencies about one third of the local plate natural frequencies between the stiffeners. Beyond that, influence of local plate vibration becomes more significant in panel vibration, making the ESL-theory based element limited. With the applied correction method, the validity of the ESL-model can be extended to approximately two thirds of the local plate natural frequency.

Keywords: equivalent single layer (ESL) theory, global finite element model, coarse mesh, vibration analysis, cruise ship.
1 INTRODUCTION

Stricter environmental regulations are driving the demand for lighter ships. Less steel mass means more payload and lower production cost. For cruise ships several challenging untraditional design solutions have been investigated recently. For example, longitudinal strength of a cruise ship with narrow superstructure was studied by Bergström in [1]. Feasibility of a 425.5 m cruise ship concept was studied by Tsitsilonis et al in [2]. Modularisation of passenger ship cabin area, which leads to non-load-carrying accommodation decks was analysed by Parmasto in [3]. Lillemäe investigated utilization of 3 mm thin superstructure decks and smaller HP-profiles on hull girder response in [4]. The recent trend in industry is also to reduce the scantling of primary T-girders to achieve lower mass and smaller distance between superstructure decks. Altogether, lower centre of gravity will be achieved, which can be utilized for increasing passenger experience by adding features like rollercoasters, water parks or even kart tracks. While strength-wise these solutions may be feasible, they will significantly impact the hull global and local vibration performance and make its prediction for such prototype vessels extremely important. The calculation methods must be computationally light, but still capable to describe the vibration response of entire vessel. Without sufficient analysis, it might be difficult to achieve the comfort class [5] that matches owner expectations and contract, resulting in possible penalties for the shipyard.

Complete ship vibration analysis consists of three different types of analysis [6]: 1) wave-induced hull girder, 2) propeller-, main engines- and thrusters-induced and 3) local machinery-induced vibration. Typically wave-induced vibration analysis includes springing and whipping, see example studies in [7], [8] and [9], for which accurate evaluation of hull girder global natural frequencies is required. Ship propeller, main engines and thrusters are usually main source of such vibration that passengers and crew will notice, and this affects the comfort class of the ship. In practical design, this kind of vibration is treated as a steady state problem and solved in frequency domain [10], [13]. First order propeller blade passing frequencies are obtained by multiplying the shaft rpm with propeller blade number. For 120 rpm 4-6 blade propeller, it gives frequency range of 8-12 Hz. Higher-order excitation frequencies are obtained simply by multiplying the first order with harmonic number. Medium speed engines of large cruise ship are generally working with speed between 500-750 rpm [11], [12], which results in frequency range of 8.33-12.5 Hz and as for propeller, also second-, third- and other higher-order excitations occur. However, considering such high frequencies in forced vibration analysis of entire ship is computationally very expensive. Also, their influence on overall vibration response is typically less compared to first order excitations. Therefore, they are not commonly considered in cruise ship global structural analysis. Most noticeable short-term vibration is caused by bow and aft thrusters, mainly because their working frequency of approximately 11-15 Hz is closest to the natural frequency of the deck structure. In addition to these three main sources, there might also be some relevantly powerful machinery on board e.g., sea water pumps, scrubber deplume unit, FlowRider, etc. which are working at higher than 15 Hz. In this case noticeable local vibration might occur and separate analysis using global or local calculation model needs to be performed.

Today, 3D fine mesh FE-analysis is regarded as the most reliable method for evaluation of ship global and local vibration response [14]. An extensive overview with guidelines about creating fine mesh models for free and forced vibration analysis is shown in [15], where recommendation is to have a mesh size of at least two 4-node elements.
between adjacent stiffeners. Coarser mesh model should be used only in analysis that are limited to hull girder modes. Girders and main stiffeners should be modelled using shell elements for web and beam elements for flange. When higher frequencies are of interest, then model should have 4 elements per stiffener spacing, which is sufficient to describe bending behaviour between stiffeners [15]. Due to this precise modelling, a lot of degrees of freedom (DOF) are introduced, which makes this method computationally expensive. Therefore, fine mesh is usually applied for small vessels. For example, Boote et al. [16], Macchiavello & Tonelli [17] and Pais [18] performed the vibration analysis of a superyacht and Lin et al. [19] examined the low and high frequency vibration response of a 30 m long crew vessel and Moro et al. [20] the river boat.

In cruise ship design process general arrangement often changes, and large number of analyses must be performed within the limited time frame. Utilization of fine mesh leads to millions of elements and thousands of extra design and computational hours. To reduce the modelling and calculation time, the ship global FE-model needs to be created using coarse mesh. The standard mesh arrangement according to DNV-GL [14] is normally one 4-node element per web frame and deck height. Ship door and window openings are modelled directly or using orthotropic plate element techniques e.g., [21], [22]. Due to large participation in overall stiffness, the primary beams, i.e., web frames and girders, are explicitly modelled using offset beam elements. Ship secondary stiffeners are incorporated into plate or shell element formulation so that equivalent stiffness and mass is achieved. There are several modelling techniques available. According to DNV-GL [14] or Hughes [23], stiffeners can be lumped to the nearest mesh-line using truss or beam elements, where the cross-section and mass equals to the sum of lumped stiffeners. With this approach membrane stiffness is presented, but bending properties are neglected. Since mesh size equals to web-frame spacing, then only two elements are used to describe the deck vibration mode between pillars. Also, the mass will be concentrated in an unrealistic way. As a result, this simple method leads to inaccuracy in higher frequency vibration analysis [24]. Similar conclusion was also drawn in [10].

To use denser mesh than one element per web frame, stiffened panel bending properties need to be included. Avi et al. in [24] introduced three-layer laminate element modelling principle, where the first layer represents the deck plate, the second layer the stiffener web and the third the stiffener flange. The element follows Equivalent Single Layer (ESL) theory, which was extended from asymmetric sandwich panel by Romanoff & Varsta [25] to stiffened panel. Thus, the element considers the full stiffness matrix i.e., membrane, membrane-bending coupling, bending and follows first-order shear deformation theory (FSDT). Tilander et al [9] utilized the element for cruise ship springing analysis and in [24] cabin deck natural frequencies were analysed. In lower modes particularly good agreement with 3D fine mesh FE model was obtained, however in the higher modes where plate between the stiffeners deforms locally as part of global vibration modes, see Figure 1, larger error was observed. In classical ESL-theory, the homogenized stiffness properties are used i.e., only the average response of the panel is considered and therefore the behaviour between the stiffeners is neglected. For stiffened panel only, such modes can be calculated using e.g., Brubak et al. [26] presented semi-analytical approach. Similar local effects also occur in buckling modes, and for web-core sandwich panels they were captured using correction for ESL-FSDT in [27] or more advanced micropolar ESL-FSDT plate model [28], [29], which is based on non-classical continuum mechanics where microrotations and -inertia are included. In [30], natural frequencies of stiffened panel were obtained by combining ESL-model frequencies with the
local plate between stiffener frequencies, using the assumption that stiffeners with plate between them act as springs and masses in series. The discussed methods are currently limited for stiffened panel-level only and are difficult to apply for larger ship structure using commercial software. The extension of correction to real ship deck structure was done by Laakso et al. in [31]. The modification of natural frequencies obtained by ESL model was carried out using kinetic and strain energies of local deformations, which are assumed to be induced only by translational vibration of ESL elements in their normal direction. As a result, error was reduced to less than 2 % from initial 5-10 % error of uncorrected ESL results.

Despite that several papers have investigated ship hull girder low and high frequency vibration response, using various modelling techniques and mesh sizes, the boundaries of where each FE-model is valid has not been clearly communicated, similarly as it was done for fine mesh models in [15]. Here, the applicability of commonly used method, i.e., the mesh size equal to web frame spacing and only membrane stiffness considered, is systematically studied and compared to ESL models. In addition, the correction method [31] is applied to forced vibration analysis and the importance to include tertiary vibration effects between the stiffeners is discussed.

Figure 1. Cabin deck structure vibration mode analysed with fine mesh and ESL-theory based model.
2 THEORY

2.1 Modelling stiffened panel using equivalent single layer (ESL) theory

According to [24], a stiffened panel can be modelled using three-layered laminated shell element, which follows Equivalent Single Layer and First-order Shear Deformation Theory (ESL-FSDT or in terms of simplicity ESL). The plate layer thickness $t_p$ equals to the deck plate, web and flange layer thicknesses $h_w$ and $h_f$ correspond to the height of the stiffener web and flange, respectively. The x-direction of the stiffened panel is taken parallel to the stiffener orientation while z-direction is normal to the deck plate. It should be noted that commonly the element’s reference plane is taken at the geometrical mid-plane, see e.g. [25], [32]. However, to represent the stiffened panel membrane-bending stiffness and couplings and mass distribution between T-girder correctly, the reference plane of ESL-theory based element should be offset from geometrical mid-plane to the interface of the deck plate and stiffener web as it is shown in Figure 2.

![Figure 2. Stiffened panel division into three-layer laminate element and reference plane offset [33].](image)

The equivalent element x, y and z displacements are denoted by $u$, $v$ and $w$, respectively, and can be divided into global and the local plate deflection. Thus, the displacements are given as:

$$ u = u_0(x, y, t) + z\phi_x(x, y, t) + z_l\theta_x(x, y, x_l, y_l, t), $$

(1)

$$ v = v_0(x, y, t) + z\phi_y(x, y, t) + z_l\theta_y(x, y, x_l, y_l, t), $$

(2)

$$ w = w_0(x, y, t) + w_l(x, y, x_l, y_l, t), $$

(3)

where $t$ represents time, subscript 0 denotes displacements at reference plane of the plate stiffener assembly and subscript $l$ the local plate-layer deflection. The distance $z$ is the distance from the laminate element reference plane, while $z_l$ from the deck plate mid-plane. The rotations for stiffened panel and plate between stiffeners, denoted with $\phi$ and $\theta$ respectively, are taken as:

$$ \phi_x = y_{xz} - \frac{d w_0}{d x}, \quad \phi_y = y_{yz} - \frac{d w_0}{d y}, \quad \theta_x = -\frac{d w_l}{d x_l}, \quad \theta_y = -\frac{d w_l}{d y_l}, $$

(4)

meaning that the entire plate stiffener assembly bends according to the FSDT and the plate layer as Classical Plate Theory (CPT, Kirchhoff plate theory). The equations of motion are:

$$ \frac{d^2 N_{xx}}{d x} + \frac{d^2 N_{xy}}{d y} = I_0 \frac{d^2 u_0}{d t^2} + I_1 \frac{d^2 \phi_x}{d t^2} + I_{11} \frac{d^2 \theta_x}{d t^2} $$

(5)
\[
\frac{dN_{yy}}{dy} + \frac{dN_{xy}}{dx} = I_0 \frac{d^2 \psi_0}{dz^2} + I_1 \frac{d^2 \phi_x}{dz^2} + I_{1,t} \frac{d^2 \theta_y}{dz^2}
\]
\[
\frac{dQ_x}{dx} + \frac{dQ_y}{dy} = -q + I_0 \frac{d^2 w_0}{dz^2} + I_{0,t} \frac{d^2 w_1}{dz^2}
\]
\[
\frac{dM_{xx}}{dy} + \frac{dM_{xy}}{dx} = I_2 \frac{d^2 \phi_y}{dx^2} + I_1 \frac{d^2 w_0}{dz^2} + I_{2,t} \frac{d^2 \theta_x}{dt^2}
\]
\[
\frac{dM_{xy}}{dy} + \frac{dM_{yy}}{dx} = I_2 \frac{d^2 \psi_y}{dy^2} + I_1 \frac{d^2 v_0}{dt^2} + I_{2,t} \frac{d^2 \theta_x}{dt^2}
\]

where internal forces $N, M, Q$ refer to normal force, bending moment and out-of-plane shear force, respectively, and subscripts $xx, yy$ and $xy$ describe the plane and load direction, see Figure 3. It should be noted that the local, underlined, terms only contribute through rotations and $I_0, I_1$ and $I_2$ are the mass moments of inertia, which for stiffened panel and local plate bending are found from following:

\[
\begin{align*}
\{I_0\} &= \int_{-t_p/2}^{h+w+h_f/2} \rho \frac{z_1}{z^2} \rho dz, \\
\{I_1\} &= \int_{-t_p/2}^{h+w+h_f/2} \rho \frac{z_2}{z^2} \rho dz.
\end{align*}
\]

Figure 3. Stiffened panel (a) global and (b) local plate between stiffeners rotations and (c) homogenized stiffened panel internal forces.

The differential equations of ESL-FSDT are given in Appendix B. Commercial FE-solvers are based on laminate theory where the local terms in equilibrium and differential equations are omitted, i.e., the underlined terms are set to zero. This can be achieved by smearing the stiffeners to equivalent plate properties. This calls for two simplifications:

1. The local bending moments $M_{xx,l}, M_{xy,l}$ and $M_{yy,l}$ must vanish at the left hand side of equation. This can be achieved by assuming that the plate fields between the webs undergo cylindrical bending due to local uniform pressure. For this deformation shape the volume average of the bending moments is zero (see e.g. [34]). As shown by Avi et al [24] this local moment can be recovered from sub-model analysis to yield very accurate stress prediction for static response.

2. The local inertia terms on the right-hand side of equation, i.e. with $I_{0,t}, I_{1,t}$ and $I_{2,t}$, must vanish. This can be achieved by assuming that the stiffener spacing $S$ is small in relation to the characteristic length of the panel.
deformation, i.e., S/B or S/L→0 where B and L are stiffened panel breath and length, respectively. In passenger ships these dimensions can be in the range of: S=500...700 mm, B=3...7 m and L=2...3 m. The assumption is valid in classical continuum mechanics formulations where the material point is assumed to be infinitesimal. Clearly the assumption is violated to some extent and an error is introduced which needs to be corrected.

Another simplification can be made with the 1st and 2nd order rotary inertia terms $I_1$ and $I_2$, which depend on element $z$-coordinate mass distribution. Such layer-wise mass distribution in $z$-direction is important only in analysing such high frequencies where local stiffener torsional modes occur ([35], page 125). Therefore, in practical ship design problems, these terms can be neglected. These simplifications will bring significant savings to solve the problem numerically and theory can be applied to the laminated shell element, which is supported by common FE-software e.g. Nastran, ANSYS, ABAQUS.

In 3-layer laminate element, the elasticity $[E]_p$ of the plate layer is described as 2D isotropic, while the web $[E]_w$ and flange $[E]_f$ are described as 2D orthotropic material, where the components are found by applying the Rule of Mixtures:

$$[E]_p = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}, [E]_w = \frac{E_w}{s} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, [E]_f = \frac{E_f}{s} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

where $s$ is stiffener spacing and $E$ is Young’s modulus of structural material. Using the definition of strains with these elasticity matrices per layer and carrying out the through thickness integration for normal forces and bending moments, and including FSDT, following relation between the strains and stress resultants can be obtained:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ Q_{Qx} \\ Q_{Qy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 & 0 & 0 \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 & 0 & 0 \\ 0 & 0 & A_{33} & 0 & 0 & B_{33} & 0 & 0 \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 & 0 & 0 \\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 & 0 & 0 \\ 0 & 0 & B_{33} & 0 & 0 & D_{33} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & D_{Qx} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & D_{Qy} \end{bmatrix} \begin{bmatrix} \epsilon^0_x \\ \epsilon^0_y \\ \gamma_{x2} \\ \gamma_{x3} \\ \gamma_{y3} \\ \gamma_{12} \end{bmatrix},$$

where $\{N\}$ is normal force, $\{M\}$ moment and $\{Q\}$ shear force vectors, which are related to strain $\{\epsilon\}$, rotations $\{\kappa\}$ and out-of-plane shear strain $\{\gamma\}$ vectors by multiplying with stiffness matrices. $A_{ij}, B_{ij}$ and $D_{ij}$ matrix components represents membrane, membrane-bending and bending stiffness and are found through layer thickness integration:

$$[A] = \int_{-t/2}^{0} [E]_p \, dz + \int_0^{h_w} [E]_w \, dz + \int_0^{h_w+h_f} [E]_f \, dz,$$

$$[B] = \int_{-t/2}^{0} [E]_p \, dzd + \int_0^{h_w} [E]_w \, dzd + \int_0^{h_w+h_f} [E]_f \, dzd,$$

$$[D] = \int_{-t/2}^{0} [E]_p \, \gamma^2 \, dz + \int_0^{h_w} [E]_w \, \gamma^2 \, dz + \int_0^{h_w+h_f} [E]_f \, \gamma^2 \, dzd.$$

Out-of-plane shear stiffness components represent stiffener direction $D_{Qx}$ and transverse to stiffener direction $D_{Qy}$ and are found from next equations:

$$D_{Qx} = k_{xx}(G_p t_p + G_w h_w + G_f h_f),$$

$$D_{Qy} = k_{yz}(G_p t_p).$$
where shear moduli for the plate $G_p$, web $G_w$ and flange layer $G_f$ are found similarly like in Eq. 11. $k_{xz}$ is the shear correction factor in the $xz$-plane, which relates the maximum shear stress ($\tau_{xz})_{\text{max}}$ to the average shear stress ($\tau_{xz})_{\text{avg}}$ and typically varies between 0.7 to 0.8 [24]. The shear correction factor $k_{yz}$ follows Reissner-Mindlin plate theory and is 5/6.

### 2.2 Local plate vibration between the stiffeners in free vibration

As Chapter 2.1 shows, stiffener web and flange layers are described by homogenized stiffness properties, which allow to use coarser mesh than stiffener spacing and ensure high computational efficiency. Downside is that local plate bending effects between the stiffeners is neglected. For frequencies where local plate vibration between the stiffeners will not significantly interact with the global mode, this simplification is justified [24]. However, at higher frequencies, this interaction effect becomes more intense i.e., modal generalized mass increases relatively more than generalized stiffness resulting in lower frequencies and wrong response.

In paper [30], interaction between stiffened panel global vibration modes and local vibration effects between stiffeners were studied for clamped and pinned stiffened panel. The results are summarized in Figure 4, where the difference between ESL results with and without correction as a function of ratio between natural frequencies of local plate and ESL-theory based stiffened panel model are presented. Different cases were analysed in [30], but the results followed similar pattern, which can be expressed by following power function, which covers the range from $0.4 < \frac{\omega_{\text{local}}}{\omega_{\text{ESL}}} < 15$:

$$\text{ESL}_{\text{error}, \%} = 31.3 \left(\frac{\omega_{\text{local}}}{\omega_{\text{ESL}}}\right)^{-1.95},$$  \hspace{1cm} (18)

where $\text{ESL}_{\text{error}, \%}$ is the % difference between ESL results compared to ESL with correction. $\omega_{\text{ESL}}$ is natural frequency of panel obtained from ESL model and $\omega_{\text{local}}$ is the natural frequency of plate between the stiffeners, which can be obtained by either analytical, e.g., [36], or numerical methods.

According to Figure 4 and Eq. 18, if the ratio between the natural frequency of plate between the stiffeners and ESL is less than 2.5, then the error caused by exclusion of interaction in natural frequency analysis is within 5%. For forced vibration analysis, where different modes influence each other and resonance peak has also certain bandwidth, it is recommended to increase this ratio limit to 3.0, i.e. $\omega < \omega_{\text{local}}/3$, to be conservative. For traditional passenger ship structures, where natural frequency of local plate between stiffeners is more than 45 Hz and most of the outfitting mass is carried by stiffeners and girders, the limit of ESL model is around 16-20 Hz. Beyond that, coupling between local and global mode needs to be included.
The coupling between ESL and local vibration modes can be recreated using energy-based correction method presented by Laakso, et al. [31]. Accordingly, the displacement mode shape $\Psi_m(x, y, s, \omega_m)$ of global mode $m$ can be divided into sum of global reference plane mode shape $\Psi_{gm}(x, y)$ and local plate deformation shape $\Psi_{lm}(x, y, s, \omega_m)$, where $s$ represents local $y$-directional coordinate between stiffener, see Figure 5. It is assumed that stiffened panel global flexural waves are long in comparison to stiffener spacing, where locally only clamped-clamped 1 half wave mode occurs, which is excited only in its normal direction by the global reference plane through deck plate and stiffener connection points. Therefore, local amplitude is directly proportional to the amplitude of global reference plane and mode shape can be described as:

$$\Psi_{lm}(x, y, s, \omega_m) = \Psi_{gm}(x, y)\Psi_{lr}(s, \omega_m)$$  \hspace{1cm} (19)

Considering this, total mode shape can be written as:

$$\Psi_m(x, y, s, \omega_m) = \Psi_{gm}(x, y)[1 + \Psi_{lr}(s, \omega_m)]$$  \hspace{1cm} (20)
The natural modes in terms of their frequencies $\omega$ and corresponding mode shapes $\Psi$ are obtained by solving an eigenvalue problem of the free vibration equation:

$$[\omega^2 (M + M_a) - (K + K_G)]\Psi = 0,$$

where $M_a$ is fluid added mass matrix and $M$ is structural mass matrix, which also include the ship non-structural mass. $K$ is structural stiffness matrix and $K_G$ is geometric stiffness matrix, which represents stress stiffening effects due to preloading condition. For lightly damped structure, such as ships, free vibration modes are orthogonal i.e., each individual mode can be treated as generalized single degree of freedom system, which natural frequency $\omega_m$ is linked with the mode shape by corresponding generalized stiffness $K_m$ and generalized mass $M_m$ as follows:

$$\omega_m = \sqrt{\frac{K_m}{M_m}}. \tag{22}$$

Considering the mode shape division presented by Eq. 20, corrected generalized mass $M_m(\omega_i)$ and general stiffness $K_m(\omega_i)$ for mode $m$ can be found by adding local plate vibration kinetic and strain energy respectively to ESL model solution. The brief overview about their calculation procedure is given in Appendix C and D and more extensive overview in [31]. If both generalised properties are obtained, corresponding modal angular frequency can be found from Eq. 22. However, as $M_m(\omega_i)$ and $K_m(\omega_i)$ functions are frequency dependant, several iterations need to be performed. Figure 6 shows the flowchart of the correction method, where in each iteration loop, $M_m(\omega_i)$ and $K_m(\omega_i)$ for mode $m$ are calculated, from where angular frequency for the next iteration step $i+1$ is found. Iteration continues until the desired user-defined convergence limit $\delta$ is achieved. As case study of ref. [31] showed, the method is computationally very effective, and $10^{-5}$ Hz convergence was already achieved in step $i=4$. 

Figure 5. Coordinate system with global and local mode shape [31].
2.1 Forced vibration by modal method

In modal response calculation, response of the complete system is calculated as sum of individual modal responses. Equation of motion for mode $m$ is the following:

$$M_m \ddot{U} + C_m \dot{U} + K_m U = F_m$$  \hspace{1cm} (23)

where $\ddot{U}$, $\dot{U}$ and $U$ are acceleration, velocity, and displacement vectors, respectively, and $F$ is generalized force. $C$ is viscous damping, which is here treated as proportional damping as product of damping ratio $\zeta$ and critical damping $c_c$. In ship structure, damping ratio $\zeta$ is increasing with the frequency and depends on vessel type, it varies from 0.7…3% [37]. Critical damping coefficient for each mode $m$ is found by multiplying mass with natural frequency $\omega_m$ i.e.: $c_c = 2M_m\omega_m$. To calculate the forced response for node $n$, Eq. 23 can be simplified into single degree of freedom system [38]. Normalization of the mode shapes are now fixed by setting generalized mass $M_m$ to unity. Therefore, amplitude for generalized coordinate $X_{m,n}$ and phase angle $\phi_m$ for mode $m$, at excitation frequency $\omega$ and under sinusoidal excitation force, $F(t) = F_0 \sin \omega t$, can be found from following equations:

$$X_{m,n} = \frac{F_{0,n} \omega_m}{\sqrt{(1 - \frac{\omega^2}{\omega_m^2})^2 + (2\zeta \frac{\omega}{\omega_m})^2}} \Psi_{m,n}^2,$$  \hspace{1cm} (24)

$$\phi_m = \tan^{-1} \frac{-2\zeta \frac{\omega}{\omega_m}}{1 - \frac{\omega^2}{\omega_m^2}}$$  \hspace{1cm} (25)

where angular frequency $\omega_m$ and general stiffness $K_m$ are found from Eq. 22 and D1 respectively.
In order to properly consider the phase differences of individual modes in summation of the modal responses, the responses are transformed into complex format i.e., \( X = X_{\text{RE}} + iX_{\text{IM}} \), where \( X_{\text{RE}} \) is real and \( iX_{\text{IM}} \) imaginary part, which are found from amplitude \( X_{m,n} \) and phase angle \( \phi_m \):

\[
X_{\text{RE},m,n} = X_{m,n} \cos \phi_m, \quad (26)
\]
\[
iX_{\text{IM},m,n} = X_{m,n} \sin \phi_m, \quad (27)
\]

Total corrected response \( X_{\text{tot},n} \) and phase angle \( \phi_{\text{tot},n} \) at excitation frequency \( \omega \) can now be found by superimposing generalized modal responses of real and imaginary parts:

\[
X_{\text{tot},n} = \sqrt{\left( \sum_{m=1}^{M} X_{\text{RE},m,n} \right)^2 + \left( \sum_{m=1}^{M} iX_{\text{IM},m,n} \right)^2}, \quad (28)
\]
\[
\phi_{\text{tot},n} = \tan^{-1} \frac{\sum_{m=1}^{m_{\text{tot}}} iX_{\text{IM},m,n}}{\sum_{m=1}^{M} X_{\text{RE},m,n}}, \quad (29)
\]

where \( m_{\text{tot}} \) is the number of modes which are included in the forced vibration analysis.

In [31] the presented correction method for local plate deformation was applied for free vibration analysis. As can be seen from Eq. 24, in case of forced response, the correction of ESL-results is done in two directions. Updated angular frequency \( \omega_m \) moves the curve along horizontal axis and the corrected generalized mass and stiffness change the results in vertical direction, see Figure 7.

![Figure 7. Forced vibration response correction of ESL-model using energy-based method.](image)

3 CASE STUDIES

3.1 General

The target of the case studies is to investigate the application limits of common coarse mesh global FE-modelling techniques in vibration analysis. Two types of FE-models are created. At first, only the membrane stiffness of stiffened panel is considered. This is a common technique for creating coarse mesh global FE-model, which is also recommended by classification rules [14] and used in studies e.g., [7]. In the second type of model, bending properties of stiffened panel are included using ESL-theory [24] presented in Chapter 2.1. Three cases are considered. First,
natural frequencies of prismatic cruise vessel hull girder are analysed up to 7.0 Hz. Only dry modes are considered to avoid any disturbance in the results from added water effects. The target is to test the model’s suitability for wave-induced hull girder vibration analysis. In the second case, models are tested for propeller, main engine and thruster-induced first order vibration analysis. In this frequency range the ESL model should be sufficient also without applying the correction. A typical cabin area is forced by unit displacement of 1 mm to imitate the situation in global model when vibration is traveling through the bulkheads and pillars and excites the deck structure. Third case represents higher frequency response analysis, where homogenized ESL-theory is limited and correction needs to be applied. Deck structure is excited with the unit point load and equivalent element models are tested for response analysis up to 55Hz. All cases are validated against 3D fine mesh model.

FE analysis is carried out using Nastran 2020.1 software and the pre- and post-processing has been done with FEMAP 2020.2. In natural frequency calculations, lumped mass matrices are used, which provide higher computational economy due to diagonal mass matrix. In forced vibration analysis response is calculated at every 0.025 Hz step and uniform damping ratio of 2% is used.

3.1.1 A- and ABD-matrix stiffness-based models

In the first model, named as A-matrix model, only the membrane property of stiffened panel is considered. According to commonly used approach, e.g., [14], [23], secondary stiffeners can be lumped to the edges of the deck plate shell elements using truss or beam elements with relevant cross-section area. In practice, this lumping technique has a strong shortage. In addition to stiffener type, the lumped beam element is also a mesh size dependent, which introduces extra properties and results in tedious additional modelling work. To overcome the problem, this type of equivalent element can be modelled using two-layer laminate element, see Figure 8. The first layer represents the deck plating and is described by 2D isotropic material. For the second layer 2D orthotropic material is used, which describes stiffeners, that are smeared evenly along the plating and has stiffness only in one direction. The layer thickness, $t_s$, is found by dividing the cross section of stiffener, $A_s$, with stiffener spacing $S$:

$$ t_s = \frac{A_s}{S}, $$

Figure 8. Stiffened panel division into A-matrix and ESL-theory based laminate element.
In the second model, named as ESL model, the stiffened panel is modelled using 3-layer laminated shell elements and full $ABD$-matrix is exploited with the out-of-plane shear stiffness as it is described in Chapter 2.1. In both type of models NX Nastran PCOMP (Layered Composite Element Property) entity is used to define the composite laminate on a ply-by-ply basis. Plate layer is described using MAT1 (isotropic) and stiffener layer(s) with MAT8 (planar orthotropic) material model. Based on defined PCOMP entity, NX Nastran calculates A, B, D and $D_Q$ components and inertia terms according to Eq. 13-15 and outputs stiffness matrix presented Eq. 12 in the form of one equivalent PSHELL and four MAT2 (anisotropic material) entries, which are representing membrane, bending, thick plate theory and coupling between membrane forces and bending moments. CQUAD4 (quadrilateral) 4-node and 24 DOF shell elements are used to represent PSHELL entries. It is important to notice that element’s material orientation and normal direction should be defined so that it follows the stiffener directions in $x$- and $z$-coordinate, as it is shown in Figure 8.

In both models the girders, web frames and pillars are explicitly modelled using CBEAM beam element, which includes extension, torsion, bending in two perpendicular planes and out-of-plane shear response. As girders are located on bottom of the stiffened panel, element’s membrane [A], membrane-bending [B], and bending [D] stiffness need to be multiplied with offset transformation matrix, where $d_{offs}$ represents the distance from the beam neutral axis to the bottom of the deck plate i.e. laminate element reference plane, see Figure 2:

$$[ABD]' = \begin{bmatrix} 1 & 0 & 0 \\ d_{offs} & 1 & 0 \\ 0 & D & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A \\ d_{offs} \\ D \\ 1 \end{bmatrix} = \begin{bmatrix} A + Ad_{offs}^2 \\ Ad_{offs} \\ D + Ad_{offs}^2 \end{bmatrix}.$$  \hspace{1cm} (31)

### 3.1.2 3D fine mesh models

Validation is done using 3D fine mesh model, which is created according to recommendations given in [15]. Plating, stiffener, and girder webs are modelled using QUAD4 elements. CBEAM are used to represent stiffeners and girder flanges. In prismatic cruise ship model, 300 mm general mesh size is used. It means 2 elements per stiffener spacing, 2 elements per girder web and 1 element per stiffener web in height (z) direction. For HP100x6 stiffeners it gives aspect ratio of $\approx 1:3$, which is sufficient to avoid shear locking effects. The example of model is shown in Figure 10. In total 3 281 366 shell and 1 076 966 beam elements are used, which leads to 18 917 634 DOF. For cabin area model, two times denser mesh size is used i.e., 4 elements per stiffener spacing, 4 elements per girder and 2 elements per stiffener web height, see Figure 13. The local model contains of 27 682 shell and 5612 beam elements and 166 470 DOF.

### 3.2 Ship global hull girder vibration

#### 3.2.1 Case description

First ten dry natural frequency modes of prismatic cruise ship, see Figure 9, are calculated. The length of the vessel is $L=286.944$ m, breadth $B=35.8$ m and draught $T=8.05$ m. The ship is made of steel, with Young’s modulus of 206 GPa, Poisson ratio 0.3 and density 7850 kg/m$^3$. She has 13 decks with the total height of 43.7 m. Deck plating stiffener spacing is 640 mm. The frame and web frame spacings are 854 mm and 2562 mm, respectively. Depending on location,
plating thickness varies from 5...16 mm, which is reinforced using HP profiles from 100x6 to 180x8. Typical girder
size in superstructure part is T-440x7+FB150x10 and in hull section T-530x7+FB150x10. Fire bulkheads are located
at every 40.992 m and they are made of 6 mm plate, which is stiffened using HP-120x6 profiles and girders T-
250x8+150x10. Three different mesh densities are used for A-matrix and ESL theory-based models: 1, 2 and 4
elements per web frame, see Figure 10.

Figure 9. (a) prismatic FE-model of a cruise ship with (b) midframe.

Figure 10. Mesh size of (a) fine mesh model, (b) 1 element (c) 2 elements and (d) 4 elements per web frame model.

3.2.2 Results

The obtained mode shapes are presented in Figure 11 and the results are listed in Table 1. Results indicate that
already 1 element per web frame is sufficient mesh size to capture global vibration modes with less than 1% difference
compared to fine mesh model. Also results obtained from A-matrix and ESL-model are similar, which means that
including stiffened panel membrane stiffness only is sufficient to analyse hull girder response due to wave loads.
However, when membrane type equivalent elements are used, the nodes should always be connected to those elements,
which have relevant bending stiffness e.g., girders, side shell, bulkhead etc. Otherwise, a lot of unrealistic modes due
to element low bending stiffness will occur.
Figure 11. Calculated first 10 natural frequency dry modes of prismatic ship (fine mesh model).

Table 1. Natural frequencies [Hz] of first 10 dry modes of fine mesh, A-matrix and ESL-theory based model.

<table>
<thead>
<tr>
<th></th>
<th>Fine mesh</th>
<th>1 el/web A-matrix</th>
<th>diff. [%]</th>
<th>1 el/web ABD-matrix</th>
<th>diff. [%]</th>
<th>2 el/web ABD-matrix</th>
<th>diff. [%]</th>
<th>4 el/web ABD-matrix</th>
<th>diff. [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. mode</td>
<td>1.89</td>
<td>1.89</td>
<td>-0.1</td>
<td>1.89</td>
<td>-0.1</td>
<td>1.89</td>
<td>0.1</td>
<td>1.89</td>
<td>-0.1</td>
</tr>
<tr>
<td>2. mode</td>
<td>1.96</td>
<td>1.97</td>
<td>0.0</td>
<td>1.97</td>
<td>0.1</td>
<td>1.96</td>
<td>-0.2</td>
<td>1.95</td>
<td>-1.0</td>
</tr>
<tr>
<td>3. mode</td>
<td>2.13</td>
<td>2.12</td>
<td>-0.7</td>
<td>2.12</td>
<td>-0.5</td>
<td>2.10</td>
<td>-1.5</td>
<td>2.07</td>
<td>-2.8</td>
</tr>
<tr>
<td>4. mode</td>
<td>3.56</td>
<td>3.54</td>
<td>-0.5</td>
<td>3.55</td>
<td>-0.3</td>
<td>3.53</td>
<td>-0.8</td>
<td>3.49</td>
<td>-1.9</td>
</tr>
<tr>
<td>5. mode</td>
<td>4.10</td>
<td>4.08</td>
<td>-0.5</td>
<td>4.09</td>
<td>-0.2</td>
<td>4.05</td>
<td>-1.1</td>
<td>4.01</td>
<td>-2.1</td>
</tr>
<tr>
<td>6. mode</td>
<td>4.46</td>
<td>4.46</td>
<td>-0.1</td>
<td>4.46</td>
<td>0.0</td>
<td>4.46</td>
<td>0.0</td>
<td>4.45</td>
<td>-0.2</td>
</tr>
<tr>
<td>7. mode</td>
<td>5.15</td>
<td>5.12</td>
<td>-0.5</td>
<td>5.13</td>
<td>-0.4</td>
<td>5.10</td>
<td>-0.8</td>
<td>5.05</td>
<td>-1.8</td>
</tr>
<tr>
<td>8. mode</td>
<td>6.05</td>
<td>6.02</td>
<td>-0.5</td>
<td>6.04</td>
<td>-0.3</td>
<td>6.00</td>
<td>-0.8</td>
<td>5.97</td>
<td>-1.4</td>
</tr>
<tr>
<td>9. mode</td>
<td>6.41</td>
<td>6.37</td>
<td>-0.7</td>
<td>6.38</td>
<td>-0.5</td>
<td>6.35</td>
<td>-0.9</td>
<td>6.30</td>
<td>-1.7</td>
</tr>
<tr>
<td>10. mode</td>
<td>6.78</td>
<td>6.80</td>
<td>0.3</td>
<td>6.83</td>
<td>0.8</td>
<td>6.76</td>
<td>-0.2</td>
<td>6.75</td>
<td>-0.5</td>
</tr>
</tbody>
</table>
3.3 Propeller, main engine and thruster-induced vibration

3.3.1 Case description

In this case, a typical superstructure deck structure is excited by harmonic enforced displacement of 1 mm amplitude, see Figure 12, to imitate the situation when propeller, main engine or thruster induced vibration is traveling through the bulkheads and pillars and excites the deck structure. The non-structural mass of 100 kg/m² for deck is used, where 50% of the mass is carried by the transversal and longitudinal T-girders and the rest by deck plating. Model size in length direction is 4 web frame spacings and structure continuity is established using symmetric boundary conditions (BC). Thickness of the deck plating is 6.0 mm and it is stiffened using HP 120x6 profiles with spacing of 640 mm. Deck T-girder size is T-440x7+150x10 and web frame spacing 2562 mm. Pillars (RHS-150x150x12.5) are located at every second frame. All structural parts are made of steel with Young’s modulus of 206 GPa, Poisson ratio of 0.3 and mass density of 7850 kg/m³.

Figure 12. Scantlings and applied harmonic enforced displacement for cabin area model.

Four different mesh densities are used for A-matrix and ESL models: 1, 2, 4 and 8 elements per web frame spacing. Applied laminate elements material properties for both models are shown in Table 2. For A-matrix elements, two types of models are created for mesh size 2 or more elements per web frame. In the first type, elements match with ESL model i.e., there are nodes between girders, see Figure 13c, e, g. In the second type, the equivalent element nodes are connected to hard points i.e., there is only one element in longitudinal direction, see Figure 13d, f, h. Frequency/harmonic response analysis type is used with modal solution. Response is calculated for the range 7.0…30 Hz and natural frequencies up to 60 Hz are considered, while the main interest is in frequencies up to $\omega = \omega_{\text{local}}/3$ i.e. 18.3 Hz. Therefore, in this case study, the correction method for ESL model is not applied.
Table 2. Material properties of laminate elements of deck structure model.

### A-matrix model:

<table>
<thead>
<tr>
<th>Layer name</th>
<th>Thickness [mm]</th>
<th>E [GPa]</th>
<th>G [GPa]</th>
<th>ν</th>
<th>ρ [kg/m³]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>x</td>
<td>y</td>
<td>xy</td>
<td>xz</td>
</tr>
<tr>
<td><strong>Deck plate: t=6.0; S=640; HP 120x6</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plate layer</td>
<td>6.0</td>
<td>206</td>
<td>206</td>
<td>79.2</td>
<td>79.2</td>
</tr>
<tr>
<td>Stiffener layer</td>
<td>1.3</td>
<td>206</td>
<td>-</td>
<td>-</td>
<td>79.2</td>
</tr>
<tr>
<td><strong>Longitudinal bulkhead: t=7.0 mm; S=675; HP 120x6</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plate layer</td>
<td>7.0</td>
<td>206</td>
<td>206</td>
<td>79.2</td>
<td>79.2</td>
</tr>
<tr>
<td>Stiffener layer</td>
<td>1.38</td>
<td>206</td>
<td>-</td>
<td>-</td>
<td>79.2</td>
</tr>
</tbody>
</table>

### ESL-theory based model:

<table>
<thead>
<tr>
<th>Layer name</th>
<th>Thickness [mm]</th>
<th>E [GPa]</th>
<th>G [GPa]</th>
<th>ν</th>
<th>ρ [kg/m³]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>x</td>
<td>y</td>
<td>xy</td>
<td>xz</td>
</tr>
<tr>
<td><strong>Deck plate: t=6.0; S=640; HP 120x6</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plate layer</td>
<td>6.0</td>
<td>206</td>
<td>206</td>
<td>79.2</td>
<td>79.2</td>
</tr>
<tr>
<td>Web layer</td>
<td>110</td>
<td>1.92</td>
<td>-</td>
<td>-</td>
<td>0.70</td>
</tr>
<tr>
<td>Flange layer</td>
<td>10.0</td>
<td>86.2</td>
<td>-</td>
<td>-</td>
<td>3.16</td>
</tr>
<tr>
<td><strong>Longitudinal bulkhead: t=7.0 mm; S=675; HP 120x6</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plate layer</td>
<td>7.0</td>
<td>206</td>
<td>206</td>
<td>79.2</td>
<td>79.2</td>
</tr>
<tr>
<td>Web layer</td>
<td>110</td>
<td>1.83</td>
<td>-</td>
<td>-</td>
<td>0.65</td>
</tr>
<tr>
<td>Flange layer</td>
<td>10.0</td>
<td>82.4</td>
<td>-</td>
<td>-</td>
<td>2.94</td>
</tr>
</tbody>
</table>
Figure 13. Example of mesh size of (a) fine mesh model, (b) 1 element, (c) 2 elements, (e) 4 elements and (g) 8 elements per web frame for A-matrix type 1 and ESL-theory based equivalent element. Type 2 A-matrix model mesh for (d) 2 elements, (f) 4 elements and (h) 8 elements per web frame model.

3.3.2 Results

Between frequencies 7...30 Hz, two resonance peaks occur, which are mainly influenced by cabin deck natural frequency modes $A$, $B$ and $C$, $D$ respectively, see Appendix A. The results are plotted for $z$-directional translation in the middle of transversal girder, the exact location is shown in Figure 12.

Results of A-matrix type 1 model are shown in Figure 14. Mesh density of 1 element per web frame leads to wrong estimation of the vibration, especially in higher frequencies. This is because only two elements are used to describe the vibration shapes and mass distribution is represented in too coarse way. Increasing the mesh size only seemingly improves the results, but actually makes the response unpredictable i.e., convergence due to decreasing mesh size is missing. Since bending stiffness of stiffened panel is not correctly considered and acts almost as plate without the stiffeners, lots of disturbance from local unrealistic modes occurs, see Figure 17. For current cabin structure with mesh density of 2 and 8 elements per web frame spacing, 131 and 945 modes, respectively, were found in the frequency range up to 60 Hz. For entire cruise ship the number of unrealistic modes becomes so large that it will be computationally impossible to carry out the analysis. This can be solved by modifying the mesh by connecting the equivalent elements to hardpoints i.e., girders and bulkheads, like it is done in A-matrix model type 2. Calculated response of such models is shown in Figure 15. As can be seen, undesired local modes disappear and with finer mesh convergence occurs, but the error remains large, especially in higher frequencies i.e., working range of thrusters and higher.

![Figure 14](image-url)
Figure 15. Response of cabin area under harmonic enforced displacement of 1 mm of A-matrix type 2 models.

Results of ESL-theory based uncorrected model are shown in Figure 16. Already 1 element per web frame model performs better than similar size A-matrix model. Increasing the mesh size to 2 elements per web frame significantly improves results and very good correspondence between fine mesh is obtained in typical working range of propeller, main engine and thruster. For higher modes, plate vibration between stiffeners becomes more active and error increases and corrections needs to be applied. It can also be noticed that despite of relatively coarse mesh, 2 elements per web frame gives already good results without correction. This is due to lumped mass matrix error-cancelling effect. The mass is applied to nodes, which in case of 2 elements per web frame means that there is more mass in the middle of the stiffened panel, and this lowers the natural frequencies. At the same time local deck plate vibration effects are not considered, which makes the structure stiffer. These two errors with opposite signs compensate each other. Same conclusion was also found in [24]. It should be noted that this error-cancelling effect will not occur when consistent mass matrix is used. As can be seen in Figure 16, when 4 or more elements per web frame are used, error-cancellation effect disappears, and the results converge.

Figure 16. Response of cabin area under harmonic enforced displacement of 1 mm of ESL-theory based model.
3.4 **Higher frequency vibration caused by local excitation force**

3.4.1 **Case description**

In this case, ESL-theory based model is tested for local response analysis, where correction is needed. Cabin deck structure, see Figure 12, is excited with z-directional harmonic point force of 10 kN, Figure 18. This imitates the condition where some higher frequency working machinery locally influences the deck structure and vibration response needs to be separately investigated. Response calculations are performed until 55 Hz, which corresponds to the natural frequency of local plate between the stiffeners, and modes up to 110 Hz are included. ESL models with three different discretization levels, with and without the correction method, are analysed: 2, 4 and 8 elements per web frame spacing. In these models, T-girders are represented using off-set beam elements, see Figure 13, which means that there is no relative flexibility between the girder web and flange. Therefore, in the fourth ESL model, T-girder webs are modelled using shell and flanges with beam elements, similarly as it is done in the fine mesh model. This enables to see how much the modelling technique of primary beams influences the overall deck structure response and how much error comes from the ESL and correction method.

3.4.2 **Results**

Response of z-directional translation is calculated in the same location where the harmonic point force is applied and the results are shown in Figure 19. As in previous case, ESL-model enables to evaluate forced response with very high accuracy until the investigated frequencies are less than \( \omega = \omega_{\text{local}}/3 \) i.e. 18.3 Hz. Beyond that, correction method needs to be applied and mesh density increased at least to 4 elements per web frame spacing. After that very good correspondence with the fine mesh results can be observed until \( \omega_{\text{local}}/1.5 \) i.e. 37 Hz. The modes that contribute...
the most up to this limit are shown in Appendix A and their corresponding frequencies in Table 3. For Mode A the natural frequency is $\omega_{\text{local}}/2.8$ and the ESL model also without the correction is rather accurate, differing approximately 3% from the fine mesh results. However, if modes are getting higher, error increases and for mode F, where $\omega = \omega_{\text{local}}/1.85$, the error becomes already close to 10%. After applying the correction method, error drops to less than 1%. As can be seen in Figure 19c, some small error remains in the ESL model when resonance peaks are evaluated. To reduce this error, T-girder needs to be remodelled with shell and beam elements, see Figure 19d. However, beyond 37 Hz also this type of model’s accuracy becomes insufficient. Several local modes start to dominate, which cannot be predicted by homogenized ESL model and the correction method, see Figure 18c.

![Figure 18](image_url)

**Figure 18.** Fine mesh and ESL-theory based model responses at 20 Hz and 45 Hz under 10 kN harmonic point force.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Fine mesh</th>
<th>ESL, 2 el/web</th>
<th>ESL, 4 el/web</th>
<th>ESL, 8 el/web</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[Hz]</td>
<td>error [%]</td>
<td>[Hz]</td>
<td>error [%]</td>
</tr>
<tr>
<td>Mode A</td>
<td>19.9</td>
<td>-</td>
<td>20.1</td>
<td>-</td>
</tr>
<tr>
<td>Mode B</td>
<td>19.7</td>
<td>-1.1</td>
<td>19.9</td>
<td>-0.8</td>
</tr>
<tr>
<td>Mode C</td>
<td>19.2</td>
<td>-3.8</td>
<td>19.4</td>
<td>-3.5</td>
</tr>
<tr>
<td>Mode D</td>
<td>20.3</td>
<td>2.0</td>
<td>20.6</td>
<td>2.3</td>
</tr>
<tr>
<td>Mode E</td>
<td>19.7</td>
<td>-1.0</td>
<td>19.9</td>
<td>-0.7</td>
</tr>
<tr>
<td>Mode F</td>
<td>20.4</td>
<td>2.5</td>
<td>20.7</td>
<td>3.1</td>
</tr>
</tbody>
</table>

![Figure 18](image_url)

**Figure 18.** Fine mesh and ESL-theory based model responses at 20 Hz and 45 Hz under 10 kN harmonic point force.

Table 3. Deck structure natural frequencies of six most significant vibration modes. Mode shapes are shown in Appendix A.
Figure 19. Cabin area forced vibration response under 10 kN harmonic point force using ESL-theory based model with and without correction.
4 DISCUSSION

The aim of the case studies was to present the range of validity of two common equivalent element techniques for creating ship global FE-model for vibration analysis. The findings are summarised in Figure 20. For wave-induced vibration it is sufficient that only membrane stiffness of the stiffened panel is considered and mesh density can be 1 element per web frame spacing, as recommended by current ship classification guidelines, e.g. DNV-GL [14]. However, this type of simplification is not enough for local vibration analysis, where stiffened panel bending properties need to be considered and at least two elements per web frame spacing is required to describe local mode shapes correctly. For that ESL-theory based element can be used. In forced vibration analysis excellent agreement with fine mesh model can be achieved until frequencies are 3 times smaller than local plate frequency between the stiffeners. This means that typical cruise ship propeller, engines and thrusters first order frequencies are well covered. Beyond that limit, small deviation from the fine mesh results starts to occur, which noticeably increased in second and even more in the third vibration peak, see Figure 16 and Figure 19. With increasing frequencies, the ratio between the global and local modes gets smaller, plate vibration between stiffeners becomes more active and interaction with the global panel modes gets stronger. To overcome this problem, a correction method presented for free vibration analysis in [31] was applied to forced vibration and very good correspondence with the fine mesh model until 37 Hz i.e., $\omega_{\text{local}}/1.5$ was observed, Figure 19. Beyond that, several local modes start to occur, which cannot be calculated by homogenized stiffness properties of ESL model. For local cabin model, 468 modes were found from 8 el/web ESL model and 1498 modes were extracted from fine mesh model, when range of 0…110 Hz was investigated. Not all these extra modes are relevant, but some of them will still influence the secondary response scale, see Figure 18c, and therefore phase interaction between modes is not sufficiently considered, see Eq. 28 and Eq. 29. However, it is important to notice that overall response level at these frequencies is significantly lower compared to first two vibration peaks. General stiffness gets higher and much more energy is needed to create undesired vibration levels.

Figure 20. Validity range of A-matrix and ESL-theory based FE-model.
Based on the case studies, the most optimal mesh density for creating global FE-model of a large vessel seems to be two 4-node elements per web frame spacing. This recommendation is denser than given in the classification society rules [14], but it is justified as then the same FE-model can be utilized also in propeller/machinery induced vibration analysis and number of DOF remains still relatively low, see Figure 21. Compared to 1 el/web model, the DOF will be 2-3 time higher, which reflects only in few extra hours for complete cruise ship forced vibration analysis. When higher frequency response analysis together with correction method needs to be performed, mesh density should be increased at least to 4 elements per web frame spacing. The minimum mesh requirements could be reduced by utilizing higher order i.e., 8-node shell elements instead, where in addition to corner nodes also mid-side nodes are presented. Then mesh size of 1 el/web seems suitable to perform propeller/machinery induced vibration analysis and 2 el/web to investigate higher modes. Despite that ESL model requires creating of several additional material properties for stiffener layers, this shortage is justified as rest of the modelling is very convenient. Compared to lumped beam approach, see [14], [23], less element properties need to be created. Also, material properties for stiffener layers can be found from simple equations, see Eq. 11, and scantlings can be changed without re-meshing the model. Whole modelling process can be made faster by creating appropriate macros in FE pre- and post-processing environment. Also, utilization of correction method can be speeded up with dedicated macros, as for now significant time for post-processing is required.

![Figure 21. Number of DOF of (a) prismatic ship and (b) deck structure models.](image)

This paper is focused on cruise ships, but the introduced vibration analysis procedure can also be utilized for other ship types e.g., container and RoRo ships, bulkers and tankers. Especially attractive would be to implement ESL-theory based element in the design of cargo ship accommodation structure to improve the wellbeing of crew by lowering the vibration levels. These analyses with the ship type dependant case structures are left future work. Also, due to laminate element layer-wise formulation, it would be possible to add deck concrete and floating floor or another floor outfitting layer to stiffened panel. To investigate their modelling principle and their effect on local vibration
levels would be an attractive topic for study.

5 CONCLUSION

The objective of this paper was to present the modelling procedure for performing global and local vibration analysis using coarse mesh ship global FE-model. Based on the results, following conclusions can be drawn:

- Mesh size of 1 element per web frame is suitable for global FE-model when vibration analysis is limited to hull girder modes. It is sufficient that only membrane stiffness of stiffened panel is included. For propeller-, engines-, and thruster-induced vibration response analysis, global FE-model should have at least 2 elements between web frames and bending properties of stiffened panel together with out-of-plane shear stiffness should be included correctly. Presented ESL-theory based element fulfill these requirements.

- Forced vibration analysis using ESL-theory based model gave excellent correspondence with fine mesh model until investigated frequencies were 3 times smaller than local plate frequencies between the stiffeners, i.e., $\omega = \omega_{local}/3$. With the correction method presented in this paper the limit was extended to $\omega = \omega_{local}/1.5$.

6 ACKNOWLEDGEMENTS

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Appendix A: Deck structure most significant vibration modes of first three peaks

- Fine mesh model: Mode A
- ESL, B el/web: Mode A
- Fine mesh model: Mode B
- ESL, B el/web: Mode B
- Fine mesh model: Mode C
- ESL, B el/web: Mode C
- Fine mesh model: Mode D
- ESL, B el/web: Mode D
- Fine mesh model: Mode E
- ESL, B el/web: Mode E
- Fine mesh model: Mode F
- ESL, B el/web: Mode F
Appendix B: Differential equations of ESL-FSDT

The differential equations of ESL-FSDT are obtained by substituting the constitutive equations to strain definitions and further the result to equilibrium equations:

$$
A_{11} \frac{\partial^2 w_0}{\partial x^2} + A_{12} \frac{\partial^2 u_0}{\partial x \partial y} + B_{11} \frac{\partial^2 \phi_x}{\partial y^2} + B_{12} \frac{\partial^2 \phi_y}{\partial x \partial y} + B_{66} \left( \frac{\partial^2 \phi_x}{\partial y^2} + \frac{\partial^2 \phi_y}{\partial x \partial y} \right) + A_{66} \left( \frac{\partial^2 u_0}{\partial y^2} + \frac{\partial^2 v_0}{\partial x \partial y} \right) = I_0 \frac{\partial^2 w_0}{\partial t^2} + I_1 \frac{\partial^2 w_0}{\partial t^2} + I_{1,1} \frac{\partial^2 \phi_x}{\partial t^2},
$$

(B1)

$$
A_{22} \frac{\partial^2 u_0}{\partial y^2} + A_{12} \frac{\partial^2 u_0}{\partial x \partial y} + B_{22} \frac{\partial^2 \phi_x}{\partial x^2} + B_{12} \frac{\partial^2 \phi_y}{\partial x \partial y} + B_{66} \left( \frac{\partial^2 \phi_x}{\partial x^2} + \frac{\partial^2 \phi_y}{\partial x \partial y} \right) + A_{66} \left( \frac{\partial^2 u_0}{\partial x^2} + \frac{\partial^2 v_0}{\partial x \partial y} \right) = I_0 \frac{\partial^2 u_0}{\partial t^2} + I_1 \frac{\partial^2 \phi_x}{\partial t^2} + I_{1,1} \frac{\partial^2 \phi_y}{\partial t^2},
$$

(B2)

$$
D_{Q3} \left( \frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 \phi_x}{\partial x^2} \right) + D_{Qy} \left( \frac{\partial^2 w_0}{\partial y^2} + \frac{\partial \phi_y}{\partial y} \right) + q = I_0 \frac{\partial^2 w_0}{\partial t^2} + I_{0,0,1} \frac{\partial^2 \phi_x}{\partial t^2}.
$$

(B3)

$$
B_{11} \frac{\partial^2 u_0}{\partial x^2} + B_{12} \frac{\partial^2 u_0}{\partial x \partial y} + D_{11} \frac{\partial^2 \phi_x}{\partial x^2} + D_{12} \frac{\partial^2 \phi_y}{\partial x \partial y} + D_{66} \left( \frac{\partial^2 \phi_x}{\partial x^2} + \frac{\partial^2 \phi_y}{\partial x \partial y} \right) + B_{66} \left( \frac{\partial^2 u_0}{\partial x^2} + \frac{\partial v_0}{\partial x \partial y} \right) - D_{Qx} \left( \frac{\partial w_0}{\partial x} + \phi_x \right) -
$$

$$
\frac{\partial M_{xyl}}{\partial x} + \frac{\partial M_{xyl}}{\partial y} = I_2 \frac{\partial^2 \phi_x}{\partial t^2} + I_1 \frac{\partial^2 u_0}{\partial t^2} + I_{2,1} \frac{\partial^2 \phi_x}{\partial t^2}.
$$

(B4)

$$
B_{22} \frac{\partial^2 u_0}{\partial y^2} + B_{12} \frac{\partial^2 u_0}{\partial x \partial y} + D_{22} \frac{\partial^2 \phi_x}{\partial x^2} + D_{12} \frac{\partial^2 \phi_y}{\partial x \partial y} + D_{66} \left( \frac{\partial^2 \phi_x}{\partial x^2} + \frac{\partial^2 \phi_y}{\partial x \partial y} \right) + B_{66} \left( \frac{\partial^2 u_0}{\partial x^2} + \frac{\partial v_0}{\partial x \partial y} \right) - D_{Qy} \left( \frac{\partial w_0}{\partial y} + \phi_y \right) -
$$

$$
\frac{\partial M_{xyl}}{\partial y} + \frac{\partial M_{xyl}}{\partial x} = I_2 \frac{\partial^2 \phi_y}{\partial t^2} + I_1 \frac{\partial^2 v_0}{\partial t^2} + I_{2,1} \frac{\partial^2 \phi_y}{\partial t^2}.
$$

(B5)

ESL-FSDT is applied to commercial finite element software Femap with NX Nastran by using laminated shell element. According to that the underlined moment and inertia terms, which are associated with the local bending of the plate between stiffeners, are set to zero. Also the inertia terms of local plate between stiffeners, $I_{0,1}, I_{1,1},$ and $I_{2,1},$ are omitted together with the 1st and 2nd order rotary inertia terms $I_1$ and $I_2,$ which describe the stiffened panel mass distribution trough the stiffness.

Appendix C: Generalized mass of stiffened panel

Generalized mass from kinetic energy of stiffened panel is found from following equation [31]:

$$
M_m(\omega_m) = \frac{2}{\omega_m^2 A_m} \left[ T_{cm}^{\text{peak}}(\omega_m) + T_{L,b}(\omega_m) \sum_{n=1}^{N} (\alpha_n \Psi_{Gmn}^2) \right],
$$

(C1)

where $A_m$ is generalized amplitude of mode $m,$ which can be taken as 1. $\alpha_n$ is an effective elements area which corresponds to a single node $n$ with global deformation mode shape $\Psi_{Gmn}.$ $T_{cm}^{\text{peak}}(\omega_m)$ represents peak value of all translation kinetic energy of the model, except z-component of the deck plate. Its initial value is obtained from ESL-model and during the iteration, the term changes in relation to squares of the frequencies as follows:

$$
T_{cm}^{\text{peak}}(\omega_{i+1}) = \frac{\omega_{i}^2}{\omega_{ESL,m}^2} T_{cm}^{\text{peak}}(\omega_{ESL,m}),
$$

(C2)

where:

$$
T_{cm}^{\text{peak}}(\omega_{ESL,m}) = \frac{M_m \omega_{ESL,m}^2}{2} - T_{Dcm}(\omega_m, t) = \frac{\omega_{ESL,m}^2}{2} (1 - \frac{m_{dp}}{2} \sum_{n=1}^{N} (\alpha_n \Psi_{Gmn}^2)),
$$

(C3)
where $\omega_{\text{ESL},m}$ is angular frequency for $m$-mode obtained from ESL-model and $M_{m,\text{ESL}}$ is corresponding generalized mass, which is 1, if the normalization of ESL mode shape is done by setting generalized mass into unity. $T_{Dm}(\omega_m,t)$ is deck plate z-directional part of kinetic energy and $m_{dp}$ deck plate mass per area.

$T_{ls}$ is local kinetic energy factor, which represents the kinetic energy of unit deck area under the enforced excitation of unit amplitude of the global reference plane:

$$T_{LR}(\omega_m) = \frac{\omega_m^2 m_{dp}}{2s} \int_0^S \left(1 + \varphi_{LR}(s,\omega_m)\right)^2 ds,$$  \hspace{1cm} (C4)

where coordinate $s$ describes the local distance from the stiffeners, see Figure 5. By considering that only the lowest clamped-clamped mode is active, Eq. C4 can be simplified into [31]:

$$T_{LR}(\omega_m) \approx \omega_m^2 m_{dp} \left(\frac{\omega_m}{2} + 0.523164 r_d(\omega) + 0.198239 r_d^2(\omega)\right),$$  \hspace{1cm} (C5)

where $r_d(\omega_m)$ is dynamic response of the midspan:

$$r_d(\omega_m) = \frac{r_d(\omega_m)}{1-\omega_m^2},$$  \hspace{1cm} (C6)

where $\omega_m$ is the lowest modal frequency of the clamped-clamped plate and can be calculated from the following equation [36]:

$$\omega_m = \frac{\beta^2}{S^2} \sqrt{\frac{D}{m_x}},$$  \hspace{1cm} (C7)

where $\beta \approx 4.73$ for clamped plate, $S$ is stiffener spacing and $m_x$ mass for unit area together with the non-structural mass and $D$ represents the plate flexural rigidity. Static response $r_s$ in Eq. C6 can be found by considering static bending in midspan that uniform inertia load of unit amplitude motion would cause:

$$r_s(\omega_m) = \frac{m_{dp} \omega_m^2 S^4}{384D}.$$  \hspace{1cm} (C8)

### Appendix D: Generalized stiffness of stiffened panel

Generalized stiffness of mode $m$ is found from the strain energy by following equation [31]:

$$K_m(\omega_m) = \frac{2}{A_m^2} \left(U_{\text{cm}}^{\text{peak}}(\omega_m) + U_{LR}(\omega_m) \sum_{n=1}^N (a_n \varphi_{mn}^2)\right),$$  \hspace{1cm} (D1)

where $U_{\text{cm}}^{\text{peak}}$ represents all strain energy of uncorrected i.e., initial ESL model and are found from the following relationship:

$$U_{\text{cm}}^{\text{peak}} = \frac{1}{2} K_{\text{ESL},m} A_m^2 = \frac{\omega_{\text{ESL},m}^2}{2}. $$  \hspace{1cm} (D2)

$U_{LR}(\omega_m)$ presents the strain energy of unit deck area under the enforced excitation of unit amplitude of the global reference plane:

$$U_{LR}(\omega_m) = \frac{D}{2S} \int_0^S \left[\frac{\beta^2 \varphi_{LR}(s,\omega_m)}{ds^2}\right]^2 ds \approx 99.23127 \frac{dr_d^2(\omega_m)}{S^4},$$  \hspace{1cm} (D3)

where $r_d$ is given in Eq. C6.